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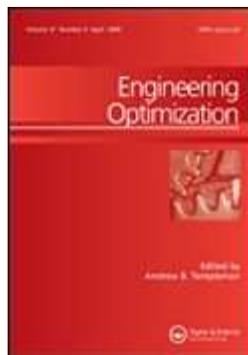
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## RESEARCH ARTICLE

An Approach for the Reliability Based Design  
Optimization of Laminated Composites

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This paper aims at optimizing laminated composite plates taking into account uncertainties in the structural dimensions. As laminated composites require a global optimization tool, the Particle Swarm (PSO) Optimization method was employed. A new reliability based design optimization (RBDO) methodology based on safety factors is presented and coupled with the PSO. Such safety factors are derived from the Karush- Kuhn-Tucker (KKT) optimality conditions of the reliability index approach and eliminate the need of the reliability analysis in the RBDO. The plate weight minimization was the objective function of the optimization process. The results showed the coupling of the evolutionary algorithm with the safety factor method proposed in this paper successfully performed the RBDO of laminated composite structures.

**Keywords:** Laminated composite, optimization, RBDO, safety factors, PSO.

## 1. Introduction

Laminated composites provide not only high structural performance, but also minimize the cost of such structures. A recent report by the US National Materials Advisory Board estimates that a 1 lb weight reduction amounts to a total saving of \$200 over the 100,000 hours life of a civil transport, increasing to \$1,000 in the case of military aircraft and reaching \$20,000 for aerospace industry (Kim *et al.* 2008). These numbers explain why structural weight is considered particularly critical in the aerospace industry. Thus, optimization techniques have been applied in order to get minimum weight structures, what corresponds to take a structure to its limit. However, the optimization of laminated composites can be classified as a non-convex and multimodal optimization problem. For such problem, the evolutionary algorithms are well-suited. Among others, the following were employed to optimize laminated composite structures: Genetic Algorithm (GA)

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5 (Le Riche and Haftka 1993, Nagendra *et al.* 1994, Lopez *et al.* 2009a,b), Particle Swarm  
6 Optimization (PSO) (Kennedy and Eberhart 1995, Omkar *et al.* 2008), and Ant Colony  
7 Optimization (ACO) (Dorigo 1992, Aymerich and Serra 2007). Known advantages of  
8 the use of evolutionary algorithms include the following: (i) they do not require gradient  
9 information and can be applied to problems where the gradient is hard to obtain or simply  
10 does not exist; (ii) if correctly tuned, they do not get stuck in local minima; (iii) they  
11 can be applied to non-smooth or discontinuous functions. On the other hand, their main  
12 drawback is the extremely large number of evaluations of the objective function to achieve  
13 optimization, which can make their use nonviable depending on the computational cost  
14 of each evaluation.

15  
16 In deterministic optimization, however, the uncertainties of the system (i.e. dimension,  
17 model, material, loads, etc) are not taken into account. Doing so, the resulting optimum  
18 solution may lead to a lower level of reliability and, as a consequence, a higher risk  
19 of failure. Thus, it is the objective of reliability based design optimization (RBDO) to  
20 optimize structures guaranteeing that its probability of failure is lower than a certain  
21 level chosen a priori by the designer.

22 This paper focus on RBDO methods that employ the first order reliability analysis  
23 (FORM) to approximate the probability of failure of the structures. Hence, other ap-  
24 proaches such as simulation methods are not discussed in the brief review of the literature  
25 presented in the sequel.

26  
27 As the reliability analysis may be an optimization procedure itself, the RBDO, in  
28 its classical version, is a double-loop strategy: structural optimization and reliability  
29 analysis. This double loop leads to a high computational cost. The reliability analysis is  
30 usually the most time consuming task in RBDO and most of the algorithms are based  
31 on the FORM using either the reliability index approach (RIA) (Hasofer and Lind 1974)  
32 or the performance measure approach (PMA) (Tu *et al.* 1999).

33  
34 To reduce the computational burden of the RBDO, several papers decoupled the struc-  
35 tural optimization and the reliability analysis. This procedure may be divided into two  
36 groups: (i) the serial single loop methods and, (ii) the unilevel methods.

37 The basic idea of the serial single loop methods is to decouple the structural optimiza-  
38 tion (outer loop) and the reliability analysis (inner loop). Each method of this group  
39 utilizes a specific strategy to decouple such loops and then, perform them sequentially  
40 until some convergence criterion is achieved. Among these methods, the following may  
41 be cited: Traditional Approximation method (Torng and Yang 1993), Single Loop Single  
42 variable (SLSV) (Chen *et al.* 1997), Sequential Optimization and Reliability Assessment  
43 (SORA) (Du and Chen 2004) and Safety Factor Approach (SFA) (Wu *et al.* 2001). For a  
44 comparison of such methods see references (Yang and Gu 2004) and (Yang *et al.* 2005).

45 The central idea of the unilevel methods (also called monolevel) is to replace the  
46 reliability analysis by some optimality criteria of the optimum (e.g. imposing it as a  
47 constraint in the outer loop). Thus, there is a concurrent convergence of the design  
48 optimization and reliability calculation, in other words, they are sought simultaneously  
49 and independently. Examples of this approach may be found in references (Kuschel and  
50 Rackwitz 2000, Agarwal *et al.* 2007, Cheng *et al.* 2006, Yi *et al.* 2008, Yi and Cheng  
51 2008).

52  
53 The main objective of this paper is the RBDO of laminated composite plates, what  
54 leads to the problem of coupling RBDO and global optimization techniques. Despite  
55 all these advances in reducing the computational cost of the RBDO, a few papers have  
56 dealt with global optimization (Antonio 2006) mainly due to its high computational  
57 cost. The coupling of standard RBDO methodologies and global optimization algorithms  
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5 would lead to a huge computational cost due to the high cost of both procedures. One  
6 way to take into account uncertainties would be the use of safety factors. However,  
7 such factors are usually based on engineering experience and/or experimental work and  
8 may lead to either high cost or low reliability levels. It would be interesting to develop  
9 safety factors that achieve both the desired reliability level and optimized structure. The  
10 main advantage of such procedure is that it requires a little additional computational  
11 effort if compared to the standard deterministic optimization and at the same time it  
12 guarantees the minimum reliability level of the structure. Thus, this paper presents a new  
13 RBDO methodology based on safety factors, which are derived from the Karush-Kuhn-  
14 Tucker (KKT) optimality conditions of the RIA. It means that such factors are able to  
15 provide a final design that respects the probabilistic constraint of the RBDO problem.  
16 Since the proposed method eliminates the need of the reliability analysis leading to a  
17 computational cost really close to the one of classical deterministic optimization methods,  
18 it is affordable to couple it with global optimization algorithms. Hence, the proposed  
19 methodology is coupled with an evolutionary algorithm: the PSO. Then, it is employed  
20 to pursue the RBDO of laminated composite plates taking into account the uncertainty  
21 on the structural dimensions.  
22

23  
24 The main contributions of the paper are: (i) the proposition of a new RBDO method-  
25 ology based on safety factors derived from the KKT optimality conditions of the RIA,  
26 which eliminates the need of the reliability analysis; (ii) the coupling of such RBDO  
27 methodology with a global optimization algorithm, the PSO; and (iii) the application of  
28 the new RBDO methodology in the RBDO of laminated composite plates.

