Nonparametric probabilistic approach of uncertainties in computational elastoacoustics of complex systems. Experimental identification and validation

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Abstract

The paper deals with the robustness of uncertain computational elastoacoustic models in low- and medium-frequency ranges. The elastoacoustic system is made up of a heterogeneous viscoelastic structure coupled with an internal acoustic cavity filled with a dissipative acoustic fluid. A reduced mean elastoacoustic model is deduced from the mean finite element model by using the modal approach with the structural modes of the structure and the acoustic modes of the acoustic cavity. Both data uncertainties and model uncertainties are taken into account by using a nonparametric probabilistic approach for the structure, for the acoustic cavity and for the vibroacoustic coupling interface. The main objectives of this paper are (1) to present an experimental validation of the nonparametric probabilistic approach of model uncertainties for computational elastoacoustics of complex systems in the low- and medium-frequency ranges and (2) to propose a method to perform the experimental identification of the probabilistic model parameters. The experimental configuration which is analyzed with the stochastic computational elastoacoustic model is a car made up of a complex heterogeneous structure coupled with a complex acoustic cavity.

1 Introduction

The paper is devoted to computational elastoacoustics in low- and medium-frequency ranges of uncertain complex systems made up of a viscoelastic heterogeneous structure coupled with an internal acoustic cavity filled with a dissipative acoustic fluid. Usually, data uncertainties are taken into account by using a parametric probabilistic approach allowing the uncertain parameters of the computational model to be modeled by random variables and many papers have been published in this subject (see for instance [1] for uncertainty in structural dynamics, [2] for a recent overview on computational methods in stochastic mechanics and reliability analysis and [3] for the stochastic finite element method).

The mathematical-mechanical modeling process of the designed elastoacoustic system is used to construct the computational model. This process introduces model uncertainties which cannot be taken into account by the parametric probabilistic approach (see [4, 5]). Consequently, we propose to use the nonparametric probabilistic approach to take into account both the data uncertainties and the model uncertainties which has recently been introduced (see [6, 7, 8] for the fundamental concept of the nonparametric approach and the developments of the theory based on the use of the random matrix theory, [9] for an extension of the theory allowing a more flexible description of the dispersion levels, [10] for linear dynamical systems in the medium-frequency range, [11, 12, 13, 14, 15, 16] for the experimental identification and the experimental validation of the nonparametric probabilistic approach of model uncertainties in structural dynamics, [15, 17, 18, 19, 20, 21] for the experimental identification and the experimental validation in structural acoustics, [22, 23, 24] for nonlinear dynamical systems).

The main objectives of this paper are (1) to present an experimental validation of the nonparametric probabilistic approach of model uncertainties for computational elastoacoustics of complex systems in the lowand medium-frequency ranges and (2) to propose a method to perform the experimental identification of the probabilistic model parameters. The experimental configuration which is analyzed with the stochastic computational elastoacoustic model is a car made up of a complex heterogeneous structure coupled with a complex acoustic cavity. Experimental measurements have been carried out for 22 manufactured cars of the same type with optional extra [17, 18].

2 Uncertainties in the predictive model of a real elastoacoustic system

The designed elastoacoustic system is the system conceived by the designers and analysts. A designed elastoacoustic system, made up of a structure coupled with an internal acoustic cavity, is defined by geometrical parameters, by the choice of materials and by many other parameters. The designed elastoacoustic system can be a very complex elastoacoustic system. The real elastoacoustic system is a manufactured version of the system realized from the designed elastoacoustic system such as an automative vehicle (car). Consequently, the real elastoacoustic system is a man-made-physical system which is never exactly known due to the variability induced for instance by the process. The objective of a predictive model is to predict the output $(\mathbf{v}^{\rm exp}, p^{\rm exp})$ of the real elastoacoustic system to a given input $\mathbf{f}^{\rm exp}$ in which $\mathbf{v}^{\rm exp}$ is the response in displacement of the structure and where $p^{\rm exp}$ is the acoustic pressure inside the acoustic cavity. Such predictive models are constructed by developing mathematical-mechanical model of the designed elastoacoustic system for a given input (see Figure 1). Consequently, the mean model has an input $\underline{\mathbf{f}}$ modeling $\mathbf{f}^{\rm exp}$, an output $(\underline{\mathbf{v}}, \underline{p})$ modeling $(\mathbf{v}^{\rm exp}, p^{\rm exp})$ and exhibits a vector-valued parameter $\underline{\mathbf{s}}$ for which data has to be given. The errors are related to the construction of an approximation $(\underline{\mathbf{v}}^n, \underline{p}^n)$ of the output $(\underline{\mathbf{v}}, \underline{p})$ of the mean model for given input $\underline{\mathbf{f}}$ and parameter $\underline{\mathbf{s}}$ and have to be reduced and controled using adapted methods developed in applied mathematics and in numerical analysis. The mathematical-mechanical modeling process of the designed elastoacoustic system introduces two fundamental types of uncertainties: data uncertainties and model uncertainties. Data uncertainties are the input $\underline{\mathbf{f}}$ and the parameter $\underline{\mathbf{s}}$ of the mean model. The best

