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## Linearization of RF Power Amplifiers Using Adaptive Kalman Filtering Algorithm

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In this paper, a new linearization algorithm of Power Amplifier, based on Kalman filtering theory is proposed for obtaining fast convergence of the adaptive digital predistortion. The proposed method uses the real-time digital processing of baseband signals to compensate the nonlinearities and memory effects in radio-frequency Power Amplifier. To reduce the complexity of computing in classical Kalman Filtering, a sliding timewindow has been inserted which combines off-line measurement and on-line parameter estimation with high sampling time to track the changes in the PA characteristics. We evaluated the performance of the proposed linearization scheme through simulation and experiments. Using digital signal processing, experimental results with commercial power amplifier are presented for multicarrier signals to demonstrate the effectiveness of this new approach.

*Keywords*: Power amplifiers, digital predistortion, Kalman Filtering, parameter estimation, adaptive control.

## 1. Introduction

Nonlinear system linearization of microwave components and radio-frequency circuits becomes a challenge and potential useful problem in the radiocommunication system research areas. Interest for Radio Frequency Power Amplifier (RF PA) control is motivated by the increasing growth of the wireless communication systems which has lead to use digital modulation techniques such as (BPSK, QPSK, QAM, ...) with non-constant envelope to improve spectral efficiency <sup>1</sup>. As a result of the variable envelope modulation schemes, the improvement of the linearity of the PA becomes an objective of first importance for mobile communication systems. This is due to the nonlinear distortions and dynamical effects which generate unwanted spectrum components for the transmitted signal and lead to Adjacent Channel Power Ratio (ACPR) requirements. It also causes in-band distortion which degrades the bit error rate (BER) performance, especially for modulation with high peak-toaverage power ratio and large fluctuations in signal envelopes such as CDMA and OFDM.

A number of approaches and variations have been proposed for linearizing the

PA, which can be divided into three categories: the feedforward, Cartesian feedback, and predistortion approaches <sup>1,2,3,4</sup>. Other popular techniques are used to insure an efficient amplification by including a variety of circuit elements in transmitter such as LInear amplification with Nonlinear Components (LINC), Combined Analog-Locked Loop Universal Modulator (CALLUM), and Envelope Elimination and Restoration (EER) <sup>1,2,5</sup>.

In feedforward approach, an error amplifier and delays are used to compensate the distorted signal generated by the main amplifier. As feedforward is inherently an open-loop process, changes of components characteristics, signal properties or matching conditions are not ideally compensated <sup>2</sup>. To adaptively compensate the amplitude and delays imbalance, an on-line correction of the gain and phase weights can be added to the original structure. An example of adaptive DSP controlled with feedforward technique using LMS algorithm was published in <sup>6</sup>.

Feedback control is extensively studied in automatic domain and particularly appreciated in the control of systems with low frequencies dynamics (electrical machines, audio amplifiers, ...)<sup>7</sup>. The general principle of this approach is to force the output to follow an input reference. It can provide linearization if applied directly to the amplifier in the form of Radio or Intermediate Frequency feedback, harmonic feedback or envelope feedback <sup>1</sup>. In all cases, a portion of the output signal from feed-backed amplifier is fed back through a voltage divider, subtracted from the input signal, and the PA is driven with this error signal. Typical results with this approach are: an improvement of 10dB of two-tone IMD for Envelope feedback <sup>8</sup>, around 35dB for Polar feedback with a narrow-band PA <sup>1</sup> and for Cartesian feedback with high nonlinear PA (Class-C) <sup>9,10</sup>.

Unfortunately, the disadvantage of feedback is that the large bandwidth of the PA signal induces stability problem. The feedback changes the input to output relationship and induces a new dynamic mode which can affect the stability criteria defined by gain and phase margins. The current solution is based on the addition of new control strategies using advanced signal processing  $^{8,10,11}$ .

One of the most promising linearization methods for nonlinear PA is to predistort the baseband drive signal <sup>14,15,16,17,18</sup>. This technique is based on off-line estimation of inverse characteristics of the amplifier to be linearized. If accurate predistortion is required, it is necessary to adjust in real-time the predistorter characteristics so that it can track changes in amplifier characteristics <sup>4,10,19,20</sup>. Kalman Filter (KF) algorithm is one of the most popular adaptive filtering techniques in nonstationary environments and real-time estimation <sup>21,22</sup>. This algorithm, originally developed for linear systems, is generalized for a nonlinear case, called Extended Kalman Filter (EKF) <sup>7</sup>. However, the EKF has some inherent limitations mainly due to calculation of complicated analytical derivatives for linearizing the nonlinear model <sup>23</sup>. This is a major constraint for the implementation in adaptive predistortion using nonlinear models with memory. In this article, new approach based on identification by a sliding time-window is proposed which has less training complexity than EKF algorithm. Both analytical and simulation results using memory polynomial predistorter are presented to demonstrate the feasibility and performance of this approach to adaptive predistortion. Also, this paper presents preliminary results achieved with an experimental system based on digital processing system and a Class AB amplifier.