29  
30 The paper is structured as follows: the laminated composite optimization problem to be  
31 solved is now presented in Section 2. The basic RBDO material is presented in Section 3.  
32 Moreover, in this section, the deterministic problem of section 2 has been converted into a  
33 RBDO problem which is the main objective of the paper. One of the main contributions  
34 of the paper, the new RBDO methodology, is fully presented in Section 4. The PSO  
35 method is briefly described in section 5. The safety factors method is demonstrated on  
36 classical studies in sections 6 and 7. Numerical examples on the laminate composites are  
37 in Section 8. Finally, comments about the extension of the methodology to more complex  
38 structures and its limitations are in Section 9, which reports the main conclusions drawn  
39 from the work.  
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## 2. Optimization of Laminated Composites

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51 A laminated composite is usually tailored according to the designer's needs by choosing  
52 the thickness and orientation of the laminae. Thus, ply thickness and orientation angles  
53 are usually the design variables of a laminated composite optimization problem. In this  
54 paper, the goal is the minimization of the weight of laminated composite plates having as  
55 constraint either the first ply failure criterion of Tsai-Wu or the buckling failure factor.  
56 Thus, the general formulation of the optimization problem is presented here. The details  
57 of the mechanics of laminated composite plates are not presented in this paper, yet the  
58 interested reader is referred to (Jones 1999) for a basic textbook and to (Reddy 2004)  
59 for a more advanced material.  
60

## 2.1. Objective Function

Consider a simply supported glass-epoxy laminated plate, subjected to compressive in-plane loads  $N_x$  and  $N_y$ , as shown in Figure 1. Each layer is  $t_i$  thick and the length and width of the plate are  $l_p$  and  $w_p$ , respectively. The elastic materials properties of the layers are: elastic modulus  $E_1$  and  $E_2$ , shear modulus  $G_{12}$ , Poisson's ratio  $\nu_{12}$  and specific weight  $\rho$ . Thus, the weight of the structure is given by:

$$W(t) = \rho \cdot w_p \cdot l_p \cdot \sum_{i=1}^{N_p} t_i \quad (1)$$

Note that except  $t_i$ , all variables in equation (1) are constant in the optimization problem. Thus, the problem is re-defined as minimizing the total thickness of the laminate. The laminate is composed by  $N_p$  laminae whose orientation angles and thicknesses are  $\theta_i$  and  $t_i$  ( $i = 1$  to  $N$ ), respectively. Since the orientation angle of each layer plays an important role in the determination of the stiffness of the laminate, they are also considered as design variables. For convenience of notation, all the design variables (thickness and orientation of each lamina) are grouped into the design vector  $\mathbf{d} = [\mathbf{t} \ \boldsymbol{\theta}] = [t_1, \dots, t_{N_p}, \theta_1, \dots, \theta_{N_p}]$ .

## 2.2. Optimization Constraints

As commented before, two types of limit state functions are considered in this paper: (i) the buckling failure factor and (ii) the first ply failure criterion of Tsai-Wu. Both are described in the sequel.

In all the numerical analysis of this paper, the plates are analyzed using the classical lamination theory (Jones 1999). Thus, the buckling failure factor is given by (see reference (Gurdal *et al.* 1999)):

$$\lambda_{buckling}(\mathbf{d}) = \min_{p,q} \left( \frac{\pi \left[ D_{11}(\mathbf{d}) \left( \frac{p}{l_p} \right)^4 + 2(D_{12}(\mathbf{d}) + 2D_{66}(\mathbf{d})) \left( \frac{p}{l_p} \right)^2 \left( \frac{q}{w_p} \right)^2 + D_{22}(\mathbf{d}) \left( \frac{q}{w_p} \right)^4 \right]}{\left( \frac{p}{l_p} \right)^2 N_x + \left( \frac{q}{w_p} \right)^2 N_y} \right) \quad (2)$$

where  $D_{ij}(\mathbf{d})$  are coefficients of the laminate bending stiffness matrix, which are functions of both the thickness and the orientation angle of each lamina;  $p$  and  $q$  determine the amount of half waves in the plane of the plate, and  $\lambda_{buckling}(\mathbf{d})$  is buckling failure factor, which is the buckling load divided by the applied load. Notice that equation (2) input requires positive values for compressive and negative values for tensile forces. The failure occurs when the  $\lambda_{buckling}(\mathbf{d})$  is lower than one. Thus, the limit state function can be written as:

$$G_{buckling}(\mathbf{d}) = \lambda_{buckling}(\mathbf{d}) - 1 \quad (3)$$

where negative values imply failure.

The first ply failure criterion of Tsai-Wu follows the Von Mises yield criterion adapted to orthotropic materials (Jones 1999). The failure factor of the Tsai-Wu criterion is given by:

$$\lambda_{TW}(\mathbf{d}) = F_{11}\sigma_1(\mathbf{d})^2 + 2F_{12}\sigma_1(\mathbf{d})\sigma_2(\mathbf{d}) + F_{22}\sigma_2(\mathbf{d})^2 + F_{21}\tau_{12}(\mathbf{d})^2 + F_1\sigma_1(\mathbf{d}) + F_2\sigma_2(\mathbf{d}) \quad (4)$$

where  $\sigma_1(\mathbf{d})$  and  $\sigma_2(\mathbf{d})$  are the normal stresses in the direction 1 and 2, respectively,  $\tau_{12}(\mathbf{d})$  is the shear stress in the elastic symmetry plane 1-2.  $F_{ij}$  are parameters function of the strength properties  $X_T, X_C, Y_T, Y_C$  and  $S_{12}$  (see reference (Jones 1999)).  $X_T$  and  $X_C$  are the tensile and compressive strengths parallel to the fibre direction, respectively,  $Y_T$  and  $Y_C$  are the tensile and compressive strengths normal to the fibre direction, respectively,  $S_{12}$  is the shear strength. Note that  $X_T, X_C, Y_T, Y_C$  and  $S_{12}$  are positive quantities.

The failure occurs when  $\lambda_{TW}(\mathbf{d})$  is higher than one. Thus, the limit state function can be written as:

$$G_{TW}(\mathbf{d}) = 1 - \lambda_{TW}(\mathbf{d}) \quad (5)$$

where negative values imply failure.

### 2.3. Deterministic Optimization Problem

After defining the objective function and constraints, the deterministic laminated composite optimization problem is posed as:

$$\begin{aligned} & \text{Minimize : Weight } W(\mathbf{d}) \\ & \text{subject to : Tsai - Wu failure criterion } G_{TW}(\mathbf{d}) \\ & \quad \text{Buckling failure } G_{buckling}(\mathbf{d}) \\ & \quad t_i \in R^+, \theta_i \in [-90^\circ, 90^\circ], i = 1 \text{ to } N_p \end{aligned} \quad (6)$$

Thus, the optimization algorithm aims at finding the thickness and orientation angle of each one of the  $N_p$  laminas that minimize the weight of the plate, not letting the structure fail accordingly to the chosen failure mode. The solution of the equation (6) will be noted  $\mathbf{d}^{optimal}$  in the sequel.

