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Abstract:

This paper aims at modeling of linear behavior of heterogeneous materials by a numerical homogenization method based on the Extended Finite Element Method (XFEM) [1]. One of major obstacles in using direct numerical methods for estimation of effective properties of heterogeneous materials resides in difficulties to construct the mesh conform to complexity of their structures which needs an expensive time and calls for considerable calculating resources. In turn, using of XFEM needs only a regular mesh and some mathematical resources for structure description and is expected to match the accuracy of the classical FEM approach. During this work, a MATLAB-based XFEM code is developed and used. Some case tested are studied and compared to assess the performance of the method.

1. Introduction

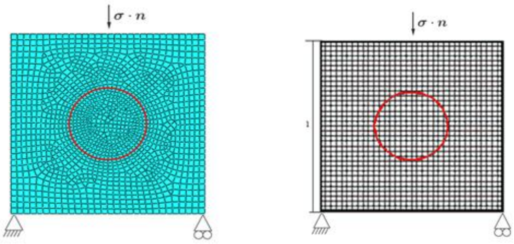
One important goal of micromechanics method consists of predicting the response of the heterogeneous material based on the geometries and the properties of the individual phases, a task known as up-scaling. Various approaches could be followed including analytical approaches and variation approaches (see for example [2], [3]), [4], [5] [6]) or FEM-based numerical approaches ([7], [8]). While the analytical and semi-analytical methods give a quick response for simple structures and inclusions shape, the numerical approaches seem well adapted to deal with more complex structures and nonlinear behavior of constituents. However, using of classical FEM needs a mesh that conforms to the surfaces of boundaries of different constituents, which in turn would lead to the high computation efforts and sometimes in the impossibility to perform analysis in a normal PC machine. As discussed in ([1]) the XFEM would be a serious alternative to classical FEM for this kind of problems. In this paper, we use the idea of ([1]) to propose a homogenization procedure based on the XFEM approach (FEM extended [1],[9]). This paper is limited in 2D case only. After a brief description of basic ideas of XFEM some examples are presented to show the performance and the accuracy of the method. As opposed to classical FEM numerical analysis with a limited number of inclusions we uses in this paper a great number of inclusions in some standard spatial distributions (spherical, ellipsoidal etc) for which analytical estimators exist from various authors ([4], [5]).

2. Principle of the X-FEM

Compared with the classical FEM approach, where the meshes have to be conformed to the surfaces of discontinuity (Fig.1.a), in the method XFEM, it is possible to make the integration of equation using a uniform mesh (Fig. 1.b). In the same time the surfaces of boundaries among different constituents described by level sets functions and classical approximation of

FEM is replaced by an enriched interpolation function formulation, i.e (see for example [1]):

$$u^{X-FEM} = \sum_{i \in I} N_i(x) u_i + \sum_{J} a_J N_J(x) F(x) \quad (1)$$



(a).FEM (b). XFEM

Figure 1 : Mesh in the FEM and XFEM methods

where N_i is classical shape function, J is the set of all enriched nodes, a_j are new degrees of freedom and $F(x)$ is the enrichment function, dependent on the type of analyses. In this paper we used the enriched function proposed by ([1]),

$$F(x) = \sum_I \left|\phi_I N_I(x)\right| - \left|\sum_I \phi_I N_I(x)\right| \quad (3)$$

with ϕ_I being the nodal value of level set function ([10]), it approximated by some interpolated functions ([11]):

$$\Phi(x) = \sum_I N_I(x)\phi_I \quad (2)$$

3. Results and discussion

The results presented here are obtained by a home-made MATLAB-based code. The validation of code is made through a number of classical problems for which the analytical solution are known. As an validation example, in the Fig.2 the variation of stress following a diametric line in the case of isolated 3D inclusion obtained by this analysis, are compared by those given by analytical solution of Eshelby and FEM commercial software (ABAQUS) and a good agreement is observed.

