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► **To cite this version:**

| Jamel Ghannouchi. The theorem of the primal radius. 2011. <hal-00677734>

**HAL Id: hal-00677734**

**<https://hal.science/hal-00677734v1>**

Preprint submitted on 9 Mar 2012

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# The theorem of the primal radius

Jamel Ghanouchi

## Abstract

(MSC=11) The present algebraic development begins simply by an exposition of the data of the problem. Our calculus is supported by a reasoning which must conduct to an impossibility. We define the primal radius : For all  $x$  an integer greater or equal to 3, we define a primal number  $r$  for which  $x - r$  and  $x + r$  are prime numbers. We see then that Goldbach conjecture would be verified because  $2x = (x + r) + (x - r)$ . We prove the existence of  $r$  for all  $x \geq 3$ . We prove also the existence, for all  $x'$  an integer, of a primal radius  $r'$  for which  $x' + r'$  and  $r' - x'$  are prime numbers strictly greater than 2. De Polignac conjecture would be quickly verified because  $2x' = (x' + r') - (r' - x')$ .

## Introduction

Goldbach and de Polignac conjectures seem actually impossible to be solved. Everyone has remarked the similarity between them. One stipulates that each even is the sum of two primes when the other stipulates that it is always the difference of two primes and that there is an infinity of such couples. In fact, we have used that similarity to solve those very old problems, we have considered in this research the conjectures as two faces of the same problem. For this, we have defined a notion : the primal radius. It allowed finally to prove the conjectures.

## The Goldbach conjecture

Goldbach conjecture, fruit of personal works and correspondences between the mathematicians of the XVIII century (Leonard Euler, who was born exactly 300 years before 2007, was one of them), stipulates that an even number is always equal to the sum of two prime numbers. Let an integer  $x ; x \geq 3$ , and  $p_1 \geq 3, p_2 \geq 3$  prime numbers. We can pose for  $p_1$  and  $p_2$  distinct prime numbers verifying  $p_1 > x > p_2$ .

$$2x = p_1 + p_2 + 2b$$

$b$  depends of  $p_1$  and  $p_2$ . Then

$$x = \frac{p_1 + p_2}{2} + b$$

And for all  $x, p_1, p_2$ , exists  $y$  whose expression is

$$y = \frac{p_1 - p_2}{2} + b$$

We pose

$$\begin{cases} x_1 = p_1 + 2b \\ x_2 = p_2 - 2b \\ x_3 = p_2 + 2b \\ x_4 = p_1 - 2b \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{p_1+p_2}{2} + b = \frac{p_1+x_2}{2} + 2b = \frac{x_1+p_2}{2} = \frac{x_1+x_2}{2} + b \\ = \frac{p_1+x_3}{2} = \frac{x_1+x_3}{2} - b = \frac{x_4+x_3}{2} + b = \frac{x_4+x_2}{2} + 3b \\ y = \frac{p_1-p_2}{2} + b = \frac{p_1-x_2}{2} = \frac{x_1-p_2}{2} = \frac{x_1-x_2}{2} - b \\ = \frac{p_1-x_3}{2} + 2b = \frac{x_1-x_3}{2} + b = \frac{x_4-x_3}{2} + 3b = \frac{x_4-x_2}{2} + b \\ x_1 + x_2 = p_1 + p_2 \end{cases}$$

**LEMMA 1** The following formula

$$\begin{cases} x = \frac{p_1+p_2}{2} + b = \frac{p_1+x_2}{2} + 2b = \frac{x_1+p_2}{2} = \frac{x_1+x_2}{2} + b \\ = \frac{p_1+x_3}{2} = \frac{x_1+x_3}{2} - b = \frac{x_4+x_3}{2} + b = \frac{x_4+x_2}{2} + 3b \\ y = \frac{p_1-p_2}{2} + b = \frac{p_1-x_2}{2} = \frac{x_1-p_2}{2} = \frac{x_1-x_2}{2} - b \\ = \frac{p_1-x_3}{2} + 2b = \frac{x_1-x_3}{2} + b = \frac{x_4-x_3}{2} + 3b = \frac{x_4-x_2}{2} + b \\ x_1 + x_2 = p_1 + p_2 \end{cases}$$

