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To cite this version:

M. Pellissitti, Evangéline Capiez-Lernout, H. Pradlwarter, G.I. Schueller, Christian Soize. Data and model uncertainties in complex aerospace engineering systems. 6th International Conference on Structural Dynamics, Université de Marne-la-Vallée, Sep 2005, Paris, France. pp.Pages: 677-682. hal-00686194

HAL Id: hal-00686194

https://hal.science/hal-00686194

Submitted on 8 Apr 2012

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Data and model uncertainties in complex aerospace engineering systems

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ABSTRACT: The dynamical analysis of complex mechanical systems is in general very sensitive to random uncertainties. In order to treat the latter in a rational way and to increase the robustness of the dynamical predictions, the random uncertainties can be represented by probabilistic models. The structural complexity of the dynamical systems arising in these fields results in large finite element models with significant random uncertainties. Parametric probabilistic models capture the uncertainty in the parameters of the numerical model of the structure, which are often directly related to physical parameters in the actual structure, e.g. Young's modulus. Model uncertainties would have to be modeled separately. On the other hand, the proposed nonparametric model of random uncertainties represents a global probabilistic approach which, in addition, takes directly into account model uncertainty, such as that related to the choice of a particular type of finite element. The uncertain parameters of the structure are not modeled directly by random variables (r.v.'s); instead, the probability model is directly introduced from the generalized matrices of a mean reduced matrix model of the structure by using the maximum entropy principle (Soize 2001). In this formulation the global scatter of each random matrix is controlled by one real positive scalar called dispersion parameter.
An example problem from aerospace engineering, specifically the FE model of the scientific satellite INTEGRAL of the European Space Agency (ESA) (Alenia 1998) is used to elucidate the two approaches. First the analysis based on the parametric formulation is carried out; the associated results are then used to calibrate the dispersion parameters and to construct the reduced matrices of the non-parametric model.

1 INTRODUCTION

Numerical models have become a vital source of information for the manufacturers of complex structural systems. With these models the dynamical behavior of the manufactured structural systems can be predicted beforehand. In practice, the accuracy of every manufacturing process is limited. Consequently, the manufactured system is different from the designed system. These differences can have significant effects on the dynamics of the structure. For this reason a deterministic model, hereafter referred to as the *mean model*, is usually not sufficient for a robust prediction of the dynamic response of the structure. The robustness of the predictions is however an indispensable prerequisite for its practical application. To increase the robustness of the predictions, the mean model can be extended to construct a probabilistic model.
In this paper, two probabilistic approaches for modeling random uncertainties are considered, namely the parametric and the nonparametric approach.
The parametric probabilistic approach allows data uncertainties to be modeled by considering the uncertain physical parameters of the mechanical-numerical model as random quantities. Such uncertain parameters are the geometrical parameters, the components of the elasticity tensor or the boundary conditions. Parametric approaches have been shown to be efficient for modeling data uncertainties and are widely used in computational mechanics (see for instance (Ibrahim 1987, Singh and Lee 1993, Lin and Cai 1995, Schuëller (Ed.) 1997, Schuëller 2001, Schenk and Schuëller 2003)). While some attempts have been made to incorporate model uncertainties with the parametric approach (cf. e.g. (Menezes and Schuëller 1997, Menezes and Brenner 1994)), it is typically focused to model the scatter in the parameters of a given model.
On the other hand, the nonparametric approach aims to take into account model uncertainties to begin with. Its theoretical concepts have been developed in (Soize 2000, Soize 2001) and experimental validation has been carried out in (Chebli and Soize 2004). In the nonparametric probabilistic approach, the generalized

matrices issued from a mean reduced matrix model of the structure are replaced by random generalized matrices. The probabilistic description of these random matrices is constructed by using the maximum entropy principle under constraints defined by the available information and yields a new class of random matrices called the "positive definite ensemble" (Soize 2005a, Soize 2005b). With such a formulation, the global dispersion level of each random matrix is controlled by a unique positive scalar which is called the dispersion parameter.
In this paper, the parametric approach is used to construct a reference solution of the probabilistic response. This solution is used to calibrate the nonparametric model, from which response predictions are derived. A test example from aerospace engineering involves the frequency response analysis of the INTEGRAL satellite of the European Space Agency.

