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Survey on Some Automotive Semi-Active Suspension Control Methods: a Comparative Study on a Single-Corner Model

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Abstract: In this paper, we propose an overview and a benchmark of the most classic semi-active suspension control laws. Based on a recent result where the optimal semi-active performance trade-off was derived (see Poussot-Vassal et al. (2010)), here, a complete benchmark to evaluate the main semi-active suspension control design methods is proposed. The present paper gives a quick picture of the present state of the art in the semi-active suspension control field in terms of *comfort* and *road-holding* trade-off.

Keywords: Semi-active suspension, Nonlinear control, LPV control, Performance evaluation, Benchmark.

1. INTRODUCTION

In the automotive field, semi-active suspensions have received a lot of attention since they seem to provide the best compromise between cost (energy-consumption, actuators/sensors hardware) and performance. The research in this field follows two mainstreams: the study of new technologies of semi-active actuation of damping (like electro-hydraulic, electro-rheological and magneto-rheological damper), and the design of semi-active control strategies (see *e.g.* Karnopp et al. (1974); Tseng and Hedrick (1994); Hrovat (1997); Kawabe et al. (1998); Ahmadian and Reichert (2001); Guardabassi and Savaresi (2001); Hong et al. (2002); Giua et al. (2004); Geurts et al. (2006); Giorgetti et al. (2006a); Canale et al. (2006); Savaresi and Spelta (2009) and references therein).

In a previous work, Poussot-Vassal et al. (2010) proposed a methodology to evaluate the best performance trade-off, in term of *comfort and road-holding*, a semi-active suspension system can achieve. Based on this recent development, the main contribution of this paper is to provide a full analysis and benchmark of some of the most common and dedicated semi-active suspension control law performances, both for the *comfort* and for the *road-holding* objectives.

This work is carried out on the basis of the well known single-corner vehicle model including a semi-active suspension system (see Figure 1). Performances are then thoroughly evaluated through dedicated frequency domain simulations.

The paper is organized as follows: Section 2 presents the semi-active suspension models and recalls the performance criteria used in Savaresi et al. (2010). In Section 3 and

4, some classical dedicated semi-active control approach for *comfort* and *road-holding* performance enhancement are recalled. More generic or complex control algorithms are given in Section 5. All the proposed algorithms are evaluated with numerical simulations and compared to the optimal and passive performances (obtained in Poussot-Vassal et al. (2010)), using both frequency responses and performance index, providing then a(n almost) complete semi-active suspension control benchmark. Conclusions are gathered in Section 7.

2. SINGLE-CORNER MODEL AND PERFORMANCE DEFINITIONS

2.1 Single-corner model

The single-corner car model is the basic model used for suspension analysis. It consists in double mass / spring / damper system describing the dynamics of the chassis and the center of the wheel. The suspension system is modeled by a spring and a damping system, and the tire is often reduced to a spring element. Figure 1 illustrates this single-corner model, defined through relation (1).

$$\begin{cases} M\ddot{z} = -k(z - z_t - \Delta_s) - c(\dot{z} - \dot{z}_t) - Mg \\ m\ddot{z}_t = k(z - z_t - \Delta_s) + c(\dot{z} - \dot{z}_t) \\ \quad - k_t(z_t - z_r - \Delta_t) - mg \\ \dot{c} = -\beta(c - c_{in}) \\ z_t - z_r < \Delta_t \end{cases} \quad (1)$$

where z , z_t , and z_r are the vertical positions of the body, of the unsprung mass, and of the road profile respectively. M is the single-corner body mass; m is the unsprung mass (tire, wheel, brake calliper, suspension links, etc.).

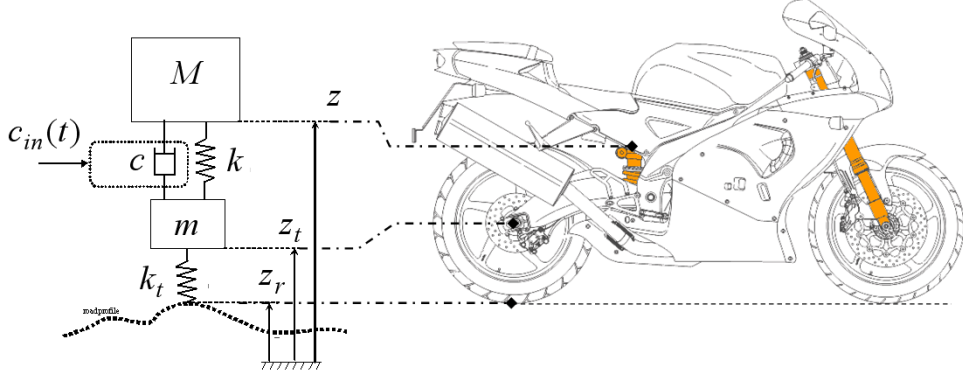


Fig. 1. Single-corner vehicle model.

