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# Erratum to "Scheduling of a two-machine flowshop with availability constraints on the first machine" [International Journal of Production Economics 99 (2006) 16 – 27]

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## Abstract

In Allaoui et al. (2006) (Allaoui H., Artiba A., Elmaghraby S.E., Riane F. Scheduling of a two-machine flowshop with availability constraints on the first machine. International Journal of Production Economics 99 (2006): 16 – 27) two conditions of optimality for the Johnson sequence are given. We establish in this note that these conditions are false, under the resumable and the non-resumable case. We also point out two incorrect proofs and give evidence that the time complexity of their dynamic approach is very unlikely to be correct.

## Optimality condition for the Johnson order

Allaoui et al. (2006) give in their Section 3 & 4 two conditions of optimality for the Johnson order (JO) and the Modified Johnson order (MJO) in presence of an unavailability period  $[s, t]$  on the first machine. Considering the first machine, let  $a$  be the first job completed after the unavailability period, and let  $B^H$  be the set of jobs completed before time  $s$ . The following propositions are stated in Allaoui et al. (2006), with the makespan minimization as objective:

**Proposition 2.** *If the completion time of set  $B^H$  on the second machine is larger than  $t + p_{1,a}$ , then JO is optimal.*

**Proposition 3.** *If no idle time appears on the first machine, then MJO is optimal.*

The following example proves that both propositions are false, under the resumable and the non-resumable case. The unavailability period on the first machine is  $[5, 6]$  and we have 5 jobs to schedule, whose processing times are given below:

Jobs	1	2	3	4	5
$p_{1,i}$	1	2	2	3	4
$p_{2,i}$	4	2.1	2.1	3.1	3.9

The JO (and MJO) sequence is (1, 2, 3, 4, 5) and is unique. The resulting makespan is 16.9. Observe that set  $B^H = \{1, 2, 3\}$  completes exactly at the beginning of the unavailability period, at time  $s = 5$ . Thus the JO schedule fulfills the condition of Proposition 3. Also observe that set  $B^H$  completes at time 9.2 on the second machine, while job 4, which plays the role of job  $a$ , completes at time 9 on the first machine. Thus the JO schedule also fulfills the condition of Proposition 2. However this schedule is not optimal: sequence (1, 5, 2, 3, 4) results in a makespan of 16.2. Notice that on the example we obtain the same JO schedule in the resumable and the non-resumable case.

This example shows that even if a JO schedule fulfills conditions of Proposition 2 and Proposition 3, it is not necessarily optimal. The following simpler example establishes that if a JO schedule satisfies only condition of Proposition 3, it can be asymptotically as bad as twice the optimal value, which is the worst possible performance ratio for a JO schedule ! Consider an unavailability period of length  $g$  ( $g \geq 3$ ) starting at time 3, and 3 jobs to schedule, whose processing times are given below:

Jobs	1	2	3
$p_{1,i}$	1	2	3
$p_{2,i}$	2	3	$g + 1$

The JO (and MJO) sequence is (1, 2, 3) and is unique. Notice that this is also the only optimal sequence relaxing the unavailability period. The makespan of the

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resulting JO schedule is  $2g + 7$ , and clearly no idle time appears on the first machine, i.e. the condition of Proposition 3 is fulfilled. However sequence  $(3, 2, 1)$  has a makespan of only  $g + 9$ . The performance ratio tends to 2 when  $g$  tends to infinity.

### Johnson rule as heuristic

The authors state in Lemma 6 that the makespan of the JO schedule in the resumable case is at most the optimal makespan without the unavailability period plus the duration  $g$  of the period. In the notations of the paper,  $C_{\max}^H(G) \leq C_{\max}^H(\emptyset) + g$ . The result is correct, but not the proof given. Let  $b$  be the last job that completes on the first machine before the unavailability period. In case 1, the authors claim that if a condition very closed to Proposition 2 is fulfilled, namely  $C_{2,b}^H \geq C_{1,a}^H$ , then the equality  $C_{\max}^H(G) = C_{\max}^H(\emptyset)$  holds. This claim is false, as proved by the following example: We have an availability period  $[1, 2]$  and 3 jobs to schedule:

Jobs	1	2	3
$p_{1,i}$	1	2	4
$p_{2,i}$	3	3	1

The (unique) JO sequence is  $(1, 2, 3)$ . The resulting makespan without the unavailability period is 8. With the availability period, jobs 1 plays the role of job  $b$  and job 2 the role of job  $a$ . It is easy to check that the condition is fulfilled, since  $C_{2,1}^H = C_{1,2}^H = 4$ . However the makespan of the JO schedule is now 9.

We give a short correct proof. Let  $G^H(\emptyset)$  be the disjunctive graph oriented according to the JO sequence. It is folklore that the longest path in  $G^H(\emptyset)$  is the makespan  $C_{\max}^H(\emptyset)$  of the schedule. We define accordingly the execution graph  $G^H(G)$ , where the operation of  $a$  on the first machine is replaced by an operation  $o'_{1,a}$  of duration  $p_{1,a} + g$ . Now consider the longest path of  $G^H(G)$ , whose length is clearly equal to  $C_{\max}^H(G)$ . This path may or may not pass through  $o'_{1,a}$  (i.e. the unavailability period is or is not on the critical path), but in any case its length is at most  $g$  plus its length in graph  $G^H(\emptyset)$ . By definition this latter quantity is bounded by the length of the longest path in  $G^H(\emptyset)$ . The result follows.

Section 8 presents Lemma 9, the counter part of Lemma 6 for the non-resumable case. The proof is also incorrect: the same example as for Lemma 6 shows that the argument in Case 1 does not hold. The correct proof of Lemma 6 given above can be easily adapted by increasing the processing time of job  $a$  on the first ma-

chine by  $g + \delta$ , with  $\delta$  the idle occurring on the first machine before the unavailability period.

### A dynamic programming model

In Section 5, Allaoui et al. (2006) propose a dynamic programming approach to solve the problem in the non-resumable case. They claim to obtain a time complexity in  $O(2^k n \log n)$ , with  $k$  the maximal number of jobs that can be completed before the unavailability period in any schedule. As the authors noticed, we can assume that  $k$  is not larger than  $n/2$ . The details of the approach are hard to understand due to loose definitions, but the final result is quite surprising. Indeed the SubSetSum problem is a particular case of this flowshop scheduling problem, considering instances with zero processing time on the second machine. The time complexity of the algorithm in  $O(2^{n/2} n \log n)$  challenges the best exact algorithm known for the SubSetSum problem, see Woeginger (2003) and references within. Moreover, consider the  $k$ -SubSetSum problem, a variant of the SubSetSum problem with the additional cardinality constraint to pick exactly  $k$  items. The DP proposed by Allaoui et al. (2006) can solve it to optimality in time complexity  $O(2^k n \log n)$ . This time complexity would demonstrate that  $k$ -SubSetSum is by definition fixed parameter tractable, with  $k$  the parameter of the problem (for an introduction to parametrized complexity, see Downey & Fellows (1995)). However Downey & Fellows (1995) establish that problem  $k$ -SubSetSum is hard for class  $W[1]$ , see their Theorem 4.3. Thus the DP of Allaoui et al. (2006) would involve that  $FTP = W[1]$  and the collapse of the parameterized complexity hierarchy, which is very unlikely to happen.

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