2 PARAMETRIC PROBABILISTIC APPROACH FOR DYNAMICAL SYSTEM WITH RANDOM UNCERTAINTIES

2.1 *Mean finite element model of the dynamical system*

Based on the theory of the linear viscoelasticity without memory, the mean finite element matrix equation of the structure is written for all ω in band $\mathcal{B}$,

$$\big(-\omega^2 \, [\underline{M}] + i\,\omega \, [\underline{D}] + [\underline{K}] \big) \, \underline{\mathbf{u}}(\omega) = \underline{\mathbf{f}}(\omega) \quad , \quad (1)$$

in which $\underline{\mathbf{u}}(\omega)$ and $\underline{\mathbf{f}}(\omega)$ are the $\mathbb{C}^m$ vectors of the DOFs and of the external forces, respectively. In the above equation and in the sequel an underscore denotes mean matrices and vectors. Since the structure has a free boundary, the mean mass matrix $[\underline{M}]$ is a positive-definite symmetric $(m \times m)$ real matrix and the mean damping and stiffness matrices are positive semidefinite symmetric $(m \times m)$ real matrices. Furthermore, it is assumed that the kernel of the mean matrices $[\underline{D}]$ and $[\underline{K}]$ is identical, constituted of r rigid-body modes with $0 \leq r \leq 6$ denoted as $\underline{\boldsymbol{\varphi}}_1, \ldots, \underline{\boldsymbol{\varphi}}_r$.

2.2 *Parametric model of random uncertainties*

Let $\mathbf{X} = (X_1, \ldots, X_\mu)$ be the random $\mathbb{R}^\mu$-valued vector whose components are independent Gaussian r.v.'s and describe mechanical parameters such as geometrical parameters of the structure, coefficients of the elasticity tensor, mass density etc. Clearly, the randomness propagates to the finite element mass, damping and stiffness matrices. The random finite element model is then written as

$$\big(-\omega^2 \, [\mathbf{M}^{\text{par}}] + i\,\omega \, [\mathbf{D}^{\text{par}}] + [\mathbf{K}^{\text{par}}] \big) \, \mathbf{U}^{\text{par}}(\omega) = \underline{\mathbf{f}}(\omega) \quad , \quad (2)$$

in which $\mathbf{U}^{\text{par}}(\omega)$ is the $\mathbb{C}^m$-valued vector of the DOFs and where $[\mathbf{M}^{\text{par}}] = [M(\mathbf{X})]$, and $[\mathbf{D}^{\text{par}}] = [D(\mathbf{X})]$, $[\mathbf{K}^{\text{par}}] = [K(\mathbf{X})]$ are the random finite element mass and damping, stiffness matrices with values in the set of the positive-definite symmetric $(m \times m)$ real matrices and in the set of the positive semidefinite symmetric $(m \times m)$ real matrices.

3 NONPARAMETRIC APPROACH FOR DYNAMICAL SYSTEMS WITH RANDOM UNCERTAINTIES

The main idea of the nonparametric approach (Soize 2000, Soize 2001) consists in replacing the generalized matrices of a mean reduced matrix model of the structure by random matrices.

