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Abstract

Aircraft maintenance costs are becoming an important issue in the aeronautical industry. In this paper we present a tactical planning model for an helicopter maintenance center. The objective is to guarantee a good service level, i.e. to limit aircraft visit duration. Difficulties come from the numerous uncertainties on maintenance activities: additional activities, procurement delays... To cope with these uncertainties, a fuzzy multiproject planning model is proposed. From the task fuzzy dates and durations, a periodic load chart can be established. Then a parallel algorithm is adapted to this model in order to solve capacity problems.

Key words: Maintenance, Multiproject, Fuzzy, Capacity planning

1 Introduction

Aircraft maintenance is a highly regulated activity, due to the potential criticality of the failures. Aircrafts must follow a maintenance program in which several levels of inspection appear, from light maintenance that can be performed daily at the aircraft's basis, to heavy maintenance that can last several months and requires specific equipment. Our work focuses on the organization of a helicopter maintenance center where heavy maintenance visits (HMV) are performed.

An HMV contains planned maintenance tasks and also corrective maintenance tasks because problems are discovered during the inspection of the helicopter at the beginning of the visit. Even planned tasks may differ from one helicopter to another, according to equipment, conditions of use, etc. Precedence constraints exist, due to technical or accessibility considerations. Hence a HMV may be seen as a project involving various resources as operators, equipment and spare parts. Minimizing the overall visit duration give a competitive advantage to the company. Consequently, the management of a maintenance center is viewed as multiproject management, where every project duration should be minimized while respecting capacity constraints. A particularity of these project is the level of uncertainty, mainly due to unexpected failures that induce additional work and procurement delays. In case of important homogeneous fleets, a global optimization of maintenance visits can be done, guaranteeing a general helicopter availability level [8]. This is not the case in our project dedicated to civil customers whose mean number of helicopter is between two and three, with a great heterogeneity in the equipments and conditions of use.

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This paper deals with tactical planning for an helicopter maintenance center. At a tactical level, maintenance operations are gathered into macro-tasks, and resources are considered by period in a tactical horizon that may cover several projects from their beginning to their end. Given the level of uncertainty and the lack of historical information, a fuzzy model of the task dates and durations has been chosen [12]. This paper aims at solving capacity problems at the tactical level, based on a fuzzy representation of resource workload. Section 2 recalls fuzzy project planning and describes the starting date and task duration models chosen for our maintenance projects. In Section 3 we present the way to build the resource workload charts from the task fuzzy dates and durations. Section 4 presents a parallel algorithm adapted to our fuzzy model in order to solve capacity issues. Finally, section 5 presents an example of application on three helicopter maintenance projects.

2 Multiproject planning under uncertainty

2.1 Fuzzy project planning

Zadeh [13] has defined a fuzzy set $\widetilde{A}$ as a subset of a referential set X, whose boundaries are gradual rather than abrupt. Thus, the *membership function* $\mu_{\widetilde{A}}$ of a fuzzy set assigns to each element $x \in X$ its degree of membership $\mu_{\widetilde{A}}(x)$ taking values in $[0;1]$.

To generalize some operations from classical logic to fuzzy sets, Zadeh has given the possibility to represent a fuzzy profile by an infinite family of intervals called α-cuts. Hence, the fuzzy profile $\widetilde{A}$ can be defined as a set of intervals $A^\alpha = [A^\alpha_{\min}; A^\alpha_{\max}] = \{x \in X/\mu_{\widetilde{A}}(x) \geq \alpha\}$ with $\alpha \in (0;1]$. It became consequently easy to utilize classical interval arithmetic and adapt it to fuzzy profiles. Dubois and Prade [5] and Chen [2] have defined mathematical operations that can be performed on trapezoidal fuzzy sets.

Fortin *et al.* [6] describe the algorithms to adapt Critical Path Method to fuzzy numbers. In the fuzzy case, forward propagation is done using fuzzy arithmetics, leading to fuzzy earliest dates and a fuzzy end-of-project event. Unfortunately, classical backward propagation is no longer applicable because uncertainty would be taken into account twice. Fortin *et al.* propose algorithms and show that some problems (e.g. minimal float determination) become NP-hard. A recent review of these concepts can be found in Dubois *et al.* [4].

2.2 Project release date

Figure 1 presents an example of an equipment inspection date determination from helicopter exploitation assumptions, flight hours and calendar limits. From the update, flight hours evolve in a range going from no exploitation to the physical limits of the aircraft, through pessimistic and optimistic exploitation values. Intersections of these lines with calendar and flight hours limits define the four points a_H, b_H, c_H and d_H of the trapezoidal fuzzy number $\widetilde{H}$, inspection date according to the flight hours. It is the same for flight cycles.