The paper is organized as follows. In Section 2, the adaptive system, based on the indirect learning, and the parametric model of nonlinear predistortion are defined. The KF theory is revised in section 3 and applied to the PA linearization problem. The performance of the linearized transmitter system is investigated through simulations and experiments in Section 4 from different digitally modulated signals and a two-tone test. And, finally, discussions and conclusions are given in Section 5.

## 2. System Model

A commonly used method to reduce distortions and fluctuations in systems affected by static nonlinearities and short/long term memory is to use the inverse model. An overview of such methods is given by Åström and Witenmark <sup>24</sup>, Goodwin and al. <sup>25</sup> (see also <sup>26</sup>). Most of the methods for on-line identification can be classified into two main groups: direct learning and indirect learning <sup>7</sup>. In an indirect learning mechanism, an inverse PA model with memory is computed in quasi-open loop and applied as a feedforward controller. It has been demonstrated that this indirect approach is more efficient than a direct learning for linearization structure using Volterra models and its variants (Hammerstein, Wiener, polynomial models, ...) <sup>12,19</sup>. Hence, only nonlinear adaptive predistorter with indirect learning architecture is considered in this work.

## 2.1. Predistorter based on the indirect learning

The block diagram of indirect learning adaptation is shown in Fig. 1. All signal designations refer either to complex baseband signals, sampled at the period  $T_s$ , and does not depend on the modulation format.

The predistorter creates a complex predistorted version  $V_{pre_k} = I_{pre_k} + j.Q_{pre_k}$ of the transmitted input signal  $V_{in_k} = I_{in_k} + j.Q_{in_k}$ , based on amplifier output  $V_{out_k} = I_{out_k} + j.Q_{out_k}$ . In the identification part, input and output complex envelopes are sampled for the real-time estimation of the PA inverse function. The input and output signals of the predistorter model are respectively  $V_{out_k}/G$  and  $\hat{V}_{pre_k}$ , where G is the PA gain. The feedback path called "Recursive identification" is the predistorter training based on minimization of the IQ errors  $\varepsilon_I$  and  $\varepsilon_Q$  for a set of K input/output data. The identification algorithm converges when the multivariable quadratic criterion  $J = \sum_{k=1}^{K} \varepsilon_{I_k}^2 + \varepsilon_{Q_k}^2$  is minimized. After identification, the new predistorter parameters are uploaded into predistorter which becomes an exact copy of predistorter model.

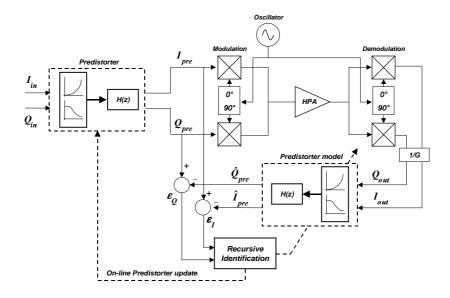


Fig. 1. Baseband equivalent scheme of the adaptive digital predistortion

## 2.2. Predistortion model

Volterra series are used in nonlinear model with memory and applied in system modeling and analysis like channel identification, PA characterization, echo cancellation <sup>1,23,27</sup>. The main advantage of such models is that they are linear-in-parameters allowing Least Mean Square (LMS) estimation techniques. However, there are severe drawbacks, especially for on-line identification, such as the large number and complexity of coefficients depending on the number of kernels (memory and the degree of nonlinearity). A special case of Volterra series is to consider a diagonal representation of their Kernels corresponding to Hammerstein model. This model, used intensively in literature <sup>19,20,23</sup>, can be interpreted by a memoryless nonlinearity block followed by a discrete filter (usually a Finite Impulse Response Filter FIR). In this paper, the Hammerstein model used for the predistorter block is described as:

$$V_{pre_k} = \sum_{q=0}^{Q-1} \sum_{p=0}^{P} \alpha_{q,2p+1} \cdot |V_{in_{k-q}}|^{2p} \cdot V_{in_{k-q}}$$
(1)

where P is the nonlinearity order, Q represents the memory length of the power amplifier and  $\alpha_{q,2p+1}$  are the predistortion complex coefficients. For parameters estimation, the model (1) is expressed in linear regression system such as:

$$V_{pre_k} = \underline{\varphi}_k^T \cdot \underline{\theta} \tag{2}$$

where  $\underline{\varphi}_k^T$  is the transposed regression vector of input signal and  $\underline{\theta}$  is the vector of  $\alpha_{q,2p+1}$  coefficients to be estimated:

$$\underline{\varphi}_{k} = \begin{bmatrix} V_{in_{k}} \\ \vdots \\ |V_{in_{k}}|^{2P} \cdot V_{in_{k}} \\ \vdots \\ V_{in_{k-Q+1}} \\ \vdots \\ |V_{in_{k-Q+1}}|^{2P} \cdot V_{in_{k-Q+1}} \end{bmatrix} \quad \text{and} \quad \underline{\theta} = \begin{bmatrix} \alpha_{0,1} \\ \vdots \\ \alpha_{0,2P+1} \\ \vdots \\ \alpha_{0,2P+1} \\ \vdots \\ \alpha_{Q-1,1} \\ \vdots \\ \alpha_{Q-1,2P+1} \end{bmatrix}$$

The objective of identification procedure is to obtain recursively the optimal estimates noted  $\hat{\underline{\theta}}$  of the vector  $\underline{\theta}$  which minimize a quadratic IQ errors.