3.1 *Mean reduced matrix model*

Since we are interested in the elastic motion of the structure, we then introduce the $(m \times n)$ real matrix $[\underline{\Phi}]$ whose columns are the $n \lll m$ eigenvectors $\underline{\boldsymbol{\varphi}}_\alpha$ related to the n strictly positive lowest eigenfrequencies $\underline{\lambda}_\alpha = \underline{\omega}_\alpha^2$. The mean reduced matrix model is written as

$$\underline{\mathbf{u}}(\omega) = [\underline{\Phi}] \, \underline{\mathbf{q}}(\omega) \quad , \quad (3)$$

in which $\underline{\mathbf{q}}(\omega)$ is the $\mathbb{C}^n$ vector of the generalized coordinates solution of the mean reduced equation

$$\big(-\omega^2 \, [\underline{M}_{\text{red}}] + i\,\omega \, [\underline{D}_{\text{red}}] + [\underline{K}_{\text{red}}] \big) \, \underline{\mathbf{q}}(\omega) = \underline{\mathcal{F}}(\omega) \quad , \quad (4)$$

in which $\underline{\mathcal{F}}(\omega) = [\underline{\Phi}]^T \, \underline{\mathbf{f}}(\omega)$ is the $\mathbb{C}^n$ vector of the generalized forces and where the mean reduced mass, damping and stiffness matrices $[\underline{M}_{\text{red}}] = [\underline{\Phi}]^T \, [\underline{M}] \, [\underline{\Phi}]$, $[\underline{D}_{\text{red}}] = [\underline{\Phi}]^T \, [\underline{D}] \, [\underline{\Phi}]$ and $[\underline{K}_{\text{red}}] = [\underline{\Phi}]^T \, [\underline{K}] \, [\underline{\Phi}]$ are positive-definite symmetric $(n \times n)$ real matrices.

3.2 *Construction of the nonparametric model of random uncertainties*

For linear elastodynamics in the low frequency range the nonparametric model of random uncertainties yields the random matrix equation (Soize 2000, Soize 2001),

$$\big(-\omega^2 \, [\mathbf{M}_{\text{red}}^{\text{npar}}] + i\,\omega \, [\mathbf{D}_{\text{red}}^{\text{npar}}] + [\mathbf{K}_{\text{red}}^{\text{npar}}] \big) \, \mathbf{Q}(\omega) = \underline{\mathcal{F}}(\omega) \quad , \quad (5)$$

in which $[\mathbf{M}_{\text{red}}^{\text{npar}}]$, $[\mathbf{D}_{\text{red}}^{\text{npar}}]$ and $[\mathbf{K}_{\text{red}}^{\text{npar}}]$ are positive-definite symmetric $(n \times n)$ real-valued matrices corresponding to the random reduced mass, damping and stiffness matrices and where $\mathbf{Q}(\omega)$ is the $\mathbb{C}^n$-valued random vector of the random generalized coordinates. The $\mathbb{C}^m$-valued random vector $\mathbf{U}^{\text{npar}}(\omega)$ is thus reconstructed by

$$\mathbf{U}^{\text{npar}}(\omega) = [\underline{\Phi}^n] \, \mathbf{Q}(\omega) \quad . \quad (6)$$

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3.2.1 *Probability model of the random matrices*

The nonparametric probabilistic approach requires the normalization of the mean reduced matrices such that $[\underline{M}_{\text{red}}] = [\underline{L}_M]^T [\underline{L}_M]$, $[\underline{D}_{\text{red}}] = [\underline{L}_D]^T [\underline{L}_D]$ and $[\underline{K}_{\text{red}}] = [\underline{L}_K]^T [\underline{L}_K]$, in which $[\underline{L}_M]$, $[\underline{L}_D]$ and $[\underline{L}_K]$ are diagonal $(n \times n)$ real matrices. Each random matrix is written as

$$[\mathbf{M}_{\text{red}}^{\text{npar}}] = [\underline{L}_M]^T [\mathbf{G}_M] [\underline{L}_M] \tag{7}$$

$$[\mathbf{D}_{\text{red}}^{\text{npar}}] = [\underline{L}_D]^T [\mathbf{G}_D] [\underline{L}_D] \tag{8}$$

$$[\mathbf{K}_{\text{red}}^{\text{npar}}] = [\underline{L}_K]^T [\mathbf{G}_K] [\underline{L}_K] \quad . \tag{9}$$

The probability distribution of random matrices $[\mathbf{G}_M]$, $[\mathbf{G}_D]$ and $[\mathbf{G}_K]$ is derived from the maximum entropy principle issued from the information theory (Shannon 1948) with the available information (Soize 2000). It can be shown that random matrices $[\mathbf{G}_M]$, $[\mathbf{G}_D]$ and $[\mathbf{G}_K]$ are independent r.v.'s whose dispersion level can be controlled by the positive real parameters δ_M, δ_D and δ_K which are independent of the dimension n.