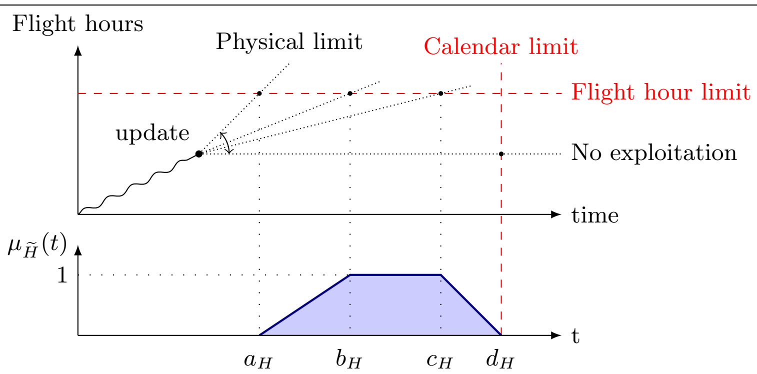

Fig. 1. Equipment fuzzy inspection date

The fuzzy release date of the project is the fuzzy minimum of the inspection dates of critical equipments listed in the maintenance program, and of the helicopter itself. The uncertainty on this date decreases along the time, as information on actual exploitation increases, so periodic updates should be done.

2.3 Macro task durations

At the tactical level, uncertainty on macro task duration is mainly due to unexpected corrective maintenance. These additional tasks (work and delays) can represent an important part of the total project duration. They generally appear during the structural inspection macro tasks, but the whole project is impacted. Procurement for corrective maintenance may introduce delays in the planning. As the equipments to be purchased are not known before inspection, we consider scenarios: the equipment is available on site, at an European supplier, at a foreign supplier, or it may be found after some research, or it is obsolete and must be manufactured again. According to the information on the helicopter (age of the aircraft, conditions of use, etc.), some scenarios can be discarded from the beginning (e.g. new helicopter ⇒ no obsolescence) and task durations can be refined. Hence task durations will be represented by trapezoidal fuzzy numbers that may change at the planning updates.

3 Resource load charts

3.1 Possibility theory

To cope with decision making on fuzzy area, Zadeh [14] developed the concept of the possibility approach based on fuzzy subsets. The possibility theory introduces both a possibility measure (denoted Π) and a necessity measure (denoted N), in order to express plausibility and certainty of events [5].

Let τ be a variable in the fuzzy interval $\widetilde{A}$ and t be a real value. To measure the truth of the event $\tau \leq t$, equivalent to $\tau \in (-\infty; t]$, we need the couple $\Pi(\tau \leq t)$ and $N(\tau \leq t)$ (Fig. 2). Thus :

$$\Pi(\tau \leq t) = \sup_{u \leq t} \mu_{\widetilde{A}}(u) = \mu_{[\widetilde{A};+\infty)}(t) = \sup_{u} \min(\mu_{\widetilde{A}}(u), \mu_{(-\infty;t]}(u)) \quad (1)$$

$$N(\tau \leq t) = 1 - \sup_{u > t} \mu_{\widetilde{A}}(u) = \mu_{]\widetilde{A};+\infty)}(t) = \inf_{u} \max(1 - \mu_{\widetilde{A}}(u), \mu_{(-\infty;t]}(u)) \quad (2)$$

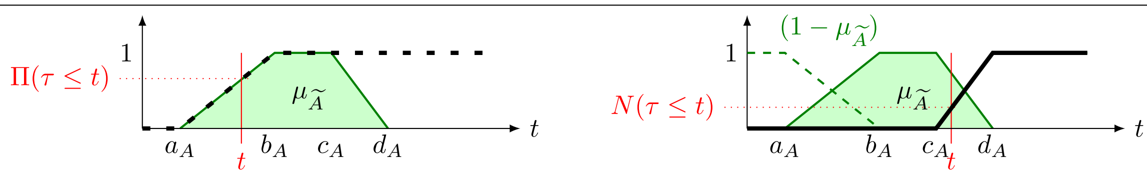

Fig. 2. Possibility and Necessity of $\tau \leq t$ with $\tau \in \widetilde{A}$.

Consequently, let τ and σ two variables in respectively fuzzy intervals $\widetilde{A}$ and $\widetilde{B}$ and t a real value. To measure the truth of the event "t between τ and σ" we need both $\Pi(\tau \leq t \leq \sigma)$ and $N(\tau \leq t \leq \sigma)$. Thus:

$$\Pi(\tau \leq t \leq \sigma) = \mu_{[\widetilde{A};\widetilde{B}]}(t) = \mu_{[\widetilde{A};+\infty)\cap(-\infty;\widetilde{B}]}(t) = \min(\mu_{[\widetilde{A};+\infty)}(t), \mu_{(-\infty;\widetilde{B}]}(t)) \quad (3)$$

$$N(\tau \leq t \leq \sigma) = \mu_{]\widetilde{A};\widetilde{B}[}(t) = \mu_{]\widetilde{A};+\infty)\cap(-\infty;\widetilde{B}[}(t) = \min(\mu_{]\widetilde{A};+\infty)}(t), \mu_{(-\infty;\widetilde{B}[}(t)) \quad (4)$$

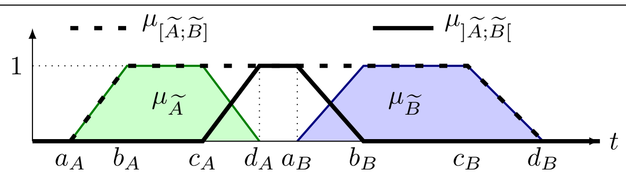

Fig. 3. Possibility and Necessity of t being between $\widetilde{A}$ and $\widetilde{B}$.