However, the uncertainties on the dimensions of the structure are taken into account in this paper. Thus, the deterministic thickness values  $t_i$  are replaced by the random variables  $T_i$ . For convenience of notation, such random variables are grouped into the vector  $\mathbf{X} = [\mathbf{T}] = [T_1, \dots, T_{N_p}]$ . In accordance with this modification, the deterministic design vector  $\mathbf{d}$  is replaced by the reliability design vector  $\mathbf{m}$  which is composed by the mean values  $m_{T_i}$  of the random thicknesses  $T_i$  and the deterministic orientation angles  $\theta_i$  of each layer of the laminate:  $\mathbf{m} = [m_{T_1}, \dots, m_{T_{N_p}}, \theta_1, \dots, \theta_{N_p}]^T$ .

Hence, in the next section, the classical strategy of RBDO is presented to treat the constraint in a probabilistic sense.

## 3. Reliability Based Design Optimization

Converting the optimization problem of equation (6) into a RBDO problem, leads at looking for  $\mathbf{m}^{reliable}$  the solution of the equation (7):

$$\begin{aligned} & \text{Minimize : } W(\mathbf{m}) \\ & \text{subject to : } P_f(G(\mathbf{m}, \mathbf{X}) \leq 0) \leq P_f^{allowed} \\ & \quad m_{T_i} \in R^+, \theta_i \in [-90^\circ, 90^\circ], i = 1 \text{ to } N_p \end{aligned} \quad (7)$$

Notice that the limit state function  $G(\mathbf{m}, \mathbf{X})$  is function of the random variables  $\mathbf{X}$ . The constraint of the optimization problem is now defined in terms of the probability of not being fulfilled, the so-called probability of failure  $P_f$ . Therefore,  $P_f^{allowed}$  represents the maximum allowed probability of failure of the structure and  $P_f$  is given by:

$$P_f(\mathbf{m}) = P_f(G(\mathbf{m}, \mathbf{X}) \leq 0) = \int_{G(\mathbf{m}, \mathbf{X}) \leq 0} f(\mathbf{X}) d\mathbf{X} \quad (8)$$

where  $f(\mathbf{X})$  is the joint probability density function of  $\mathbf{X}$ . The probability of failure as well as the maximum probability of failure can be approximated using the FORM of the RIA (Hasofer and Lind 1974). Such approximation is given by:

$$\begin{aligned} P_f(\mathbf{m}) &\approx \Phi(-\beta(\mathbf{m})) \\ P_f^{allowed} &\approx \Phi(-\beta^{target}) \end{aligned} \quad (9)$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function;  $\beta(\mathbf{m})$  is the so-called reliability index, which is defined in the sequel, and  $\beta^{target}$  is the target reliability index of the problem. The use of the equation (9) to describe the probabilistic constraint of equation (7), leads to:

$$\begin{aligned} &\text{Minimize : } W(\mathbf{m}) \\ &\text{subject to : } \beta^{target} \leq \beta(\mathbf{m}) \\ &\quad m_{T_i} \in R^+, \theta_i \in [-90^\circ, 90^\circ], i = 1 \text{ to } N_p \end{aligned} \quad (10)$$

To measure the reliability index  $\beta(\mathbf{m})$ , one needs to transform the original random vector  $\mathbf{X}$  (given in the so-called physical space) into a standard Gaussian vector  $\mathbf{U}$  (given in the so-called standard space) (Lemaire *et al.* 2005). First, vectors  $\mathbf{x}$  and  $\mathbf{u}$ , which are realizations of the random vectors  $\mathbf{X}$  and  $\mathbf{U}$ , respectively, are introduced.

Thus, the transformation between the two spaces of each realization is expressed as:

$$\mathbf{u} = T(\mathbf{m}, \mathbf{x}) \text{ or } \mathbf{x} = T^{-1}(\mathbf{m}, \mathbf{u}) \quad (11)$$

Then, the limit state function can be written as  $G(\mathbf{m}, \mathbf{X}) = G(\mathbf{m}, T^{-1}(\mathbf{m}, \mathbf{U})) = g(\mathbf{m}, \mathbf{U})$ . Notice that  $g(\mathbf{m}, \mathbf{U})$  is the limit state function in the standard space and the previous equalities relation hold for all the realizations  $\mathbf{x}$  and  $\mathbf{u}$ . The reliability index  $\beta(\mathbf{m})$  can be obtained from the following optimization problem in the  $\mathbf{U}$ -space:

$$\begin{aligned} &\text{for a given } \mathbf{m} : \\ &\text{Minimize : } \|\mathbf{u}\| \\ &\text{subject to : } g(\mathbf{m}, \mathbf{u}) \leq 0 \end{aligned} \quad (12)$$

where the solution of equation (12)  $\mathbf{u}^*(\mathbf{m})$  is the most probable failure point (MPP) which is defined as the realization of the random vector  $\mathbf{U}$  on the limit state surface closer to the origin of the standard space (Hasofer and Lind 1974), and reliability index  $\beta(\mathbf{m}) = \|\mathbf{u}^*(\mathbf{m})\|$  is defined as the distance of the origin of the standard space to the MPP. Notice that  $\mathbf{x}^*(\mathbf{m})$  is the equivalent of  $\mathbf{u}^*(\mathbf{m})$  in the physical space and it can be obtained applying the transformation of equation (11).

In the classical form, the RBDO is performed by nesting two optimization problems, the structural optimization and the reliability analysis. For instance, solve the equation

(10) by any sequential approximation method gives:

$$\begin{aligned}
 & \text{for } k = 1 \text{ to } n_{it} \\
 & \text{Minimize : } W(\mathbf{m}^k) \\
 & \text{subject to : } \beta^{target} \leq \beta(\mathbf{m}^k) \\
 & m_{T_i}^k \in R^+, \theta_i^k \in [-90^\circ, 90^\circ], i = 1 \text{ to } N_p
 \end{aligned} \tag{13}$$

where  $n_{it}$  is the total number of iteration to solve the problem. In other words, for each step of the optimization problem  $k$  an entire reliability analysis (equation (12)) must be performed to compute the MPP  $\mathbf{u}^*(\mathbf{m}^k)$  and consequently  $\beta(\mathbf{m}^k)$ . As commented in the introduction of this paper, this leads to a high computational cost, especially if  $n_{it}$  ends up being high, which is generally the case when we apply any global optimization method.

In order to overcome such problem, a new RBDO methodology based on safety factors derived from the first order KKT optimality conditions of the RIA is presented in the next section. Such methodology, as it is shown in the sequel, can be easily coupled with any global optimization algorithm and possesses a computational cost equivalent to that of the deterministic global optimization, since it eliminates the need of the reliability analysis. Hence, it is suitable to pursue the RBDO of laminated composite structures.

#### 4. Proposed RBDO Methodology and its coupling with Global Optimization Algorithms

As the goal is to couple reliability analysis with global optimization algorithms to pursue the RBDO of laminated composite plates, a new RBDO methodology that works differently from the one described in the Section 3 is proposed here. Its main idea is to estimate the MPP of an optimal design (i.e., approximate  $\mathbf{u}^*$  or  $\mathbf{x}^*$  of  $\mathbf{m}^{optimal}$ ) obtained using any global optimization algorithm and, then, with this information, calculate safety factors  $S_F$  to be applied to such point, obtaining the final design  $\mathbf{m}^{reliable}$  that guarantees the prescribed reliability level of the structure. Such methodology is described in the sequel.