The formal probabilistic description of the random positive-definite symmetric real matrix $[\mathbf{G}]$ is described in (Soize 2000, Soize 2001). For numerical calculations, i.e. Monte Carlo simulation, the following procedure has been proposed to generate realizations of the random matrix $[\mathbf{G}]$,

$$[\mathbf{G}] = [\mathbf{L}_G]^T [\mathbf{L}_G] \quad , \tag{10}$$

In the above equation $[\mathbf{L}_G]$ is an $(n \times n)$ upper triangular random matrix resulting from the Cholesky factorization such that,

(1) r.v.'s $\{[\mathbf{L}_G]_{jj'}, j \leq j'\}$ are independent;

(2) for $j < j'$, real-valued r.v. $[\mathbf{L}_G]_{jj'}$ can be written as $[\mathbf{L}_G]_{jj'} = \sigma_n U_{jj'}$ in which $\sigma_n = \delta(n+1)^{-1/2}$ and where $U_{jj'}$ is a real-valued Gaussian r.v. with zero mean and variance equal to 1;

(3) for $j = j'$, positive-valued r.v. $[\mathbf{L}_G]_{jj}$ can be written as $[\mathbf{L}_G]_{jj} = \sigma_n \sqrt{2V_j}$ in which σ_n is defined above and where V_j is a positive-valued gamma random variable whose probability density function $p_{V_j}(v)$ with respect to dv is written as,

$$p_{V_j}(v) = {}_{\mathbb{R}^+}(v) \, \frac{1}{\Gamma(\alpha_{n,j})} \, v^{\alpha_{n,j}-1} \, e^{-v} \quad , \tag{11}$$

where $\alpha_{n,j} = \frac{n+1}{2\delta^2} + \frac{1-j}{2}$.

4 IDENTIFICATION OF THE DISPERSION PARAMETERS FOR THE NONPARAMETRIC APPROACH

In this section the identification of the dispersion parameters δ_M, δ_D and δ_K is outlined, based on the parametric probabilistic model. Let $\Lambda_\alpha^{\text{par}}$ and $\Lambda_\alpha^{\text{npar}}$ be the non zero first random eigenvalues related to the real random generalized eigenvalue obtained with the parametric and the nonparametric model of random uncertainties. The probability density functions of random eigenvalues Λ_1^{par} and Λ_1^{npar} denoted as $p_{\Lambda_1^{\text{par}}}(\lambda)$ and $p_{\Lambda_1^{\text{npar}}}(\lambda)$ are then compared in the least square sense. The function $J(\delta_M, \delta_K)$ is introduced,

$$J(\delta_M, \delta_K) = \frac{||p_{\Lambda_1^{\text{npar}}}(\delta_M, \delta_K) - p_{\Lambda_1^{\text{par}}}||_{\mathcal{L}^2}}{||p_{\Lambda_1^{\text{par}}}||_{\mathcal{L}^2}} \quad , \tag{12}$$

in which the norm $||f||_{\mathcal{L}^2}$ is given by

$$||f||_{\mathcal{L}^2} = \Big(\int_{\mathbb{R}} |f(\mathbf{x})|^2 \, d\mathbf{x} \Big)^{1/2} \quad . \tag{13}$$

The identification is then carried out such that parameters δ_M and δ_K are solution of the equation

$$\min_{\delta_M, \delta_K} J(\delta_M, \delta_K) \quad . \tag{14}$$

Dispersion parameter δ_D is identified separately by using the identification method proposed in (Capiez-Lernout and Soize 2004). Let $[\mathbf{D}_{\text{red}}^{\text{par}}]$ be the random reduced dissipation matrix from the parametric probabilistic model. The dispersion parameter δ_D is written as

$$\delta_D = \sqrt{\frac{W_D^{\text{par}}(n+1)}{\text{tr}([\underline{D}_{\text{red}}])^2 + \text{tr}([\underline{D}_{\text{red}}]^2)}} \quad , \tag{15}$$

where $W_D^{\text{par}} = E\{||[\mathbf{D}_{\text{red}}^{\text{par}}] - [\underline{D}_{\text{red}}]||_F^2\}$,
and $||[A]||_F = tr([A]\,[A]^T)$.