g is the gravitational constant. $k \in \mathbb{R}^+$ and $k_t \in \mathbb{R}^+$ are the stiffness of the suspension spring and of the tire, respectively; $\Delta_s \in \mathbb{R}^+$ and $\Delta_t \in \mathbb{R}^+$ are the length of the unloaded suspension spring and tire, respectively. $c \in \mathbb{R}^+$ and $c_{in} \in \mathbb{R}^+$ are the actual and the requested damping coefficients of the shock-absorber, respectively. The damping-coefficient variation is ruled by a 1st-order dynamic, where $\beta \in \mathbb{R}^+$ is the bandwidth. The actual damping coefficient c always remains in that interval: $c_{min} \leq c \leq c_{max}$, where $c_{min} \in \mathbb{R}^+$ and $c_{max} \in \mathbb{R}^+$ are the shock-absorber technological limitations. This last inequality is the so-called "passivity-constraint" of a semi-active suspension - guaranteeing that the actuator only dissipates energy (see *e.g.* Savaresi and Spelta (2009); Savaresi et al. (2010)). Since the control signal c_{in} modifies the damping coefficient c , a state variable, model (1) is nonlinear. When a passive suspension is considered, (1) is reduced to a 4th-order linear system (by simply setting $\dot{c} = 0$ and $c = \text{constant}$ - *e.g.* nominal damping).

For nonlinear control design, the following model is commonly used:

$$\begin{cases} M\ddot{z} = -k(z - z_t) - c(\dot{z} - \dot{z}_t) \\ m\ddot{z}_t = k(z - z_t) + c(\dot{z} - \dot{z}_t) - k_t(z_t - z_r) \\ \dot{c} = \beta(c_{in} - c) \end{cases} \quad (2)$$

Similarly, in order to apply linear control techniques, a linear-like semi-active suspension model is (commonly) defined as follows:

$$\begin{cases} M\ddot{z} = -k(z - z_t) - c^0(\dot{z} - \dot{z}_t) - F_d \\ m\ddot{z}_t = k(z - z_t) + c^0(\dot{z} - \dot{z}_t) - k_t(z_t - z_r) + F_d \\ \dot{F}_d = -\beta(F_d - u) \end{cases} \quad (3)$$

With reference to models (1) and (2), the linearised model (3) includes symbols with the following meaning: c^0 denotes the nominal damping (which can be used as a design parameter when synthesizing the controller); F_d is an additional damping force commanded by the control variable u , according to a 1st order actuation dynamic.

In order to be fully equivalent to (1) and (2), the control signal u should respect the passivity-constraint. In models (1) and (2) this constraint is described as $c_{min} \leq c_{in} \leq c_{max}$. It is straightforward to see that in model (3) this constraint is recast as follows: the additional force reference u should satisfies $u \in \mathcal{D}(c_{min}, c_{max}, c^0) \subseteq \mathbb{R}$, where the dissipative $\mathcal{D}(c_{min}, c_{max}, c^0)$ set is defined as follows

(see Figure 2):

$$\left\{ \begin{array}{l} \forall (U, V) \in \mathbb{R} \times \mathbb{R} \\ (U - (c_{max} - c^0)V)((c_{min} - c^0)V - U) \geq 0 \end{array} \right\} \quad (4)$$

where c_{min} and c_{max} are the minimal and maximal damping factors of the considered controlled damper, normalized around $c^0 \in \mathbb{R}^+$. Note that in formulation (3) of the single-corner model, the system with no control ($u = 0$) is damped thanks to c^0 (hence stable and damped). This remark is practically important for numerical reasons when some control approaches (such as LMI-based, MPC, ...) are applied.

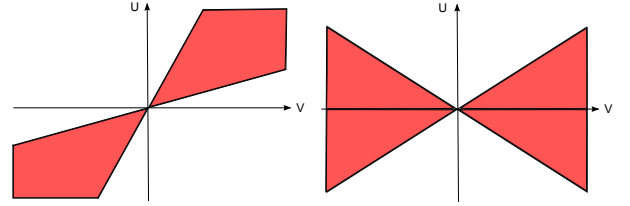


Fig. 2. Illustration of the dissipative $\mathcal{D}(c_{min}, c_{max}, c^0)$ set as a function of c^0 . Left: $c^0 = 0$, right: $c^0 = \frac{c_{min} + c_{max}}{2}$.

