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For Peer Review

# Arbitrage Costs and Nonlinear Adjustment in the G7 Stock Markets

Fredj JAWADI<sup>+</sup> and Georges PRAT<sup>∅</sup>

**Abstract** - This paper aims to study stock price adjustments toward fundamentals due to the existence of arbitrage costs defined as the sum of transaction costs and a risky arbitrage premium associated with the uncertainty characterizing the fundamentals. Accordingly, it is shown that a two-regime STECM (Smooth Transition Error Correction Model) is appropriate to reproduce the dynamics of stock price deviations from fundamentals in the G7 countries during the period 1969-2005. This model takes into account the interdependences or contagion effects between stock markets. Deviations appear to follow a quasi random walk in the central regime when prices are near fundamentals (i.e. when arbitrage costs are greater than expected arbitrage profits, the mean reversion mechanism is inactive), while they approach a white noise in the outer regimes (i.e. when arbitrage costs are lower than expected arbitrage profits, the mean reversion is active). Interestingly, as expected when arbitrage costs are heterogeneous, the estimated STECM shows that stock price adjustments are smooth and that the convergence speed depends on the size of the deviation. Finally, using two appropriate indicators proposed by Peel and Taylor (2000), both the magnitudes of under- and overvaluation of stock price and the adjustment speed are calculated per date in the G7 countries. These indicators show that the dynamics of stock price adjustment are strongly dependent on both the date and the country under consideration.

**JEL:** C22, G15.

**Keywords:** Stock price, heterogeneous transaction costs, STECMs.

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# Arbitrage Costs and Nonlinear Adjustment in the G7 Stock Markets

## 1- Introduction

Many studies suggest that fundamentals alone cannot explain stock price dynamics since deviations between the market price and fundamentals are often large and durable (among others, see Shiller (1981), Campbell and Shiller (2001), Allen and Yang (2001), Manzan (2003), Boswijk *et al.* (2007)). Deviations may be explained in different ways. Shiller (1981) and Summers (1986) suggest that “irrational fads” generate persistent deviations between prices and fundamentals and Daniel *et al.* (1998) explain positive deviations by investor overconfidence. Barberis and Thaler (2003) suggest that investors under-react to news about fundamentals in the short term, although they gradually incorporate them in the long run. Other studies show that heterogeneity in expectations (i.e. chartists, fundamentalists and noise traders), mimetic behavior and information asymmetry may help to explain the deviations and the existence of a nonlinear mean-reverting strength that leads stock prices to converge to fundamentals (see Poterba and Summers (1988), Fama and French (1988), Cecchetti *et al.* (1990), Manzan (2003), De Grauwe and Grimaldi (2006)), Jawadi (2006), and Boswijk *et al.* (2007)). In particular, Barberis *et al.* (1998) and Boswijk *et al.* (2007) develop two-regime models describing the dynamics of stock price deviations, while distinguishing a trend regime related to “trend follower” investors and a mean-reverting regime related to “fundamentalists”. As a result, the authors show that nonlinearity characterizing the asset price adjustment dynamics can be explained by this heterogeneity in shareholder expectations.

Another approach focuses on arbitrage costs that have two components: the transaction costs and the risky arbitrage premium required due to the uncertainty characterizing the fundamentals. According to Anderson (1997), the transaction cost hypothesis alone could justify deviations and nonlinearity in stock price adjustment dynamics. Concerning the risky arbitrage hypothesis, Shleifer and Summers (1990) suggest that noise traders’ behaviour may lead to greater fundamental mispricing of an asset. Perceived stock price deviations which represent risky arbitrage opportunities may not be