5 METHODOLOGY OF RESOLUTION FOR ANALYZING THE RANDOM RESPONSE

5.1 *Convergence analysis of the stochastic system*

A stochastic convergence analysis allows to specify the number n of modes related to the mean finite element model of the satellite to be kept and to specify the number n_s of realizations used in the Monte Carlo numerical simulation. The convergence is monitored by defining the following sequence $|||\mathbf{U}^{\text{npar}}|||$ such that

$$|||\mathbf{U}^{\text{npar}}|||^2 = E\Big\{ \int_{\omega \in \mathbb{B}} ||\mathbf{U}^{\text{npar}}(\omega)||^2 \, d\omega \Big\} \quad , \tag{16}$$

in which $||\mathbf{U}^{\text{npar}}(\omega)||^2$ denotes the Hermitian norm of random vector $\mathbf{U}^{\text{npar}}(\omega)$. The norm $|||\mathbf{U}^{\text{npar}}|||$ is estimated with the function $(n_s, n) \mapsto Conv(n_s, n)$ such that,

$$Conv(n_s, n)^2 = \frac{1}{n_s} \sum_{j=1}^{n_s} \int_{\mathbb{B}} ||U^{\text{npar}}(\omega, \theta_j)||^2 \, d\omega \quad . \tag{17}$$

5.2 *Confidence region of the random response*

The confidence regions of the random response related to a given probability level α are considered. The exponents ^{npar} or ^{par} are omitted in this section, since the expressions apply in general. Let j_{obs} be the observation node. Let $\underline{\mathbf{u}}_{j_{\text{obs}}}(\omega)$ be the deterministic vector and let $\mathbf{U}_{j_{\text{obs}}}(\omega)$ be the random vector related to the three translational DOFs of node j_{obs}. We then introduce the scalar $\underline{w}_{j_{\text{obs}}}(\omega) = 20\,\log_{10}(||\underline{\mathbf{u}}_{j_{\text{obs}}}(\omega)||)$ and the random variable $W_{j_{\text{obs}}}(\omega) = 20\,\log_{10}(||\mathbf{U}_{j_{\text{obs}}}(\omega)||)$. The mean value $W^0_{j_{\text{obs}}}(\omega)$ of the random response is introduced such that

$$W^0_{j_{\text{obs}}}(\omega) = 20\,\log_{10}(|\mathcal{E}\{||\mathbf{U}_{j_{\text{obs}}}(\omega)||\}|) \quad . \tag{18}$$

Let ω fixed in . . The quantile function $Q_{W_{j_{\text{obs}}}}(\alpha;\omega)$ of random variable $W_{j_{\text{obs}}}(\omega)$ is defined such that

$$Q_{W_{j_{\text{obs}}}}(\alpha;\omega) = \inf_{w} F_{W_{j_{\text{obs}}}}(w;\omega) \geq \alpha \quad , \tag{19}$$

in which $F_{W_{j_{\text{obs}}}}(w;\omega)$ is the cumulative density function of random variable $W_{j_{\text{obs}}}(\omega)$. Let $\widetilde{W}_{j_{\text{obs}}}(\theta_1;\omega) < \ldots < \widetilde{W}_{j_{\text{obs}}}(\theta_{n_s};\omega)$ be the ordered statistic associated with $W_{j_{\text{obs}}}(\theta_1;\omega) < \ldots < W_{j_{\text{obs}}}(\theta_{n_s};\omega)$. The unbiased estimation of cumulative density function $F_{W_{j_{\text{obs}}}}(w;\omega)$ is defined as