2.2 Performance indexes definition

Since the paper aims at evaluating the *trade-off* of the semi-active suspension laws in terms of *comfort* and *road-holding*, the performance metric proposed in Poussot-Vassal et al. (2010) is used here. This criteria focuses on two specific signals, representing either the comfort or the road-holding performance (for further detail, refer to Hrovat (1997); Kiencke and Nielsen (2000) and book of the authors Savaresi et al. (2010)), namely:

- The vertical chassis acceleration \ddot{z} (or displacement z) response to road disturbances z_r , between 0 and 20Hz, representing the acceleration felt by the driver, *i.e.* the comfort specification.
- The vertical wheel deflection $z_t - z_r$ response to road disturbances z_r , between 0 and 30Hz, representing the ability of the wheel to stay in contact with the road, *i.e.* the road-holding specification.

The common objective is then to minimize either the transfer z_r to \ddot{z} - identically z - (comfort) or the transfer z_r to $z_t - z_r$ (road-holding) or a combination of these two transfer. More formally, let define the function \mathcal{C} :

$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, as $\mathcal{C}(x, \underline{f}, \bar{f}) = \int_{\underline{f}}^{\bar{f}} |x(f)|^2 df$, where $x(f)$ represents the frequency dependent signal of interest, obtained by a nonlinear frequency response algorithm (similar to the one given in Savaresi et al. (2010); Poussot-Vassal et al. (2010)); \underline{f} and \bar{f} represent the interval limits of interest. Then, the comfort and road-holding criteria are respectively defined as:

- J_z , Comfort criteria:

$$J_z = \frac{\mathcal{C}(F_z, 0, 20)}{\mathcal{C}(F_z^{nom}, 0, 20)} \quad (5)$$

- $J_{z_{def_t}}$, Road-holding criteria:

$$J_{z_{def_t}} = \frac{\mathcal{C}(F_{z_{def_t}}, 0, 30)}{\mathcal{C}(F_{z_{def_t}}^{nom}, 0, 30)} \quad (6)$$

where F_z and $F_{z_{def_t}}$ are the frequency responses of the controlled suspension; while F_z^{nom} and $F_{z_{def_t}}^{nom}$ are the frequency responses of the passive uncontrolled reference suspension system with a nominal damping $c = 1500 Nm/s$ (*i.e.* model (1) with $c = 1500$ and $c_{in} = 1500$).

3. COMFORT-ORIENTED SEMI-ACTIVE SUSPENSION CONTROL

This section focusses on the main comfort-oriented dedicated semi-active suspension control strategies. This section gathers, in an increasing complexity order, the main semi-active control laws for comfort objective.

3.1 Skyhook two-state damper control (SH-2)

The two-state Skyhook control is an on/off strategy that switches between high and low damping coefficients in order to achieve body comfort specifications. This control law is recalled as follows.

Proposition 1. (SH-2 states control). Given system (2), the SH-2 law is defined as:

$$c_{in} = \begin{cases} c_{min} & \text{if } \dot{z}\dot{z}_{def} \leq 0 \\ c_{max} & \text{if } \dot{z}\dot{z}_{def} > 0 \end{cases} \quad (7)$$

Basically, this control law consists in a switching controller which deactivates the controlled damper when the body speed and suspension deflection speed have opposite signs. The controlled damper technology only needs to have two damping coefficient states. This control strategy presents the advantage to be simple but requires two sensors.

Many studies have concerned the Skyhook control strategy since it represents a simple but efficient way to achieve good comfort requirement (see *e.g.* Simon (2001); Ahmadian et al. (2004)). Some extended versions of the Skyhook control have been also developed, such as the adaptive one in Song et al. (2007) or the gain-scheduled one in Hong et al. (2002).

3.2 Skyhook linear approximation damper control (SH-L)

An improved version of Skyhook control has been used to handle variable damping, either with discrete damping coefficients, or with continuously variable damper, as illustrated in Sohn et al. (2000); Sammier et al. (2003). The linear approximation of the Skyhook control algorithm, adapted to semi-active suspension actuators, is given as:

Proposition 2. (SH-L). Given system (2), the SH-L law is defined as:

$$c_{in} = \begin{cases} c_{min} & \text{if } \dot{z}\dot{z}_{def} \leq 0 \\ \text{sat}\left(\frac{\alpha c_{max}\dot{z}_{def} + (1-\alpha)c_{max}\dot{z}}{\dot{z}_{def}}\right) & \text{if } \dot{z}\dot{z}_{def} > 0 \end{cases} \quad (8)$$

where $\alpha \in [0;1]$ is a tuning parameter that modifies the closed-loop performances and **sat** denotes that $c_{in} \in [c_{min}; c_{max}]$.

When $\alpha = 1$, this control law is equivalent to the Skyhook two-state one. As the two-state control, the linear approximation consists in a switching controller which modifies the damping factor according to the body speed and suspension deflection speed. The innovation rely in the fact that, according to the second expression (when $\dot{z}\dot{z}_{def} > 0$), such a control provides an infinite number of damping coefficients. As a matter of fact, this control law requires a continuously variable controlled damper (*e.g.* an MR dampers). From the computational point of view, this control law also requires two measurements and is simple to implement, but suffers of \dot{z}_{def} zero crossing as well (which is practically complex to measure).