3.2 Presence of a task

The project dates and durations are represented by trapezoidal fuzzy numbers. Let $\widetilde{S}(a_S, b_S, c_S, d_S)$ be the fuzzy start date of a task T, $\widetilde{F}(a_F, b_F, c_F, d_F)$ its finish date and $\widetilde{D}(w, x, y, z)$ its duration. Relations between these values are:

$$a_F = a_S + w, \quad b_F = b_S + x, \quad c_F = c_S + y, \quad d_F = d_S + z$$

with

$$a_S \leq b_S \leq c_S \leq d_S \text{ and } w \leq x \leq y \leq z.$$

We characterize the presence of a task by the possibility (denoted $\Pi(t)$) and necessity ($N(t)$) of event t being between the start date and the finish date of the task.

Then we define the probability of presence of a task as a piecewise linear distribution $p(t)$ situated between the possibility and the necessity profile: $N(t) \leq p(t) \leq \Pi(t)$. The shapes of these profiles vary according to the overlap configuration of start and finish date (Fig. 4). Parameter H, ranging in $[0, 1]$, makes profile $p(t)$ evolve from $N(t)$ ($H = 0$) to $\Pi(t)$ ($H = 1$). The formal definitions of these profiles have been presented in [12].

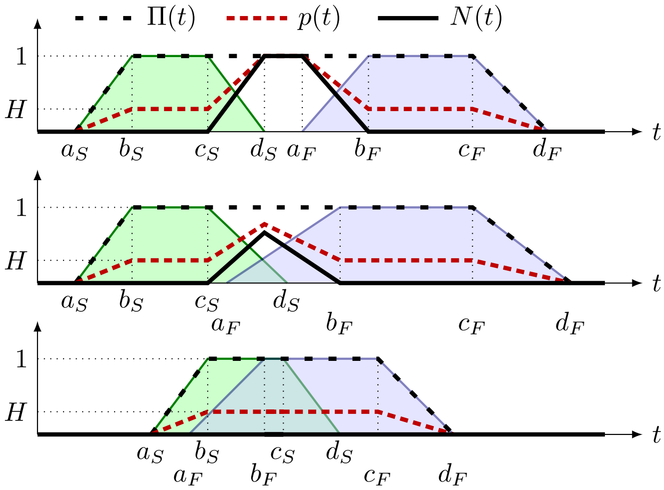

Fig. 4. Task presence profiles: without overlap (top), with small (middle) and large overlap (bottom).

3.3 Fuzzy resource usage profile

Building a relevant resource usage profile for a task with fuzzy dates and durations is not straightforward. Most of the time, the problem parameters are fixed in order to obtain a deterministic configuration. This leads to a scenario approach [10] where various significant scenarios may be compared in a decision process: lower and upper bounds, most plausible configuration, etc.

We proposed in [12] to build task resource usage profiles in a way that keeps track of uncertainty on start and finish dates. Hence the profile reflects the whole possible time interval while giving a plausible repartition of the workload according to the duration parameter value. To this aim, the resource usage profiles are defined as a projection of the task presence distributions onto the workload space. The projection of the possible profile should then give the maximal resource profile $L_\Pi(t)$, and the necessary profile the minimal resource profile $L_N(t)$. As the surface of these extreme resource profiles does generally not correspond to extreme loads $r.w$ and $r.z$ (where r is the resource requirement of the task, w the minimum and z the maximum duration), we use probability profiles to match the exact loads. Figure 5 presents the extreme resource profiles $L_w(t)$ and $L_z(t)$ for the case without overlap (the other cases can be found in [12]). To determine these profiles, parameters H_w and H_z are calculated so that $\int_0^{+\infty} r.p(t)dt$ respectively equals to $r.w$ and $r.z$.

3.4 Precedence constraints

If the tasks were independent, the sum of their resource load profiles would give the overall project load chart. However, when considering a precedence constraint between two tasks, their load profiles may not overlap because the constraint expresses the fact that the two tasks cannot be performed simultaneously.

Let us consider two tasks A and B so that A precedes B. Their resource consumptions are denoted r_A and r_B. We assume that the start date of B is equal to the finish date of A (e.g. in case of forward earliest dates calculation). This means that between the start date of A and the finish date of B, an activity will occur successively induced by A then B. So between the necessity peaks of A and B, we can affirm that an activity will necessarily occur, induced by A or B. This necessary presence of A or B is projected onto the resource load space using the minimal resource requirement $\min(r_A, r_B)$. Figure 6 presents an example of this case, and the load profiles $L_{\Pi(A\to B)}(t)$ and $L_{N(A\to B)}(t)$ for $r_A = 2$ and $r_B = 1$.