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3 arbitrated when expected gains are not large enough to compensate this risk, and this is a limitation of  
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5 the arbitrage hypothesis. In this paper, we propose considering the total arbitrage cost as the sum of  
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7 transaction costs and a risky arbitrage premium. Both transaction costs and risky arbitrage generate  
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9 similar limit to arbitrage, to instantaneous stock price adjustment and thus to the efficiency  
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11 hypotheses. With respect to the transaction costs only or the aversion to risky arbitrage only, taking the  
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13 total arbitrage cost into account increases the possibility to observe nonlinearity in stock price  
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15 adjustments. Considering heterogeneity in arbitrage costs, the main goal of this paper is to examine if,  
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17 as expected, when prices are near fundamentals (i.e. the arbitrage cost is greater than expected gains)  
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19 the deviations are durable, while when the prices are far from fundamentals (i.e. the arbitrage cost is  
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21 lower than expected gains), the mean reversion is strongly activated.  
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25 As summarized hereafter, even though some recent papers such as Kian *et al.* (2007), Bali *et*  
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27 *al.* (2008), Kim *et al.* (2009) have focused on the mean reversion in stock prices in a nonlinear  
28  
29 framework,<sup>1</sup> the literature associated with the issue of stock prices toward fundamentals is still  
30  
31 relatively scarce, probably because of the difficulty involved in representing the fundamental value. In  
32  
33 this paper, we propose an estimation of the fundamental value using the Dividend Discount Model  
34  
35 (DDM), where the expected dividends are represented by a Smooth Transition Autoregressive Model  
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37 (STAR). The deviation between stock price and fundamentals is modeled using a Smooth Transition  
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39 Error Correction Model (STECM). The paper makes several contributions to the literature. First, while  
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41 most previous studies do not investigate the adjustment toward fundamentals but rather toward some  
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43 reference stock price index (e.g. the US index), this paper focuses on stock prices adjustment toward  
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45 an explicit fundamental value. Second, the paper relates the econometric methodology of switching  
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47 models to a formal theoretical model based on heterogeneous total arbitrage costs. Third, while most  
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49 studies have focused on the American stock market, the present paper considers the G7 countries.  
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51 Fourth, our proposed model takes into account the interdependences or contagion effects between  
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53 stock markets. Finally, using indicators proposed by Peel and Taylor (2000) for the foreign exchange  
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55 market, we identify the magnitude of under- and overvaluations and the speeds of adjustment for each  
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60 <sup>1</sup> For exchange rate market, see Yoon (2010) who identified a nonlinear mean-reversion toward the power parity (i.e. the fundamentals) using an ESTAR model.

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3 country at each date. To our knowledge, these characteristics have not been collected in any previous  
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5 stock market study.  
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8 The rest of the article is organized as follows. The nonlinearity and smoothness characterizing  
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10 stock price adjustment are formally justified by the total cost of arbitrage in section 2. Section 3  
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12 presents the STECM methodology to model stock price deviations, and discusses the subsequent  
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14 empirical results. We set out our concluding remarks in section 4.  
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## 18 19 **2 - Arbitrage costs and stock price adjustments**

### 20 21 **2.1 - Theoretical framework: why do arbitrage costs cause nonlinearity in stock price** 22 23 **adjustment?** 24 25

26 According to Shleifer and Summers (1990), noise trader behaviour may lead to enlarge the  
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28 fundamental mispricing of assets. Perceived deviations of stock prices represent risky arbitrage  
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30 opportunities, but these deviations may not be arbitrated when expected gains are not large enough to  
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32 compensate the risk, and this is a limitation of the arbitrage hypothesis. Moreover, the existence of a  
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34 distribution of the degrees of risk aversion across investors suggests that arbitrage increases according  
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36 to the size of the fundamental mispricing, so that the arbitrage is more stabilizing when deviations are  
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38 large, as suggested by Gallagher and Taylor (2001). Overall, this risky arbitrage hypothesis implies  
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40 limit to arbitrage and may explain nonlinearity in stock price dynamics.  
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43 Concerning transaction costs, it is now well established that transaction costs may also  
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45 generate such nonlinearities. Dumas (1992) suggests that the presence of transaction costs may  
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47 generate nonlinear dynamics in foreign exchange rates. The author shows that these costs create two  
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49 zones. In the first zone, called “the no trade band,” arbitrages and adjustments are not active since the  
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51 expected returns are lower than the transaction costs. This means that prices can continually deviate  
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53 from their fundamental values. The deviations are left uncorrected as long as they are low with respect  
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55 to transaction costs and they follow a near-unit root process in this area. Disequilibrium is only  
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57 corrected in the second zone called the “exchange region”, when price deviations and arbitrage profits  
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59 are large enough to compensate for transaction costs. In this respect, stock price deviations are a white  
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3 noise and stock prices can join their fundamentals with a convergence speed that depends on the size  
4 of the deviation. Following Dumas (1992), several studies have confirmed that transaction costs  
5 induce some delay and persistence in the dynamics of foreign exchange rates (Michael *et al.* (1997),  
6 Peel and Taylor (2000), De Grauwe and Grimaldi (2006)), interest rates (Anderson (1997), Liu  
7 (2001)) and stock prices (Manzan (2003), Boswijk *et al.* (2007)). These studies reject the linear,  
8 symmetrical, instantaneous and continuous adjustment hypothesis. It is worth noting that, for the stock  
9 market, these costs are far from negligible. According to reports by Elkins & McSherry and  
10 Cherbonnier & Vandelanoite (2008, p.89), direct transaction costs over 2005-2006, expressed as a %  
11 of the amount of the transaction, reached 5.51, 10.23, 5.0, 6.58, 8.8, 10.65 and 5.9 for Germany,  
12 Canada, the USA (NYSE), the UK, Italy and Japan, respectively. It can be seen that transaction costs  
13 appear to be largely dependent on the country in question. For example, the USA and Japan showed  
14 the lowest transaction costs, while France took fifth position after Germany and the UK.<sup>2</sup>