3.3 Acceleration Driven Damper control (ADD)

The ADD control is a semi-active control law described in Savaresi et al. (2005), which consists in changing the damping factor using the acceleration knowledge.

Proposition 3. (ADD). Given system (2), the ADD law is defined as:

$$c_{in} = \begin{cases} c_{min} & \text{if } \ddot{z}\dot{z}_{def} \leq 0 \\ c_{max} & \text{if } \ddot{z}\dot{z}_{def} > 0 \end{cases} \quad (9)$$

This strategy shows to be optimal in the sense that it minimizes the vertical body acceleration when no road information is available. Since it requires the same number of sensors as the Skyhook two-state and the linear approximations control law, this control law is simple from the implementation point of view. Note that the control law is very similar to the two-state approximation of the Skyhook algorithm, with the difference that the switching law depends on the body acceleration (\ddot{z}), instead of the body speed (which is easier to practically measure). It is also to note that the ADD design is well adapted to comfort improvement but not to road-holding. Nevertheless, as the previous approach, the "switching dynamic" may influences the closed-loop performances, implying chattering effects.

3.4 Power Driven Damper (PDD)

In Morselli and Zanasi (2008), the authors propose a semi-active suspension control strategy using the port Hamiltonian techniques, which provide powerful tools for modeling mechatronics systems with dissipative components. Based on this observation, it is straightforward that this framework perfectly fits the semi-active suspension problem.

Proposition 4. (PDD). Given system (2), the PDD control approach is described by:

$$c_{in} = \begin{cases} c_{min} & \text{if } kz_{def}\dot{z}_{def} + c_{min}\dot{z}_{def} \geq 0 \\ c_{max} & \text{if } kz_{def}\dot{z}_{def} + c_{max}\dot{z}_{def} < 0 \\ \frac{c_{min} + c_{max}}{2} & \text{if } z_{def} \neq 0 \text{ and } \dot{z}_{def} = 0 \\ -\frac{kz_{def}}{\dot{z}_{def}} & \text{otherwise} \end{cases} \quad (10)$$

where k is the stiffness of the considered suspension.

In Morselli and Zanasi (2008) (see also Figures 3, 6 and 7), the authors show that this strategy provides results comparable to those of the ADD control law, while avoiding the chattering effect of the damping control value. The additional cost is the need for the knowledge of the spring stiffness k and a more complex rule.

3.5 Mixed Skyhook-Acceleration Driven Damper (SH-ADD)

The Mixed Skyhook-ADD rational mixes the the best behaviour of SH and ADD, without an increasing of either the computational effort or the hardware complexity. The key idea exploits a very simple but effective *frequency range selector*, which is able to distinguish the instantaneous dynamical behaviour of the suspension: in the case of low frequency dynamics the SH is selected, while the ADD is selected otherwise. The resulting control law is incredibly simple and requires the same apparatus as SH.

Proposition 5. (SH-ADD). Given system (2), the mixed SH-ADD control approach is described by:

$$c_{in} = \begin{cases} c_{max} & \text{if } \left[(\ddot{z}^2 - \alpha^2 \dot{z}^2) \leq 0 \text{ AND } \dot{z}\dot{z}_{def} > 0 \right] \text{ OR} \\ c_{min} & \text{otherwise} \end{cases} \quad (11)$$

where $\alpha \in \mathbb{R}^+$ is the tuning parameter allowing for *frequency range selector*, *i.e.* adjusts the "switch" between the SH and the ADD.

The amount $(\ddot{z}^2 - \alpha^2 \dot{z}^2)$ hence can be considered as a simple "frequency-range selector". The parameter α represents the frequency limit between the low and the high frequency ranges, and it is the only tuning knob of the control strategy (11). Specifically the value of α is set at the cross-over frequency (in rad/s) between SH and ADD. For a standard motorcycle suspension it has to be selected around 19rad/s (3Hz). A simplified version of this algorithm, employing one single sensor has also been developed, leading to very satisfactory results both in simulation and experimental benchmark (see Savaresi and Spelta (2009)).

4. ROAD-HOLDING ORIENTED SEMI-ACTIVE SUSPENSION CONTROL

Complementary to comfort-oriented control strategies, a very few studies have been devoted to the possible improvement of road-holding, using suspension actuator. Since few years, the studies on Global Chassis Control have emphasized that the suspension system may also help getting better road-holding and even handling, encouraging the community to reach this new objective.