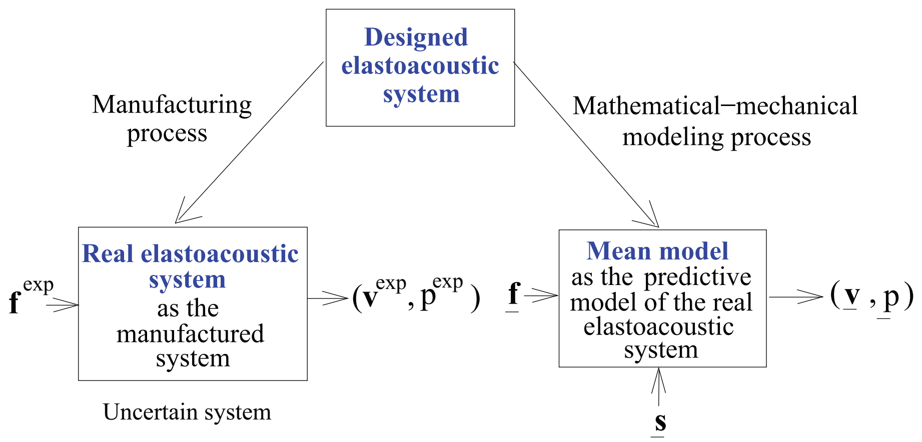

Figure 1: Designed elastoacoustic system, real elastoacoustic system and mean model as the predictive model of the real elastoacoustic system.

approach to take into account data uncertainties is the parametric probabilistic approach which consists in modeling the data of the mean model by random quantities. The mathematical-mechanical modeling process induces model uncertainties with respect to the designed elastoacoustic system. This type of uncertainties is mainly due to the introduction of simplifications in order to decrease the complexity of the mean model which is constructed. For instance, a slender cylindrical elastic structural element will be modeled by using the beam theory, a thick rectangular plate elastic structural element will be modeled by a thick plate theory, a sound proofing scheme between the structure and the acoustic cavity will be modeled by a wall coustic impedance, the geometry of the acoustic cavity will be simplified, etc. It is clear that the introduction of such simplifications yields a mean model for which all the possible variations of its parameter $\underline{\mathbf{s}}$ do not allow the model uncertainties to be reduced. The model uncertainties have then to be taken into account to improve the predictability of the mean model. As explained above, the parametric probabilistic approach cannot be used. This is the reason why a nonparametric probabilistic approach is proposed. The error between the prediction $(\underline{\mathbf{v}}^n, \underline{p}^n)$ which is calculated with the mean model and the response $(\mathbf{v}^{\exp}, p^{\exp})$ of the real elastoacoustic system can be measured by $(\|\mathbf{v}^{\exp} - \underline{\mathbf{v}}^n\|^2 + \|p^{\exp} - \underline{p}^n\|^2)^{1/2}$ in which $\|\,.\,\|$ denotes appropriate norms. Clearly, the mean model can be considered as a predictive model if this error is sufficiently small. In general, due to data uncertainties and model uncertainties, this error is not sufficiently small and has to be reduced by taking into account data uncertainties and model uncertainties.

3 Nonparametric probabilistic approach of model uncertainties

The concept of the nonparametric probabilistic approach of model uncertainties introduced in [6, 7] is the following (see [8]). Let $\mathbf{s} \mapsto \underline{A}(\mathbf{s})$ be a linear mapping from a space $\mathcal{S}$ into a space $\mathcal{A}$ of linear operators. The space $\mathcal{S}$ represents the set of all possible values of the vector-valued parameter $\mathbf{s}$ of the boundary value problem (for instance, geometric parameters, elastic properties, boundary conditions, etc). For $\mathbf{s}$ fixed in $\mathcal{S}$, operator $\underline{A}(\mathbf{s})$ represents one operator of the boundary value problem (for instance, the stiffness operator of the structure which is assumed to be symmetric and positive, and in this case, any operator in $\mathcal{A}$ will be symmetric and positive). Let $R_{\text{par}} \subset \mathcal{A}$ be the range of the mapping $\mathbf{s} \mapsto \underline{A}(\mathbf{s})$, i.e. the subset of $\mathcal{A}$ spanned by $\underline{A}(\mathbf{s})$ when $\mathbf{s}$ runs through $\mathcal{S}$. The corresponding operator of the real elastoacoustic system is $\mathbf{A}^{\exp}$ belonging to $\mathcal{A}$. If $\mathbf{s} = \underline{\mathbf{s}}$ is the nominal value, then $\underline{\mathbb{A}} = \underline{A}(\underline{\mathbf{s}}) \in R_{\text{par}}$ is the operator of the mean model.