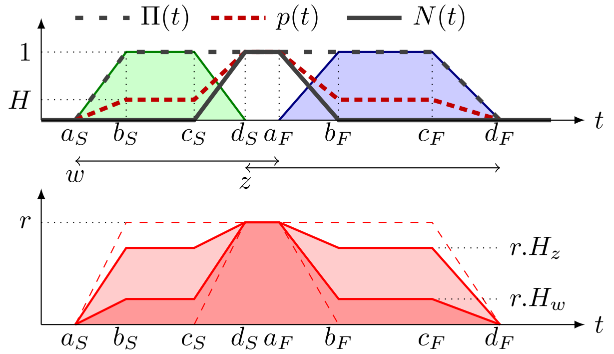

Fig. 5. Task presence (top) and resource load (bottom) profiles.

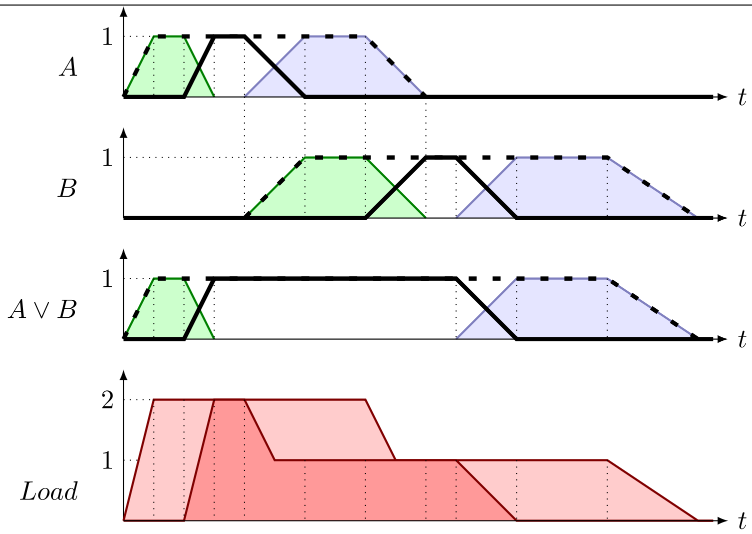

Fig. 6. Task presence (top) and resource load (bottom) profiles.

The projected necessity and possibility load profiles of the sequence $A \rightarrow B$ can be defined as follow:

$$L_{N(A\rightarrow B)}(t) = \max(r_A.N_A(t), r_B.N_B(t), \min(r_A, r_B).N_{A\vee B}(t))$$
$$L_{\Pi(A\rightarrow B)}(t) = \max(r_A.\Pi_A(t), r_B.\Pi_B(t))$$

Again, we should check if the resource load profiles match with the effective load for various durations and adapt probability profiles to this aim. Moreover, this should be checked globally on the sequence because of the area where we do not know which task is effectively executed. Consequently, the load profiles should be checked for each path in the project graph. Further work will study this aspect.

3.5 Global resource load chart

At the tactical level, planning decisions are taken according to the capacity of some critical resources: it is called rough cut capacity planning. The planning horizon is decomposed in periods on which the load is evaluated and compared to the available capacity.

The fuzzy load chart is established by periods, using the task load profiles. For each period, four values are given, corresponding to scenarios with the four duration values w, x, y, z of the tasks. Load chart can be represented like the workload plan suggested by Grabot et al. [7] for fuzzy MRPII (Fig. 7).

4 Capacity planning

Schedule Generation Schemes (SGS) are the core of many heuristics for the RCPSP. The so-called serial SGS performs activity incrementation and the parallel SGS performs time incrementation [11]. In both procedures,

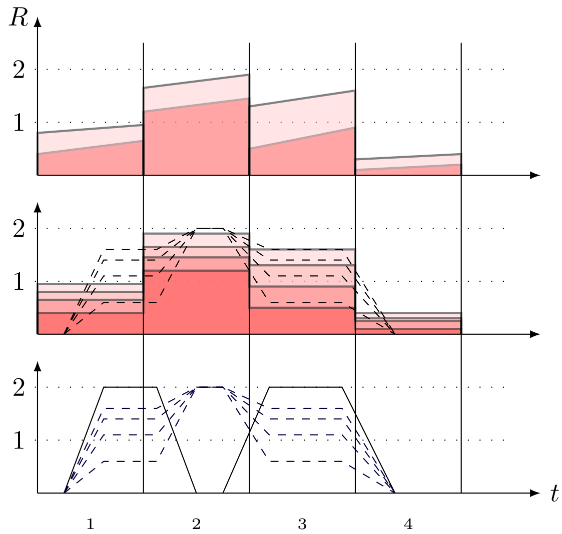

Fig. 7. Fuzzy load chart

tasks are ranked in some order and scheduled according to resources availabilities. Hapke and Slowinski [10] have proposed a parallel scheduling procedure for fuzzy projects. It is based on fuzzy priority rules and fuzzy time incrementation. The parallel procedure that we propose mainly differs from the latter on the resource availability test. Actually, as a tactical capacity planning tool, our test relies on a periodic resource load chart where availability is taken as a whole on each period.