4.1 Ground-hook 2 states (GH-2)

In a dual way to the Skyhook case, the 2-states Ground-hook control (see Valasek et al. (1998)) consists in a

switching control law depending now on the sign of the product between the suspension deflection velocity \dot{z}_{def} and the velocity of the unsprung mass \dot{z}_t , as:

Proposition 6. (GH-2). Given system (2), the GH-2 control approach is:

$$c_{in} = \begin{cases} c_{min} & \text{if } -\dot{z}_t\dot{z}_{def} \leq 0 \\ c_{max} & \text{if } -\dot{z}_t\dot{z}_{def} > 0 \end{cases} \quad (12)$$

This control has globally the same properties as the SH-2 one, but focussing around the unsprung mass.

4.2 Ground-hook linear (GH-L)

In this case, the semi-active damper allows to continuously change the damping coefficient, according to:

Proposition 7. (GH-L). Given system (2), the GH-L control approach is defined through:

$$c_{in} = \begin{cases} c_{min} & \text{if } -\dot{z}_t\dot{z}_{def} \leq 0 \\ \text{sat} \left(\frac{\alpha c_{max}\dot{z}_{def} + (1-\alpha)c_{max}\dot{z}_t}{\dot{z}_{def}} \right) & \text{if } -\dot{z}_t\dot{z}_{def} > 0 \end{cases} \quad (13)$$

where $\alpha \in [0;1]$ is a tuning parameter that modifies the closed-loop performances and **sat** denotes that $c_{in} \in [c_{min}; c_{max}]$.

5. ADVANCED SEMI-ACTIVE SUSPENSION CONTROL

Until now, the presented control approaches were completely dedicated to semi-active suspension systems and based on the nonlinear model (2). In this section we describe some of the main semi-active suspension control method, based on more classical (or usual) control tools, and relying on model (3).

5.1 Clipped approaches

Many works have concerned the application of classical control methods (*e.g.* \mathcal{H}_∞ , \mathcal{H}_2 , pole placement, disturbance rejection, optimal, active Skyhook ...). However, most of the results were obtained for active suspensions, as in Zin et al. (2008). When applied to semi-active dampers, the *dissipative constraint* of the damper is usually handled using a simple projection (*i.e.* saturation, as shown in Figure 2). Even if it is not always referred to as the "*clipped approach*", the latter is very widespread in control strategies for semi-active suspension (see *e.g.* Karnopp (1983) and Margolis (1983)). In the control step, the force applied by the semi-active damper is then chosen to be as close as to the force required by the controller for a given suspension deflection speed and for the possible range of forces the damper can deliver. This simple strategy has been then applied in many cases (see *e.g.* Rossi and Lucente (2004); Du et al. (2005); Senname and Dugard (2003)).

The question that arises is: is optimal the clipped-optimal? If not, how far is it from the real optimal one? How would look like the optimal semi-active one? Clipped approaches lead to unpredictable behaviours and ensure neither closed-loop internal stability nor performances any longer. As a matter of fact, active control applied on

a semi-active damper results in a "synthesize and try" method.

To cope with this last drawback, some modern control techniques have been applied to the specific semi-active suspension problem.

5.2 Hybrid MPC control approaches

In Giorgetti et al. (2006a), the authors introduce an hybrid model predictive optimal controller (using receding horizon). They solve an off-line optimization process which is a finite horizon optimal regulation problem *s.t.*:

$$J^*(\xi, x(k)) = \min_{\xi} \left[x^T(N)Q_N x(N) + \sum_{k=1}^{N-1} x^T(k)Qx(k) + y^2(k) \right] \quad (14)$$

subject to,

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \\ 0 \leq u(k) \leq z_{def}(k) \\ |u(k)| \leq \Lambda \end{cases} \quad (15)$$

where Q is a performance index and Q_N is the final weight, as in the optimal control theory. Matrices A , B , C and D in (15) define the LTI single-corner model (3), Λ is the maximal force allowed by the considered controlled damper and $u(k)z_{def}(k) \geq 0$ guarantees the *passivity constraint*. ξ is a vector composed by the sequence of control signals (from 0 to $N-1$) to be applied, where, N is the prediction horizon. Giorgetti et al. (2006a) show that choosing $N = 1$ leads to performances that are identical to those of the *clipped-optimal* approach, and by increasing N (*e.g.* until 40), the performances can be significantly improved. The implemented control law does not involve any optimization procedure since the control algorithm provides a collection of affine gains over a polyhedral partition of the system states x (*e.g.* Borrelli et al. (2003)). By the way, this approach exhibits notable drawbacks, such as high complexity and switching between control regions and requires a complete full-state measurement.

5.3 LPV semi-active control

In an other philosophy, the "LPV semi-active" control adjustment, introduced in Poussot-Vassal et al. (2008), is a robust semi-active suspension control design using a Linear Parameter Varying (LPV) approach. More specifically, the proposed semi-active suspension control strategy is designed so that it minimizes the \mathcal{H}_∞ performance criteria while guaranteeing the *dissipative constraint* thanks to a specific parameter dependent structure and a scheduling strategy design. The "LPV semi-active" controller design is summarized in the following algorithm (for \mathcal{H}_∞ performances).