First, the safety factors are deduced from the KKT optimality conditions of the RIA (equation (12)), which are given by (Cheng *et al.* 2006):

$$\begin{cases} \nabla_u (\|\mathbf{u}^*(\mathbf{m})\|) + \lambda \nabla_u g(\mathbf{m}, \mathbf{u}^*(\mathbf{m})) = 0 \\ \lambda g(\mathbf{m}, \mathbf{u}^*(\mathbf{m})) = 0 \\ \lambda \geq 0 \\ g(\mathbf{m}, \mathbf{u}^*(\mathbf{m})) \leq 0 \end{cases} \tag{14}$$

where  $\lambda$  is the Lagrange multiplier allowing to take into account of the restriction  $g(\mathbf{m}, \mathbf{u}) \leq 0$ . We first assume  $\mathbf{u}^*(\mathbf{m}) \neq 0$  ( meaning  $\beta(\mathbf{m}) > 0$ , i.e. a probability of failure less then 0.5), which does not impose any restriction on practical problems. Then, equation (14) can be rewritten as:

$$\begin{cases} \frac{\mathbf{u}^*(\mathbf{m})}{\|\mathbf{u}^*(\mathbf{m})\|} = -\frac{\nabla_u g(\mathbf{m}, \mathbf{u}^*(\mathbf{m}))}{\|\nabla_u g(\mathbf{m}, \mathbf{u}^*(\mathbf{m}))\|} \\ g(\mathbf{m}, \mathbf{u}^*(\mathbf{m})) = 0 \end{cases} \tag{15}$$

where the gradient of the constraint is defined in the standard space. The Jacobian of the transformation  $T(\cdot)$ , which is defined in equation (11), may be used to obtain their values in the physical space, such as:

$$\nabla_u g(\mathbf{m}, \mathbf{u}^*(\mathbf{m})) = (\nabla_{\mathbf{m}\mathbf{x}})^T (\nabla_{\mathbf{x}} G(\mathbf{m}, \mathbf{x}^*(\mathbf{m}))) \quad (16)$$

where  $\nabla_{\mathbf{m}\mathbf{x}}$  is the Jacobian of the transformation between the two spaces.

In order to pursue the RBDO of a given structure, the maximum probability of failure of such structure or its target reliability index  $\beta^{target}$  is defined. At the end of the RBDO process such reliability index must be achieved. Therefore, the optimum verify  $\|\mathbf{u}^*(\mathbf{m}^{reliable})\| = \beta^{target}$ . Substituting this relation, and equation (16), into equation (15), gives:

$$\mathbf{u}^*(\mathbf{m}^{reliable}) = -\beta^{target} \frac{(\nabla_{\mathbf{m}\mathbf{x}})^T (\nabla_{\mathbf{x}} G(\mathbf{m}^{reliable}, \mathbf{x}^*(\mathbf{m}^{reliable})))}{\|(\nabla_{\mathbf{m}\mathbf{x}})^T (\nabla_{\mathbf{x}} G(\mathbf{m}^{reliable}, \mathbf{x}^*(\mathbf{m}^{reliable})))\|} \quad (17)$$

Here, it is considered that the random design variables of the problem are Gaussian (in the case of other types of random variables, the procedure would be the same, only equation (18) would be the one corresponding to the transformation of such random variables to the standard normal space). The transformation from the physical to the standard normal space is given by:

$$u_i = \frac{x_i - m_i}{s_i} \quad (18)$$

where  $s_i$  is the standard deviation of the  $i^{th}$  random variable. Consider that  $x_i^*$  can be related to  $m_i$  using safety factors by the following relation:

$$m_i = S_{F_i} x_i^* \quad (19)$$

Now, substituting equation (18) and (19) into (17), leads to:

$$S_{F_i} = 1 + \beta^{target} \frac{s_i}{x_i^*(\mathbf{m}^{reliable})} \frac{\left[ (\nabla_{\mathbf{m}\mathbf{x}})^T (\nabla_{\mathbf{x}} G(\mathbf{m}^{reliable}, \mathbf{x}^*(\mathbf{m}^{reliable}))) \right]_i}{\|(\nabla_{\mathbf{m}\mathbf{x}})^T (\nabla_{\mathbf{x}} G(\mathbf{m}^{reliable}, \mathbf{x}^*(\mathbf{m}^{reliable})))\|} \quad (20)$$

where  $S_{F_i}$  is the safety factor of the  $i^{th}$  design variable ( $i = 1$  to  $Np$ ).

Assuming that the problem has only one constraint and considering uncertainties only on the design variables of the problem, the vector  $\mathbf{x}^*(\mathbf{m}^{reliable})$  may be estimated by solving the deterministic optimization of the structural problem (equation (6)). In other words, it is assumed that  $\mathbf{x}^*(\mathbf{m}^{reliable}) \approx \mathbf{x}^*(\mathbf{m}^{optimal})$

Then, applying the safety factors to the vector  $\mathbf{x}^*(\mathbf{m}^{optimal})$  (equation (19)), the result of the RBDO problem  $\mathbf{m}^{reliable}$  is obtained guaranteeing the target reliability of the structure. As it has been shown in the derivation of the safety factors, the presented methodology does not make use of the reliability analysis, reducing then the computational cost of the RBDO.

Thus, the main steps for the implementation of the methodology are:

- (1) Estimate the MPP on the physical space of the optimal design  $\mathbf{m}^{optimal}$  by solving the deterministic optimization of the structural problem (equation (6)). Any deterministic optimization algorithm may be employed to obtain  $\mathbf{x}^*(\mathbf{m}^{optimal})$  making it possible to couple the RBDO methodology with any global optimization algorithm. In this paper the PSO is used to pursue this step.
- (2) After a sensitivity analysis on  $\mathbf{x}^*(\mathbf{m}^{optimal})$ , compute the safety factors  $\mathbf{S}_F$  using equation (20).
- (3) Calculate the optimal solution: the safety factors are applied to the vector  $\mathbf{x}^*(\mathbf{m}^{optimal})$ , using equation (19), finding then, the final design  $\mathbf{m}^{reliable}$  that guarantees the minimum allowed reliability level of the structure.

As mentioned before, the main advantage of this method would be that the computational effort is really close to the standard deterministic optimization. For instance, in the case of the evolutionary algorithm PSO, the extra computation effort to perform the RBDO is the one used in step 2 to pursue the sensitivity analysis. However, the methodology possesses some limitations such as: (i) only the design variables can be treated as random variables, (ii) in the form the methodology was presented, it is limited to one active probabilistic constraint and, (iii) it is limited to the cases where the precision of the FORM approach is enough.

## 5. Particle Swarm Optimization

The optimization algorithm PSO used in this paper is the same that have been employed in (Lopez *et al.* 2008), where a convergence analysis of the deterministic optimization of laminated composite plates was performed. Such analysis showed that the PSO achieved a better performance when compared to other evolutionary algorithms such as the GA and the ACO. The basic concepts of the PSO method are presented in the sequel.

The social psychologist James Kennedy and the electrical engineer Russel Eberhart introduced the PSO (Kennedy and Eberhart 1995), as emerged from experiences with algorithms inspired in the social behaviour of some bird species.