4.1 Parallel algorithm

The fuzzy parallel procedure is adapted from the parallel SGS by considering fuzzy dates and fuzzy number comparison. It can be described as follows:

Begin
Choose a priority rule;
Initialize (resource, period) capacity value;
Initial time $\tilde{t}$:= project starting date;
Repeat
Step 1: compose the set $Q(\tilde{t})$ of tasks ready for scheduling at $\tilde{t}$;
Step 2: Schedule at $\tilde{t}$, according to the priority rule, each task from $Q(\tilde{t})$ that respects resource availability;
When a task is scheduled, calculate its finishing date, update the earliest starting date of its direct successors and the resource availabilities;
Step 3: increase time $\tilde{t}$;
Until all tasks are scheduled
End

4.1.1 Fuzzy priority rules

Priority heuristics using crisp or fuzzy time parameters were found efficient by many researchers either for one project or multiproject scheduling [11,10,1]. It is generally useful to perform parallel scheduling with a set of rules instead of one as the computational complexity is low [9,10]. Some rules that appears to be good in minimizing makespan are presented in Table 3. The aim of this paper is not to find the best rule, otherwise many other interesting rules could be used, like the Minimum Worst Case Slack (*MINWCS*), the Minimum Total Work Content(*MINTWC*) and some dynamic and combined rules presented in [1].

4.1.2 Time incrementation and resource availability

A task is ready to schedule at time $\tilde{t}$ when all its predecessors have been completed at time $\tilde{t}$. In Deterministic parallel SGS, a dynamic time progression is used. When at time t no task can be scheduled, current time is

Table 1
Priority rules giving good results in makespan minimisation

Rule	Name	Formula	Rule	Name	Formula
EST	Early Start Time[1]	$min(\widetilde{E}_j^s)$	LIS	Least Immediate Succesors[1]	$\min(\|S_j\|)$
EFT	Early Finish Time[1]	$min(\widetilde{E}_j^f)$	MIS	Most Immediate Succesors[1]	$\max(\|S_j\|)$
LST	Late Start Time[123]	$min(\widetilde{L}_j^s)$	MTS	Most Total Successors[23]	$\max(\|\overline{S_j}\|)$
LFT	Late Finish Time[123]	$min(\widetilde{L}_j^f)$	GRD	Greatest Resource Demand[1]	$\widetilde{p_j}\sum_{k=1}^{K} r_{jk}$
MINSLK	Minimum slack[123]	$min(\widetilde{f}_j)$	SASP	Shortest Activity from Shortest Project[3]	$min(\widetilde{p}_{jl})$
MAXSLK	Maximum slack[3]	$max(\widetilde{f}_j)$	LALP	Longest Activity from Longest Project[3]	$max(\widetilde{p}_{jl})$
SPT	Shortest Processing Time[123]	$min(\widetilde{p}_j)$	GRPW	Greatest Rank Positional Weight[123]	$max(\widetilde{p}_j + \sum_{i\in S_j}\widetilde{p}_i)$
LPT	Longuest Processing Time[13]	$min(\widetilde{p}_j)$	LRPW	Least Rank Positional Weight[1]	$min(\widetilde{p}_j + \sum_{i\in S_j}\widetilde{p}_i)$

[1]: used by Slowinski in [10] for a Fuzzy RCPSP, [2]: used by Kolish in [11] for Deterministic RCPSP,
[3]: used by Browning in [1] for Multi-projects RCPSP (RCMPSP),
$\widetilde{p}_j$:duration, $\widetilde{L}_j^f$:Last finishing, $\widetilde{E}_j^f$:Earliest finishing, $\widetilde{L}_j^s$:Last starting, $\widetilde{E}_j^s$:Earliest starting, $\widetilde{f}_j$:Margin
r_{jk}: is the requirement for resource R_k, S_j: direct successors, $\overline{S_j}$: total successors

increased to the least value from the finishing times of scheduled tasks that are not yet finished and the starting times of tasks that are candidates for scheduling. To cope with fuzzy scheduling, Hapke and Slowinski have generalized the deterministic time progression to fuzzy area and compare fuzzy numbers using weak and strong inequalities. A first assumption gives that a task has been completed at time $\tilde{t}$ if $\tilde{t}$ is strongly greater than or equal to the fuzzy finish time of the task. When at time $\tilde{t}$ no task can be scheduled, the time is increased to the earliest date, in the sense of weak inequality, from the set of those ones in which any resource is released or any task is ready to be scheduled [10]. Moreover, due to their resource representation, Hapke and Slowinski may implicitly add precedence constraints between tasks to solve capacity problems. However in a tactical point of view, it can sometimes be accepted to maintain a set of tasks scheduled simultaneously when uncertainty is large, without deciding too early which sequence will be respected. So in our algorithm, resource needs are considered globally on the time periods, using task resource load profiles.