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29 Interestingly, Anderson (1997) proposed a model in which, due to transaction costs, the  
30 adjustment process of the US Treasury Bills rate toward its equilibrium value can be represented  
31 empirically by a STECM. The STECM was introduced by Granger and Teräsvirta (1993) and was  
32 more recently developed by Van Dijk *et al.* (2002). In this paper, we start with Anderson's model but  
33 adapt it to study stock market dynamics. In particular, and contrary to Anderson, we take into account  
34 both transaction costs and the risky character of arbitrage opportunities. Let  $z_t = p_t - f_t$  be the  
35 actual deviation between the market log-price  $p_t$  of a portfolio of equities and its fundamental log-  
36 value  $f_t$  perceived by all investors. In the absence of transaction costs and arbitrage opportunities,  
37 any investor can make a profit from a stock price deviation. When  $z_t = 0$ , there are no arbitrage  
38 opportunities, but when  $z_t > 0$  (respectively  $z_t < 0$ ), the portfolio is overvalued (respectively  
39 undervalued) and the incentive of arbitrage is based on expected profits. In this case, the adjustment  
40 process bringing the stock price toward fundamentals is classically continuous and linear with a  
41 constant speed of adjustment:

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<sup>2</sup> Direct or explicit costs are largely composed of taxes, regulation costs and other commissions. They generally depend on the nature of the type of broker, the nature of the order, and the stock market.



$$\Delta z_t = -\rho z_{t-1} + \Phi(L) \Delta z_{t-1} + \nu_t \quad (1)$$

where  $\Phi(L)$  represents the distributed lag polynomial,  $\Delta$  the first difference and  $\nu_t$  a white noise.

Nevertheless, it can be seen that the existence of transaction costs and risk aversion reduces arbitrage opportunities. Let  $\tau_t$  represent at time  $t$  the sum of transaction costs and of a required premium due to the risky character of arbitrage opportunities resulting from uncertainty about the fundamental value perceived, both expressed in percent of the price. Accordingly,  $\tau_t$  may be viewed as the total cost of arbitrage that we will hereafter call the “cost of arbitrage”. Suppose in a first instance  $\tau_t$  to be homogeneous across agents. When  $z_t > \tau_t$  or when  $z_t < -\tau_t$ , the investor is incited to raise his/her detention of equities, while when  $-\tau_t < z_t < \tau_t$ , this arbitrage opportunity disappears. Here, the expected profits are clearly limited by arbitrage costs. It is worth noting that, in such a configuration, equation (1) is no longer adequate to reproduce the stock price adjustment dynamics, since it fails to replicate this discontinuity of arbitrages. In this case, the adjustment process takes into account both the no-trade zone and the arbitrage opportunity zone, and can be written as:

$$\Delta z_t = -\rho \Omega(|z_{t-1}|) z_{t-1} + \Phi(L) \Delta z_{t-1} + \varepsilon_t \quad (2)$$

$$\text{where } \Omega(|z_{t-1}|) = 1 \text{ if } |z_{t-1}| > \tau_t$$

$$\Omega(|z_{t-1}|) = 0 \text{ if } |z_{t-1}| \leq \tau_t$$

where  $0 < \Omega(\cdot) < 1$  represents a transition function, allowing us to characterize which regime of the adjustment process holds at each date.

Moreover, stock market transaction costs are heterogeneous since they depend on many factors such as the amount of the transaction, the investor type, etc.<sup>3</sup> The appreciation of risk associated with arbitrage opportunities is also agent-dependant since risk aversion is an individual

<sup>3</sup> In particular, spreads between transaction costs supported by individual investors and those supported by institutional investors contribute to heterogeneity.

