



**HAL**  
open science

# $H_\infty$ Control of Delayed Teleoperation Systems under Polytopic-Type Uncertainties

Bo Zhang, Alexandre Kruszewski, Jean-Pierre Richard

► **To cite this version:**

Bo Zhang, Alexandre Kruszewski, Jean-Pierre Richard.  $H_\infty$  Control of Delayed Teleoperation Systems under Polytopic-Type Uncertainties. 20th Mediterranean Conference on Control and Automation, Jul 2012, Barcelone, Spain. hal-00684710

**HAL Id: hal-00684710**

**<https://hal.science/hal-00684710>**

Submitted on 2 Apr 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# $H_\infty$ Control of Delayed Teleoperation Systems under Polytopic-Type Uncertainties

Bo Zhang, Alexandre Kruszewski and Jean-Pierre Richard

**Abstract**—In this paper, the  $H_\infty$  control design under time-varying delays and polytopic-type uncertainties, which ensures the stability and performance (synchronization/transparency) between the master and slave manipulators, is proposed. With this objective, the design of the controller based on our proposed control scheme is performed by using Linear Matrix Inequality (LMI) optimization based on Lyapunov-Krasovskii functionals (LKF) and  $H_\infty$  control theory. The solution is efficient for different working conditions, e.g. abrupt tracking and wall contact motion, and this is illustrated by a final example.

## I. INTRODUCTION

Bilateral teleoperation is the extension of a person's sensing and manipulation capability to a remote environment, a typical form of which is composed of the human operator, the master robot, the communication medium, the slave robot and the environment. The remote slave robot tracks the motion of the master robot commanded by the human operator. Conversely, the corresponding data of the slave robot is also transmitted back to the master robot, in particular, to improve the task performance, force feedback from the slave to master, representing contact motion, provides a more extensive feeling of telepresence [1], [2].

In bilateral teleoperation, the master and slave are coupled via the communication medium, which incurs the additional dynamics represented by time-varying delays (especially with the Internet) [3]. In order to avoid a severe deterioration of the global performance, these delays must be considered at the control design stage [4]. Recently, various methods addressed the stability and performance issues, most of them concern *passivity-based control* under variable delays, including the scattering and wave variables, the readers can refer to the survey [5] and the references therein; based on the energy and power considerations, time domain passivity control [6], [7] without the transformation of wave variables have been proposed. For *non-passive control*, [8] and [9] summarized many teleoperation control strategies and schemes. These methods can resolve the stabilization problem under constant or time-varying delays, but the system performance is not guaranteed.

Our latest research [10] presented a force-reflecting proxy control scheme, which guarantees the stability and the position/force tracking of the closed-loop system under time-varying delays by using Lyapunov-Krasovskii functionals (LKF) and  $H_\infty$  control, which can be solved by Linear

Matrix Inequality (LMI) optimization. Thus, in this paper, based on the control scheme proposed in [10], we handle the teleoperation under time-varying delays and model uncertainties. Here, the uncertainties are expressed under the form of time-varying parameters, but could also correspond to nonlinear effects. The design approach proposed in this paper can be summarized:

- Control scheme in [10] is utilized, but the technique is extended to the case of polytopic-type models of master, proxy of master (a remote observer of the master used at the slave side to reduce the impact of the communication delays) and slave.
- Local controllers of master, proxy and slave are designed by using Lyapunov functions and LMI optimization.
- Slave controller used to eliminate the impact of asymmetric time-varying delays are fixed by LKF,  $H_\infty$  control and LMI [11]. Local and slave controllers mentioned above will be explained later.

This paper is organized as follows: Section 2 introduces the theorems to be used later. Some details of the control scheme and the problems to be resolved are presented in Section 3. Our main results are given in Section 4. Results of simulation are presented in Section 5. Finally we conclude in Section 6.

## II. PRELIMINARIES

This section is devoted to the general stability analysis with the  $H_\infty$  performance index for time delay system with time-varying polytopic-type uncertainties. It is considered the following system with time-varying matrices  $A_0$ ,  $A_i$ ,  $i = 1, 2, \dots, n$  and  $B$ ,

$$\begin{cases} \dot{x}(t) = A_0x(t) + \sum_{i=1}^n A_i x(t - \tau_i(t)) + Bw(t), \\ z(t) = Cx(t), \\ x(t_0 + \theta) = \phi(\theta), \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \theta \in [-h_2, 0], \end{cases} \quad (1)$$

where,  $x(t) \in \mathbf{R}^n$ ,  $w(t) \in \mathbf{R}^l$  is some exogenous disturbance signals, while  $z(t) \in \mathbf{R}^q$  is seen as the objective control output.  $\phi(\theta)$  is the initial state function, and  $\tau_i(t) \in [h_1, h_2]$ ,  $h_1 \geq 0$ ,  $i = 1, 2, \dots, n$ , are time-varying delays. There is no particular assumption on  $\dot{\tau}_i(t)$ .  $C$  is the constant matrix, and  $A_0$ ,  $A_i$ ,  $i = 1, 2, \dots, n$ ,  $B$  are subject to time-varying uncertainties and satisfy the real convex polytopic model,