$$\hat{F}_{W_{j_{\text{obs}}},n_s}(w;\omega) = \frac{1}{n_s}\sum_{k=1}^{n_s} H^0(w - \widetilde{W}_{j_{\text{obs}}}(\theta_k;\omega)) \quad , \tag{20}$$

in which H^0 is such that $H^0(x) = 1$ if $x \geq 0$ and $H^0(x) = 0$ if not. Let $w_{j_{\text{obs}},+}$ and $w_{j_{\text{obs}},-}$ be the α-quantile and the $(1-\alpha)$-quantile. Using Eqs (19) and (20) yields

$$w_{j_{\text{obs}},+}(\omega) = \widetilde{W}_{j_{\text{obs}}}(\theta_{k_+};\omega)\,, \qquad k_+ = \text{fix}(\text{n}_\text{s}\,\alpha)\,,$$

$$w_{j_{\text{obs}},-}(\omega) = \widetilde{W}_{j_{\text{obs}}}(\theta_{k_-};\omega)\,, \qquad k_- = \text{fix}(\text{n}_\text{s}\,(1-\alpha))\,,$$

in which fix(x) is the integer part of real x.

6 NUMERICAL EXAMPLE

6.1 *Mean finite element model of the free satellite*

The mean finite element model of the satellite is a three dimensional mesh with 120 000 DOFs (see Figure 1). Let j_{exc} be the x-translational DOF of the satellite structure subjected to a deterministic load. The force vector is then written as $\underline{\mathbf{f}}(\omega) = i\,a(\omega)\,M\,\underline{\mathbf{g}}$, in which the vector $\underline{\mathbf{g}} = (g_1,\ldots,g_m)$ is such that $\underline{g}_k = \delta_{j_{\text{exc}}\,k}\,,\ \forall k \in \{1,\ldots,m\}$, where the concentrated mass $M = 3.826\,10^9\,kg$ and where $a(\omega)$ is a prescribed acceleration such that $a(\omega) = 9.81\,m.s^{-2}$ if $\omega < 157\,rad.s^{-1}$ and $a(\omega) = 7.84\,m.s^{-2}$ if $\omega \geq 157\,rad.s^{-1}$. Mean reduced damping matrix $[\underline{D}_{\text{red}}]$ is such that $[\underline{D}_{\text{red}}]_{\alpha\beta} = 2\sqrt{\underline{\lambda}_\alpha}\,\underline{\xi}_\alpha\,\delta_{\alpha\beta}$, in which $\underline{\xi}_\alpha$ is the modal damping ratio related to eigenmode $\underline{\boldsymbol{\varphi}}_\alpha$ such that $\underline{\xi}_\alpha = 0.015$ if $\underline{\omega}_\alpha < 188.5\,rad.s^{-1}$ and $\underline{\xi}_\alpha = 0.025$ if $\underline{\omega}_\alpha \geq 188.5\,rad.s^{-1}$.

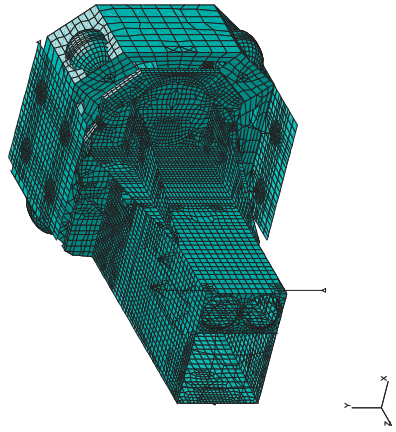

Figure 1. Finite element model of the satellite (provided by ESA/ESTEC).