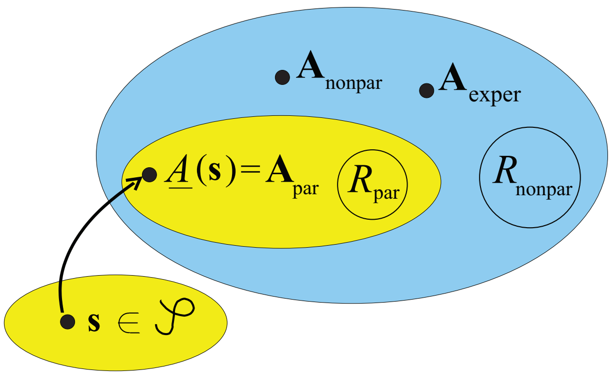

Figure 2: Parametric and nonparametric probabilistic approaches of random uncertainties.

Parametric probabilistic model of the operator. The parametric probabilistic approach for the operator consists in modeling the parameter $\mathbf{s}$ by a vector-valued random variable $\mathbf{S}$ whose probability distribution $P_{\mathbf{S}}(d\mathbf{s})$ has a support which is $\mathcal{S}$. Then the operator $\underline{\mathbb{A}}$ of the mean model is replaced in the BVP by the random operator $\mathbf{A}_{\text{par}}$ such that $\mathbf{A}_{\text{par}} = \underline{A}(\mathbf{S})$. The probability distribution $P_{\mathbf{A}_{\text{par}}}$ of the random operator $\mathbf{A}_{\text{par}}$ is $P_{\mathbf{A}_{\text{par}}} = \underline{A}(P_{\mathbf{S}})$ and its support is the set $R_{\text{par}} \subset \mathcal{A}$ (see Figure 2). Clearly, the probability $P_{\mathbf{A}_{\text{par}}}$ on R_{par} allows data uncertainties to be taken into account, but $\mathbf{A}^{\exp}$ may not be in R_{par} due to model uncertainties.

Nonparametric probabilistic model of the operator. The nonparametric probabilistic approach for the operator consists in replacing the operator $\underline{\mathbb{A}}$ of the mean model by a random operator $\mathbf{A}_{\text{nonpar}}$ whose probability distribution $P_{\mathbf{A}_{\text{nonpar}}}$ has a support which is $R_{\text{nonpar}} = \mathcal{A}$. Since $\mathbf{A}^{\exp}$ belongs to $\mathcal{A}$ and since the support of $P_{\mathbf{A}_{\text{nonpar}}}$ is also $\mathcal{A}$, model uncertainties can be taken into account by the nonparametric approach (see Figure 2). Of course, $P_{\mathbf{A}_{\text{nonpar}}}$ cannot be arbitrary chosen with support R_{nonpar}, but has to be constructed using the available information. Such a methodology has been developed in [6, 7, 8] using the information theory.

Methodology. The methodology of the nonparametric probabilistic approach of uncertainties is as follows. (1) Developement of a finite element model of the designed elastoacoustic system. Such a model will be called the mean model (or the nominal model). (2) Construction of a reduced mean model from the mean model. (3) Construction of a stochastic reduced model from the reduced mean model using the nonparametric probabilistic approach which allows the probability distribution of each random generalized matrix to be constructed. (4) Construction and convergence analysis of the stochastic solution.

Experimental identification. The level of uncertainties is controlled by the dispersion parameter of each random matrix introduced in the nonparametric probabilistic approach. In this paper, we present a method to perform the experimental identification of the dispersion parameters.

4 Stochastic model of uncertain elastoacoustic systems

4.1 Reduced mean model of the elastoacoustic system

The elastoacoustic system is made up of a viscoelastic structure coupled with an internal acoustic cavity filled with a dissipative acoustic fluid. The usual formulation in "structural displacement" - "acoustic pressure" is used to construct the mean finite element model of the elastoacoustic system (see for instance [25]). Let $\underline{\mathbf{u}}(\omega)$ be the $\mathbb{C}^{n_s}$-vector of the n_s degrees of freedom (DOF) of the structure and let $\underline{\mathbf{p}}(\omega)$ be the $\mathbb{C}^{n_f}$-vector corresponding to the the n_f DOF of the acoustic cavity. Let $\{\underline{\varphi}_1,\ldots,\underline{\varphi}_{N_s}\}$ be the N_s first structural modes of the structure in vacuo and calculated at zero frequency (not including rigid body modes if there are). Let $\{\underline{\psi}_1,\ldots,\underline{\psi}_{N_f}\}$ be the N_f first acoustic modes of the acoustic cavity with a rigid fluid-structure coupling interface (including the constant pressure mode if the acoustic cavity is closed). The reduced mean model is obtained by the projection of the mean finite element model on the subspace $V_{N_s}\times V_{N_f}$ of $\mathbb{R}^{n_s}\times\mathbb{R}^{n_f}$ in which V_{N_s} is spanned by $\{\underline{\varphi}_1,\ldots,\underline{\varphi}_{N_s}\}$ and V_{N_f} is spanned by $\{\underline{\psi}_1,\ldots,\underline{\psi}_{N_f}\}$. The reduced mean model can then be written as

$$\underline{\mathbf{u}}(\omega)=\sum_{\alpha=1}^{N_s}\underline{q}^s_\alpha(\omega)\,\underline{\varphi}_\alpha\quad,\quad \underline{\mathbf{p}}(\omega)=\sum_{\beta=1}^{N_f}\underline{q}^f_\beta(\omega)\,\underline{\psi}_\beta\quad. \tag{1}$$