{Bo Zhang, Alexandre Kruszewski and Jean-Pierre Richard} are with Université Lille Nord de France, Ecole Centrale de Lille - 59651 Villeneuve d'Ascq, France and LAGIS, CNRS UMR 8219, Laboratoire d'Automatique, Génie Informatique et Signal. J.-P. Richard is also with INRIA NON-A. Email: {bo.zhang, alexandre.kruszewski, jean-pierre.richard}@ec-lille.fr.

$$[A_0, A_i, B] \in \Omega, \quad i = 1, 2, \dots, n,$$

$$\Omega \triangleq [A_0(\rho(t)), A_i(\rho(t)), B(\rho(t))] = \sum_{j=1}^N \rho_j(t) [A_{0j}, A_{ij}, B_j],$$

$$\sum_{j=1}^N \rho_j(t) = 1, \quad \rho_j(t) \geq 0], \quad (2)$$

where,  $A_{0j}, A_{ij}, B_j, i = 1, 2, \dots, n, j = 1, 2, \dots, N$ , are constant matrices of appropriate dimension and  $\rho_j(t), j = 1, 2, \dots, N$ , are time-varying uncertainties. Note that, in the following, all systems mentioned satisfy the polytopic-type condition as in (2).

Considering the Lyapunov-Krasovskii functional as follow [12],

$$V(t, x(t), \dot{x}(t)) = \sum_{j=1}^N V_j(t, x(t), \dot{x}(t)),$$

$$V_j(t, x(t), \dot{x}(t)) = x(t)^T P_j x(t)$$

$$+ \int_{t-h_1}^t x(s)^T S_j x(s) ds + \int_{t-h_2}^t x(s)^T S_{aj} x(s) ds \quad (3)$$

$$+ h_1 \int_{-h_1}^0 \int_{t+\theta}^t \dot{x}(s)^T R_j \dot{x}(s) ds d\theta$$

$$+ \sum_{i=1}^n (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{x}(s)^T R_{aij} \dot{x}(s) ds d\theta.$$

According to  $H_\infty$  control theory, the performance will be studied by checking  $H_\infty$  performance index  $J(w) < 0$  for a positive scalar  $\gamma$ ,

$$J(w) = \int_0^\infty (z(t)^T z(t) - \gamma^2 w(t)^T w(t)) dt < 0, \quad (4)$$

and then, from [10],  $J(w) < 0$  can be assured if,

$$\dot{V}(t, x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0. \quad (5)$$

**Theorem 1:** Suppose there exist matrices  $P_j > 0, R_j > 0, S_j > 0, S_{aj} > 0, R_{aij} > 0, P_{2j}, P_{3j}, Y_{1j}, Y_{2j}, i = 1, 2, \dots, n, j = 1, 2, \dots, N$ , and a positive scalar  $\gamma$ , such that the condition (6) with notations (7),  $j = 1, 2, \dots, N$ , is feasible. Then the system (1) is robustly stable with  $J(w) < 0$  for time-varying delays  $\tau_i(t) \in [h_1, h_2], i = 1, 2, \dots, n$ .

**Proof.** From [13], the condition in (5) can be relaxed to a set of  $N$  LMIs as follows,  $j = 1, 2, \dots, N$ ,

$$\dot{V}_j(t, x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0. \quad (8)$$

The proof of (8) can be found in [10], the readers can refer to [10] for more details.

Our theorem can also be derived to the delay-free case (without  $\sum_{i=1}^n A_i x(t - \tau_i(t)), i = 1, 2, \dots, n$ ) as follow,

$$\begin{cases} \dot{x}(t) = A_0 x(t) + B w(t), \\ z(t) = C x(t). \end{cases} \quad (9)$$

**Corollary 1:** Suppose there exist matrices  $P_j > 0, P_{2j}, P_{3j}, j = 1, 2, \dots, N$ , and a positive scalar  $\gamma$ , such that

the condition (10) with the notation (11),  $j = 1, 2, \dots, N$ , is feasible. Then the system (9) is robustly stable with  $J(w) < 0$ .

$$\Gamma^{2j} = \begin{pmatrix} \Gamma_{11}^{2j} & P_j - P_{2j}^T + A_{0j}^T P_{3j} & P_{2j}^T B_j \\ * & -P_{3j} - P_{3j}^T & P_{3j}^T B_j \\ * & * & -\gamma^2 I \end{pmatrix} < 0, \quad (10)$$

$$\Gamma_{11}^{2j} = A_{0j}^T P_{2j} + P_{2j}^T A_{0j} + C^T C. \quad (11)$$

*Remark 1:* Theorem 2 in [10] can also be directly applied to system (1) (One matrix variable replaces  $N$  matrix variables, e.g.  $P$  replaces  $P_j, j = 1, 2, \dots, N$ ), and then, an polytopic method is applied to verify the LMI condition. This method can also make the system robustly stable, but leads to more conservative, which will be proved later by the simulation.

### III. PROBLEM FORMULATION

The force-reflecting proxy control scheme is presented in Fig. 1. Let us give a description of the control scheme:

- $F_m(t)$  and  $F_s(t)$  are the actuated inputs of the master and the slave.

- $F_h(t)$  and  $F_e(t)$  are the forces of the human operator and environment on the system.  $\hat{F}_h(t)$  and  $\hat{F}_e(t)$  are the estimations of these two forces, which can be obtained by adding the perturbation observers in reality.