6.2 *Description of the parametric probabilistic model*

The uncertain parameters of the satellite are modeled by 1 318 independent, Gaussian r.v.'s with coefficients of variation between 4 and 12 %. The randomness of the dissipation is introduced from the mean reduced damping matrix $[\underline{D}_{\text{red}}]$. Modal damping ratios are modeled by independent r.v.'s. Random matrix $[\mathbf{D}^{\text{par}}_{\text{red}}]$ is such that $[\mathbf{D}^{\text{par}}_{\text{red}}]_{\alpha\beta} = 2\sqrt{\Lambda^{\text{par}}_\alpha}\,\Xi_\alpha\,\delta_{\alpha\beta}$, in which Ξ_α is the random modal damping ratio whose probability distribution is Lognormal with 40 % of standard deviation around its mean value $\underline{\xi}_\alpha$.

6.3 *Estimation of the dispersion parameters for the nonparametric probabilistic approach*

The probability distribution $\lambda \mapsto p_{\Lambda^{\text{par}}_1}(\lambda)$ is estimated with $n_s = 1500$ realizations, cf. Figure 2. The minimization of function $J(\delta_M,\delta_K)$ yields $\delta_M = 0.14217$ and $\delta_K = 0.13487$. The approximate pdf's from the two approaches match reasonably well. Figure 3

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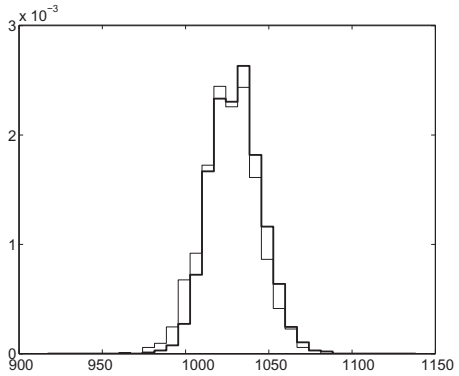

Figure 2. Identification of dispersion parameters δ_M and δ_K: Graph of the probability distributions $\lambda \mapsto p_{\Lambda_1^{\text{par}}}(\lambda)$ for the parametric approach (thin line) and $\lambda \mapsto p_{\Lambda_1^{\text{npar}}}(\lambda)$ for the nonparametric approach (thick line) with dispersion parameters such that $\delta_M = 0.14217$ and $\delta_K = 0.13487$.

shows the estimation of δ_D with respect to the number n_s of realizations used for the Monte Carlo simulation. It is seen that a good convergence is observed for $n_s = 300$ and yields $\delta_D = 0.4166$.

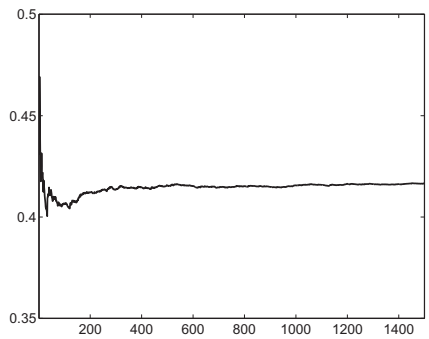

Figure 3. Identification of dispersion parameter δ_D: Graph of $n_s \mapsto \delta_D$

6.4 *Convergence analysis of the stochastic system*

A Monte-Carlo simulation has been performed with n_s realizations and observing the function $n_s \mapsto 20\log_{10}(Conv(n_s, n))$ for $n \leq 294$ and $n_s \leq 1500$. The convergence analysis showed that with $n_s = 750$ and $n = 150$ the resulting approximation is adequate.

6.5 *Confidence region comparison related to the random elastic response of the free satellite*

To simplify the notations, indicial exponents ^{npar} and ^{par} are omitted. We consider the random response of the free satellite at node j_{obs} (see figure 1) in low-frequency band . . The numerical calculations are carried out with $n = 150$ and $n_s = 1500$. Figures 4

