HAL
open science

A new formulation of differential radiosity and a rendering application

Didier Arquès, Venceslas Biri, Olivier Derpierre

To cite this version:

Didier Arquès, Venceslas Biri, Olivier Derpierre. A new formulation of differential radiosity and a rendering application. WSCG 2005, Mar 2005, Czech Republic. pp.177-180. hal-00681561

HAL Id: hal-00681561

https://hal.science/hal-00681561

Submitted on 21 Mar 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A new formulation of differential radiosity and a rendering application

D. Arquès V. Biri O. Derpierre
Gaspard Monge Institute
University of Marne-la-Vallée
6, cours du Danube
77 000 SERRIS, FRANCE
{arques,biri,oderpier}@univ-mlv.fr

ABSTRACT

Among all the existing realistic lighting methods, the radiosity method is the only one that gives precisely an analytic solution of diffuse light exchanges. The gradient of this solution have been studied but not often used in a context of pure rendering. We present in this article a method to render a surface using the radiosity contour levels. First, we define a differential formulation of the radiosity equation which leads us to a new expression of the gradient of radiosity. We deduce from this general equation a simpler equation of this gradient in the case of a planar surface lighted by a light source reduced to a point. Then we present our method to render planar surfaces using a radial mesh that follows the contour levels of the radiosity. This method is shown to improve the quality of the rendering and decrease the number of vertices used for rendering.

Keywords

Differential radiosity, Radiosity gradient, Global illumination, Rendering.

1. INTRODUCTION

The generation of photo-realistic images using just a scane description requires a precise and realist lighting model. The radiosity method, initially proposed by Goral et al. [Gor84] uses a faithful model of diffuse light exchanges. This method belongs to the global illumination methods that strive to describe the light interactions as precisely as possible. Though this approach tends to be less widely used today, it is still the only one that gives a true analytic solution of the illumination solution on the surfaces forming the virtual scene. Moreover the use of radiosity is still perfectly adequate in the case of architectural scenes where its ability to handle multiple light sources and indirect lighting allows the generation of truly amazing images.

Netherveless, the initial approach of Goral et al. requires to mesh precisely the surfaces composing the scene and to consider a constant radiosity function on every single single generated face. Therefore the computer graphics community had to elaborate different methods to solve these problems. Hierarchical methods were form the first of these answers. These methods use meshes that adapt themselves to the radiosity function and to the shadows cast on objects which reduces considerably the number of generated faces. A second answer was found in progressive algorithms that are able to compute quickly a reconstruction of the illumination function by considering first the main light transfers. But the two answers were mainly algorithmic solutions to an analytic problem. Research based on the analytic formulation of the radiosity equation finally gave birth to the higher order radiosity methods that separate the radiosity function from its geometric support by projecting it on higher order function bases.

We follow the same strategy focusing on the radiosity function. We propose a new equation of the radiosity function gradient but instead of using it to build oracles used in the refinement process of a hierarchical method, as proposed by Holzshuch et al. [Hol95], we use it for rendering. Coupled with the property of unimodality of the radiosity function we propose a meshing method using directly the shape of the radiosity function as opposed to using the geometry support of that function.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

Conference proceedings ISBN 80-903100-7-9
WSCG'2005, January 31-February 4, 2005
Plzen, Czech Republic.
Copyright UNION Agency – Science Press

2. A NEW FORMULATION OF THE RADIOSITY GRADIENT

Let F denote the set of faces describing a scene. The exchanges of radiosity Bi between a face i belonging to F and the rest of the scene are described by the equation [Gor84] where E_i is the emittance of the face i and ρ_i its reflectivity.

$$B_i = E_i + \rho_i \sum_{j \in F} B_j F_{i,j} \tag{2.1}$$

In the case of unoccluded polygonal surfaces, we can define the oriented contour of the surface A_i as the set β_i of its oriented edges. The form factor $F_{i,j}$ between two polygonal surfaces A_i and A_j can be expressed :

$$F_{i,j} = \frac{1}{2\pi A_i} \sum_{\vec{k} \in \beta_i} \sum_{\vec{l} \in \beta_j} \underbrace{\int_{\vec{k}} \int_{\vec{l}} \ln\left(\left\|\overrightarrow{M_i M_j}\right\|\right) dM_i \, dM_j}_{L_{\vec{k},\vec{l}}} \tag{2.2}$$