- $\tau_1(t)$  (from the master to the slave) and  $\tau_2(t)$  (from the slave to the master) are the time-varying delays, which are modeled as the previous section,  $\tau_1(t), \tau_2(t) \in [h_1, h_2]$ . Master and slave clocks are synchronized thanks to time-stamped data packet exchanges between them, using a network time protocol as in [14]. Therefore,  $\hat{\tau}_1(t)$  is the estimated network delay between the master and slave, it is available at slave's side:  $\hat{\tau}_1(t) = \tau_1(t)$ .

- From the master to slave, the information transferred are the velocity/position of the master and the estimated force  $\hat{F}_h(t)$ . However, from the slave to the master, only the estimated force  $\hat{F}_e(t)$  is transferred, so the force tracking,  $F_m(t) = \hat{F}_e(t - \tau_2(t))$ , is realized, if the stability of the whole system is verified.

- $\dot{\theta}_m(t)/\theta_m(t)$  and  $\dot{\theta}_s(t)/\theta_s(t)$  are the velocities/positions of the master and slave.

The models of master and slave with polytopic-type uncertainties are described as follows,

$$(\Sigma_m) \quad \dot{x}_m(t) = (A_m(\rho_m(t)) - B_m(\rho_m(t))K_m^0)x_m(t) + B_m(\rho_m(t))(F_m(t) + F_h(t)), \quad (12)$$

$$(\Sigma_s) \quad \dot{x}_s(t) = (A_s(\rho_s(t)) - B_s(\rho_s(t))K_s^0)x_s(t) + B_s(\rho_s(t))(F_s(t) + F_e(t)),$$

where,  $x_m(t) = \dot{\theta}_m(t), x_s(t) = \dot{\theta}_s(t)$  are the states of the master and slave,

$$[A_m(\rho_m(t)), B_m(\rho_m(t))] = \sum_{j=1}^N \rho_{mj}(t) [A_{mj}, B_{mj}], \quad (13)$$

$$[A_s(\rho_s(t)), B_s(\rho_s(t))] = \sum_{j=1}^N \rho_{sj}(t) [A_{sj}, B_{sj}].$$

$$\Gamma^{1j} = \begin{pmatrix} \Gamma_{11}^{1j} & \Gamma_{12}^{1j} & R_j + \sum_{i=1}^n P_{2j}^T A_{ij} - nY_{1j}^T & nY_{1j}^T & -P_{2j}^T A_{1j} + Y_{1j}^T & \dots & -P_{2j}^T A_{nj} + Y_{1j}^T & Y_{1j}^T & \dots & Y_{1j}^T & P_{2j}^T B_j \\ * & \Gamma_{22}^{1j} & \sum_{i=1}^n P_{3j}^T A_{ij} - nY_{2j}^T & nY_{2j}^T & -P_{3j}^T A_{1j} + Y_{2j}^T & \dots & -P_{3j}^T A_{nj} + Y_{2j}^T & Y_{2j}^T & \dots & Y_{2j}^T & P_{3j}^T B_j \\ * & * & -S_j - R_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_{aj} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -R_{a1j} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \dots & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_{anj} & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -R_{a1j} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \dots & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -R_{anj} & 0 \\ * & * & * & * & * & * & * & * & * & * & -\gamma^2 I \end{pmatrix} < 0, \quad (6)$$

$$\Gamma_{11}^{1j} = S_j + S_{aj} - R_j + A_{0j}^T P_{2j} + P_{2j}^T A_{0j} + C^T C, \quad \Gamma_{12}^{1j} = P_j - P_{2j}^T + A_{0j}^T P_{3j}, \quad \Gamma_{22}^{1j} = -P_{3j} - P_{3j}^T + h_1^2 R_j + (h_2 - h_1)^2 \sum_{i=1}^n R_{aij}. \quad (7)$$

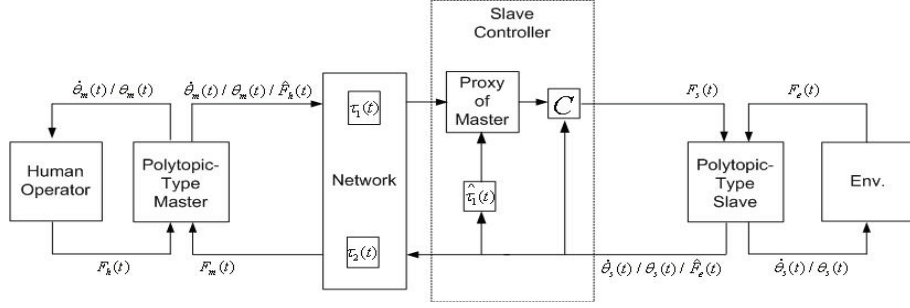


Fig. 1. Force-reflecting proxy control scheme

As mentioned above, all uncertainties proposed in this paper satisfy the polytopic-type condition as in (2).  $K_m^0$ ,  $K_s^0$  are local controllers of the master and slave ensuring the speed stability, which will be designed in next section.