## 3. Kalman filtering algorithm with sliding window

## 3.1. Linear prediction and correction approach

To introduce the KF concept, consider a general case of a linear discrete-time and multivariable system described in state space by:

$$\begin{cases} \underline{x}_{k+1} = A_k \, \underline{x}_k + B_k \, \underline{u}_k + \underline{v}_k \\ \underline{y}_k = C \, \underline{x}_k + \underline{\epsilon}_k \end{cases} \tag{3}$$

where

 $\underline{x}_{k}^{n \times 1}$  is the state space vector and n is system order,  $\underline{u}_{k}^{l \times 1}$  and  $\underline{y}_{k}^{m \times 1}$  are the input and the output of the system,  $\underline{v}_{k}^{n \times 1}$  and  $\underline{\epsilon}_{k}^{m \times 1}$  are respectively the state noise vector and the measurement noise,  $A_{k}^{n \times n}$ ,  $B_{k}^{n \times l}$  and  $C_{k}^{m \times n}$  are the system matrix that defining the model.

Application of the KF algorithm supposes that the noises  $\underline{v}_k$  and  $\underline{\epsilon}_k$  which affect the system are white and Gaussian and assumed statistically independent. Later we use the covariance matrices  $Q_k$  and  $R_k$  of  $\underline{v}_k$  and  $\underline{\epsilon}_k$ . We assume that we have an initial state estimate  $\underline{x}_0$  and its covariance  $P_{0/0}$ . If  $\mathbb{E}\{\cdot\}$  denotes statistical expectation operator, then these basic assumptions can be written as:

$$\mathbb{E}\left\{ \begin{bmatrix} \underline{x}_0 \\ \underline{v}_i \\ \underline{\epsilon}_i \end{bmatrix} \right\} = \begin{bmatrix} \underline{\hat{x}}_{0/0} \\ \underline{0} \\ \underline{0} \end{bmatrix} \quad \text{and} \quad \mathbb{E}\left\{ \begin{bmatrix} \underline{x}_0 \\ \underline{v}_i \\ \underline{\epsilon}_i \end{bmatrix} \cdot \begin{bmatrix} \underline{x}_0 \\ \underline{v}_j \\ \underline{\epsilon}_j \end{bmatrix}^T \right\} = \begin{bmatrix} P_{0/0} & 0 & 0 \\ 0 & Q_i \delta_{ij} & 0 \\ 0 & 0 & R_i \delta_{ij} \end{bmatrix}$$

where  $\delta_{ij}$  is the Kronecker symbol defined as:

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

and  $P_{0/0} > 0$ ,  $Q_i > 0$ ,  $R_i > 0$ .

Throughout the paper,  $(\cdot)^T$  denotes matrix transposition,  $(\cdot)^*$  conjugate transform and  $(\cdot)^H$  conjugate-transpose transform (i.e. hermitian transpose).  $I_n$  stands for the identity matrix of dimension  $n \times n$ .

The goal in using the Kalman filter is to estimate recursively the next state of a system  $\underline{x}_{k+1}$ , given the previous measurement  $\underline{y}_k$ . Assume we know at the  $k^{\text{th}}$  sampling time an estimate  $\underline{\hat{x}}_{k/k}$  of the actual state vector and the error covariance matrix  $P_{k/k}$  defined according to the estimation error  $\underline{e}_k = \underline{x}_k - \underline{\hat{x}}_{k/k}$  such as:

$$P_{k/k} = \mathbb{E}\left\{\underline{e}_k \, \underline{e}_k^T\right\} \tag{4}$$

hence from the first relation of (3), it is possible to derive a predicted value of the state at the  $(k + 1)^{\text{th}}$  sampling time noted  $\underline{\hat{x}}_{k+1/k}$ :

$$\underline{\hat{x}}_{k+1/k} = A_k \, \underline{\hat{x}}_{k/k} + B_k \, \underline{u}_k \tag{5}$$

and a predicted error covariance matrix defined by  $^{21,26}$ 

$$P_{k+1/k} = \mathbb{E}\left\{\left(\underline{x}_{k+1} - \underline{\hat{x}}_{k+1/k}\right) \left(\underline{x}_{k+1} - \underline{\hat{x}}_{k+1/k}\right)^T\right\}$$
$$= A_k P_{k/k} A_k^T + Q_k$$
(6)