Our objective is to find an equation describing the gradient of the radiosity function. So we consider an oriented square discretization of all the surfaces describing the scene and we study the difference of radiosity between two adjacent faces i and i' compared to the radiosity of every other face j in the scene. Starting from equation (2.1) and using the form factor on polygonal contours (2.2) we obtain the following equation :

$$B_i = E_i + \frac{\rho_i}{2\pi A_i} \sum_{\vec{k} \in \beta_i} \sum_{j \in F} \sum_{\vec{l} \in \beta_j} B_j L_{\vec{k},\vec{l}} \tag{2.3}$$

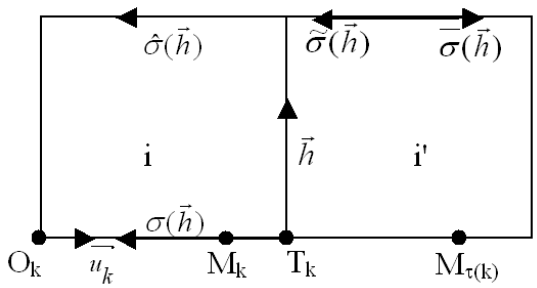

Figure 1 : Notation of a face *i* and its neighbor *i'*

We first subtract the radiosity $B_{i'}$ to the radiosity B_i where i' is the neighboring face of i along the edge h. Every internal edge of the scene i is counted twice in the sum (2.3), once positively and once negatively. We group this terms together to get :

$$\begin{aligned} dB_{\vec{h}} &= B_i - B_{i'} \\ &= dE_{\vec{h}} + \frac{\rho_i}{2\pi A_i} \sum_{\vec{k} \in \beta(i)} \sum_{\vec{l} \in B} L_{\vec{k},\vec{l}} \, dB_{\vec{l}} - \frac{\rho_{i'}}{2\pi A_{i'}} \sum_{\vec{k} \in \beta(i')} \sum_{\vec{l} \in B} L_{\vec{k},\vec{l}} \, dB_{\vec{l}} \end{aligned} \tag{2.4}$$

We then separate the edges of the discretization that form the contour of the surfaces from the internal edges. Let B_e denote the first set of edges and B_i the second one. Equation (2.4) can then be rewritten as follows:

$$dB_{\vec{h}} = dE_{\vec{h}} + \sum_{\vec{l} \in B = Bi \cup Be} dB_{\vec{l}} \left(\sum_{\vec{k} \in \beta(i)} \frac{\rho_i}{2\pi A_i} L_{\vec{k},\vec{l}} - \sum_{\vec{k} \in \beta(i')} \frac{\rho_{i'}}{2\pi A_{i'}} L_{\vec{k},\vec{l}} \right)$$

Using the following notations :

$$\begin{cases} \overrightarrow{O_{\vec{k}} T_{\vec{k}}} = dk \, \overrightarrow{u_{\vec{k}}} \\ \overrightarrow{O_{\vec{l}} T_{\vec{l}}} = dk \, \overrightarrow{u_{\vec{l}}} \\ M_{\vec{k}} = O_{\vec{k}} + \lambda \overrightarrow{O_{\vec{k}} T_{\vec{k}}} \\ M_{\vec{l}} = O_{\vec{l}} + \lambda \overrightarrow{O_{\vec{l}} T_{\vec{l}}} \end{cases}$$

we get, after some computations [Der04] :

$$dB_{\vec{h}} = dE_{\vec{h}} + \sum_{\vec{l} \in B} dB_{\vec{l}} \, S_{\vec{l},\vec{h}} \tag{2.5}$$

with :

$$S_{\vec{l},\vec{h}} = \sum_{\vec{k} \in \beta(i)} \frac{\rho_i}{2\pi A_i} \int_0^1 \int_0^1 \log\left(\frac{\left\|\overrightarrow{M_{\vec{k}} M_{\vec{l}}}\right\|}{\left\|\overrightarrow{M_{\tau(\vec{k})} M_{\vec{l}}}\right\|} \right) \left(\overrightarrow{u_{\vec{k}}} . \overrightarrow{u_{\vec{l}}} \right) dk \; dl \; d\lambda \; d\mu \tag{2.6}$$