imply that  $\exists p_1, p_2$  prime numbers which verify

$$b = 0$$

**Proof of lemma 1** If  $x$  is a prime number,  $2x = x + x$  is the sum of two primes, then

$$p_1 - p_2 \neq 0$$

we will suppose firstly that

$$(x_1 - x_2)(x_1 + x_3) \neq 0$$

let

$$\begin{cases} \frac{x_1-x_2}{p_1-p_2} = \frac{p_1-p_2+4b}{p_1-p_2} = 1 + \frac{4b}{p_1-p_2} \\ \frac{p_1-p_2}{x_1-x_2} = \frac{x_1-x_2-4b}{x_1-x_2} = 1 - \frac{4b}{x_1-x_2} \end{cases}$$

we pose

$$\begin{cases} k = \frac{2b}{p_1-p_2} \\ k' = -\frac{2b}{(p_1-p_2+4b)} \end{cases}$$

if

$$kk' = 0 \Rightarrow b = 0$$

we will suppose

$$kk' \neq 0$$

But  $\forall x, y, \exists \phi$  verifying  $x = \phi y$

$$x + y = (\phi + 1)y = x_1 \neq 0$$

$$x - y = (\phi - 1)y = p_2 \neq 0$$

and  $\forall k, k', \exists \alpha$  verifying  $k = \alpha k'$

$$\begin{aligned} \Rightarrow k &= \frac{2b}{p_1 - p_2} = \alpha \frac{-2b}{x_1 - x_2} \\ \Rightarrow x_1 - x_2 &= -\alpha(p_1 - p_2) \\ \Rightarrow x_1 - x_2 - p_1 + p_2 &= 4b = -(\alpha + 1)(p_1 - p_2) \\ \Rightarrow b &= \frac{-(\alpha + 1)}{4}(p_1 - p_2) \\ \Rightarrow x &= \frac{p_1 + p_2}{2} + b = \frac{p_1 + p_2}{2} - \frac{\alpha + 1}{4}(p_1 - p_2) \\ \Rightarrow y &= \frac{p_1 - p_2}{2} + b = \frac{p_1 - p_2}{2} - \frac{\alpha + 1}{4}(p_1 + p_2) \end{aligned}$$

or

$$x = \frac{(1 - \alpha)p_1 + (3 + \alpha)p_2}{4} = \frac{\phi}{\phi - 1}p_2$$

and

$$y = \frac{(1 - \alpha)(p_1 - p_2)}{4} = \frac{1}{\phi - 1}p_2$$

let

$$\begin{cases} \frac{x_1 + x_3}{p_1 + p_2} = \frac{p_1 + p_2 + 4b}{p_1 + p_2} = 1 + \frac{4b}{p_1 + p_2} \\ \frac{p_1 + p_2}{x_1 + x_3} = \frac{p_1 + p_2 - 4b}{x_1 + x_3} = 1 - \frac{4b}{x_1 + x_3} \end{cases}$$

we pose

$$\begin{cases} m = \frac{2b}{p_1 + p_2} \\ m' = -\frac{2b}{x_1 + x_3} \end{cases}$$

$$mm' \neq 0$$

and  $\forall m, m', \exists \beta$  verifying  $m = \beta m'$

$$\begin{aligned} \Rightarrow m &= \frac{2b}{p_1 + p_2} = \beta \frac{-2b}{x_1 + x_3} \\ \Rightarrow x_1 + x_3 &= -\beta(p_1 + p_2) \\ \Rightarrow x_1 + x_3 - p_1 - p_2 &= 4b = -(\beta + 1)(p_1 + p_2) \\ \Rightarrow b &= \frac{-(\beta + 1)}{4}(p_1 + p_2) \\ \Rightarrow x &= \frac{p_1 + p_2}{2} + b = \frac{p_1 + p_2}{2} - \frac{(\beta + 1)}{4}(p_1 + p_2) \\ \Rightarrow y &= \frac{p_1 - p_2}{2} + b = \frac{p_1 - p_2}{2} - \frac{(\beta + 1)}{4}(p_1 + p_2) \end{aligned}$$