Because the proxy acts as a remote observer of the master, the proxy model is the same as that of the master, but under different polytopic-type uncertainty,

$$(\Sigma_p) \quad \dot{x}_p(t) = (A_p(\rho_p(t)) - B_p(\rho_p(t))K_m^0)x_p(t) + B_p(\rho_p(t))(\hat{F}_e(t) + \hat{F}_h(t - \tau_1(t)) - F_p(t - \tau_1(t))), \quad (14)$$

$$[A_p(\rho_p(t)), B_p(\rho_p(t))] = \sum_{j=1}^N \rho_{pj}(t)[A_{mj}, B_{mj}], \quad (15)$$

where,  $x_p(t) = \hat{\theta}_p(t)$  is the state of the proxy,  $\hat{\theta}_p(t)/\theta_p(t)$  is the velocity/position,  $F_p(t - \tau_1(t))$  is the correction term of the proxy of master,

$$F_p(t - \tau_1(t)) = L \begin{pmatrix} \hat{\theta}_p(t - \tau_1(t)) \\ \hat{\theta}_m(t - \tau_1(t)) \\ \theta_p(t - \tau_1(t)) - \theta_m(t - \tau_1(t)) \end{pmatrix}, \quad (16)$$

$$L = (L_1 \ L_2 \ L_3).$$

Here,  $L$  is the gain of proxy of master, next,  $K = (K_1 \ K_2 \ K_3)$  is the gain of the controller  $C$ ,

$$F_s(t) = -K \begin{pmatrix} \hat{\theta}_s(t) \\ \hat{\theta}_p(t) \\ \theta_s(t) - \theta_p(t) \end{pmatrix}. \quad (17)$$

Thus, our following works are to solve:

**Problem 1:** the local controllers of master, proxy and slave,  $K_m^0$  and  $K_s^0$ , are designed to make the master, proxy

and slave robustly stable with respect to the polytopic-type uncertainties.

**Problem 2:**  $L$ ,  $K$  (as the slave controller) will be designed in order to provide the stability and performance guarantee for teleoperation system under time-varying delays and polytopic-type uncertainties.

#### IV. MAIN RESULTS

The objective of this section is to solve the problems mentioned above.

##### A. Problem 1: Local Controller Design

The local controllers are designed by a Lyapunov functional and LMI, taking the master as an example, and considering the Lyapunov function  $V(t, x_m(t)) = \sum_{j=1}^N V_j(t, x_m(t))$ ,  $V_j(t, x_m(t)) = x_m(t)^T P_j x_m(t)$ ,  $P_j = P_j^T > 0$ ,  $j = 1, 2, \dots, N$ . In order to apply LMI condition,  $\dot{V}(t, x_m(t)) < 0$  is relaxed to a set of  $\dot{V}_j(t, x_m(t)) < 0$ ,  $j = 1, 2, \dots, N$ . We introduce free weighting matrices  $P_2$ ,  $P_3$  into  $\dot{V}_j(t, x_m(t))$ ,  $j = 1, 2, \dots, N$  [13],

$$2[x_m(t)^T P_2^T + \dot{x}_m(t)^T P_3^T] \underbrace{[(A_{mj} - B_{mj}K_m^0)]}_{\bar{A}_{mj}} x_m(t) - \dot{x}_m(t) = 0. \quad (18)$$

We set  $\eta(t) = \text{col}\{x_m(t), \dot{x}_m(t)\}$  and get,  $j = 1, 2, \dots, N$ ,

$$\begin{pmatrix} \bar{A}_{mj}^T P_2 + P_2^T \bar{A}_{mj} & P_j - P_2^T + \bar{A}_{mj}^T P_3 \\ * & -P_3 - P_3^T \end{pmatrix} < 0. \quad (19)$$

Multiplying (19) by  $\text{diag}\{P_2^{-T}, P_2^{-T}\}$  at the left side,  $\text{diag}\{P_2^{-1}, P_2^{-1}\}$  at the right side, then  $K_m^0$  can be obtained by defining  $P_3 = \xi P_2$  and  $N_m = K_m^0 P_2$ . The result  $K_m^0 = N_m P_2^{-1}$  follows,  $j = 1, 2, \dots, N$ ,

$$\begin{pmatrix} A_{mj}P_2 - B_{mj}N_m + P_2^T A_{mj}^T - N_m^T B_{mj}^T & P_j - P_2 + \xi P_2^T A_{mj}^T - \xi N_m^T B_{mj}^T \\ * & -\xi P_2 - \xi P_2^T \end{pmatrix} < 0. \quad (20)$$

*Remark 2:* The proxy has the same local controller with master ( $K_m^0$ ), and the local controller of slave ( $K_s^0$ ) can be obtained by the same procedure.

### B. Problem 2: Slave Controller Design

Firstly, we will design the proxy of master by Lyapunov-Krasovskii functionals,  $H_\infty$  control and LMI. Considering the models of the master and the proxy as the following system,

$$\begin{cases} \dot{x}_{mp}(t) = A_{mp}^0(\rho_{mp}(t))x_{mp}(t) + A_{mp}^1(\rho_{mp}(t))x_{mp}(t - \tau_1(t)) \\ \quad + B_{mp}(\rho_{mp}(t))w_{mp}(t), \\ z_{mp}(t) = C_{mp}x_{mp}(t), \end{cases} \quad (21)$$