We study then what happens to equation (2.6) when the distances dk and dl decrease toward 0. By grouping the terms of both the "vertical" and the "horizontal" edges, we get the following equation [Der04] :

$$S_{\vec{l},\vec{h}} = \frac{\rho_i}{2\pi d_{hl}^2} dk \, dl \left[\left(\overrightarrow{u_{\vec{h}}^{\perp}} . \overrightarrow{u_{\vec{l}}} \right) + 2 \left(\overrightarrow{u_{hl}} . \overrightarrow{u_{\sigma^{-1}(\vec{h})}} \right) \left((\overrightarrow{u_{\vec{l}}} \wedge \overrightarrow{u_{hl}}) . \overrightarrow{u_n} \right) \right]$$

Let d_{hl} denotes the distance between the points M_k and M_l, $\overrightarrow{u_{hl}}$ the normalized direction between these points, and $\overrightarrow{u_n}$ the normal to face i on edge h. We can now express the gradient of the radiosity [Der04] in the equation (2.7) that follows :

$$\begin{aligned} \overrightarrow{grad \; B_i} = \overrightarrow{grad \; E_i} &- \frac{\rho_i}{2\pi} \sum_{j \in F} \iint_{A_j} \frac{1}{d_{hl}^2} \left(\overrightarrow{Grad \, B_j}^{\perp} \wedge \overrightarrow{u_n} \right) dA_j \\ &- \frac{\rho_i}{\pi} \sum_{j \in F} \iint_{A_j} \frac{1}{d_{hl}^2} \left(\overrightarrow{u_{hl}} - \left(\overrightarrow{u_{hl}} . \overrightarrow{u_n} \right) \overrightarrow{u_n} \right) \left(\left(\overrightarrow{Grad \, B_j}^{\perp} \wedge \overrightarrow{u_{hl}} \right) . \overrightarrow{u_n} \right) dA_j \\ &- \sum_{j \in F} \int_{C_j} \frac{B_j \rho_i}{2\pi d_{hl}^2} \left[\left(\overrightarrow{u_{\vec{l}}} \wedge \overrightarrow{u_n} \right) + 2 \left(\overrightarrow{u_{hl}} - \left(\overrightarrow{u_{hl}} . \overrightarrow{u_n} \right) \overrightarrow{u_n} \right) \left((\overrightarrow{u_{\vec{l}}} \wedge \overrightarrow{u_{hl}}) . \overrightarrow{u_n} \right) \right] dl \end{aligned}$$

We also check that this equation is still valid even in the case of a discretization using parallelograms instead of squares. We get, instead of expressions in [Hol95], a gradient contained in the plane since we consider the radiosity as a two dimension function. But equations are close especially in the simple case that we will study in the next section. This confirms the two different approaches.

The gradient being orthogonal to the contour levels, it can prove interesting to consider the cross product of the previous equation with the normal to the surface to obtain the tangent to the contour levels :

$$\overrightarrow{grad\,B_i}^{\perp} = \overrightarrow{grad\,E_i}^{\perp} - \frac{\rho_i}{2\pi}\sum_j \iint_{surface} \frac{1}{d_{hl}^2}\left(\overrightarrow{GradB_j}^{\perp} \wedge \vec{u_n}\right)\wedge \vec{u_n}\, dA_j$$
$$- \frac{\rho_i}{\pi}\sum_j \iint_{surface} \frac{1}{d_{hl}^2}\left(\vec{u_{hl}} \wedge \vec{u_n}\right)\left(\left(\vec{u_{hl}} \wedge \vec{u_n}\right).\overrightarrow{GradB_j}^{\perp}\right) dA_j$$
$$- \frac{\rho_i}{2\pi}\sum_j \int_{bord} \frac{B_j}{d_{hl}^2}\left[\left(\vec{u_l} \wedge \vec{u_n}\right)\wedge \vec{u_n} + 2\left(\vec{u_{hl}} \wedge \vec{u_n}\right)\left(\left(\vec{u_{hl}} \wedge \vec{u_n}\right).\vec{u_l}\right)\right] dl \qquad (2.8)$$

3. GRADIENT IN A SIMPLE CASE

We will show here an interesting use of the radiosity gradient in a simple case : a disc lighting a plane. This simple case allows us to compute in any point of the plane an analytic expression of the radiosity and also an analytic expression of the radiosity gradient.

Analytic equation of radiosity

We consider now the simple case of a disc of center *O* emitting light and illuminating a surface contained in an infinite plane *P* (cf. figure 2). Moreover, we will consider that the radiosity of the disc will be constant and equal to its emittance.