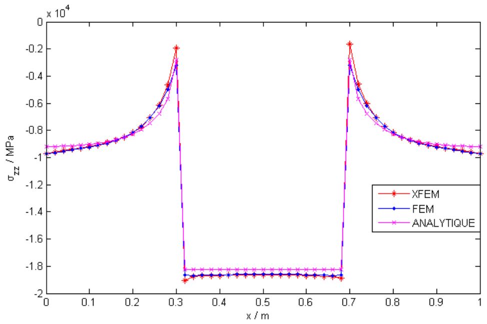


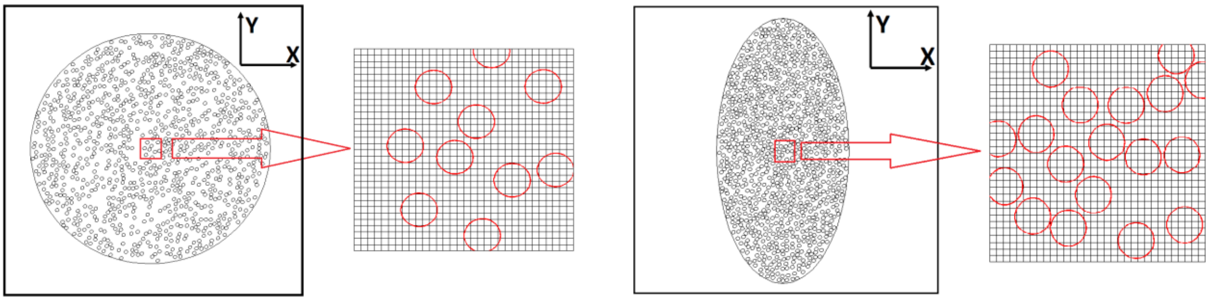
Figure 2: Comparison of stresses in a horizontal section (section BB) in the middle of the cube containing an inclusion

More concluding results are obtained in the study of the influence of the spatial distribution of inclusions in effective properties, through two 2D examples as illustrated in the Fig. 3.a and 3.b. For two calculations, the same elastic properties are used for the matrix and inclusions (presented in the Tab1) and a volume fraction of 11% are chosen.

Table 1: The elastic properties of the constituents

Elastic properties	Matrix	Inclusion
Young's modulus	7600 MPa	84000 MPa
Poisson's ratio	0.1	0.3

Each inclusion has a circular section with radius of 0.0059 mm. In both cases of figures 3.a and 3.b 1000 inclusions are generated randomly inside of predefined regions of circular and elliptical shape respectively. The ratio of diameters of the elliptical region in figure 3.b is 0.44. In figure 3 are also shown the background mesh and local details of inclusion distributions.



a. Circular spatial distribution b. Elliptical spatial distribution

Figure 3: Two examples of random distributions of inclusions in the isotropic matrix

Table 2: Effective stiffness of materials simulated by XFEM method (a) circular distribution of circular inclusions in the isotropic matrix

Effective properties	X-FEM	Mori-Tanaka	Differences between two methods (%)
C_{1111} (MPa)	9170.68	9133.27	0.4%
C_{2222} (MPa)	9172.75	9133.27	0.4%
C_{1122} (MPa)	1232.16	1227.46	0.4%

Table 3: Effective stiffness of materials simulated by XFEM method (b) elliptical distribution of circular inclusions in the isotropic matrix

Effective properties	X-FEM	Two step Mori-Tanaka	Differences between two methods (%)
C_{1111} (MPa)	9068.97	9078.29	0.1%
C_{2222} (MPa)	9430.46	9309.91	1.3%
C_{1122} (MPa)	1229.69	1265.72	2.8%

The effective mechanical properties obtained in the two examples are represented in the Tab 2 and Tab 3 and compared with predictions of Mori-Tanaka schema. For elliptical case Mori-Tanaka has been used in a two step homogenization schema when firstly the effective parameters of ellipse are calculated and then the overall properties are obtained in the second step. As it is known, for the first case (circular distribution) the Mori-Tanaka scheme is a bound and XFEM is in full accord with theoretical predictions. For the second case as expected the direct Mori-Tanaka predicts isotropic properties, while XFEM-method and two-step Mori-Tanaka up-scaling indicate an anisotropic behavior due to spatial distribution of inclusions. The good agreement between analytical and numerical predictions validates the proposed XFEM approach in a problem where the number of inclusions makes almost impossible using of direct FEM method. In particular the XFEM method seems to catch quite well the interaction among inclusions supposed to be more important when the structure of material becomes complex and the number of inclusions increases. The method XFEM will become a valuable tool to consider these interactions.

4. Conclusion

In this paper, we have proposed a numerical homogenization procedure by using the XFEM method in the context of the linear mechanical behavior of geomaterials. Some applications have validated the development of the XFEM in MATLAB and demonstrated the capacity of this method to predict the effective properties of heterogeneous materials such as geomaterials. The XFEM provides similar results to those obtained by the classical finite element method and analytical method. The immediate continuity of this work is to study the materials with much more complex structures such as the multi-phases materials, a random distribution, complex shapes and a more realistic nonlinear behavior of components.

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