A predicted measurement  $\underline{\hat{y}}_{k+1/k}$  is derived with  $\underline{\hat{x}}_{k+1/k}$  from the second relation of (3):

$$\underline{\hat{y}}_{k+1/k} = C_{k+1} \underline{\hat{x}}_{k+1/k} \tag{7}$$

After prediction step defined above, we proceed to the correction step of state in which we use the measurement  $\underline{y}_{k+1}$  to improve the estimation error such as:

$$\underline{\hat{x}}_{k+1/k+1} = \underline{\hat{x}}_{k+1/k} + K_{k+1} \left( \underline{y}_{k+1} - \underline{\hat{y}}_{k+1/k} \right)$$
(8)

where the matrix gain  $K_{k+1}^{n \times m}$  called *Kalman gain*, is computed in order to minimize the trace of the error covariance matrix defined as the sum of the elements on its main diagonal. Then it can be shown that  $K_{k+1}$  and  $P_{k+1/k+1}$  are given by <sup>26</sup>

$$K_{k+1} = P_{k+1/k} C^T (C P_{k+1/k} C^T + R_{k+1})^{-1}$$
(9)

and

$$P_{k+1/k+1} = (I_n - K_{k+1}C) P_{k+1/k}$$
(10)

State vector estimation using Kalman filter requires an initial values  $\underline{\hat{x}}_{0/0} = \underline{\hat{x}}(0)$ and computation of the corresponding error covariance matrix  $P_{0/0}$ . Usually initial values of state vector are chosen to have a relative stability at the beginning of the system adaptation and  $P_{0/0}$  are the identity matrix with high values of the diagonal terms to ensure convergence and unbiased estimates <sup>23</sup>. In practice, the Kalman filter stabilizes progressively during iterations and converge to an optimal values in spite of initialization errors.

To summarize the recursive algorithm we state all the formulas for the Kalman filter one last time :

- Initialization step : Select initial values of  $\underline{\hat{x}}_{0/0}$  and  $P_{0/0}$
- Prediction Step: Compute the evolution model estimate and covariance:

$$\frac{\hat{x}_{k+1/k}}{\hat{y}_{k+1/k}} = A_k \, \underline{\hat{x}}_{k/k} + B_k \, \underline{u}_k \\
\underline{\hat{y}}_{k+1/k} = C_{k+1} \, \underline{\hat{x}}_{k+1/k} \\
P_{k+1/k} = A_k \, P_{k/k} \, A_k^T + Q_k$$
(11)

• Correction Step: Correct a state estimate and covariance:

$$K_{k+1} = P_{k+1/k} C^T (C P_{k+1/k} C^T + R_{k+1})^{-1}$$
  

$$\underline{\hat{x}}_{k+1/k+1} = \underline{\hat{x}}_{k+1/k} + K_{k+1} (\underline{y}_{k+1} - \underline{\hat{y}}_{k+1/k})$$
  

$$P_{k+1/k+1} = (I_n - K_{k+1} C) P_{k+1/k}$$
(12)

• Update k = k + 1 and return to Prediction step.

## 3.2. Kalman filtering and parameters estimation

The problem of interest is to extract recursively the predistorter parameters  $\underline{\theta}$  composed from the complex coefficients  $\alpha_{q,2p+1}$  (Eqs. 1-2) using sampled input and output envelope in baseband format. In the indirect learning approach described in Fig. (1), we consider the classical problem of inverse model computation describing the output envelope  $V_{out}$  to predistorted envelope  $V_{pre}$  relationship. At each iteration, the Kalman filter solves the problem of estimating the parameters of Hammerstein with memory model to minimize the quadratic error using PA's output  $V_{out}$  as an input signal <sup>19</sup>. In this formulation, the parameters are considered as state variables described by a quasi-stationary evolution, thus the discrete system defined in (3) becomes a parameter evolution and predistortion system represented by a particular case of single output system with  $A_k = I_n$ ,  $B_k = 0$  and  $C = \underline{\varphi}^T$ , given by:

$$\begin{cases} \underline{\theta}_{k+1} = \underline{\theta}_k + \underline{\upsilon}_k \\ V_{pre_k} = \hat{F}_{pre} \left( \underline{\theta}_k, \, V_{out} \right) + \underline{\epsilon}_k = \underline{\varphi}^T \cdot \underline{\theta} + \epsilon_k \end{cases}$$
(13)

Because we are in inverse model formulation, the regressor vector  $\underline{\varphi}$  is obtained by replacing  $V_{in}$  by  $V_{out}$  in relations (1) and (2).

Using the Kalman filter developed in previous section in this case, the updating of the parameters vector is carried out:

$$\begin{cases} \frac{\hat{\theta}_{k+1} = \hat{\theta}_k + K_{k+1} \left( V_{pre_{k+1}} - \hat{V}_{pre_{k+1}} \right) \\ K_{k+1} = P_k \, \underline{\varphi} \left( \underline{\varphi}^T \, P_k \, \underline{\varphi} + R_{k+1} \right)^{-1} \\ P_{k+1} = \left( I_n - K_{k+1} \, \underline{\varphi}^T \right) \left( P_k + Q_k \right) \end{cases}$$
(14)

where  $\underline{\hat{\theta}}_{k+1} = \underline{\hat{\theta}}_{k+1/k} = \underline{\hat{\theta}}_{k+1/k+1}$  according to the first relation of (13) which shows that predicted estimates (*a priori*)  $\underline{\hat{\theta}}_{k+1/k}$  and corrected one (*a posteriori*)  $\underline{\hat{\theta}}_{k+1/k+1}$ are equal. This simplify a previous notations used in section (3.1).