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3 preference parameter. These disparities between individual transaction costs and individual  
4 appreciation of arbitrage risk may generate different arbitrage thresholds, so that the model (2) is no  
5 longer appropriate to describe the stock price adjustment. In this case, as shown by Anderson, the  
6 adjustment becomes gradual rather than sudden. Let  $\tau_{jt}$  be the price of arbitrage associated with the  
7 purchase of a portfolio unity by investor  $j$  at time  $t$ . A rational investor reacts to a price deviation only  
8 if  $\tau_{jt}$  is such as  $\tau_{jt} < z_{t-1} < -\tau_{jt}$ . Let  $H(|z_{t-1}|)$  be the cumulative density function of all investors'  
9 expenses, which represents the proportion of equities for which investors expect a benefit for time  $t$   
10 due to the price deviation. Formally, the introduction into equation (2) of heterogeneity in transaction  
11 costs and risky arbitrage premia implies the following adjustment process:  
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$$\Delta z_t = -\rho H(|z_{t-1}|) z_{t-1} + \Phi(L)\Delta z_{t-1} + \varepsilon_t \quad (3)$$

26  
27 where the cumulative density function  $H(|z_{t-1}|)$ , ranging between 0 and 1, is an exponential function  
28 defined as:  
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$$H(|z_{t-1}|) = H(\tau_t) = 1 - \exp[-\beta \tau_t^2], \quad \beta > 0 \text{ and } \tau_t \geq 0 \quad (4)$$

36  
37 with  $\beta$  the transition speed and  $\tau_t$  the average arbitrage costs at time  $t$ . Note that  $H(|z_{t-1}|)$  equals  
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39  $H(\tau_t)$  since, at the equilibrium price, expected gains resulting from price deviations  $|z_{t-1}|$  just  
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41 compensate the total cost of arbitrage  $\tau_t$ . The structural model given by equations (3) and (4) can be  
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43 empirically estimated using a STECM, where  $H(\tau_t)$  represents a smooth exponential transition  
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45 function.  
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## 51 **2.2 - Empirical evidence of nonlinearity in stock price adjustment: surveying the** 52 53 **literature** 54 55