thus

$$\begin{aligned} x &= \frac{1 - \beta}{4}p_1 + \frac{1 - \beta}{4}p_2 = \frac{\phi}{\phi - 1}p_2 \\ y &= \frac{1 - \beta}{4}p_1 - \frac{\beta + 3}{4}p_2 = \frac{1}{\phi - 1}p_2 \end{aligned}$$

we resume

$$x = \frac{(1 - \alpha)p_1 + (3 + \alpha)p_2}{4} = \frac{1 - \beta}{4}p_1 + \frac{1 - \beta}{4}p_2 = \frac{\phi}{\phi - 1}p_2$$

and

$$y = \frac{(1 - \alpha)(p_1 - p_2)}{4} = \frac{1 - \beta}{4}p_1 - \frac{\beta + 3}{4}p_2 = \frac{1}{\phi - 1}p_2$$

$$b = \frac{-(\alpha + 1)}{4}(p_1 - p_2) = \frac{-(\beta + 1)}{4}(p_1 + p_2)$$

$$\Rightarrow \frac{\beta - \alpha}{4}p_1 = \frac{-2 - \alpha - \beta}{4}p_2$$

$$\Rightarrow \frac{\beta - \alpha}{p_2} = \frac{-2 - \alpha - \beta}{p_1}$$

but

$$(2k + 1)(2k' + 1) = 1 \Rightarrow 2k'k + k + k' = 0 \Rightarrow (2k + 1)k' = -k$$

And

$$2k + 1 = \frac{-k}{k'} = \frac{k + 1}{k' + 1} = \frac{a(k + 1) + a'(2k + 1)}{a(k' + 1) + a'}$$

and

$$2k' + 1 = \frac{-k'}{k} = \frac{k' + 1}{k + 1} = \frac{c(k' + 1) + c'(2k' + 1)}{c(k + 1) + c'}$$

$\forall a, a', c, c'$ , but

$$\frac{2k + 1}{2k' + 1} = \frac{((a + 2a')k + a + a')(ck + c + c')}{((c + 2c')k' + c + c')(ak' + a + a')}$$

$$= \frac{(ca + 2a'c)k^2 + (ac' + 3a'c + 2ac + 2a'c')k + ac + ac' + a'c' + a'c}{(ac + 2ac')k'^2 + (3c'a + ca' + 2ac + 2a'c')k' + ac + a'c' + ac' + a'c}$$

and

$$\frac{2k + 1}{2k' + 1} - 1 = (k - k')\frac{2}{2k' + 1}$$

$$= \frac{ac(k^2 - k'^2) + 2a'ck^2 - 2ac'k'^2 + (ac' + a'c + 2ac + 2a'c')(k - k') + 2a'ck - 2ac'k'}{(ac + 2ac')k'^2 + (3c'a + ca' + 2ac + 2a'c')k' + ac + a'c' + ac' + a'c}$$

$\forall a, a', c, c'$ , particularly  $a, a', c, c'$  verifying  $ac' = \delta k^2$  et  $a'c = \delta k'^2$ , then

$$(k - k')\frac{2}{2k' + 1}$$

$$= (k - k')\frac{-2ack'k + 2\delta(k^2k'^2 - k'^2k^2) + (ac' + a'c + 2ac + 2a'c') - 2\delta kk'}{(ac + 2ac')k'^2 + (3c'a + ca' + 2ac + 2a'c')k' + ac + a'c' + ac' + a'c}$$

$$= (k - k')\frac{-2ack'k + (ac' + a'c + 2ac + 2a'c') - 2\delta kk'}{(ac + 2ac')k'^2 + (3c'a + ca' + 2ac + 2a'c')k' + ac + a'c' + ac' + a'c}$$