### 3.3. Adaptation to linearization problem

The KF algorithm described above is based on the discrete state space model describing the future evolution of the system when the input is given. In identification approach, the Kalman gain corrects iteratively the estimate according to the error between measured output and desired input <sup>21</sup>. The advantage of this technique is that the estimate is corrected recursively at each iteration. However, there are severe drawbacks, not acceptable in real-time estimation, such as the great number of calculations with complex data and matrix to obtain an appropriate KF gain <sup>23</sup>. To reduce and simplify these computations, the proposed method is based on the description of a sliding time-window. In this case, the time domain is decomposed into several data sets as shown in Fig. 2. At the end of each set composed by  $N_w$  input and output data, the vector of parameters is corrected according to KF algorithm, which amounts to introduce a new sampling period greater than  $T_s$  and equal to  $N_w \times T_s$ .

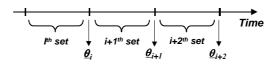


Fig. 2. On-line estimation with sliding window

According to linearization scheme in Fig. (1) and given  $N_w$  measured values of predistorted signal  $V_{pre}$  and output envelope signal  $V_{out}$ , we can analyze for one set an off-line situation where we can write the  $N_w$  equations:

$$\begin{cases}
V_{pre_1} = F_{pre_1}(\underline{\theta}, V_{out}) + \epsilon_1 = \underline{\varphi}_1^T \cdot \underline{\theta} + \epsilon_1 \\
\vdots \\
V_{pre_k} = F_{pre_k}(\underline{\theta}, V_{out}) + \epsilon_k = \underline{\varphi}_k^T \cdot \underline{\theta} + \epsilon_k \\
\vdots \\
V_{pre_{N_w}} = F_{pre_{N_w}}(\underline{\theta}, V_{out}) + \epsilon_{N_w} = \underline{\varphi}_{N_w}^T \cdot \underline{\theta} + \epsilon_{N_w}
\end{cases}$$
(15)

which can be re-written in linear matrix regression model such as:

$$\underline{V}_{pre_{i}}^{Nw \times 1} = \phi_{i}^{Nw \times N} \cdot \underline{\theta}^{N \times 1} + \underline{\epsilon}_{i}^{Nw \times 1} \quad \text{with} \quad \phi_{i} = \begin{bmatrix} \underline{\varphi}_{1}^{T} \\ \vdots \\ \underline{\varphi}_{k}^{T} \\ \vdots \\ \underline{\varphi}_{Nw}^{T} \end{bmatrix}$$
(16)

and N = (P+1).Q is the number of parameters to be estimated.

If we assume that the variations of PA characteristics in each frame are negligible according to the global variation during operation, the application of KF algorithm gives the final updating system parameters by mixing off-line measurement with a fundamental sampling time equal to  $T_s$  and on-line estimation with  $N_w \times T_s$ sampling time such as:

$$\begin{cases} K_{i+1} = P_i \phi_{i+1}^H \left( R + \phi_{i+1} P_i \phi_{i+1}^H \right)^{-1} \\ P_{i+1} = P_i - K_{i+1} \phi_{i+1} P_i \\ \underline{\hat{\theta}}_{i+1} = \underline{\hat{\theta}}_i + K_{i+1} \left( \underline{V}_{pre_{i+1}} - \underline{\hat{V}}_{pre_{i+1}} (\underline{\hat{\theta}}_i, \underline{V}_{out}) \right) \end{cases}$$
(17)

where transposition  $(.)^T$  is replaced here by conjugate-transpose transform  $(.)^H$  because the predistortion parameters are a complex coefficients.

By the matrix inversion lemma detailed in  $^{26}$ , the gain matrix  $K_{i+1}$  becomes for complex coefficients:

$$\underline{K}_{i+1} = P_{i+1} \phi_{i+1}^H R^{-1} \tag{18}$$

and we obtain the final updating system:

$$\begin{cases} \underline{\hat{\theta}}_{i+1} = \underline{\hat{\theta}}_i + P_{i+1} \, \phi^H \, R^{-1} \left( \underline{\varepsilon}_{I_{i+1}} + j \underline{\varepsilon}_{Q_{i+1}} \right) \\ P_{i+1}^{-1} = P_i^{-1} + \phi_{i+1}^H \, R^{-1} \, \phi_{i+1} \end{cases}$$
(19)

When physical knowledge or prior information on parameter variations is known and modeled by parameters covariance matrix noted Q, we can rewrite the error covariance matrix P to take into account this constraint on parametric domain such as:

$$\begin{cases} \underline{\hat{\theta}}_{i+1} = \underline{\hat{\theta}}_i + P_{i+1} \, \phi^H \, R^{-1} \left( \underline{\varepsilon}_{I_{i+1}} + j \underline{\varepsilon}_{Q_{i+1}} \right) \\ P_{i+1}^{-1} = (P_i + Q)^{-1} + \phi_{i+1}^H \, R^{-1} \, \phi_{i+1} \end{cases}$$
(20)