The $\mathbb{C}^{N_s}$-vector $\mathbf{q}^s(\omega)=(q^s_1(\omega),\ldots,q^s_{N_s}(\omega))$ and the $\mathbb{C}^{N_f}$-vector $\mathbf{q}^f(\omega)=(q^f_1(\omega),\ldots,q^f_{N_f}(\omega))$ are the solution of the following matrix equation

$$\begin{bmatrix}-\omega^2[\underline{M}_s]+i\omega[\underline{D}_s(\omega)]+[\underline{K}_s(\omega)] & [\underline{C}]\\ \omega^2[\underline{C}]^T & -\omega^2[\underline{M}_f]+i\omega[\underline{D}_f]+[\underline{K}_f]\end{bmatrix}\begin{bmatrix}\underline{\mathbf{q}}^s(\omega)\\ \underline{\mathbf{q}}^f(\omega)\end{bmatrix}=\begin{bmatrix}\underline{\mathbf{f}}^s(\omega)\\ \underline{\mathbf{f}}^f(\omega)\end{bmatrix}, \tag{2}$$

in which the $(N_s\times N_s)$ real matrices $[\underline{M}_s]$, $[\underline{D}_s(\omega)]$ and $[\underline{K}_s(\omega)]$ are the generalized mass, damping and stiffness matrices of the structure, where the $(N_f\times N_f)$ real matrices $[\underline{M}_f$, $[\underline{D}_f]$ and $[\underline{K}_f]$ are the generalized mass, damping and stiffness matrices of the acoustic and where the rectangular $(N_s\times N_f)$ real matrix $[\underline{C}]$ is the generalized vibroacoustic coupling matrix. In Eq. (2) the $\mathbb{C}^{N_s}$-vector $\underline{\mathbf{f}}^s(\omega)$ and the $\mathbb{C}^{N_f}$-vector $\underline{\mathbf{f}}^f(\omega)$ are the generalized force vector of the structure and the generalized acoustic source vector of the acoustic cavity respectively.

4.2 Stochastic reduced model using the nonparametric probabilistic approach

The principle of construction of the nonparametric probabilistic approach (see [6, 7, 8]) of uncertainties in the structure, in the acoustic cavity and for the vibroacoustic coupling consists (1) in modeling the generalized mass $[\underline{M}_s]$, damping $[\underline{D}_s(\omega)]$ and stiffness $[\underline{K}_s(\omega)]$ matrices of the structure by the random matrices $[\mathbf{M}_s]$, $[\mathbf{D}_s(\omega)]$ and $[\mathbf{K}_s(\omega)]$ whose dispersion parameters are δ_{M_s}, δ_{D_s} and δ_{K_s} respectively; (2) in modeling the generalized mass $[\underline{M}_f]$, damping $[\underline{D}_f]$ and stiffness $[\underline{K}_f]$ matrices of the acoustic cavity by the random matrices $[\mathbf{M}_f]$, $[\mathbf{D}_f]$ and $[\mathbf{K}_f]$ whose dispersion parameters are δ_{M_f}, δ_{D_f} and δ_{K_f} respectively; (3) in modeling the generalized vibroacoustic coupling matrix $[\underline{C}]$ by the random matrix $[\mathbf{C}]$ whose dispersion parameter is δ_C. The explicit construction of the probability distribution of these random matrices were performed by using the maximum entropy principle and is given in [6, 7] for random matrices $[\mathbf{M}_s]$, $[\mathbf{D}_s(\omega)]$, $[\mathbf{K}_s(\omega)]$, $[\mathbf{M}_f]$, $[\mathbf{D}_f]$ and $[\mathbf{K}_f]$, and is given in [8] for random matrix $[\mathbf{C}]$. Let $[\mathbf{A}]$ be anyone of these random matrices. In this theory, the probability distribution of such a random matrix $[\mathbf{A}]$ depends only on its mean value $[\underline{A}] = E\{[\mathbf{A}]\}$ (in which E is the mathematical expectation) and on its dispersion parameter δ_A which is independent of the matrix dimension. In addition, an algebraic representation of the random matrix $[\mathbf{A}]$ has been developed and allows independent realizations to be constructed for a stochastic solver based on the use of the Monte Carlo numerical simulation. It should be noted that if $[\mathbf{A}(\omega)]$ is a symmetric positive real-valued matrix depending on ω, then the random matrix $[\mathbf{A}(\omega)]$ is written as $[\mathbf{A}(\omega)] = [\underline{L}_A(\omega)]^T\,[\mathbf{G}]\,[\underline{L}_A(\omega)]$ in which $[\underline{A}(\omega)] = [\underline{L}_A(\omega)]^T\,[\underline{L}_A(\omega)]$ and where the random matrix germ $[\mathbf{G}]$ is independent of ω. The dispersion parameter δ_A must be taken independent of ω. Using such an approach, the stochastic reduced model of the uncertain elastoacoustic system for which the reduced mean model is defined by Eq. (2) is written, for all ω fixed in the frequency band of analysis $B = [\omega_0, \omega_1]$ with $0 < \omega_0 < \omega_1$, as