Consider the following situation: a swarm of birds is searching for food around a delimited area. Suppose that there is just one place where food can be found and the birds do not know where it is. Then, if one bird is successful in its search, it can attract other birds, and as a result of this social behaviour, the others will also find the food. From the socio-cognitive viewpoint this means that mind and intelligence are social features. Following this principle, each individual learns (and contributes) primarily to the success of its neighbours. This fact requires the balance between exploration (the capacity of individual search) and exploitation (the capacity of learning from the neighbours).

The essence is the possibility of learning from the experience of other individuals. From the optimization viewpoint, finding the food is similar to reaching the optimum. In this sense, the adjustment between exploration (the act of travelling around a place in order to learn about it) and exploitation (taking advantage of someone else's success) is required. If there is little exploration, the birds will all converge on the first good place encountered. On the other hand, if there is little exploitation, the birds will never converge or they will try alone to find food.

As described before, the main idea of the PSO is to mimic the social behaviour of birds. This is achieved by modelling the flight of each particle by using a velocity vector, which

considers a contribution of the current velocity, as well as two other parts accounting for the self knowledge of the particle and of the knowledge of the swarm about the search space. This way, the velocity vector is used to update the position of each particle in the swarm (Kennedy and Eberhart 1995).

The outline of a basic PSO algorithm is as follows:

- (1) Define the PSO parameters (inertia, self trust, swarm trust, etc.).
- (2) Create an initial swarm, randomly distributed throughout the design space.
- (3) Update the velocity vector of each particle.
- (4) Update the position vector of each particle.
- (5) Go to step 3 and repeat until the stop criteria is achieved.

## 6. Optimum column design

This example will demonstrate the capabilities and shortcomings of the proposed strategy. A short column, having a rectangular cross section with dimensions  $b$  and  $h$ , is optimized in order to minimize its cross section. The column is subjected to bending moments  $M_1 = 250kNm$  and  $M_2 = 125kNm$ , and to an axial force  $F = 2500kN$ . A limit state function is written in terms of the design vector  $\mathbf{d} = (b, h)$ :

$$G(\mathbf{d}) = 1 - \frac{4M_1}{bh^2Y} - \frac{4M_2}{hb^2Y} - \frac{F^2}{(bhY)^2} \quad (21)$$

where  $Y = 40MPa$  is the yield stress of the column material. In this problem, uncertainties are considered on the dimensions  $b$  and  $h$  of the column. The variables have normal distributions with standart deviations of  $0.03m$ . Thus, the RBDO problem may be stated as:

$$\begin{aligned} \text{Minimize : } & f(\mathbf{m}) = h \cdot b \\ \text{subject to : } & P_f(G(\mathbf{m}, \mathbf{X}) \leq 0) \leq P_f^{allowed} \\ & 0 \leq b \\ & h/2 \leq b/h \leq 2 \end{aligned} \quad (22)$$

The design variables were assembled into the vector  $\mathbf{m}$  and the RBDO of the column was pursued by three different methods: the proposed method, Classical RBDO based on RIA and PMA. The target reliability index  $\beta^{target}$  of this structure is equal to 3. All the final designs, presented in Table 1, have been verified by a  $10^6$  sample MCS in order to compute the final and reference probability of failure.

The proposed method obtained a final structure whose reliability index matches with the target one. Although the final result obtained is different from the ones obtained by RBDO-RIA/PMA, their probability of failure, calculated using the MCS, are alike. However, the computational costs of the strategies, represented by the number of calls of the cost function (NCC), are very different. The proposed strategy, based on safety factors, only needs a few more evaluations after the deterministic optimization. The classical RBDO have to pursue the complete optimization several times.

Note that the result achieved by RBDO-RIA and RBDO-PMA was the same and its final area was smaller than the one yielded by the proposed method. It is explained by the fact that, although the deterministic optimisation pursued in the proposed method

obtained a smaller area than the one of the MPP of the RBDO-RIA/PMA, when applying the safety factors, the final result ended up being higher than the safe design of the RBDO-RIA/PMA methods. In other words, the MPP of the RBDO-RIA/PMA designs is different from the one of the proposed method, resulting in different final designs. However, if the correction method is applied to the MPP of the RBDO-RIA/PMA, the same final result is obtained.

This example has been evaluated for different values of the target reliability index  $\beta^{target}$ . Results, presented in Table 2, show that the computational cost of the proposed method does not depend of the reliability target. Classical strategies may have their costs seriously impacted. However, the safety factor method always gives a higher final design area.

The following conclusions may be drawn from this example: (i) the reliability level of the final design provided by the proposed method is the same as the target reliability level; (ii) the numerical cost of the proposed method is much lower than the ones of the RBDO methods; (iii) although the method is able to provide a final design fulfilling the target reliability index of the structure, it may not yield the same optimum as RBDO based on RIA or PMA.

## 7. Reliable design of a plate

The case of a 2D square plate with a quarter of circle retired from a corner (see Figure 2) has been simulated by finite elements. The material is a steel with a Young modulus  $E = 200GPa$  and a yield limit stress  $\sigma_Y = 200MPa$ . The plate is clamped in his lower boundary and loaded in his left boundary with a distributed load with a total magnitude of  $800N$ . The design variables are the thickness of the plate  $h$  and the radius of the hole  $r$ . The plate is optimized in order to minimize its volume under the constraint of having an elastic behaviour, meaning a maximum stress below the yield limit.

$$G(h, r) = 1 - \frac{\sigma_{max}}{\sigma_Y} \quad (23)$$

In this case, uncertainties are considered on the dimensions  $h$  and  $r$ . These variables have normal distributions with standart deviations of  $s_h = 0.1mm$  and  $s_r = 4.0mm$ . The length of the border of the plate is fixed to  $l = 1m$ . Thus, the RBDO problem may be stated as:

$$\begin{aligned} \text{Minimize : } f(\mathbf{m}) &= \left( l^2 - \frac{\pi \cdot r^2}{4} \right) \cdot h \\ \text{subject to : } P_f(G(\mathbf{m}, \mathbf{X}) \leq 0) &\leq P_f^{allowed} \\ 1mm &\leq m_h \\ 20mm &\leq m_r \leq 60mm \end{aligned} \quad (24)$$

A convergence study lead to a mesh with 1352 elements and 1458 nodes. Stress are evaluated on Gauss integration points. The normal stress in the  $s$ -direction is used for the evaluation of the limit state function  $G(h, r)$  (Figure 3). Starting from an initial design  $(h, r) = (1.50mm, 50.00mm)$ , a SQP optimization algorithm lead to an optimal conception  $(h^*, r^*) = (1.00mm, 48.00mm)$ .

The reliability study has been pursued with a target reliability index  $\beta^{target}$  equal to 3. All the final designs are presented in Table 3. The conclusions are the same as those of the previous example. The proposed safety factors method gives a worse design than RIA

and PMA. However, the computational cost of the proposed strategy is really lower than the others (over 60%). It should be remarked that the cost reduction is very significant in this case due to the use of finite element analysis for the calculation of the limit state function.

## 8. Numerical Results of the composite study

In this section, two laminated composite RBDO problems are solved to demonstrate the effectiveness of the proposed methodology.