and if

$$\frac{2}{2k' + 1} \neq \frac{-2ack'k + (ac' + a'c + 2ac + 2a'c') - 2\delta kk'}{(ac + 2ac')k'^2 + (3c'a + ca' + 2ac + 2a'c')k' + ac + a'c' + ac' + a'c}$$

$$\begin{aligned}\Rightarrow k = k' &\Rightarrow -2k'k = -2k^2 = -2k'^2 = k + k' = 2k = 2k' \\ &\Rightarrow k = k' = -1\end{aligned}$$

consequently

$$\begin{aligned}k &= \frac{2b}{p_1 - p_2} = k' = -\frac{2b}{x_1 - x_2} = -1 \\ \Rightarrow 2b &= p_2 - x_2 = x_1 - x_2 = x_1 - p_1 = p_2 - p_1 \\ &\Rightarrow 2x = x_1 + p_2 = 2p_2\end{aligned}$$

trivial solution ( $x$  prime  $2x = x + x$ ), thus

$$\frac{2}{2k' + 1} = \frac{-2ack'k + (ac' + a'c + 2ac + 2a'c') - 2\delta kk'}{(ac + 2ac')k'^2 + (3c'a + ca' + 2ac + 2a'c')k' + ac + a'c' + ac' + a'c}$$

also

$$2m + 1 = \frac{-m}{m'} = \frac{m + 1}{m' + 1} = \frac{a(m + 1) + a'(2m + 1)}{a(m' + 1) + a'}$$

and

$$2m' + 1 = \frac{-m'}{m} = \frac{m' + 1}{m + 1} = \frac{c(m' + 1) + c'(2m' + 1)}{c(m + 1) + c'}$$

$\forall a, a', c, c'$ , but

$$\begin{aligned}\frac{2m + 1}{2m' + 1} &= \frac{((a + 2a')m + a + a')(cm + c + c')}{((c + 2c')m' + c + c')(am' + a + a')} \\ &= \frac{(ca + 2a'c)m^2 + (ac' + 3a'c + 2ac + 2a'c')m + ac + ac' + a'c' + a'c}{(ac + 2ac')m'^2 + (3c'a + ca' + 2ac + 2a'c')m' + ac + a'c' + ac' + a'c}\end{aligned}$$

and

$$\begin{aligned}\frac{2m + 1}{2m' + 1} - 1 &= (m - m')\frac{2}{2m' + 1} \\ &= \frac{ac(m^2 - m'^2) + 2a'cm^2 - 2ac'm'^2 + (ac' + a'c + 2ac + 2a'c')(m - m') + 2a'cm - 2ac'm'}{(ac + 2ac')m'^2 + (3c'a + ca' + 2ac + 2a'c')m' + ac + a'c' + ac' + a'c}\end{aligned}$$

$\forall a, a', c, c'$ , particularly  $ac' = \delta k^2 = \gamma m^2$  et  $a'c = \delta k'^2 = \gamma' m'^2$ , hence

$$\begin{aligned}(m - m')\frac{2}{2m' + 1} \\ &= (m - m')\frac{-2acm'm + 2\frac{(\gamma - \gamma')}{m - m'}m^2m'^2 + (ac' + a'c + 2ac + 2a'c') - 2(\gamma\frac{m}{m - m'} - \gamma'\frac{m'}{m - m'})mm'}{(ac + 2a'c)m'^2 + (3c'a + ca' + 2ac + 2a'c')m + ac + a'c' + ac' + a'c}\end{aligned}$$