$$\mathbf{U}(\omega) = \sum_{\alpha=1}^{N_s} Q_\alpha^s(\omega)\,\boldsymbol{\varphi}_\alpha \quad , \quad \mathbf{P}(\omega) = \sum_{\beta=1}^{N_f} Q_\beta^f(\omega)\,\boldsymbol{\psi}_\beta \quad , \tag{3}$$

in which, for ω fixed in B, the $\mathbb{C}^{N_s}$-valued random variable $\mathbf{Q}^s(\omega) = (Q_1^s(\omega), \ldots, Q_{N_s}^s(\omega))$ and the $\mathbb{C}^{N_f}$-valued random variable $\mathbf{Q}^f(\omega) = (Q_1^f(\omega), \ldots, Q_{N_f}^f(\omega))$ are the solution of the following random matrix equation

$$\begin{bmatrix} -\omega^2[\mathbf{M}_s] + i\omega[\mathbf{D}_s(\omega)] + [\mathbf{K}_s(\omega)] & [\mathbf{C}] \\ \omega^2[\mathbf{C}]^T & -\omega^2[\mathbf{M}_f] + i\omega[\mathbf{D}_f] + [\mathbf{K}_f] \end{bmatrix} \begin{bmatrix} \mathbf{Q}^s(\omega) \\ \mathbf{Q}^f(\omega) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{f}}^s(\omega) \\ \underline{\mathbf{f}}^f(\omega) \end{bmatrix} \quad . \tag{4}$$

4.3 Construction and convergence of the stochastic solution

For all ω fixed in B, it can be proven that the probability model constructed for the random matrices is such that Eq. (4) has a unique second-order solution (see the methodology presented in [7]), i.e., $E\{\|\mathbf{Q}^s(\omega)\|^2\} \leq c_1 < +\infty$ and $E\{\|\mathbf{Q}^f(\omega)\|^2\} \leq c_2 < +\infty$. Concerning the stochastic solver, for all ω fixed in B, the stochastic solution of Eq. (4) is constructed by using the Monte Carlo numerical simulation with m independent realizations. Using the usual statistical estimator of the mathematical expectation E, the convergence of the stochastic solution with respect to N_s, N_f and m is studied in constructing the functions $(N_s, m) \mapsto \text{conv}^s(N_s, m)$ and $(N_f, m) \mapsto \text{conv}^f(N_f, m)$ such that

$$\text{conv}^s(N_s, m) = \{\frac{1}{m}\sum_{k=1}^{m}\int_B \|\mathbf{U}(\omega, \theta_k)\|^2\, d\omega\}^{1/2} \quad , \tag{5}$$

$$\text{conv}^f(N_f, m) = \{\frac{1}{m}\sum_{k=1}^{m}\int_B \|\mathbf{P}(\omega, \theta_k)\|^2\, d\omega\}^{1/2} \quad , \tag{6}$$

in which $\mathbf{U}(\omega, \theta_1), \ldots, \mathbf{U}(\omega, \theta_m)$ and $\mathbf{P}(\omega, \theta_1), \ldots, \mathbf{P}(\omega, \theta_m)$ are m independent realizations of $\mathbf{U}(\omega)$ and $\mathbf{P}(\omega)$ respectively.

5 Identification of the probabilistic model parameters from experiments

The problem to be solved is related to the experimental identification of the vector-valued dispersion parameter $\boldsymbol{\delta} = (\delta_{M_s}, \delta_{D_s}, \delta_{K_s}, \delta_{M_f}, \delta_{D_f}, \delta_{K_f}, \delta_C)$ introduced in the nonparametric probabilistic approach of data and model uncertainties. Let $\mathbf{Y}(\omega, \boldsymbol{\delta}) = (Y_1(\omega, \boldsymbol{\delta}), \ldots, Y_\mu(\omega, \boldsymbol{\delta}))$ be the $\mathbb{R}^\mu$-valued random variable corresponding to μ observations of the stochastic reduced model which will be measured for all ω belonging to the frequency band B. This vector-valued random variable depends on the vector-valued dispersion parameter $\boldsymbol{\delta}$ which has to be identified using measurements. The mean value $\underline{\mathbf{Y}}(\omega, \boldsymbol{\delta})$ of the random vector $\mathbf{Y}(\omega, \boldsymbol{\delta})$ is then such that

$$\underline{\mathbf{Y}}(\omega, \boldsymbol{\delta}) = E\{\mathbf{Y}(\omega, \boldsymbol{\delta})\} \quad . \tag{7}$$