Tests are realised on a plate having the following dimensions :  $l_p = 1.00m$  and  $w_p = 1.00m$ . The numerical values of the material properties are:  $E_1 = 45000MPa$ ,  $E_2 = 12000MPa$ ,  $G_{12} = 4500MPa$ ,  $\nu_{12} = 0.3$  and  $\rho = 1900kg/m^3$ . The strength properties  $X_T$ ,  $X_C$ ,  $Y_T$ ,  $Y_C$  and  $S_{12}$  of the lamina are shown in Table 4.

### 8.1. Example 1 - Buckling Constraint

In this example, the weight of a laminated composite plate having as constraint the buckling failure factor has to be minimized. The laminate is subjected to compressive in-plane loads  $N_x$  and  $N_y$  equal to  $500 N/mm$  and it is comprised by 12 laminas, thus,  $N_p = 12$ . Due to the symmetric expression of the chosen failure mode, only half of the laminate are considered as variables. The problem is posed as:

$$\begin{aligned} & \text{Minimize : } W(\mathbf{m}) \\ & \text{subject to : } P_f(G_{buckling}(\mathbf{m}, \mathbf{X}) \leq 0) \leq P_f^{allowed} \\ & \quad m_{T_i} \in R^+, \theta_i \in [-90^\circ, 90^\circ], i = 1 \text{ to } \frac{N_p}{2} \end{aligned} \quad (25)$$

The probability of failure  $P_f$  of the structure is assumed to be lower than 0.16%, which leads to  $\beta^{target} = 3.00$ . Furthermore, it is assumed that the random design variables follow the normal distribution, such as  $T_i \sim N(m_{T_i}, s_{T_i})$ , where  $s_{T_i} = 0.02mm$ .

Now, the first step to apply the proposed RBDO methodology is to pursue the deterministic optimization of the problem to estimate  $\mathbf{m}^{optimal}$ . Thus, the following problem is solved (it is equivalent to the problem of equation (6)):

$$\begin{aligned} & \text{Minimize : } W(\mathbf{m}) \\ & \text{subject to : } G_{buckling}(\mathbf{m}) \leq 0 \\ & \quad m_{T_i} \in R^+, \theta_i \in [-90^\circ, 90^\circ], i = 1 \text{ to } \frac{N_p}{2} \end{aligned} \quad (26)$$

where  $\mathbf{m} = [m_{T_1}, \dots, m_{T_6}, \theta_1, \dots, \theta_6]^T$ . Notice that since the orientation angles are deterministic design variables, their final values are determined in this step of the method.

The optimization problem of equation (26) was solved using the PSO algorithm having as stopping criterion 3000 objective function evaluations and 20 individuals as population. The result of problem is shown in Table 5.

In that particular situation, the constraint ( $G_{buckling}(\mathbf{m}^{optimal}) = 0$ ) is saturated. Thus the optimal design corresponds to the most probable point of failure :  $\mathbf{x}^*(\mathbf{m}^{optimal}) = \mathbf{m}^{optimal}$ .

Now that the MPP have been estimated, that allows to pursue the next step of the proposed RBDO methodology, which is the computation of the safety factors using equa-

tion (20). To accomplish that, a sensitivity analysis must be pursued at  $\mathbf{x}^*$ . Here, a finite difference method was employed leading to a cost of extra 10 function evaluations. The resulting safety factors are shown in Table 6.

In the last step, the final design  $\mathbf{m}^{reliable}$  of the structure is calculated applying the safety factors to  $\mathbf{m}^{optimal}$  using equation (19). The final design is shown in Table 7.

To validate the reliability constraint of the final design, a reliability analysis using the RIA and a Monte Carlo Simulation using  $10^6$  samples were pursued. The resulting reliability index and probability of failure were 3.01 and 0.16%, respectively, which agree with the prescribed reliability and the target reliability index of the problem.

This example showed: (i) the step by step procedure to employ the proposed new RBDO methodology based on safety factors; (ii) the capability of the proposed RBDO method to take into account uncertainties on the design variables of the structure with a little extra computational effort if compared to the standard deterministic optimization (the 10 extra function evaluation of the sensitivity analysis); (iii) the validation of the proposed RBDO methodology applying the reliability analysis and the MCS to the final design  $\mathbf{m}^{reliable}$ .

## 8.2. Example 2 - Tsai-Wu Constraint

The problem proposed in this example is to minimize the weight of a laminated composite plate having as constraints the symmetry and balance of the laminate as well as the first ply failure criterion of Tsai-Wu. The laminate is subjected to in-plane loads  $N_x = 2500N/mm$  and  $N_y = -1500N/mm$  and it is comprised by 48 laminas whose thicknesses  $T_i$  are all the same. In this example, the thickness is also considered as random variable, being its mean value one of the design variables of the optimization problem.

The symmetry and balance of the laminate were handled using a data structure strategy, which consists in coding only half of the laminate and making each stack of the laminate be composed by two laminas of the same orientation with opposite signs. It is a classical way to deal with such constraints, for instance, see (Le Riche and Haftka 1993, Lopez *et al.* 2009a,b). The domain of the orientation angles becomes  $\theta_i \in [0^\circ, 90^\circ]$ . Thus, the number of design variables is reduced to 13 and the design vector  $\mathbf{m} = (m_T, \theta_1, \dots, \theta_{12})$  is composed by the mean thickness  $m_T$  and the 12 deterministic orientation angles. The problem is posed as:

$$\begin{aligned} & \text{Minimize : } W(\mathbf{m}) \\ & \text{subject to : } P_f(G_{TW}(\mathbf{m}, \mathbf{X}) \leq 0) \leq P_f^{allowed} \\ & \quad m_T \in R^+, \theta_i \in [0^\circ, 90^\circ], i = 1 \text{ to } 12 \end{aligned} \quad (27)$$

Here again, it is assumed that the probability of failure  $P_f$  of the structure has to be lower than 0.16%, which leads to  $\beta^{target} = 3.00$ . Furthermore, it is assumed that the random design variable follows the normal distribution, such as  $T \sim N(m_T, s_T)$ , where  $s_T = 0.02mm$ .

The same three steps of the proposed RBDO methodology were employed here. In the first step, the deterministic optimization was pursued using the PSO algorithm with 3000 function evaluations as stopping criterion. The cost of the sensitivity analysis was 2 function evaluations. The result of the deterministic optimization  $\mathbf{m}^{optimal}$ , the safety factors  $\mathbf{S}_F$  as well as the final design  $\mathbf{m}^{reliable}$  using the PSO are shown in Table 8.

A  $10^6$  sample Monte Carlo Simulation of the final design was performed and their results indicate the reliability of the design around 99.78%, meaning a probability of

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5 failure of 0.22%, which relatively agrees with the target reliability index imposed on the  
6 RBDO problem.