4.2 Fuzzy task preemption

Preemption can be a way to solve resource capacity problems at an aggregated level of planning. In case of deterministic projects, preemption is provided by cutting macro-tasks into elementary work parts [3]. Obviously, the elementary duration value is unique in the deterministic case and is equal to 1. Thus, any deterministic duration is a multiplication of 1. In the same way, any trapezoidal fuzzy number $\widetilde{A} = [a, b, c, d]$ is equal to a unique linear combination of the elementary numbers $\widetilde{I_0}$=[1, 1, 1, 1], $\widetilde{I_1}$=[0, 1, 1, 1], $\widetilde{I_2}$=[0, 0, 1, 1] and $\widetilde{I_3}$=[0, 0, 0, 1], listed from the most necessary to the less possible equal to 1(Fig. 8):

$$\widetilde{A} = a\widetilde{I_0} + (b-a)\widetilde{I_1} + (c-b)\widetilde{I_3} + (d-c)\widetilde{I_4} \quad (5)$$

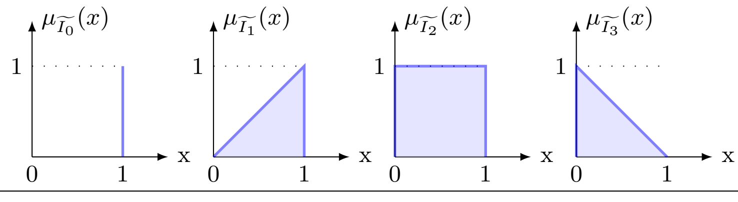

Fig. 8. Elementary trapezoidal fuzzy numbers

The decomposition formula (2) is applied to tasks fuzzy durations in AOA graph. The elementary arcs are assigned in the order of them possibility to be equal to 1. Thus, the $\widetilde{I_0}$ are assigned first, then the $\widetilde{I_1}$, after that the $\widetilde{I_2}$ and finally the $\widetilde{I_3}$(Fig. 9).

For example, the duration of task (34) on the left graph (before preemption) is equal to $[1, 2, 3, 3]$. According to the formula (5), we have $\widetilde{34} = \widetilde{I_0} + \widetilde{I_1} + \widetilde{I_2}$. Thus, the task (34) can be replaced in the right graph (after preemption) by (45), (56) and (56) with $\widetilde{45} = \widetilde{I_0}$, $\widetilde{56} = \widetilde{I_1}$ and $\widetilde{67} = \widetilde{I_2}$.

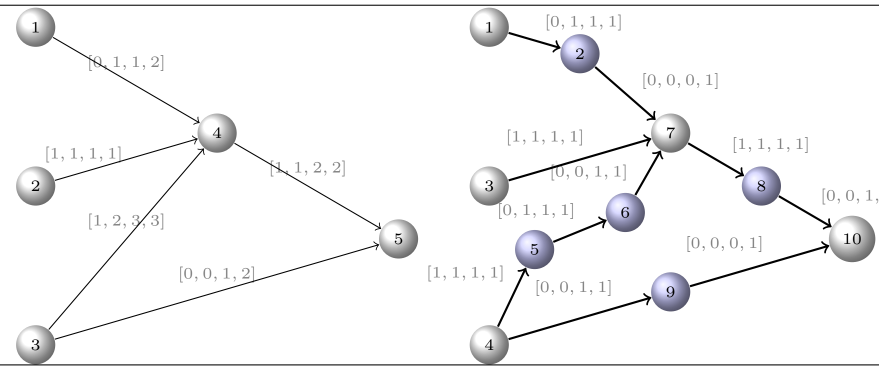

Fig. 9. Task preemption

5 Experimental results

The parallel algorithm have been applied to the helicopter maintenance planning problem. In helicopter maintenance, three categories of human resources are considered: avionics, structure and mechanics experts. They work generally on various helicopters at the same time. Table 2 contains the data of an example with three projects.

Table 2
Example of data for helicopter HMV.