The manufactured systems have a variability induced by the manufacturing process. It is assumed that the measurements are performed for ν manufactured real systems. Let $\mathbf{y}^{\exp,k}(\omega) = (y_1^{\exp,k}(\omega), \ldots, y_\mu^{\exp,k}(\omega))$ be the $\mathbb{R}^\mu$-vector of the μ measured obervations for manufactured system number k. We intoduce the experimental sampling mean value $\underline{\mathbf{y}}^{\exp}(\omega)$ such that

$$\underline{\mathbf{y}}^{\exp}(\omega) = \frac{1}{\nu} \sum_{k=1}^{\nu} \mathbf{y}^{\exp,k}(\omega) \quad . \tag{8}$$

The corresponding observations of the real systems are then modeled by a $\mathbb{R}^\mu$-valued random variable $\mathbf{Y}^{\exp}(\omega) = (Y_1^{\exp}(\omega), \ldots, Y_\mu^{\exp}(\omega))$ for which, by construction, the mean value $E\{\mathbf{Y}^{\exp}(\omega)\}$ of the random vector $\mathbf{Y}^{\exp}(\omega)$ is chosen such that

$$E\{\mathbf{Y}^{\exp}(\omega)\} = \underline{\mathbf{y}}^{\exp}(\omega) \quad . \tag{9}$$

Several methods have been recently proposed in [15] which allow the nonparametric probabilistic model of uncertainties to be experimentally identified for different situations. Below, we present only two methods which are adapted to the application analyzed in Section 6 and which will be used to identify the vector-valued dispersion parameter $\boldsymbol{\delta}$ from experiments. The first one is the mean-square identification method which can be used for a vector-valued random variable without any difficulties. This method consists in minimizing, in the mean-square sense, the distance between the computed random response and the experimental response. The second one is the maximum likelihood method which can also be used for a vector-valued random variable. Nevertheless, the computational time required by such a method is prohibitive if the vector-valued random variable has a high dimension. Consequently, we will limit the presentation of this method to a real-valued random variable. Nevertheless an extension of this method is proposed in [15] which allows high dimension cases to be treated.

5.1 Mean-square identification method

Let $\omega \mapsto \mathbf{X}(\omega) = (X_1(\omega), \ldots, X_\mu(\omega))$ be a $\mathbb{R}^\mu$-valued second-order stochastic process indexed by the frequency band B. We introduce the norm $|||\mathbf{X}|||$ of $\mathbf{X}$ such that

$$|||\mathbf{X}|||^2 = E\{||\mathbf{X}||_B^2\} \quad , \quad ||\mathbf{X}||_B^2 = \int_B ||\mathbf{X}(\omega)||^2 \, d\omega \quad , \tag{10}$$

in which $||\mathbf{X}(\omega)||^2 = X_1(\omega)^2 + \ldots + X_\mu(\omega)^2$. The mean square identification of parameter $\boldsymbol{\delta}$ consists in minimizing the cost function $J_0(\boldsymbol{\delta}) = |||\mathbf{Y}(.,\boldsymbol{\delta}) - \mathbf{Y}^{\exp}|||^2$ with respect to $\boldsymbol{\delta}$. In order to compute this cost function, we can write $|||\mathbf{Y}(.,\boldsymbol{\delta}) - \mathbf{Y}^{\exp}|||^2 = |||\mathbf{Y}(.,\boldsymbol{\delta}) - \underline{\mathbf{Y}}(.,\boldsymbol{\delta}) - (\mathbf{Y}^{\exp} - \underline{\mathbf{y}}^{\exp}) + \underline{\mathbf{Y}}(.,\boldsymbol{\delta}) - \underline{\mathbf{y}}^{\exp}|||^2$. Since $\underline{\mathbf{Y}}(.,\boldsymbol{\delta}) - \underline{\mathbf{y}}^{\exp}$ is a deterministic vector and since $\mathbf{Y}(.,\boldsymbol{\delta}) - \underline{\mathbf{Y}}(.,\boldsymbol{\delta})$ and $\mathbf{Y}^{\exp} - \underline{\mathbf{y}}^{\exp}$ are independent and centered vector-valued random variables, it can be deduced that

$$J_0(\boldsymbol{\delta}) = |||\mathbf{Y}(.,\boldsymbol{\delta}) - \underline{\mathbf{Y}}(.,\boldsymbol{\delta})|||^2 + |||\mathbf{Y}^{\exp} - \underline{\mathbf{y}}^{\exp}|||^2 + ||\underline{\mathbf{Y}}(.,\boldsymbol{\delta}) - \underline{\mathbf{y}}^{\exp}||_B^2 \quad . \tag{11}$$