7 From this example it can also be concluded that the coupling of the evolutionary  
8 optimization methods and the proposed methodology successfully performed the RBDO  
9 of the laminated composite plate taking into account uncertainties in the thickness of  
10 the structure.  
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## 12 13 9. Concluding Remarks

14  
15 In this paper, the optimization of laminated composite plates was performed taking into  
16 account uncertainties in the thickness of the structure. Safety factors based on the KKT  
17 optimality conditions of RIA were proposed as RBDO methodology. The PSO method  
18 was applied as optimization tools due to its ability of handling global optimization prob-  
19 lems. The proposed new RBDO methodology based on safety factor was employed in the  
20 optimization of laminated composite plates and validated.  
21

22 The following conclusions can be drawn from the analysis pursued in the paper:  
23

- 24 (1) The coupling of the PSO with the proposed safety factor method successfully  
25 performed the RBDO of the laminated composite plate, being the methodology  
26 validated using the Monte Carlo Simulation;
- 27 (2) Such method is able to perform the RBDO of structures with a little extra compu-  
28 tational effort than the deterministic optimization being suitable for the coupling  
29 with global optimization algorithms.  
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31 Although the mechanical model used in the numerical analysis of this paper is quite  
32 simple, the RBDO methodology can be directly extended to more complex laminated  
33 composite structures, using, for instance, the finite element method. However, the method  
34 is limited to the cases where the FORM approach can be applied, only the design variables  
35 can be treated as random variables and in the form the methodology was presented, it  
36 is limited to one active probabilistic constraint. To overcome such limitations further  
37 research has to be done.  
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Table 1. RBDO results of the column problem

$\beta = 3$	Safety Factor		RBDO-RIA		RBDO-PMA	
	b	h	b	h	b	h
$\mathbf{x}^*$	0.2538	0.5076	0.3056	0.4249	0.3057	0.4247
$m^{reliable}$	0.3343	0.5478	0.3764	0.4804	0.3765	0.4803
area ( $mm^2$ )	0.1831		0.1808		0.1808	
NCC	33		440		311	
$P_f$ by MCS	0.00449		0.00455		0.00455	

Table 2. Influence of  $\beta$  on NCC

$\beta$	$m^{reliable}$ area ( $mm^2$ ) NCC	Safety Factor		RBDO-RIA		RBDO-PMA	
		b	h	b	h	b	h
$\beta = 2$		0.3075	0.5344	0.3456	0.4714	0.3457	0.4713
		0.1643		0.1629		0.1629	
		33		684		322	
$\beta = 4$		0.3611	0.5613	0.4042	0.4937	0.4043	0.4936
		0.2027		0.1995		0.1995	
		33		432		245	
$\beta = 5$		0.3880	0.5747	0.4301	0.5094	0.4301	0.5095
		0.2230		0.2191		0.2191	
		33		560		517	

Table 3. Coupling safety factors method with finite element analysis

$\beta = 3$	Safety Factor		RBDO-RIA		RBDO-PMA	
	h	r	h	r	h	r
$\mathbf{x}^*$	1.00	48.00	0.7563	39.87	0.7563	39.87
$m^{reliable}$	1.20	41.30	1.00	34.61	1.00	34.61
volume ( $mm^3$ )	10395		9059		9059	
NCC	113		488		319	
$P_f$ by MCS	0.00449		0.00455		0.00455	

Table 4. Lamina's strength properties

$X_T$	2062 MPa	$Y_C$	240 MPa	$S_{21}$	105 MPa
$X_C$	1701 MPa	$Y_T$	70 MPa		

Table 5. Result of the deterministic optimization problem of equation (26) (angles in degrees and thicknesses in mm)

	$m_{T_1}$	$m_{T_2}$	$m_{T_3}$	$m_{T_4}$	$m_{T_5}$	$m_{T_6}$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$
$m^{optimal}$	0.1036	0.1098	0.1106	0.1000	0.1236	0.1025	45	-45	45	-45	45	-45

Table 6. Safety factors of example 1 computed using equation (20) (thicknesses in mm)

	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$
$S_F$	1.2363	1.2229	1.2213	1.2449	1.1980	1.2388

Table 7. Final design of example 1 computed using equation (19) (angles in degrees and thicknesses in mm)

	$m_{T_1}$	$m_{T_2}$	$m_{T_3}$	$m_{T_4}$	$m_{T_5}$	$m_{T_6}$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$
$m^{reliable}$	0.1281	0.1343	0.1351	0.1244	0.1481	0.1270	45	-45	45	-45	45	-45

Table 8. Results of the RBDO using PSO (population 20, 3000 FE, thickness in mm and angles in degrees)

	$m_T$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$	$\theta_7$	$\theta_8$	$\theta_9$	$\theta_{10}$	$\theta_{11}$	$\theta_{12}$
$m^{optimal}$	0.2012	2.67	0.839	0.966	0.0626	0.835	3.94	0.0952	1.60	3.23	0.913	0.0334	89.2
$S_F$	1.30	-	-	-	-	-	-	-	-	-	-	-	-
$m^{reliable}$	0.2616	2.67	0.839	0.966	0.0626	0.835	3.94	0.0952	1.60	3.23	0.913	0.0334	89.2

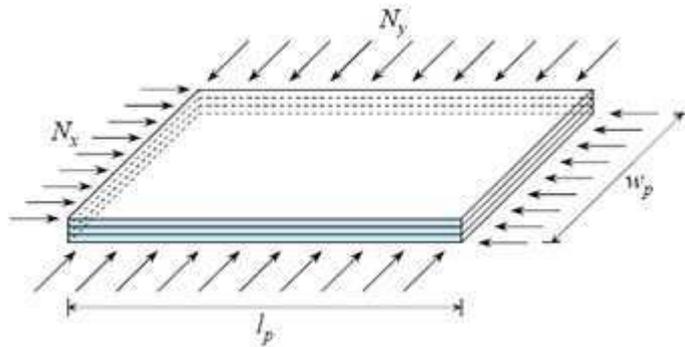


Figure 1. Laminated composite plate subjected to in-plane loads.

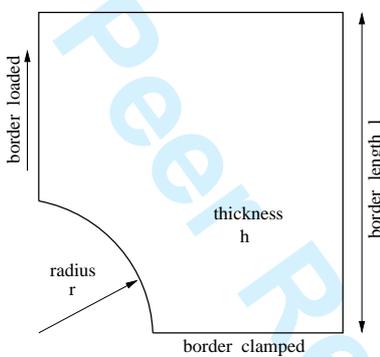


Figure 2. Reliable design of a plate.

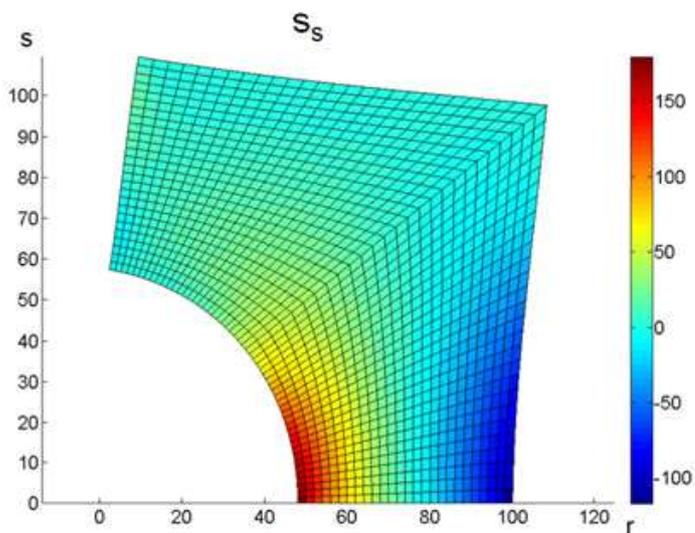


Figure 3. Stress distribution (in MPa) in the plate.