Tasks Name	Tasks Id	Project 1	Project 2	Project 3	Predecessors	Resources
		Duration	Duration	Duration		R1 R2 R3
Waiting for the release date	A	[7, 8, 9, 10]	[10, 11, 12, 13]	[17, 18, 19, 20]	-	0 - 0 - 0
First check when receiving the helicopter	B	[1]	[1]	[1]	A	2 - 0 - 0
Removal structural and mechanical parts	C	[3]	[3]	[3]	B	3 - 0 - 0
Removal avionics	D	[3]	[3]	[3]	B	0 - 1 - 0
Supplying procedure for finishing	E	[3,3,5,6]	[1,1,2,2]	[1,2,2,3]	C	0 - 0 - 0
First part of mechanical inspection	F	[7]	[5]	[3,3,3,3]	C	1 - 1 - 0
Supplying procedure for assembling task	G	[5,5,5,6]	[1,2,2,3]	[2,3,4,4]	C	0 - 0 - 0
Supplying procedure during structural inspection	H	[5,5,5,6]	[1,2,2,3]	[2,3,4,4]	C	0 - 0 - 0
Subcontracted structure-cleaning	I	[1]	[1]	[1]	C	0 - 0 - 0
Subcontracted avionic tests and repairs	J	[2,3,4,5]	[2,3,4,5]	[2,3,4,5]	D	0 - 0 - 0
First part of structural inspection	K	[11]	[7]	[5]	I	0 - 0 - 2
Second part of structural inspection	L	[1,1,3,4]	[1, 1, 3, 4]	[1, 1, 3, 4]	H-K	0 - 0 - 2
Subcontracted painting	M	[1]	[1, 1, 2, 2]	[1]	L	0 - 0 - 0
Second part of mechanical inspection	N	[1]	[1,1,1,2]	[1]	F	1 - 1 - 0
Assemble helicopter parts	O	[1]	[1]	[1]	G-J-M-N	2 - 1 - 0
Finishing before fly test	P	[1]	[1]	[1]	E-O	1 - 1 - 0
Test before delivering helicopter	Q	[1]	[1]	[1]	P	1 - 0 - 0
Possible additional work on helicopter	R	[1, 2, 2, 3]	[1, 2, 2, 3]	[1, 2, 2, 3]	Q	1 - 1 - 0

The objective is to minimize the immobilization of helicopters i.e. the makespan of each project. The capacity is constant at each period and equal to 3, 2 and 3 for, respectively, $R1$ (Mechanics expert), $R2$(Avionics expert) and $R3$(Structure expert). The use of the 16 fuzzy priority rules presented in Table 3 gives different planning with different project makespans. Planning is performed with preemption and without preemption for optimistic (smallest durations) and pessimistic (largest durations) cases. The Fig. 10 shows the load chart of the earliest planning with infinite capacity and the best result obtained for each case using our parallel algorithm.

6 Conclusion

This paper have presented a fuzzy capacity planning approach for helicopter maintenance. A parallel algorithm has been adapted to consider resource constraints through a periodic workload representation that accounts for the task fuzzy dates and durations. We also proposed to include fuzzy task preemption to the algorithm.

Future work will be dedicated to model improvement and validation. At first, resource load profiles should consider precedence constraints in order to get more realistic load charts. Then the capacity planning approach will be used with real data from Helimaintenance project in order to compare the results, following the updates of the planning along the time. Finally, this approach should be included in a broader decision support system for an helicopter maintenance center.

Table 3
Example of task start and finish times for MIS rule in pessimistic case without preemption.

Task Id	Fuzzy start time			fuzzy finish time		
	project1	project2	project3	project1	project2	project3
A	[0, 0, 0, 0]	[0, 0, 0, 0]	[0, 0, 0, 0]	[7, 8, 9, 10]	[10, 11, 12, 13]	[17, 18, 19, 20]
B	[7, 8, 9, 10]	[10, 11, 12, 13]	[17, 18, 19, 20]	[8, 9, 10, 11]	[11, 12, 13, 14]	[18, 19, 20, 21]
C	[8, 9, 10, 11]	[13, 15, 17, 19]	[23, 24, 25, 26]	[11, 12, 13, 14]	[16, 18, 20, 22]	[26, 27, 28, 29]
D	[8, 9, 10, 11]	[11, 12, 13, 14]	[18, 19, 20, 21]	[11, 12, 13, 14]	[14, 15, 16, 17]	[21, 22, 23, 24]
E	[11, 12, 13, 14]	[16, 18, 20, 22]	[26, 27, 28, 29]	[14, 15, 18, 20]	[17, 19, 22, 24]	[27, 29, 30, 32]
F	[12, 13, 14, 15]	[20, 21, 22, 23]	[26, 27, 28, 29]	[19, 20, 21, 22]	[25, 26, 27, 28]	[29, 30, 31, 32]
G	[11, 12, 13, 14]	[16, 18, 20, 22]	[26, 27, 28, 29]	[16, 17, 18, 20]	[17, 20, 22, 25]	[28, 30, 32, 33]
H	[11, 12, 13, 14]	[16, 18, 20, 22]	[26, 27, 28, 29]	[16, 17, 18, 20]	[17, 20, 22, 25]	[28, 30, 32, 33]
I	[11, 12, 13, 14]	[16, 18, 20, 22]	[26, 27, 28, 29]	[12, 13, 14, 15]	[17, 19, 21, 23]	[27, 28, 29, 30]
J	[11, 12, 13, 14]	[14, 15, 16, 17]	[21, 22, 23, 24]	[13, 15, 17, 19]	[16, 18, 20, 22]	[23, 25, 27, 29]
K	[12, 13, 14, 15]	[17, 20, 22, 25]	[31, 32, 36, 38]	[23, 24, 25, 26]	[24, 27, 29, 32]	[36, 37, 41, 43]
L	[25, 26, 27, 28]	[29, 30, 31, 32]	[35, 37, 41, 44]	[26, 27, 30, 32]	[30, 31, 34, 36]	[36, 38, 44, 48]
M	[26, 27, 30, 32]	[30, 31, 34, 36]	[36, 38, 44, 48]	[27, 28, 31, 33]	[31, 32, 36, 38]	[37, 39, 45, 49]
N	[19, 20, 21, 22]	[27, 28, 29, 30]	[29, 30, 31, 32]	[20, 21, 22, 23]	[28, 29, 30, 32]	[30, 31, 32, 33]
O	[27, 28, 31, 33]	[31, 32, 36, 38]	[37, 39, 45, 49]	[28, 29, 32, 34]	[32, 33, 37, 39]	[38, 40, 46, 50]
P	[28, 29, 32, 34]	[32, 33, 37, 39]	[38, 40, 46, 50]	[29, 30, 33, 35]	[33, 34, 38, 40]	[39, 41, 47, 51]
Q	[29, 30, 33, 35]	[33, 34, 38, 40]	[39, 41, 47, 51]	[30, 31, 34, 36]	[34, 35, 39, 41]	[40, 42, 48, 52]
R	[30, 31, 34, 36]	[34, 35, 39, 41]	[40, 42, 48, 52]	[31, 33, 36, 39]	[35, 37, 41, 44]	[41, 44, 50, 55]