In this algorithm, ρ is a parameter function of the type of semi-active actuator. It varies as a function of the deflection velocity and the required damping force (for more details refer to Poussot-Vassal et al. (2008); Savaresi et al. (2010)). The main interest of such approach is that it presents a large flexibility concerning the performances and the type and number of involved sensors. Additionally, it provides a robustness certificate thanks to the polytopic

Algorithm 1 \mathcal{H}_∞ "LPV semi-active"

- (1) Based on initial system (3), and ρ -dependent performance weighting functions, construct the LPV polytopic system. Note that to complete this step, weighting filters should be defined and parametrized (refer to Poussot-Vassal et al. (2008) and Chapter 8 of Savaresi et al. (2010), for details).
- (2) Compute the \mathcal{H}_∞ LMI based "LPV semi-active" feasibility conditions.
- (3) Reconstruct the LPV controller and obtain two dynamical full order controllers are obtained which are solution of the robust \mathcal{H}_∞ "LPV semi-active" control problem.

$$C(\underline{\rho}) = \begin{bmatrix} A_c(\underline{\rho}) & B_c(\underline{\rho}) \\ C_c(\underline{\rho}) & 0 \end{bmatrix} \quad \text{and} \quad C(\bar{\rho}) = \begin{bmatrix} A_c(\bar{\rho}) & B_c(\bar{\rho}) \\ C_c(\bar{\rho}) & 0 \end{bmatrix} \quad (16)$$

- (4) Apply control law (where ρ is varying):

$$u = \left[\frac{|\rho - \bar{\rho}|}{\bar{\rho} - \underline{\rho}} C(\underline{\rho}) + \frac{|\rho - \underline{\rho}|}{\bar{\rho} - \underline{\rho}} C(\bar{\rho}) \right] z_{def} \quad (17)$$

approach. The inherent counterpart of this flexibility is an important complexity in the design step.

6. NUMERICAL SIMULATIONS

In order to benchmark the performances of all these algorithms, the criteria presented in Savaresi et al. (2010) and recalled in Section 2 is applied. All simulation are carried out using a motorcycle parameter set (available in Poussot-Vassal et al. (2010) and Savaresi et al. (2010)). Note that in the following we do not simulate neither the Clipped approach (because of the nearly infinite way of adjusting) nor the MPC approach (complex to adjust due to multiple parameters, as well), but interested reader should refer to Tseng and Hedrick (1994); Giorgetti et al. (2006b).

6.1 Frequency domain analysis

First, we compare the frequency responses of F_z , the transfer between z_r and z (comfort), and $F_{z_{def}}$, the transfer from z_r to $z_{def} = z_t - z_r$. On Figure 3, the SH 2-states, SH linear (with $\alpha = 0$), ADD and PDD are compared with the passive ones with either c_{min} or c_{max} , illustrating to attenuate well the F_z transfer while degrading the $F_{z_{def}}$ one.

Similarly, Figure 4 compares the performance attenuation of the two road-holding control laws, namely, GH 2-states and GH linear (with $\alpha = 0$) with the passive ones with either c_{min} or c_{max} .

It clearly emphasizes the improvement on $F_{z_{def}}$ while degrading the F_z one. Moreover, it confirms the observation made in Morselli and Zanasi (2008) concerning the performances of the ADD and the PDD approaches. Finally, the Mixed SH-ADD (with α adjusted to 20Hz) and "LPV semi-active" (with either a comfort or road-holding adjustment) frequency responses are compared to the passive cases on Figure 5.

On this last Figure, it is interesting to notice that the SH-ADD approach provides very good results in term of comfort performances. Concerning the LPV semi-active one,