where,

$$\begin{aligned} x_{mp}(t) &= \begin{pmatrix} \hat{\theta}_p(t) \\ \hat{\theta}_m(t) \\ \theta_p(t) - \theta_m(t) \end{pmatrix}, \quad w_{mp}(t) = \begin{pmatrix} \hat{F}_c(t) + \hat{F}_h(t - \tau_1(t)) \\ F_m(t) + F_h(t) \end{pmatrix}, \\ z_{mp}(t) &= (\theta_p(t) - \theta_m(t)), \end{aligned} \quad (22)$$

$$\begin{aligned} A_{mp}^0(\rho_{mp}(t)) &= \begin{pmatrix} A_{mp}^0(\rho_{mp}(t))(1,1) & 0 & 0 \\ 0 & A_{mp}^0(\rho_{mp}(t))(2,2) & 0 \\ 1 & -1 & 0 \end{pmatrix}, \\ A_{mp}^0(\rho_{mp}(t))(1,1) &= A_p(\rho_p(t)) - B_p(\rho_p(t))K_m^0, \\ A_{mp}^0(\rho_{mp}(t))(2,2) &= A_m(\rho_m(t)) - B_m(\rho_m(t))K_m^0, \\ A_{mp}^1(\rho_{mp}(t)) &= \\ &= \begin{pmatrix} -B_p(\rho_p(t))L_1 & -B_p(\rho_p(t))L_2 & -B_p(\rho_p(t))L_3 \\ 0 & 0 & 0 \end{pmatrix}, \\ B_{mp}(\rho_{mp}(t)) &= \\ &= \begin{pmatrix} B_p(\rho_p(t)) & 0 \\ 0 & B_m(\rho_m(t)) \end{pmatrix} = (B_{mp}^1(\rho_{mp}(t)) \ B_{mp}^2(\rho_{mp}(t))), \\ C_{mp} &= (0 \ 0 \ 1). \end{aligned} \quad (23)$$

According to the control scheme, considering any two subsystems in the master, proxy of master and slave, or the whole system, the polytopic-type model can be found. Thus, we get the parameter matrices satisfy the real convex polytopic models,

$$\begin{aligned} A_{mp}^0(\rho_{mp}(t)) &= \sum_{j=1}^N \rho_{mpj}(t) A_{mpj}^0, \\ B_{mp}(\rho_{mp}(t)) &= \sum_{j=1}^N \rho_{mpj}(t) B_{mpj} = \sum_{j=1}^N \rho_{mpj}(t) (B_{mpj}^1 \ B_{mpj}^2). \end{aligned} \quad (24)$$

*Theorem 2:* Suppose there exist matrices  $P_j > 0$ ,  $R_j > 0$ ,  $S_j > 0$ ,  $S_{aj} > 0$ ,  $R_{a1j} > 0$ ,  $P_2$ ,  $Y_{1j}$ ,  $Y_{2j}$ ,  $M$ ,  $j = 1, 2, \dots, N$ , and positive scalars  $\gamma$  and  $\xi$ , such that the condition (26) with notations (27),  $j = 1, 2, \dots, N$ , is feasible. Then the system (21) is robustly stable with  $J(w) < 0$  for time-varying delays  $\tau_1(t) \in [h_1, h_2]$  and with the following proxy control gain:

$$L = MP_2^{-1}. \quad (25)$$

**Proof.** We use *Theorem 1* on system (21), a series of steps is made to obtain LMI condition [15], we define  $P_{2j} = P_2$ ,  $P_{3j} = \xi P_2$ ,  $j = 1, 2, \dots, N$ , multiply  $\Gamma^{1j}$ , by  $\text{diag}\{P_2^{-T}, \dots, P_2^{-T}, I\}$  at the left side,  $\text{diag}\{P_2^{-1}, \dots, P_2^{-1}, I\}$  at the right side, then make the transformation  $A_{mp}^1(\rho_{mp}(t)) = -B_{mp}^1(\rho_{mp}(t))L$ , choose  $M = LP_2$ , apply Schur formula, finally the result follows.

The position tracking between the master and the proxy of master has been achieved, and then, the position tracking between the proxy of master and the slave is assured by the controller  $C$ . The model of the system containing the proxy of master, the controller  $C$  and the slave, is given as follow,

$$\begin{cases} \dot{x}_{ps}(t) = A_{ps}(\rho_{ps}(t))x_{ps}(t) + B_{ps}(\rho_{ps}(t))w_{ps}(t), \\ z_{ps}(t) = C_{ps}x_{ps}(t). \end{cases} \quad (28)$$

where,

$$\begin{aligned} x_{ps}(t) &= \begin{pmatrix} \hat{\theta}_s(t) \\ \hat{\theta}_p(t) \\ \theta_s(t) - \theta_p(t) \end{pmatrix}, \quad z_{ps}(t) = (\theta_s(t) - \theta_p(t)), \\ w_{ps}(t) &= \begin{pmatrix} F_c(t) \\ \hat{F}_c(t) + \hat{F}_h(t - \tau_1(t)) - F_p(t - \tau_1(t)) \end{pmatrix}. \end{aligned} \quad (29)$$

$$\begin{aligned} A_{ps}(\rho_{ps}(t)) &= \\ &= \begin{pmatrix} A_{ps}(\rho_{ps}(t))(1,1) & -B_s(\rho_s(t))K_2 & -B_s(\rho_s(t))K_3 \\ 0 & A_{ps}(\rho_{ps}(t))(2,2) & 0 \\ 1 & -1 & 0 \end{pmatrix}, \\ A_{ps}(\rho_{ps}(t))(1,1) &= A_s(\rho_s(t)) - B_s(\rho_s(t))K_s^0 - B_s(\rho_s(t))K_1, \\ A_{ps}(\rho_{ps}(t))(2,2) &= A_p(\rho_p(t)) - B_p(\rho_p(t))K_m^0, \\ B_{ps}(\rho_{ps}(t)) &= \begin{pmatrix} B_s(\rho_s(t)) & 0 \\ 0 & B_p(\rho_p(t)) \end{pmatrix} = (B_{ps}^1(\rho_{ps}(t)) \ B_{ps}^2(\rho_{ps}(t))), \\ C_{ps} &= (0 \ 0 \ 1), \end{aligned} \quad (30)$$