Noted that if the process noise variance called  $\sigma_b^2$  is known, the output matrix variance R can be replaced by  $R = \sigma_b^2 I_n$ , which gives the same updating system as in reference <sup>13</sup>. As mentioned in previous sections, these design parameters will considerably modify the performance of the Kalman algorithm. Hence, these parameters must assume appropriate values to achieve an optimal tracking and fast

convergence <sup>20</sup>. The choice of the number of input and output data  $N_w$  is important in on-line identification procedure. This parameter is principally determined by the time-constant of the PA dynamics and must be chosen greater than its timetransient to insure convergence of regressors vectors  $\underline{\phi}_k$  defined in relations (15) and (16).

To start the optimization, the initial conditions of the predistorter have to be defined. The initialization is very important to insure stability and high speed convergence <sup>7</sup>. For an unknown amplifier characteristics, we can initialize the vector of predistorter coefficients  $\underline{\theta}$  at unity gain values, i.e.:

$$\underline{\theta}_{0} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^{T}$$

$$(21)$$

Another solution is to perform an off-line identification of the predistorter using LMS algorithm <sup>26</sup>. The calculated parameters can be downloaded on the digital signal processor (DSP) and serve as initial values for the predistorter.

## 4. Simulation and experimental results

In this section, we illustrate through simulations and experiments, performance of the memory polynomial predistorter identified using modified KF algorithm.

## 4.1. Simulation results

The proposed digital predistortion technique is used to linearize an actual model from Class AB PA (HEMT ZJL-3G), at the frequency of 2.1 GHz designed with MATLAB/SIMULINK SOFTWARE. The model is composed of gain and phase nonlinearities described by Saleh equations followed by a Finite Impulse Response filter with complex coefficients <sup>19</sup>. The test signal is a 16-QPSK digitally modulated signal at rate of 5 Mb/s and shaped with a raised cosine filter having a rolloff factor of 0.25. We compare the power spectral density (PSD) of the output signals to evaluate the effectiveness of the predistorter in reducing spectral regrowth. In this part, the predistorter (Eq. 1) is defined with two delay taps (Q = 2) and 5<sup>th</sup> odd-order nonlinearity (P = 2). The power amplifier was driven to the 1 dB compression point. All results are given with a vector of parameters initialized using LMS algorithm.

## 4.1.1. Convergence and linearity indicators

Fig. 3 shows the performance improvement in terms of spectral regrowth. The complex predistorter with memory could achieve 20 dB reduction in spectral distortion.

To investigate the real-time convergence of the parametric space under transmitter variations, we simulate a modification of the PA characteristics during transmission. In practice, a bias of  $\pm 25\%$  is introduced to the AM/AM and AM/PM parameters of the used PA at 500  $\mu s$ . Figures (4.a) and (4.b) give respectively the evolution of real and imaginary parts of predistorter parameters during linearization procedure. In figure (4.a), the curves from the top to the bottom after convergence

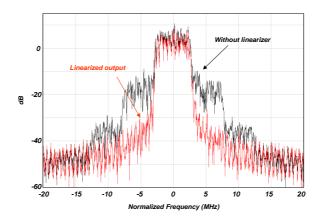


Fig. 3. Power spectral density of output signal

are, respectively, the real parts of  $\alpha_{0,1}$ ,  $\alpha_{0,5}$ ,  $\alpha_{1,3}$ ,  $\alpha_{1,1}$ ,  $\alpha_{0,3}$  and  $\alpha_{1,5}$ . In figure (4.b), the curves from the top to the bottom are, respectively, the imaginary parts of  $\alpha_{1,5}$ ,  $\alpha_{1,1}$ ,  $\alpha_{0,3}$ ,  $\alpha_{0,1}$ ,  $\alpha_{1,3}$  and  $\alpha_{0,5}$ .

The system identification results reveal that the change of the PA characteristics affect the parameters estimates. That explains why the fixed parameter predistorter is not appropriate when variations are occurring in system transmission. In the adaptive control case, the on-line scheme effectively uses system parameter estimation to adjust the predistorter parameters in real time according to PA changes. Noted that new predistorter coefficients corresponding to the modified amplifier model are achieved in only 300  $\mu s$  corresponding to 15 iterations with  $N_w = 500$ data for each time-window ( $T_s = 0.04 \, \mu s$ ).

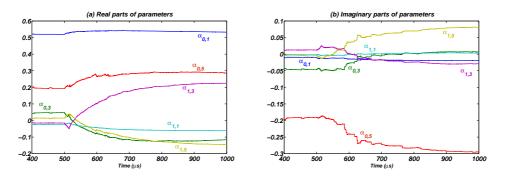


Fig. 4. Evolution of parameters during estimation procedure

In term of adjacent spectral regrowth, fig. (5) gives the simulated output spectral density before, after model variations and after parameters convergence. As we

can see, the variation induces a transient deterioration of the spectral response, corrected after by the adaptive predistorter.