In the right-hand side of Eq. (11), the first, the second and the third terms represent the variance of the random response of the stochastic model, the variance of the real system induced by its variability and the bias between the model and the real system, respectively. It should be noted that the second term is independent of $\boldsymbol{\delta}$. Consequently, the cost function $J_0(\boldsymbol{\delta})$ can be replaced by a cost function $J_1(\boldsymbol{\delta})$ obtained by removing the second term. Consequently, the mean-square identification of parameter $\boldsymbol{\delta}$ consists in solving the following optimization problem

$$\boldsymbol{\delta}^{\text{opt}} = \arg\min_{\boldsymbol{\delta}} J_1(\boldsymbol{\delta}) \quad , \tag{12}$$

in which the cost function $J_1(\boldsymbol{\delta})$ is written as

$$J_1(\boldsymbol{\delta}) = |||\mathbf{Y}(.,\boldsymbol{\delta}) - \underline{\mathbf{Y}}(.,\boldsymbol{\delta})|||^2 + ||\underline{\mathbf{Y}}(.,\boldsymbol{\delta}) - \underline{\mathbf{y}}^{\text{exp}}||_B^2 \quad . \tag{13}$$

5.2 Maximum likelihood method

For the maximum likelihood method, we introduce the real-valued random variable $Z(\boldsymbol{\delta})$ for which the ν independent realizations $z^{\text{exp},1}, \ldots, z^{\text{exp},\nu}$ correspond to the ν manufactured real systems. Let $p_Z(z,\boldsymbol{\delta})\,dz$ be the probability distribution on $\mathbb{R}$ of $Z(\boldsymbol{\delta})$ represented by a probability density function $p_Z(z,\boldsymbol{\delta})$ which depends on the dispersion parameter $\boldsymbol{\delta}$. This random variable is defined by

$$Z(\boldsymbol{\delta}) = \int_B \text{dB}(\omega,\boldsymbol{\delta})\,d\omega \quad , \quad \text{dB}(\omega,\boldsymbol{\delta}) = 10\,\log_{10}\left(w_{\text{ref}}^2 \frac{1}{\mu}\sum_{j=1}^{\mu} |Y_j(\omega,\boldsymbol{\delta})|^2\right) \quad , \tag{14}$$

in which w_{ref} is a constant of normalization. It should be noted that, for all z fixed in $\mathbb{R}$, the probability density function $p_Z(z,\boldsymbol{\delta})$ can easily be estimated with Eqs. (3) and (4) using the Monte Carlo method and the mathematical statistics. For $k = 1, \ldots, \nu$, the corresponding realization $z^{\text{exp},k}$ is written as

$$z^{\text{exp},k} = \int_B \text{dB}^{\text{exp},k}(\omega)\,d\omega \quad , \quad \text{dB}^{\text{exp},k}(\omega) = 10\,\log_{10}\left(w_{\text{ref}}^2 \frac{1}{\mu}\sum_{j=1}^{\mu} |y_j^{\text{exp},k}(\omega)|^2\right) \quad . \tag{15}$$

The use of the maximum likelihood method (see [26]) leads us to the following optimization problem

$$\boldsymbol{\delta}^{\text{opt}} = \arg\max_{\boldsymbol{\delta}} \mathcal{L}(\boldsymbol{\delta}) \quad , \tag{16}$$

in which $\mathcal{L}(\boldsymbol{\delta})$ is written as

$$\mathcal{L}(\boldsymbol{\delta}) = \sum_{k=1}^{\nu} \log_{10}(p_Z(z^{\text{exp},k},\boldsymbol{\delta})) \quad . \tag{17}$$

6 Analyzing experimental configurations

We present the experimental validation of the stochastic computational elastoacoustic model for the prediction of internal noise in a car due to engine excitation applied to the engine supports (booming noise) (see [17, 18, 19, 20]). The mean finite element model is shown in Figure 3. The structure is modeled with $n_s = 978,733$ DOF in displacement and the acoustic cavity with $n_f = 8,139$ DOF in pressure. The frequency band of analysis is $B = [33, 200]$ Hz and corresponds to $[1000, 6000]$ rpm (engine rotation per minute). The convergence of the stochastic reduced model over the frequency band B is obtained for $N_s = 1722$ structural modes, for $N_f = 57$ acoustic modes and for $m = 600$ realizations. The experimental identification of the dispersion parameters are performed in three steps as follows (see [17, 18]). For the first step, the acoustic pressures have been measured inside the acoustic cavity for a given acoustic source inside the cavity. Then the maximum likelihood method described in Section 5.2 has been used taking $\delta_{M_f} = \delta_{D_f} = \delta_{K_f}$, where $Y_j(\omega,\boldsymbol{\delta}) = P_{\ell_j}(\omega)$ in which $P_{\ell_1}(\omega), \ldots, P_{\ell_\mu}(\omega)$ are the observed acoustic pressures which are measured inside the cavity and where $w_{\text{ref}} = 1/P_{\text{ref}}$ in which P_{ref} is a reference pressure.
For the second step, the structural accelerations have been measured in the structure for driven forces applied to the engine supports. Then the mean-square identification method described in Section 5.1 has been used