Thus, we get,

$$\begin{aligned} A_{ps}(\rho_{ps}(t)) &= A_{ps}^0(\rho_{ps}(t)) + A_{ps}^1(\rho_{ps}(t)) = \\ &= \sum_{j=1}^N \rho_{psj}(t) (A_{psj}^0 + A_{psj}^1), \\ A_{psj}^0 &= \begin{pmatrix} A_{sj} - B_{sj}K_s^0 & 0 & 0 \\ 0 & A_{mj} - B_{mj}K_m^0 & 0 \\ 1 & -1 & 0 \end{pmatrix}, \\ A_{psj}^1 &= \begin{pmatrix} -B_{sj}K_1 & -B_{sj}K_2 & -B_{sj}K_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ B_{ps}(\rho_{ps}(t)) &= \sum_{j=1}^N \rho_{psj}(t) B_{psj} = \sum_{j=1}^N \rho_{psj}(t) (B_{psj}^1 \ B_{psj}^2). \end{aligned} \quad (31)$$

With the transformation of  $A_{psj}^1 = -B_{psj}^1 K$ ,  $j = 1, 2, \dots, N$ , we get the following theorem.

*Theorem 3:* Suppose there exist matrices  $P_j > 0$ ,  $P_2$ ,  $W$ , and positive scalars  $\gamma$  and  $\xi$ ,  $j = 1, 2, \dots, N$ , such that the condition (33) with notations (34),  $j = 1, 2, \dots, N$ , is feasible. Then the system (28) is robustly stable with  $J(w) < 0$  and with the control gain of the controller  $C$ :

$$K = WP_2^{-1}. \quad (32)$$

$$\Gamma^{4j} = \begin{pmatrix} \Gamma_{11}^{4j} & \Gamma_{12}^{4j} & B_{psj} & P_2^T C_{ps}^T \\ * & \Gamma_{22}^{4j} & \xi B_{psj} & 0 \\ * & * & -\gamma_j^2 I & 0 \\ * & * & * & -I \end{pmatrix} < 0, \quad (33)$$

$$\Gamma^{3j} = \begin{pmatrix} \Gamma_{11}^{3j} & \Gamma_{12}^{3j} & R_j - B_{mpj}^1 M - Y_{1j}^T & Y_{1j}^T & Y_{1j}^T + B_{mpj}^1 M & Y_{1j}^T & B_{mpj} & P_{2j}^T C_{mp}^T \\ * & \Gamma_{22}^{3j} & -\xi B_{mpj}^1 M - Y_{2j}^T & Y_{2j}^T & Y_{2j}^T + \xi B_{mpj}^1 M & Y_{2j}^T & \xi B_{mpj} & 0 \\ * & * & -S_j - R_j & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_{aj} & 0 & 0 & 0 & 0 \\ * & * & * & * & -R_{a1j} & 0 & 0 & 0 \\ * & * & * & * & * & -R_{a1j} & 0 & 0 \\ * & * & * & * & * & * & -\gamma_j^2 I & 0 \\ * & * & * & * & * & * & * & -I \end{pmatrix} < 0, \quad (26)$$

$$\Gamma_{11}^{3j} = S_j + S_{aj} - R_j + P_2^T A_{mpj}^0{}^T + A_{mpj}^0 P_2, \quad \Gamma_{12}^{3j} = P_j - P_2 + \xi P_2^T A_{mpj}^0{}^T, \quad \Gamma_{22}^{3j} = -\xi P_2 - \xi P_2^T + h_1^2 R_j + (h_2 - h_1)^2 R_{a1j}. \quad (27)$$

$$\begin{aligned} \Gamma_{11}^{4j} &= P_2^T A_{psj}^0{}^T + A_{psj}^0 P_2 - W^T B_{psj}^1{}^T - B_{psj}^1 W, \\ \Gamma_{12}^{4j} &= P_j - P_2 + \xi P_2 A_{psj}^0{}^T - \xi W^T B_{psj}^1{}^T, \\ \Gamma_{22}^{4j} &= -\xi P_2 - \xi P_2^T. \end{aligned} \quad (34)$$