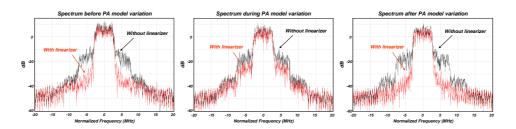


Fig. 5. On-line spectrum evolution with PA model variation

If there is a signal with non-constant envelope at the PA's input, each of its samples will be amplified with different gain and the introduced phase shift will differ according to the input signal amplitude. This nonlinear distrotion is illustrated in the case of 64-QAM modulation as shown in figure (6). The constellation point near the saturation will be more deformed in the case of PA without linearizer. These figures show too the improvement in constellation diagrams for the adaptive linearization using KF with sliding window.

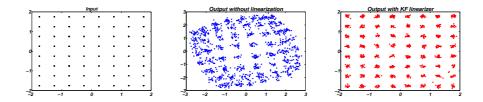


Fig. 6. Constellation diagrams for 64-QAM signal

## 4.1.2. Evaluation criteria

To carry out a more detailed study among different adaptive algorithm, we compare the proposed KF algorithm using sliding window (KF<sub>SW</sub>) with classical Extended Kalman Filter EKF (Eq. 14) and two other known techniques: Recursive Least Squares algorithm with forgetting factor  $\lambda$  (RLS algorithm) <sup>19,28</sup> and Gradient algorithm <sup>7</sup>. These approaches correct the estimate parameters according to estimation error with different gains such as:

• RLS algorithm <sup>19</sup>

$$\begin{cases} K_{k+1} = P_k \cdot \underline{\varphi}_{k+1}^* \cdot \left(\lambda + \underline{\varphi}_{k+1}^H P_k \underline{\varphi}_{k+1}\right)^{-1} \\ P_{k+1} = \lambda^{-1} \cdot \left(P_k - K_{k+1} \underline{\varphi}_{k+1}^T P_k\right) \\ \underline{\hat{\theta}}_{k+1} = \underline{\hat{\theta}}_k + K_{k+1} \left(V_{pre_{k+1}} - \hat{V}_{pre_{k+1}}\right) \end{cases}$$
(22)

• Gradient algorithm  $^{26,29}$ 

$$\begin{cases} K_{k+1} = 2 \cdot \mu \cdot \underline{\varphi}_{k+1}^* \\ \underline{\hat{\theta}}_{k+1} = \underline{\hat{\theta}}_k + K_{k+1} \left( V_{pre_{k+1}} - \hat{V}_{pre_{k+1}} \right) \end{cases}$$
(23)

The major difference between variants of KF algorithm, RLS and gradient algorithms is the degrees of freedom in covariance matrix adjustment. RLS and Gradient algorithms have only scalar parameter  $\lambda$  and  $\mu$  to adjust the speed convergence and stability contrary to KF algorithm which has  $N \times N$  weights in the monitoring matrix R (where  $N = (P + 1) \cdot Q = 6$  is the number of parameters in the vector  $\underline{\theta}$ described in section 2.2).

To give a quantitative measure of the improvement, we use the normalized mean square error (NMSE), as

$$\text{NMSE}_{dB} = 10 \, \log_{10} \left( \frac{\sum_{k=1}^{K} \left| V_{pre_k} - \hat{V}_{pre_k} \right|^2}{\sum_{k=1}^{K} \left| V_{pre_k} \right|^2} \right)$$
(24)

where K is the total number of points. We define also the number of arithmetic operations for each iteration composed by the  $N_w = 500$  data. Subtraction and division are counted respectively with addition (called Add) and with multiplication (called Mult).

|                    | $\text{NMSE}_{dB}$ | Number of operations                           |
|--------------------|--------------------|--|
| $\mathrm{KF}_{SW}$ | -44.92             | $Add = N_w . (2N^2 + 3) = 37500$               |
|                    |                    | $Mult = 4.N_w.N^3 + 4N^2 + 2N^2 = 432216$      |
| EKF                | -42.61             | $Add = N_w \cdot (8N^2 + 2N + 8) = 154000$     |
|                    |                    | $Mult = N_w \cdot (2N^4 + 4N^2 + 8) = 1372000$ |
| RLS                | -37.66             | $Add = N_w.(4N^2 + 2) = 73000$                 |
|                    |                    | $Mult = N_w.(N^4 + 4N^2 + 5N) = 735000$        |
| Gradient           | -36.94             | $Add = N_w.(2N+1) = 6500$                      |
|                    |                    | $Mult = N_w.(4N - 1) = 11500$                  |

Table 1. Comparison of  $\text{NMSE}_{dB}$  and computation complexity

The second column of Table (1) shows the NMSE<sub>dB</sub> obtained by averaging over the last  $K = 20.N_w = 10000$  error samples to reflect algorithm performances after

convergence. We can observe that the proposed  $KF_{SW}$  algorithm and classical one give more than 7dB NMSE improvement compared to classical recursive algorithms. In term of computation complexity, it is seen that the Gradient algorithm yielded the minimal result. In reference <sup>29</sup>, it is proved that Gradient algorithm is a more simplified version of Kalman filter where covariance matrix elements are reduced to one scalar coefficient equal to unity. Noted that the RLS algorithm requires two time arithmetic operations compared to  $KF_{SW}$  Algorithm.