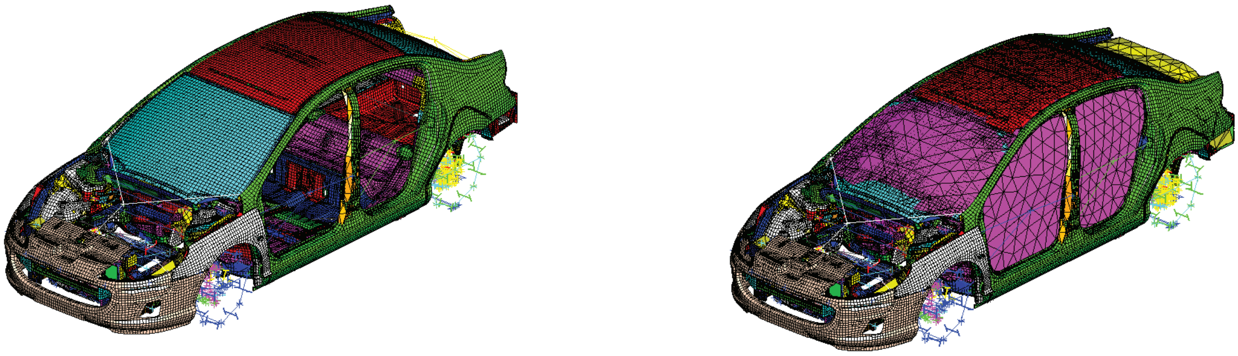

Figure 3: Finite element mesh of the structure: 978,733 DOF in displacement (left figure). Finite element mesh of the acoustic cavity: 8,139 DOF in pressure (right figure)

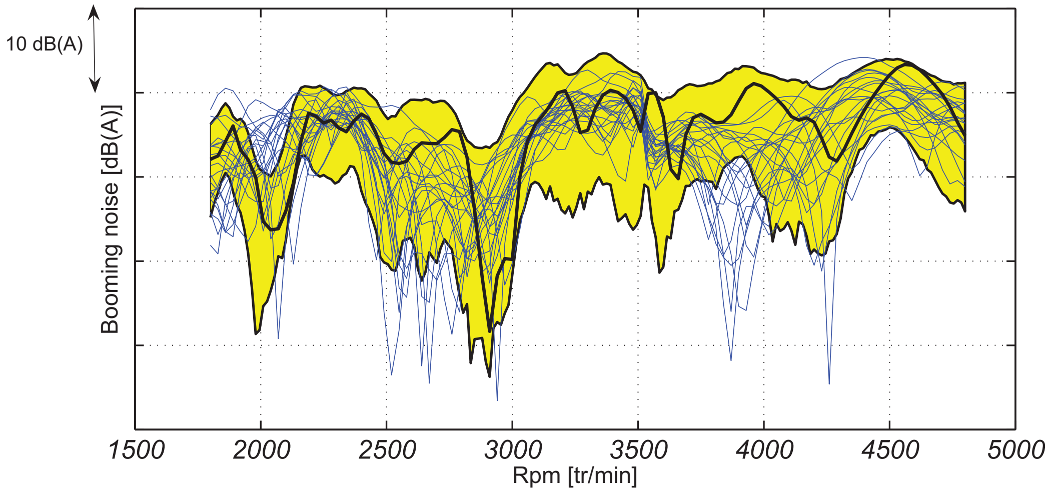

Figure 4: Experimental validation of the confidence region prediction for the random cross FRF between the input force applied to engine supports and the acoustic pressure at an observation point in the acoustic cavity for the vibroacoustic system. Horizontal axis: tr/min. Vertical axis: modulus of the acoustic pressure in dBA. 22 experimental measurements for 22 cars of the same type (22 thin solid lines). Numerical prediction of the mean reduced matrix model (thick solid line). Confidence region of the internal noise predicted with the non parametric probabilistic model and for probability level 0.95. (grey region).

with $Y_j(\omega, \boldsymbol{\delta}) = \log_{10}(w_j\,|U_{\ell_j}(\omega)|)$ in which $U_{\ell_1}(\omega), \ldots, U_{\ell_\mu}(\omega)$ are the observed displacements which are measured and where $w_1, \ldots, w_\mu$ are normalization constants such that $0 < w_j \leq 1$. In a third step, the dispersion parameter δ_C of the vibroacoustic coupling operator has been fixed at a given value. Figure 4 displays the experimental validation of the numerical prediction of internal noise due to engine excitation with structure, vibroacoustic coupling and acoustic cavity uncertainties. Taking into account the complexity of the vibroacoustic system, there is a good experimental validation of the stochastic computational elastoacoustic model with both model uncertainties and data uncertainties. The variability of the manufactured real systems is due to the process and to the extra options. The propagation of uncertainties is significant in the frequency band of analysis.

7 Conclusions

Data uncertainties and model uncertainties are be taken into account in a computational elastoacoustic model by using the nonparametric probabilistic approach. A methodology is proposed to perform the experimental identification of the dispersion parameters which controls the level of uncertainties. The approach is experimentally validated for a complex elastoacoustic system which presents variabilities.

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