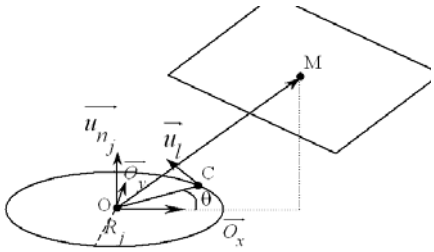

Figure 2 : Our study case : A source disc lighting a plane

We will also consider that the radius of the disc is small compared to the distance disc – plane, a property that we will use to compute difficult integrals. Finally, we do not consider any occlusion between the disc and the plane.

The case of a disc emitting light is nothing new. It was abundantly studied and multiple analytic solutions, approximate or not, were formulated [Wal89]. The contribution of the face j to the radiosity of face i is computed according to the approximation given by Wallace et al.:

$$Bi = \frac{\rho i\, Bj\, Aj}{n}\sum_{k=1}^{n}\delta k \frac{\cos(\theta kj)\,\cos(\theta ki)}{\pi\, rki + \frac{Aj}{n}} \qquad (3.1)$$

where r_{ki} is the distance from a source point to *M*, θ_{kj} (respectively θ_{ki}) is the angle between the normal to the receiver (respectively to the emitter) and the vector between this two points, and δ_k has 1 for value if the points see each other and 0 else.

Analytic equation of the gradient

We now try to use equation (2.8) to compute a simple analytic equation of the gradient of radiosity at any point of the plane, knowing that radiosity is constant on the emitter:

$$\overrightarrow{grad\ B_i}^{\perp}(M) = -\frac{\rho_i}{2\pi}\int_C \frac{B_j}{d_{hl}^2}\left[\left(\vec{u_l} \wedge \vec{u_{n_i}}\right)\wedge \vec{u_{n_i}} + 2\left(\vec{u_{hl}} \wedge \vec{u_{n_i}}\right)\left(\left(\vec{u_{hl}} \wedge \vec{u_{n_i}}\right).\vec{u_l}\right)\right] dl$$

After some manipulations and a polynomial expansion of order 2 in R_j/OM, we wet a simple analytical expression of the radiosity gradient on any point *M* of a plane lighted by a disc.

$$\overrightarrow{grad\ B_i}^{\perp}(M) = \frac{\rho_i L_j}{OM^4}\left[\left(\overrightarrow{OM}.\vec{u_{n_i}}\right)\left(\vec{u_{n_j}} \wedge \vec{u_{n_i}}\right) - 4\left(\vec{u_{n_i}}.\vec{u_{n_j}}\right)\left(\overrightarrow{OM} \wedge \vec{u_{n_i}}\right) + 4\frac{\left(\overrightarrow{OM} \wedge \vec{u_{n_i}}\right).\left(\overrightarrow{OM} \wedge \vec{u_{n_j}}\right)}{OM^2}\left(\overrightarrow{OM} \wedge \vec{u_{n_i}}\right)\right] \qquad (3.2)$$

4. RENDERING USING RADIOSITY GRADIENT

We use here our analytic expression of radiosity and its gradient in any point of a plane illuminated by a source disc. We present in this section an algorithm using this gradient that improves and speeds up the rendering of such a surface.

Unimodality of Radiosity

Radiosity is a function from $\Re^2$ to $\Re$. The idea our rendering method is to use the image space instead of the definition space as it is done in classical rendering algorithms, including the ones based on the radiosity method.

We decide to discretize the image space so that it follows closely the various values radiosity can take on a surface (cf. figure 3). This approach, though it can seems similar to the one presented in [Dre93], differs in the exact use of the gradient and the use of a radial discretization scheme.

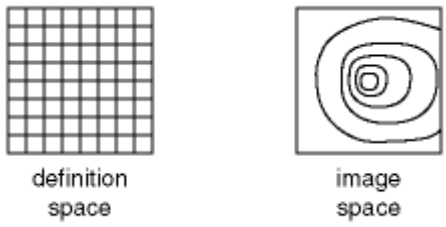

Figure 3. definition space discretization vs. image space discretization

Moreover we know, thanks to the unimodality properties of the radiosity solution, that if we consider the case of a single light source then the contour levels of the radiosity function are imbricated curves. Therefore, any line going from the maximum of radiosity toward any direction of the plane intersects every contour level of the radiosity

function whose value lies between this maximum and 0.