Priority list (rule MIS):
$C_1 C_2 C_3 B_1 B_2 B_3 A_1 D_1 E_1 F_1 G_1 H_1 I_1 J_1 K_1 L_1 M_1 N_1 0_1 P_1 Q_1 A_2 D_2 E_2 F_2 G_2 H_2 I_2 J_2 K_2 L_2 M_2 N_2 O_2 P_2 Q_2 A_3 D_3 E_3 F_3$
$G_3 H_3 I_3 J_3 K_3 L_3 M_3 N_3 O_3 P_3 Q_3 R_1 R_2 R_3$
Sequence in scheduling:
$A_1 A_2 A_3 B_1 C_1 D_1 B_2 D_2 J_1 E_1 F_1 H_1 I_1 G_1 K_1 J_2 C_2 B_3 E_2 F_2 H_2 I_2 D_3 N_1 K_2 G_2 J_3 C_3 L_1 E_3 F_3 G_3 H_3 I_3 N_2 M_1 O_1 L_2 N_3 P_1$
$Q_1 R_1 M_2 O_2 K_3 P_2 Q_2 R_2 L_3 M_3 O_3 P_3 Q_3 R_3$

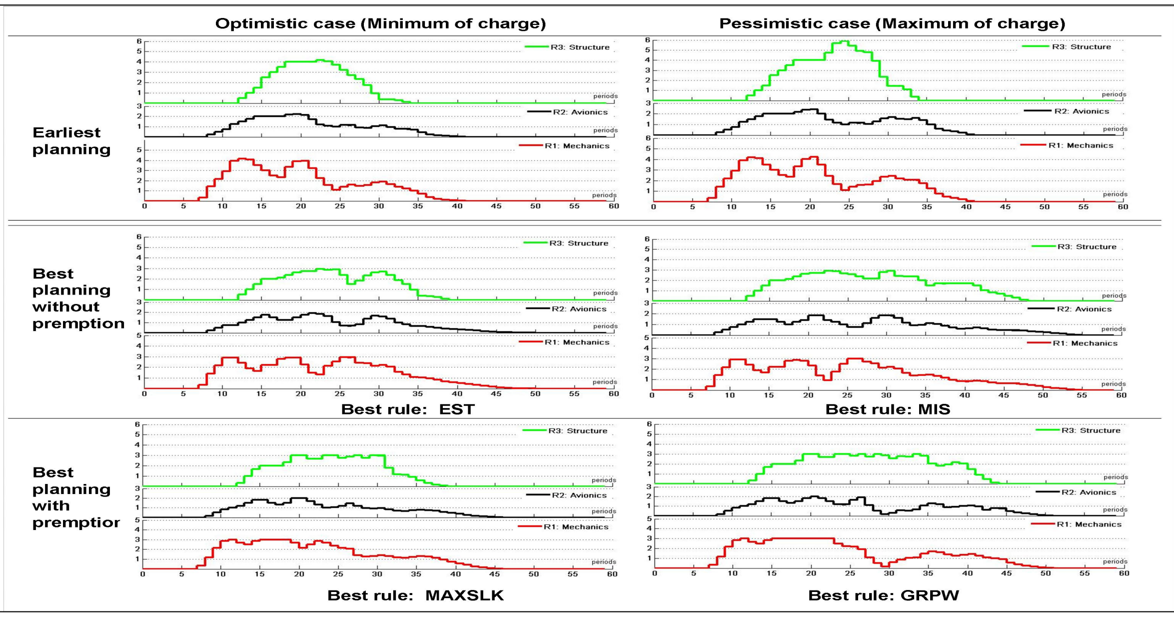

Fig. 10. Best Results with and without preemption for optimistic and pessimistic cases

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