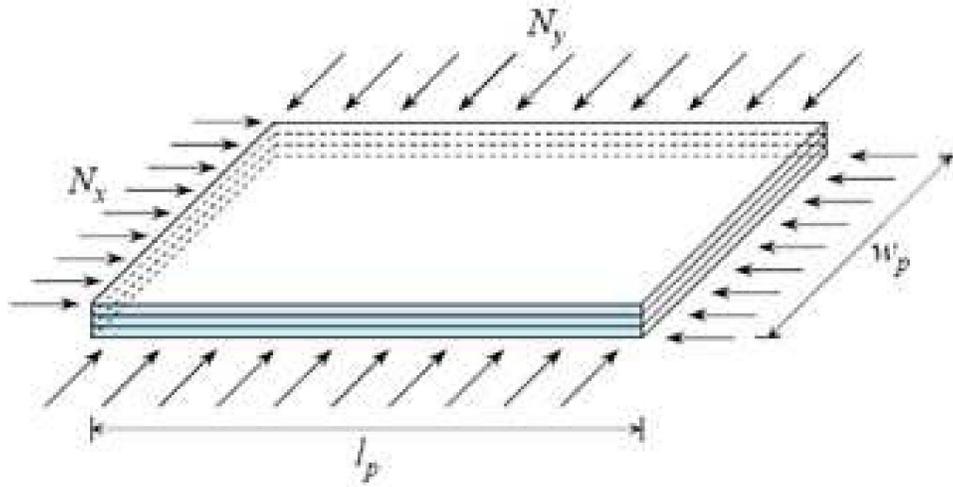


Figure 1. Laminated composite plate subjected to in-plane loads.  
93x46mm (600 x 600 DPI)

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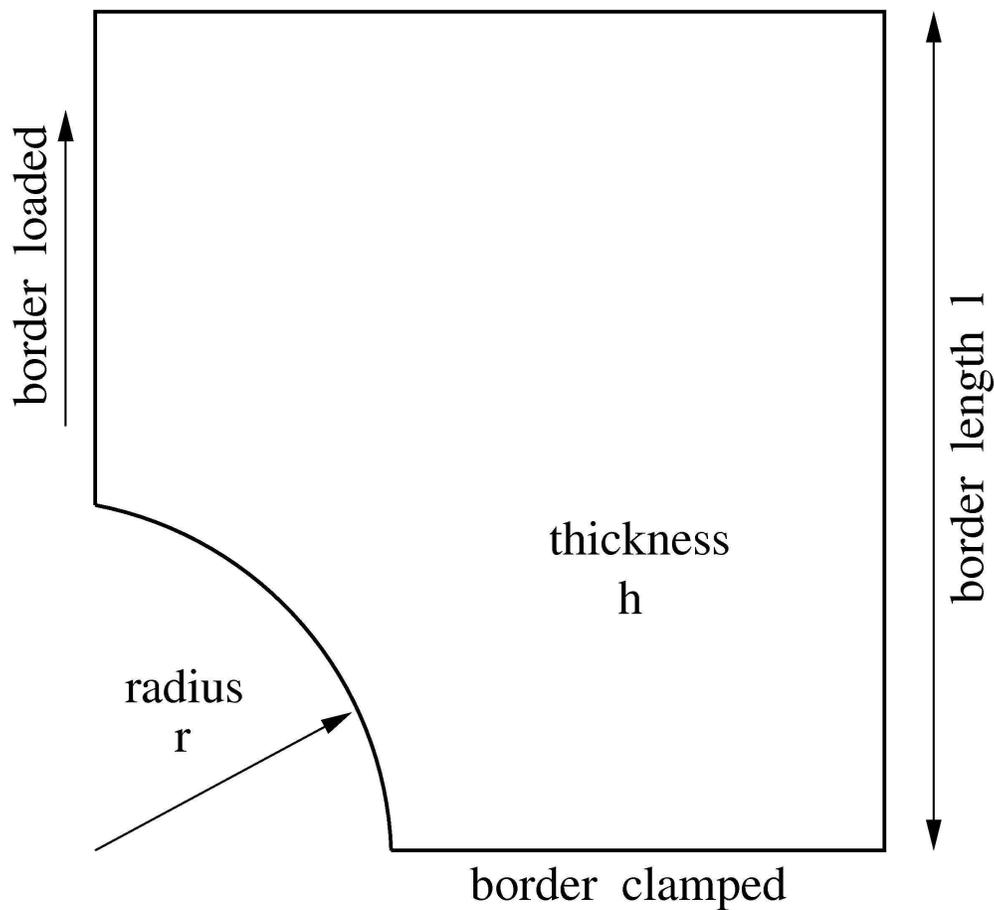


Figure 2. Reliable design of a plate.  
100x92mm (600 x 600 DPI)

Only

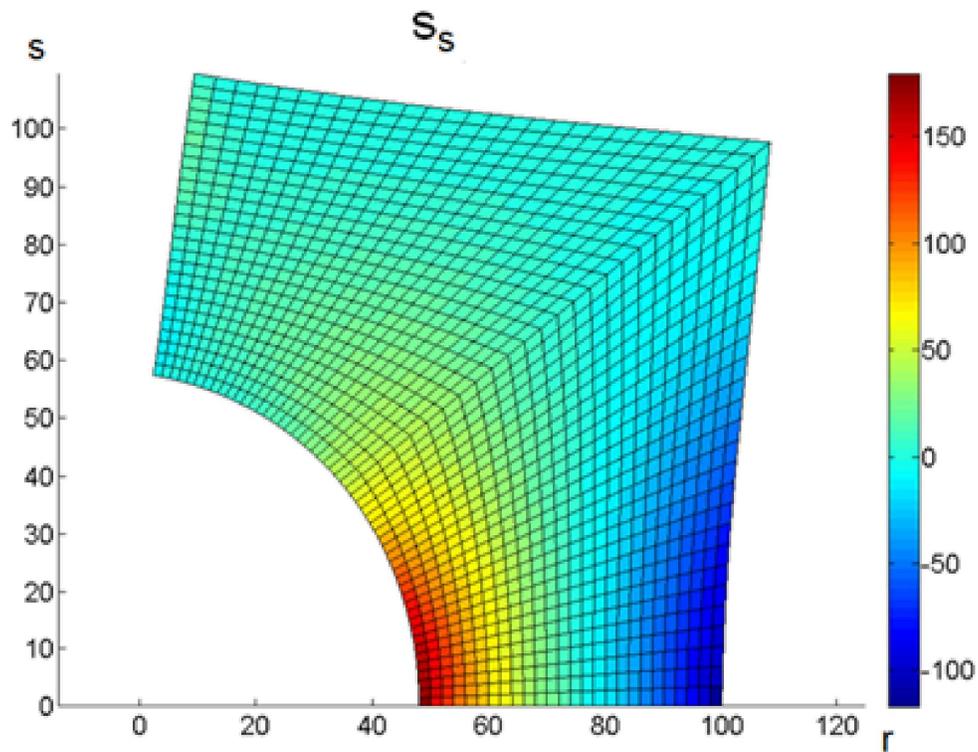


Figure 3. Stress distribution (in MPa) in the plate.  
185x143mm (600 x 600 DPI)