Discretization of the image space

To be able to discretize the image space we have to find where the maximum of the radiosity function lies. The position of this maximum is given by the following expression:

$$B(x) = \frac{h\ x\ \sin(\phi) + h^2\ \cos(\phi)}{(x2+h2)2} \qquad (4.1)$$

where h is the distance from source point O to plane P, x is the distance from point M to the projection of O on P, and ϕ is the angle between the direction of the source point and the normal to plane P.

The maximum of equation (5.1) is given by the following formula :

$$\frac{1}{6}\ \frac{h(-4+2\sqrt{4+3\tan(\phi)^2})}{\tan(\phi)} \qquad (4.2)$$

Once we know this position, we choose first the set of radiosity values we want to represent, and then a set of directions sampling uniformly all the directions. Then for every direction d and for every value of radiosity B_k, we find the intersection point $X_{k,d}$ between the line [Od) and the contour level B_k (cf. figure 4) using a variation of the classical gradient algorithm.

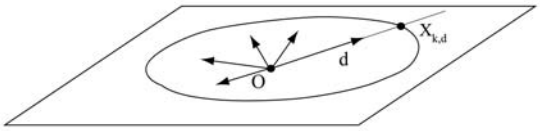

Figure 4. Intersection of a direction and a line of contour

Once we have these points, we have to restrict the rendering of the lighted plane to the boundaries of the real surface. This is done using OpenGL's stencil test to restrict the drawing to the lighted area.

5. RESULTS

We implemented the previous algorithm allowing us to discretize more adequately the illuminated surfaces. This discretization which follows the contour levels of the radiosity function, allows us to render faithfully this function at a lower cost in terms of vertices compared to a classical square discretization scheme.

Our technique following as closely as possible the contour levels of the radiosity function, we can expect visual results that are more faithful than those given by a square discretization. Figure 5 shows the results given by a 10x10 square discretization and those given by our results using 20 radial subdivisions and 5 contour levels. As expected we get a real improvement in terms of visual aspect.

Figure 5 also allows us to compare the results generated by our technique using a radial discretization scheme with 20 radial subdivisions and 5 contour levels and those generated by a 30x30 square discretization. With such discretization levels we achieve comparable results. As expected, thanks to our radiosity-following discretization scheme, we get good visual results while using fewer vertices.

6. DISCUSSION AND CONCLUSION

This article presents a new rendering technique based on a radial discretization scheme which follows the contour levels of the radiosity function. To achieve this, we present first an equation of the radiosity gradient in a simple case of a source disc lighting a plane. Then a technique similar to the gradient method is used to compute the positions of our vertices. Our method generates a radial mesh that is closer to the radiosity function and is therefore less costly in terms of vertices, allowing us to build images that are both more realistic and faster to compute.

We plan to use this rendering technique in a fast global illumination algorithm to compute coarse radiosity solution. Moreover, the equations presented in this article do not constraint in any way the formulation of the radiosity function, and therefore we want to apply this technique to higher order bases methods.

7. REFERENCES

[Der04] O. Derpierre, Details of the mathematical computation of the gradient : http://www-igm.univ-mlv.fr/~oderpier/recherche/radiosite_dif/index.html

[Dre93] G. Drettakis, E. Fiume, *Accurate and Consistent Reconstruction of Illumination Functions Using Structured Sampling*, in Eurographics'93 conf.proc., Computer Graphic Forum, vol. 12 (3), pp. 273-284, September 1993.

[Gor84] C.M. Goral, K.E. Torrance, D.P. Greenberg, B. Battaile, *Modeling the Interaction of Light between Diffuse Surfaces*, in ACM Siggraph'84 conf.proc., Computer Graphics, vol. 18 (3), pp. 213-222, July 1984.

[Hol95] N. Holzschuch, F.X. Sillion, *Accurate Computation of the Radiosity Gradient for Constant and Linear Emitter*, in 6th Eugraphics Workshop on Rendering conf.proc., Rendering Techniques'95, pp. 186-195, June 1995.

[Wal89] J.R. Wallace, K.A. Elmquist, E.A. Haines, *A Ray Tracing Algorithm for Progressive Radiosity*, in ACM Siggraph'89 conf.proc., Computer Graphics, vol. 23 (3), pp. 315-324, July 1989.