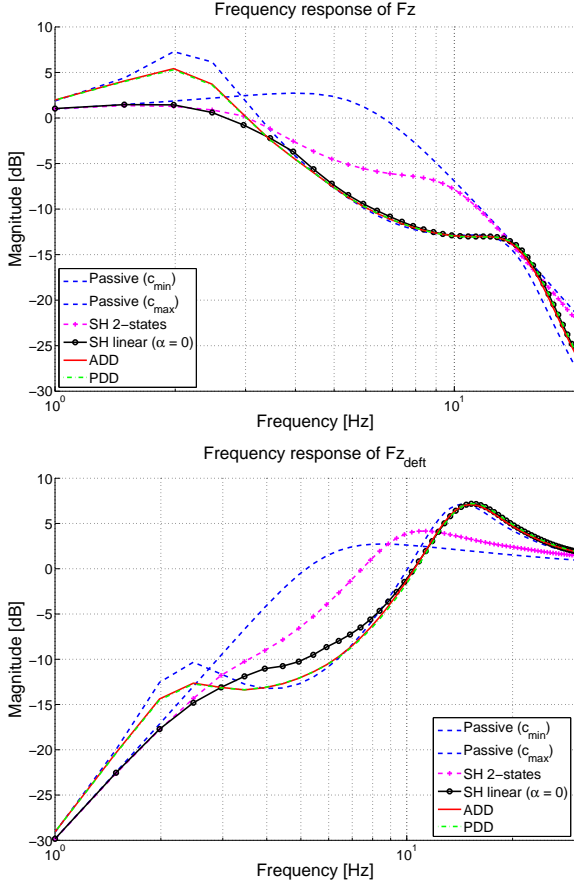


Fig. 3. Comfort oriented control law frequency response F_z (top) and $F_{z_{def_t}}$ (bottom).

the two configurations tested shows the large flexibility of the approach. In the next subsection, all these approaches are gathered and compared to the passive and optimal performances, using the index defined in Section 2.

6.2 Performance indexes & trade-off analysis

Since improvement in one criteria implies a deterioration on the other, the performance index introduced in Section 2 is also evaluated for each control strategy. On Figure 6, all the performance indexes are compared to the passive with three configurations: $c = \{c_{min}, 1500, c_{max}\}$ (note that $c = 1500$ has been selected as the nominal reference value since it provides a nice compromise between comfort and road-holding).

In order to be even more precise, on Figure 7, the trade-off between comfort and road-holding is illustrated for all the previous strategies using a diagram with Comfort in the x-axis and Road-holding in the y-axis. This figure also includes the criteria evaluation for a passive suspension with damping varying from $c = 100\text{N/m/s}$ to $c = 10000\text{N/m/s}$ and the optimal bound numerically computed using the optimization framework presented in Poussot-Vassal et al. (2010).

The interesting point of this diagram is that (i) it illustrates the interest of the control to enhance the passive performances, (ii) shows the optimal performances of the SH-ADD approach in term of comfort (blue cross close to

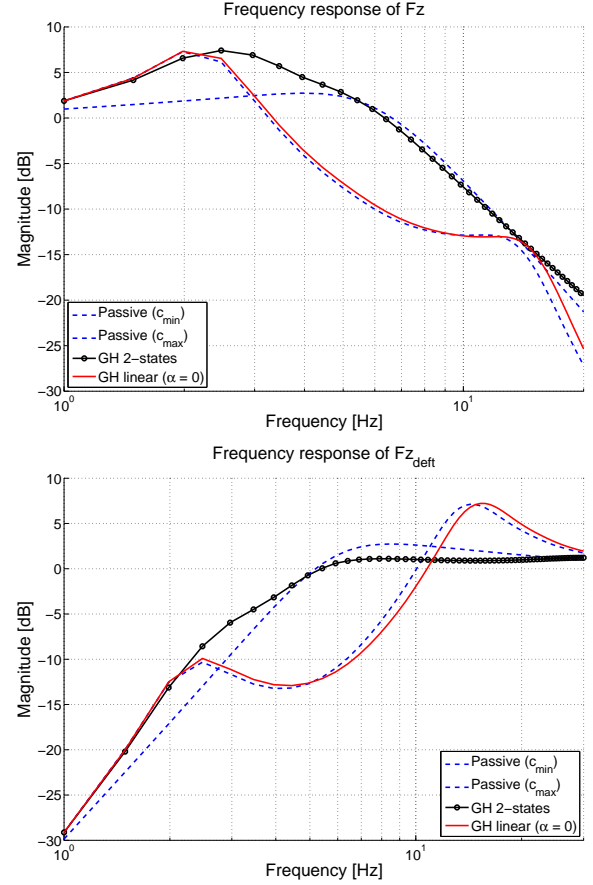


Fig. 4. Road-holding oriented control law frequency response F_z (top) and $F_{z_{def_t}}$ (bottom).

the optimal bound, in dashed red) and (iii) emphasizes the interest of the flexible LPV semi-active rule, allowing to achieve good comfort performances (red left-oriented triangle) or road-holding ones (blue right-oriented triangle).

7. CONCLUSIONS AND COMPLEXITY REMARKS

In this paper we provide a benchmark of the main semi-active suspension control strategies. The proposed benchmark is of course not exhaustive but still quite large. We illustrate the interest of the recent dedicated semi-active suspension approach (such as SH-ADD), presenting a low complexity, while providing great comfort performances. Nevertheless, other approaches (such as LPV semi-active, or Hybrid MPC) shows some interests in the fact that they are quite flexible, but far more complex to adjust and implement. The proposed benchmark also include a complete evaluation of the passive suspension and the optimal performances such a system can achieve, and thus extends the results obtained in Poussot-Vassal et al. (2010).