and if

$$\begin{aligned}\frac{2}{2m' + 1} &\neq \frac{-2acm'm + 2\frac{(\gamma - \gamma')}{m - m'}m^2m'^2 + (ac' + a'c + 2ac + 2a'c') - 2(\gamma\frac{m}{m - m'} - \gamma'\frac{m'}{m - m'})mm'}{(ac + 2a'c)m'^2 + (3c'a + ca' + 2ac + 2a'c')m + ac + a'c' + ac' + a'c} \\ \Rightarrow m = m' &\Rightarrow -2m'm = -2m^2 = -2m'^2 = m + m' = 2m = 2m' \\ &\Rightarrow m = m' = -1 \\ m &= \frac{2b}{p_1 + p_2} = m' = -\frac{2b}{x_1 + x_3} = -1\end{aligned}$$

$$\begin{aligned}\Rightarrow 2b &= p_2 - x_2 = x_1 + x_3 = x_1 - p_1 = -p_2 - p_1 \\ \Rightarrow 2x &= x_1 + p_2 = 0\end{aligned}$$

impossible ( $x \geq 3$ ), and

$$\frac{2}{2m' + 1} = \frac{-2acm'm + 2\frac{(\gamma-\gamma')}{m-m'}m^2m'^2 + (ac' + a'c + 2ac + 2a'c') - 2(\gamma\frac{m}{m-m'} - \gamma'\frac{m'}{m-m'})mm'}{(ac + 2a'c)m'^2 + (3c'a + ca' + 2ac + 2a'c')m + ac + a'c' + ac' + a'c}$$

and as we saw, it is a  $2k + 1$ -like expression, thus

$$\begin{aligned}\frac{2}{2m' + 1} &= \frac{-2acm'm + 2\frac{(\gamma-\gamma')}{m-m'}m^2m'^2 + (ac' + a'c + 2ac + 2a'c') - 2(\gamma\frac{m}{m-m'} - \gamma'\frac{m'}{m-m'})mm'}{(ac + 2a'c)m'^2 + (3c'a + ca' + 2ac + 2a'c')m' + ac + a'c' + ac' + a'c} \\ &= \frac{-2acm'm + 2\frac{(\gamma-\gamma')}{m-m'}m^2m'^2 + (ac' + a'c + 2ac + 2a'c') - 2(\gamma\frac{m}{m-m'} - \gamma'\frac{m'}{m-m'})mm'}{(ac + 2a'c)m'^2 + (3c'a + ca' + 2ac + 2a'c')m' + ac + a'c' + ac' + a'c} \\ &\Rightarrow \gamma = \gamma'\end{aligned}$$

$a, a', c, c'$ , it is always possible, are as it follows

$$\begin{aligned}ac' &= \delta k^2 = \gamma m^2 \\ a'c &= \delta k'^2 = \gamma m'^2 \\ \Rightarrow \frac{\delta}{\gamma} &= \frac{m^2}{k^2} = \frac{m'^2}{k'^2} \\ \Rightarrow \frac{k^2}{k'^2} &= (2k + 1)^2 = \alpha^2 = \frac{m^2}{m'^2} = (2m + 1)^2 = \beta^2 \\ \Rightarrow (\alpha - \beta)(\alpha + \beta) &= 0\end{aligned}$$

if

$$\begin{aligned}\alpha + \beta = 0 &\Rightarrow (\alpha - \beta)p_1 = (\alpha + \beta + 2)p_2 = 2p_2 = 2\alpha p_1 \\ \Rightarrow \alpha &= \frac{p_2}{p_1} = -\beta\end{aligned}$$

$$\begin{aligned}\Rightarrow \begin{cases} b = -\frac{1+\alpha}{4}(p_1 - p_2) = \frac{1+\frac{p_2}{p_1}}{4}(p_2 - p_1) = \frac{p_2^2 - p_1^2}{4p_1} \\ x = \frac{1-\beta}{4}(p_1 + p_2) = \frac{1+\frac{p_2}{p_1}}{4}(p_1 + p_2) = \frac{(p_1 + p_2)^2}{4p_1} \\ y = \frac{1-\alpha}{4}(p_1 - p_2) = \frac{1-\frac{p_2}{p_1}}{4}(p_1 - p_2) = \frac{(p_1 - p_2)^2}{4p_1} \end{cases} \\ \Rightarrow 4bp_1 + p_1^2 = p_2^2 &\Rightarrow 4b + p_1 = \frac{p_2^2}{p_1}\end{aligned}$$