**Proof.** We apply the system (28) in *Corollary 1*, and the proof is straightforward as *Theorem 2*.

Till now, the position tracking between the master, the proxy and slave are ensured. Finally, the objective is to ensure the global stability of the whole system described by,

$$\begin{cases} \dot{x}_{mps}(t) &= A_{mps}^0(\rho_{mps}(t))x_{mps}(t) \\ &+ A_{mps}^1(\rho_{mps}(t))x_{mps}(t - \tau_1(t)) \\ &+ B_{mps}(\rho_{mps}(t))w_{mps}(t), \\ z_{mps}(t) &= C_{mps}x_{mps}(t), \end{cases} \quad (35)$$

where,

$$\begin{aligned} x_{mps}(t) &= \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \dot{\theta}_m(t) \\ \theta_s(t) - \theta_p(t) \\ \theta_p(t) - \theta_m(t) \end{pmatrix}, \quad w_{mps}(t) = \begin{pmatrix} F_e(t) \\ \hat{F}_e(t) + \hat{F}_h(t - \tau_1(t)) \\ F_m(t) + F_h(t) \end{pmatrix}, \\ z_{mps}(t) &= \begin{pmatrix} \theta_s(t) - \theta_p(t) \\ \theta_p(t) - \theta_m(t) \end{pmatrix}. \end{aligned} \quad (36)$$

Thus, we can get,  $j = 1, 2, \dots, N$ ,

$$\begin{aligned} A_{mps}^0(\rho_{mps}(t)) &= \sum_{j=1}^N \rho_{mps_j}(t) A_{mps_j}^0, \\ A_{mps_j}^0 &= \begin{pmatrix} A_{sj} - B_{sj} K_s^0 - B_{sj} K_1 & -B_{sj} K_2 & 0 & -B_{sj} K_3 & 0 \\ 0 & A_{mj} - B_{mj} K_m^0 & 0 & 0 & 0 \\ 0 & 0 & A_{mj} - B_{mj} K_m^0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}, \\ A_{mps}^1(\rho_{mps}(t)) &= \sum_{j=1}^N \rho_{mps_j}(t) A_{mps_j}^1 = \\ &= \sum_{j=1}^N \rho_{mps_j}(t) \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -B_{mj} L_1 - B_{mj} L_2 & 0 & -B_{mj} L_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ B_{mps}(\rho_{mps}(t)) &= \sum_{j=1}^N \rho_{mps_j}(t) B_{mps_j} = \\ &= \sum_{j=1}^N \rho_{mps_j}(t) \begin{pmatrix} B_{sj} & 0 & 0 \\ 0 & B_{mj} & 0 \\ 0 & 0 & B_{mj} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad C_{mps} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned} \quad (37)$$

By *Theorem 1*, we can verify the global stability of the system. Then the force tracking,  $F_m(t) = \hat{F}_e(t - \tau_2(t))$ , is achieved.

## V. ILLUSTRATIVE EXAMPLE

To evaluate the performance of the proposed approach, different working conditions have been simulated. The maximum amplitude and sampling time of time-varying delays are  $0.2s$  (greater amplitude of delays can also be handled) and  $0.001s$ . Note that, the time-varying delays in two channels are asymmetric.

For simplicity reasons, the master, the proxy of master and the slave models can be described as  $A_m(\rho_m(t)) = A_s(\rho_s(t)) = 0$ ,  $B_m(\rho_m(t)) = B_s(\rho_s(t)) = \frac{1}{\rho(t)}$ ,  $\rho(t) \in [0.5, 1]$ . Firstly, by our local controller design,  $K_m^0 = K_s^0 = 2.6585$ . Next, the gains of the proxy of master and the controller  $C$ , the corresponding  $\gamma_{min}^L$ ,  $\gamma_{min}^K$ , and the global stability with  $\gamma_{min}^g$  are presented as follows,

$$\begin{aligned} L &= \begin{pmatrix} 1.7860 & -1.7862 & 9.1797 \end{pmatrix}, \quad \gamma_{min}^L = 0.2626, \\ K &= \begin{pmatrix} 20.4799 & -21.2537 & 575.2051 \end{pmatrix}, \quad \gamma_{min}^K = 0.0164, \\ \gamma_{min}^g &= 0.3286. \end{aligned} \quad (38)$$

For comparison reasons, by using the LMI condition proposed in *Remark 1*, we get,

$$\gamma_{min}^L = 1.399, \quad \gamma_{min}^K = 0.0954, \quad \gamma_{min}^g = 1.5681. \quad (39)$$

We can see that, the LKF proposed in this paper improves the results and reduces the conservatism.

### A. Tracking in abrupt tracking motion

Fig. 2 shows the position tracking between the master and slave, where the human operator ( $F_h(t)$ ) is modeled as the pulse generator. It is clear that our method ensures the system stability and the position tracking, even at the changing point of the position, good position convergence between the master and slave has been achieved.

### B. Tracking in Wall Contact Motion

The position tracking in wall contact motion is presented in Fig. 3. Here, the slave is driven to the hard wall with a stiffness of  $K_e = 30kN/m$  located at the position  $x = 1.0m$ . Our local and slave controllers ensure the position performance, because of the uncertainties in master and slave models and time-varying delays, there exist the small position jitters at the master and slave side, the slave side's jitter usually causes much larger jitter at the master side. The