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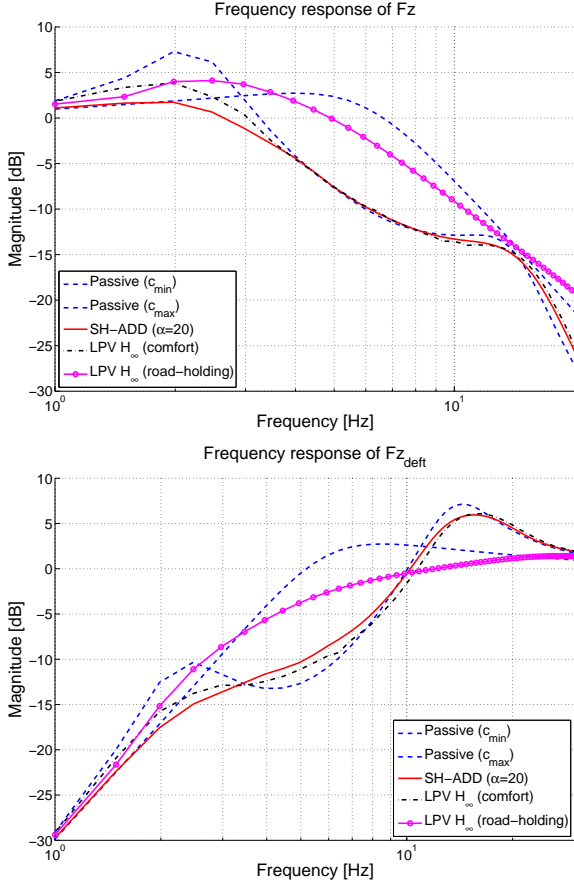


Fig. 5. Road-holding oriented control law frequency response F_z (top) and $F_{z_{def}}$ (bottom).

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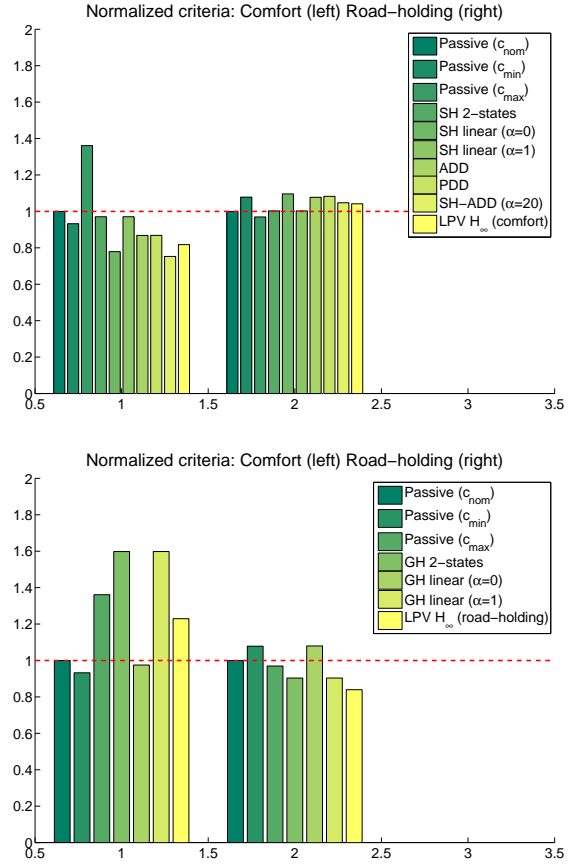


Fig. 6. Normalized performance criteria comparison for different comfort (resp. road-holding) oriented control strategies. Comfort criteria - when cost function is J_c (left histogram set) and Road-Holding criteria - when cost function is J_{rh} (right histogram set).

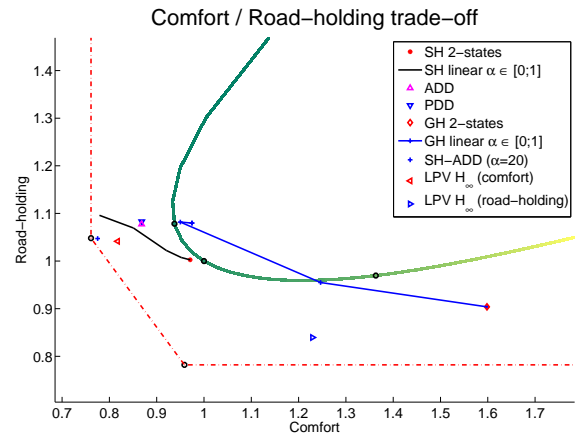


Fig. 7. Normalized performance criteria trade-off for the presented control algorithms, compared to the passive suspension system, with damping value $c \in [c_{min}; c_{max}]$ (solid line with varying color), optimal comfort and road-holding bounds (red dash dotted).

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