and it is impossible because in the left there is an integer and as  $p_1$  and  $p_2$  are primes  $\frac{p_2^2}{p_1}$  can not be an integer, thus in the right there is not an integer : it is impossible. We deduce that

$$\alpha + \beta \neq 0 \Rightarrow \alpha - \beta = 0$$

and

$$\alpha - \beta = 0 \Rightarrow (\alpha - \beta)p_1 = 0 = (\alpha + \beta + 2)p_2 = 2(\alpha + 1)p_2 = 2(\beta + 1)p_2$$

$$\begin{aligned}
&\Rightarrow \alpha = \beta = -1 \\
&\Rightarrow b = -(\beta + 1)\frac{p_1 + p_2}{4} = -(\alpha + 1)\frac{p_1 - p_2}{4} = 0 \\
&\Rightarrow x = \frac{(1 - \alpha)p_1 + (3 + \alpha)p_2}{4} = \frac{1 - \beta}{4}p_1 + \frac{1 - \beta}{4}p_2 = \frac{p_1 + p_2}{2} \\
&\Rightarrow y = \frac{(1 - \alpha)(p_1 - p_2)}{4} = \frac{1 - \beta}{4}p_1 - \frac{\beta + 3}{4}p_2 = \frac{p_1 - p_2}{2}
\end{aligned}$$

$b$  can not be different of zero for all  $p_1, p_2$ . If

$$(x_1 - x_2)(x_1 + x_3) = 0 \Rightarrow (x_4 + x_2)(x_4 - x_3) \neq 0$$

let

$$\begin{cases} \frac{x_4 + x_2}{p_1 + p_2} = \frac{p_1 + p_2 - 4b}{p_1 + p_2} = 1 - \frac{4b}{p_1 + p_2} \\ \frac{p_1 + p_2}{x_4 + x_2} = \frac{x_4 + x_2 + 4b}{x_4 + x_2} = 1 + \frac{4b}{x_4 + x_2} \end{cases}$$

we pose

$$\begin{cases} 2k + 1 = 1 - \frac{4b}{p_1 + p_2} \\ 2k' + 1 = 1 + \frac{4b}{x_4 + x_2} \end{cases}$$

let

$$\begin{cases} \frac{x_4 - x_3}{p_1 - p_2} = \frac{p_1 - p_2 - 4b}{p_1 - p_2} = 1 - \frac{4b}{p_1 - p_2} \\ \frac{p_1 - p_2}{x_4 - x_3} = \frac{x_4 - x_3 + 4b}{x_4 - x_3} = 1 + \frac{4b}{x_4 - x_3} \end{cases}$$

we pose

$$\begin{cases} 2m + 1 = 1 - \frac{4b}{p_1 - p_2} \\ 2m' + 1 = 1 + \frac{4b}{x_4 - x_3} \end{cases}$$

$\Rightarrow x = \frac{p_1 + 3p_2}{4} = \frac{p_1 + p_2}{2}$  or  $\frac{p_1 + p_2}{2} = \frac{p_1 + p_2}{4}$  and  $x = p_1 = p_2$  or  $p_1 = p_2 = 0$  it is impossible, then

$$(x_1 - x_2)(x_1 + x_3) \neq 0$$

and

$$b = 0$$

### Theorem

$$\forall x \geq 3, \exists p_1 \geq 3, p_2 \geq 3, x = \frac{p_1 + p_2}{2}$$

We did not pose any condition on  $p_1$  and  $p_2$ . As  $p_2 < x < p_1$  there are not an infinity of such primes.

**THEOREM OF THE PRIMAL RADIUS** There exists  $r$  a primal radius for which  $x + r, x - r$  are prime numbers  $\forall x \geq 3$ .