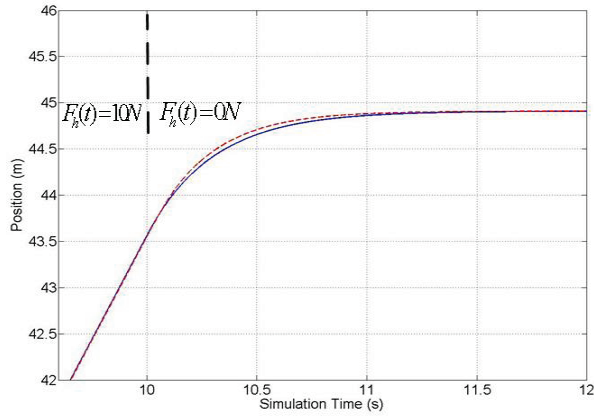


Fig. 2. Position response in abrupt changing motion

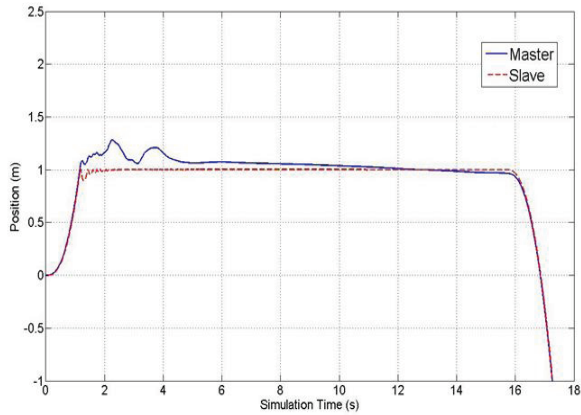


Fig. 3. Position response in wall contact motion

force tracking between  $F_m(t)$  and  $\hat{F}_e(t)$  is straightforward as in [10], thus, it is omitted here.

## VI. CONCLUSIONS

This paper focused on the slave controller design problem for teleoperation with time-varying delays and polytopic-type uncertainties. Thanks to Lyapunov functions and Lyapunov-Krasovskii functionals,  $H_\infty$  control theory and LMI, the system stability and high-quality performance are guaranteed based on our method.

The simulations performed by YALMIP and SIMULINK demonstrate that such a teleoperation system can run in different working conditions.

## REFERENCES

- [1] P. F. Hokayem and M. W. Spong, "Bilateral teleoperation: an historical survey," *Automatica*, vol. 42, no. 12, pp. 2035–2057, 2006.
- [2] H. Kawada and T. Namerikawa, "Bilateral control of nonlinear teleoperation with time varying communication delays," in *Proceedings of 2008 American Control Conference (ACC'08)*, Seattle, Washington, USA, June 2008, pp. 189–194.
- [3] J.-P. Richard, "Time delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, no. 10, pp. 1667–1694, 2003.

- [4] N. Chopra, P. Berestesky, and M. W. Spong, "Bilateral teleoperation over unreliable communication networks," *IEEE Trans. Contr. Syst. Technol.*, vol. 16, no. 2, pp. 304–313, 2008.
- [5] E. Nuño, L. Basañez, and R. Ortega, "Passivity-based control for bilateral teleoperation: A tutorial," *Automatica*, vol. 47, no. 3, pp. 485–495, 2011.
- [6] J.-H. Ryu, C. Preusche, B. Hannaford, and G. Hirzinger, "Time domain passivity control with reference energy following," *IEEE Trans. Contr. Syst. Technol.*, vol. 13, no. 5, pp. 737–742, 2005.
- [7] Y.-Q. Ye, Y.-J. Pan, and Y. Gupta, "Time domain passivity control of teleoperation systems with random asymmetric time delays," in *Proceedings of IEEE Conf. on Decision and Control, and Chinese Control Conference (CDC'09, CCC'09)*, Shanghai, China, December 2009, pp. 7533–7538.
- [8] P. Arcara and C. Melchiorri, "Control schemes for teleoperation with time delay: a comparative study," *Robotics and Autonomous Systems*, vol. 38, no. 1, pp. 49–64, 2002.
- [9] J. Chiasson and J. J. Loiseau, *Applications of Time Delay Systems*. Springer, 2007.
- [10] B. Zhang, A. Kruszewski, and J.-P. Richard, "Tracking improvement based on the proxy control scheme for bilateral teleoperation system under time-varying delays," in *2011 IEEE Int. Conf. on Emerging Technologies and Factory Automation (ETFA'11)*, Toulouse, France, September 2011.
- [11] Y. He, M. Wu, J. H. She, and G. P. Liu, "Parameter-dependent lyapunov functional for stability of time-delay systems with polytopic-type uncertainties," *IEEE Trans. Automat. Contr.*, vol. 49, no. 5, pp. 828–832, 2004.
- [12] E. Fridman, "A new Lyapunov technique for robust control of systems with uncertain non-small delays," *IMA J. Math. Contr. Infor.*, vol. 23, no. 2, pp. 165–179, 2006.
- [13] Y. He, M. Wu, J.-H. She, and G.-P. Liu, "Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties," *IEEE Trans. Automat. Contr.*, vol. 49, no. 5, pp. 828–832, 2002.
- [14] A. Kruszewski, W. J. Jiang, E. Fridman, J.-P. Richard, and A. Toguyeni, "A switched system approach to exponential stabilization through communication network," *IEEE Trans. Contr. Syst. Technol.*, to appear, 2011.
- [15] E. Fridman and U. Shaked, "New bounded real lemma representations for time-delay systems and their applications," *IEEE Trans. Automat. Contr.*, vol. 46, no. 12, pp. 1973–1979, 2001.