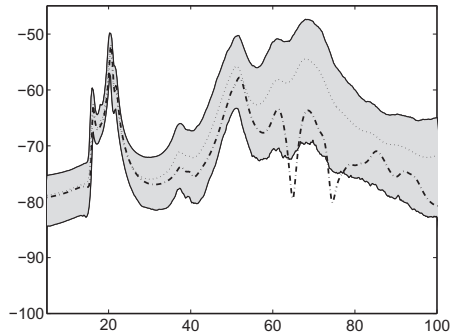

Figure 4. Confidence region of random displacement related to node j_{obs} (in dB) over a low-frequency band . $= [5, 100]\,Hz$ and obtained with the **nonparametric** probabilistic approach: deterministic response of the mean model (thick dashed-dotted line), mean of the random response for the stochastic model (thin dotted line), lower and upper envelopes of the confidence region corresponding to a probability level equal to 0.98 (dark gray filled zone).

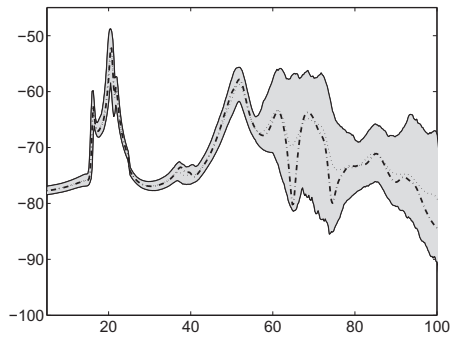

Figure 5. Confidence region of random displacement related to node j_{obs} (in dB) over a low-frequency band . $= [5, 100]\,Hz$ and obtained with the **parametric** probabilistic approach: deterministic response of the mean model (thick dashed-dotted line), mean of the random response for the stochastic model (mid thin dotted line), lower and upper envelopes of the confidence region corresponding to a probability level equal to 0.98 (dark gray filled zone).

and 5 display the graphs related to the confidence region of the random displacements of node j_{obs} obtained for a probability level equal to 0.98 and con-

structed with the quantile method. The thick dashed-dotted line shows the graph $\nu \mapsto \underline{w}_{j_{\text{obs}}}(\nu)$, in which $\nu = \omega/(2\pi)$. The thin dotted line corresponds to $\nu \mapsto W^0_{j_{\text{obs}}}(\nu)$. The confidence region corresponds to the gray filled zone whose envelopes are delimited by the mappings $\nu \mapsto w^-_{j_{\text{obs}}}(\nu)$ and $\nu \mapsto w^+_{j_{\text{obs}}}(\nu)$. Figure 4 refers to the nonparametric, figure 5 to the parametric approach. Clearly, the respective confidence regions may be compared for frequencies lower than $30\,Hz$, which justifies the relevance of the identification procedure of the dispersion parameters. But figure 4 shows that for frequencies greater than $30\,Hz$, the mean of the random response obtained with the nonparametric approach is very different from the response of the mean model. Furthermore, there exist frequencies for which the response of the mean model is outside from the confidence region. On the contrary, this phenomenon is not present for the random response obtained with the parametric probabilistic model described by figure 5. These differences are explained by the ability of the nonparametric probabilistic model to represent model uncertainties.

7 CONCLUSION

Although the dispersion level related to the random uncertainties of the satellite is the same for both probabilistic approaches, the random forced responses do not look similar. Since the two approaches focus on different facets of the problem, a direct comparison is not meaningful. Within the frequency band $[0\,,\,30\,Hz]$, for which the model uncertainties are very small (the data uncertainties being preponderant), both parametric and nonparametric approaches, which allows data uncertainties to be modeled, yield similar results (see Figures 6 and 7). For higher frequencies, however, figures 4,5, shows that the confidence regions cannot be compared. The parametric approach, as applied to this problem, models data uncertainties, while the nonparametric approach models both data and model uncertainties. Within the frequency band $[30\,,\,100\,Hz]$ (for the free satellite), the results show that the satellite structure is more sensitive to model uncertainties than to data uncertainties.

ACKNOWLEDGMENTS

This study was partially supported by the "AMADEUS" Project Nr. 22/2003 (Austria) - LAM03 (France), which is gratefully acknowledged. The authors thank ESA/ESTEC for providing the INTEGRAL satellite finite element models

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