**Proof of theorem of primal radius** For all  $x \geq 3$  exists  $p_1 \geq 3, p_2 \geq 3$  prime numbers for which

$$x = \frac{p_1 + p_2}{2}$$

if

$$r = \frac{p_1 - p_2}{2}$$

then

$$x + r = p_1$$

$$x - r = p_2$$

**Corollary** Between  $x$  and  $2x$  exists always a prime number  $x < p_1 = x + r < 2x = p_1 + p_2$ . If  $2z + 1$  is an integer strictly greater than 8, exists always a prime  $p_3$ , which can be 3, for which  $2z + 1 = p_3 + 2x = p_3 + p_1 + p_2$ .

**de Polignac conjecture** de Polignac conjecture stipulates that an even number is always equal to the difference between two prime numbers and that there is an infinity of such prime numbers. Let  $x$  an integer and  $p_1, p_2$  prime numbers strictly greater than 2.

$$2x = p_1 - p_2 + 2b_{p_1, p_2} = p_1 - p_2 + 2b$$

For commodity, we have suppressed the indexes, but  $b$  depends of  $p_1$  and  $p_2$ .

$$x = \frac{p_1 - p_2}{2} + b$$

But for all  $x, p_1, p_2, b$  exists  $y$  whose expression is

$$y = \frac{p_1 + p_2}{2} + b$$

We pose

$$\begin{cases} x_1 = p_1 + 2b \\ x_2 = p_2 - 2b \\ x_3 = p_2 + 2b \\ x_4 = p_1 - 2b \end{cases}$$

$$\Rightarrow \begin{cases} y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_2}{2} + 2b = \frac{x_1 + p_2}{2} = \frac{x_1 + x_2}{2} + b \\ = \frac{p_1 + x_3}{2} = \frac{x_1 + x_3}{2} - b = \frac{x_4 + x_3}{2} + b = \frac{x_4 + x_2}{2} + 3b \\ x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - x_2}{2} = \frac{x_1 - p_2}{2} = \frac{x_1 - x_2}{2} - b \\ = \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_4 - x_3}{2} + 3b = \frac{x_4 - x_2}{2} + b \\ x_1 + x_2 = p_1 + p_2 \end{cases}$$

**LEMMA 2** The following formula

$$\begin{cases} y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_2}{2} + 2b = \frac{x_1 + p_2}{2} = \frac{x_1 + x_2}{2} + b \\ = \frac{p_1 + x_3}{2} = \frac{x_1 + x_3}{2} - b = \frac{x_4 + x_3}{2} + b = \frac{x_4 + x_2}{2} + 3b \\ x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - x_2}{2} = \frac{x_1 - p_2}{2} = \frac{x_1 - x_2}{2} - b \\ = \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_4 - x_3}{2} + 3b = \frac{x_4 - x_2}{2} + b \\ x_1 + x_2 = p_1 + p_2 \end{cases}$$

imply that  $\exists p_1, p_2$  an infinity of couples which verify

$$b = 0$$

**Proof of lemma 2** If  $x = 0$  is a prime number,  $2x = p - p$  an infinity of couples of primes, and

$$p_1 - p_2 \neq 0$$

we will suppose firstly that

$$(x_1 - x_2)(x_1 + x_3) \neq 0$$

Now, the same calculus and reasoning than higher for Goldbach, replacing  $p_2$  by  $-p_2$  lead to  $b = 0$ . The primal radius  $r$  here is equal to

$$r = \frac{p_1 + p_2}{2}$$

with

$$x = \frac{p_1 - p_2}{2}$$

then

$$x + r = p_1$$

and

$$r - x = p_2$$

Its existence is proved.  $p_1$  and  $p_2$  are an infinity of couples of primes, because we did not specify any condition on them.

### Conclusion

The conclusion is that Goldbach conjecture and de Polignac conjecture, which seem so inaccessible, are in fact true. Because, we have defined the primal radius. The conjecture of the primal radius seems to be a consequence of Goldbach and de Polignac conjectures. In fact, if we had not its concept in mind, there would not be the present proof of those conjectures.

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