In fact, proposed  $\text{KF}_{SW}$  algorithm remains optimal in implementation because the required number of arithmetic operations are treated during  $N_w.Ts = 20\mu s$ , contrary to the other algorithms where all operations are performed at sampling time  $Ts = 0.04\mu s$ .

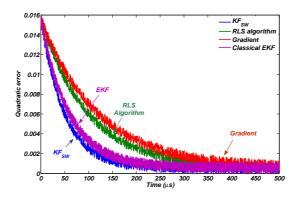


Fig. 7. Comparison of learning curves for different identification algorithm

Fig. (7) shows the learning curves of the predistorter employing the KF<sub>SW</sub> algorithm, classical EKF algorithm, RLS algorithm and Gradient algorithm where predistortion is initialized with unity gain value defined in relation (21). The evolution of mean squared error  $\varepsilon_{I_k}^2 + \varepsilon_{Q_k}^2$  during iteration are plotted. As shown, in term of speed convergence, we can see that the KF<sub>SW</sub> algorithm and classical one have similar performances. Only difference between these KF variants is the sampling time used which is greater in the case of KF with sliding window. The proposed algorithm and classical EKF converge and minimize the quadratic error faster than other algorithms.

## 4.2. Experimental results

This section describes the practical results achieved using the experimental setup shown in Fig. 8.

The power amplifier is a commercial Class AB ZHL-42 from MINI CIRCUITS manufacturer. Quadrature modulator AD8349 and demodulator AD8347 are inserted at the input and output of the PA. The DSP processor is a ADSP21161N

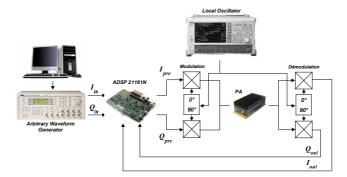


Fig. 8. Experimental setup

platform with dual DAC/ADC 4 inputs/6 outputs ports. They are standard commercial units from Analog Devices with a computational running at  $F_s = 100$  MHz.

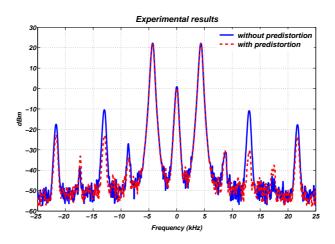


Fig. 9. Power spectral density

Fig. 9 shows the spectrum for a sinusoidal signal applied on  $I_{in}$  at  $F_m = 4.5$ kHz. The total power of the two main components at frequencies  $900 \pm F_m$  MHz is equal to 25dBm, corresponding to the 1dB compression point for a two carriers operating condition in accordance with the 1dB compression point of 28dBm for a CW signal. For such output level, a carrier to third order intermodulation ratio of 32dBc is achieved. The second curve plotted on Fig. 9 corresponds to the spectrum achieved after adaptation of the predistorter. The real time adjustment of the predistorter parameters allows ~ 15dB improvement of the third order intermodulation distortion.

Fig. 10 shows the time domain measurements of predistorted signal and residual

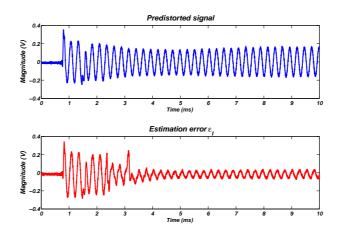


Fig. 10. Time domain measurements of predistorted signal and residual

 $\varepsilon_I$ . It's important to note that the initial values of the parameters correspond to a unity gain (Eq. 21). Hence, the adaptation algorithm is started at 0.8 ms and after a transient of 3.2 ms, we can verify that the estimation error is correctly minimized.

## 5. Conclusion

A new technique for performing baseband predistortion has been described. In this approach, an alternative Kalman Filtering algorithm is introduced to design and estimate a complex predistorter with memory. Identification algorithm has been suitably modified to insure convergence, stability and reduce number of calculations during estimation. With sliding window transformation, the resulting equation to update the covariance matrix is more simple according to the classical EKF, allowing a significant reduction of the computational complexity and numerical calculations. The technique uses the baseband transmitted signals through the PA to perform coefficients update in real-time.

The effectiveness of this approach is demonstrated through simulation results, showing that the adapted Kalman Filtering predistorter reduces the adjacent channel interference. The time domain variation of parameters illustrates the capability of this procedure to track PA changes.

Experimental setup based on DSP microprocessor has been used and showed good spectral improvement, illustrated by a reduction of  $\sim 15$ dB of IM<sub>3</sub> for a sinusoidal modulating signal. By simulation and measurement, it has been shown that the adaptive procedure is fast, even in the case of a predistorter initiliazed at unity gain. This result allows to applied different real time strategies to adapt the predistorder for example continuously during the transmission, periodically to track change in transmitter characteristics or when operating conditions or signal format are modified.

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