56 Since the end of the 1980s, mean-reversion in stock prices toward fundamentals has received  
57 significant attention in the literature (Fama and French (1988), Poterba and Summers (1988), Lo and  
58 MacKinlay (1988)). The issue of the conditions in which this phenomenon operates is important  
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3 because it contributes to define an optimal investment strategy. Indeed, when stock prices are mean-  
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5 reverting, momentum strategies are not relevant and returns may be expected to increase by short-  
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7 selling high return equities and buying poor return equities. This is the contrarian trading/arbitrage  
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9 (Balvers *et al.* (2000)), and suggests that the knowledge of the time pattern of the market price at any  
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11 date is important for a rational investor. Although previous studies have separately identified mean-  
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13 reversion and momentum, Balvers and Wu (2006) propose a mixed model combining momentum and  
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15 contrarian strategies at any date. For 18 developed equity markets (among them the US market), at a  
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17 monthly frequency over the period 1969 to 1999, the authors show that this model outperforms both  
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19 pure momentum and pure contrarian strategies. These results show the importance to knowing the type  
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21 of dynamics characterising the stock market at a given date. To make stock return forecasts, when  
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23 deviations are persistent within a central band but are mean-reverting outside, this suggests that a  
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25 momentum strategy is preferable inside the band while a contrarian strategy would be better outside  
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27 the band. Following this line, supposing that the real equilibrium value of equities is proportional to  
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29 the real dividends (the coefficient of dividends may be constant or time-varying according to the  
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31 hypothesis retained about the discount rate), Gallagher and Taylor (2001) analyse the speed of  
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33 reversion of the US stock market by considering the deviation of the aggregate log dividend-price ratio  
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35 from its equilibrium values over the period 1926-97 (quarterly data). Using an ESTAR-GARCH  
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37 model, the authors show the existence of two expectation regimes, and provide evidence supporting  
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39 the risky arbitrage hypothesis, the speed of the adjustment toward equilibrium varying according to the  
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41 size of the deviation. Black *et al.* (2003) and Bohl (2003) also suggest strong evidence of nonlinear  
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43 mean-reversion in the S&P index, while, using a nonlinear mean-reversion test, the paper by Bali *et al.*  
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45 (2008) confirms that the speed of mean-reversion is significantly higher during large falls on the US  
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47 stock market, and suggests that knowledge of these dynamics can be useful in predicting stock returns.  
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49 Boswijk *et al.* (2007) estimate a dynamic asset pricing model for the US stock market over the years  
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51 1871-2003 (S&P annual data), characterized by heterogeneous agents where the fundamental value of  
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53 equities is common knowledge (the authors suppose a constant risk-free rate and constant dividend  
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55 growth), but where the agents have different beliefs about the persistence of stock price deviations  
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57 from fundamentals, depending on transaction costs and expectation heterogeneity. According to this  
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3 model, a strategy attracts more agents if it is performed relatively well in the recent past compared to  
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5 other strategies. Using a STAR model, empirical results still reveal two expectation regimes: the  
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7 fundamentalist regime (agents believe in mean-reversion of stock prices toward fundamentals) and the  
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9 chartist regime (agents expect the deviations from the fundamental to continue). Interestingly, with  
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11 regard to the general interpretation of these nonlinearities in explaining S&P deviations, papers by  
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13 Froot and Obstfeld (1991) and by Driffill and Sola (1998) compare the bubble hypothesis with the  
14  
15 alternative of a threshold dynamic process. Both papers conclude in favor of the last hypothesis.  
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19 Although much of the research focuses on the US stock market, a growing literature about  
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21 mean reversion of stock price relates to international stock markets. Considering a panel of 18  
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23 developed stock markets, and supposing that reversions are related from one national index to another  
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25 and that the speed of reversion retains the same constant value for all markets, Balvers *et al.* (2000)  
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27 found strong evidence of a mean reverting strength in the dynamics of stock price indexes. Using a  
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29 linear model in which all the 17 emerging stock markets considered are still supposed to have the  
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31 same constant speed of convergence, Chaudhuri and Wu (2004) confirm evidence of mean reversion  
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33 in stock price indexes. In fact, these models are linear since they are based on the restrictive hypothesis  
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35 of a constant reversion speed. Kian-Ping and Khim-Sen (2007) underline the risk of drawing the  
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37 wrong inferences from mean reversion when the ADF test is applied to data governed by nonlinearity.  
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39 Using the nonlinear stationary test proposed by Kapetanios *et al.* (2003), the authors reject the  
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41 linearity hypothesis for all the Asian stock returns and find evidence of a nonlinear mean reverting  
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43 pattern represented by a STAR model.<sup>4</sup> More recently, Hyeongwoo *et al.* (2009) examined the G7  
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45 stock markets and tested whether deviations of each stock price from the reference US index are mean  
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47 reverting. Using different transition functions leading to similar results (among them, the exponential  
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49 function), the authors confirm strong evidence of nonlinear mean reversion over the period December  
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51 1969 to September 2007. Again, deviations toward the US index are found to be near a random walk  
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53 within a central band but mean reverting outside. Although the authors do not refer to a formal  
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59 <sup>4</sup> In another way, Berdin and Hyde (2005) also use STAR models to capture nonlinearity in the cyclical character of stock  
60 price dynamics for eight countries (Belgium, Canada, France, Germany, Ireland, Japan, the United Kingdom and the United States). The authors show that the process describing the stock price adjustment toward fundamentals depends on the state of the economy (two regimes are considered: growth and recession). Using STAR models, Hasanov and Omay (2008) also show strong evidence in favor of nonlinear adjustment stock returns for the Athens and the Istanbul Stock markets.

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3 theoretical model to be tested, they suggest a potential explanation based on transaction costs and risks  
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5 associated with contrarian trading/arbitrage.  
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8 Overall, these empirical results suggest that threshold models are relevant in describing stock  
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10 price adjustment dynamics. With respect to the literature, we can see that no study relates  
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12 *simultaneously* to (i) a fundamental value calculated according to an equity pricing model, (ii) an  
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14 adjustment process of stock prices toward fundamentals which is formally deduced from a theoretical  
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16 model based on both heterogeneity in risky arbitrage appreciation and on transaction costs and (iii) the  
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18 group of G7 countries in which interdependences or contagions between national stock markets are  
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20 taken into account in the adjustment process. The present paper aims to contribute to these gaps.  
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### 23 24 25 **3 - Stock price adjustment modeling in the G7 countries** 26 27

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30 We first present the fundamental value estimations (§3.1) and we then focus on the stock price  
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32 adjustment modeling (§3.2 to §3.5).  
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#### 35 36 37 **3.1 – Fundamental value estimation** 38

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40 In a world with perfect foresight and under the condition of transversality, the DDM can be  
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42 expressed by the following recurrent equation defining the fundamental value  $\bar{F}_t$  for a given  
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44 country, this value corresponding to Shiller's "rational ex-post price":  
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$$47 \bar{F}_{t+1} = \bar{F}_t (1 + i_{ot}) - D_{t+1} \quad (5)$$

48  
49 where  $i_{ot}$  is the one-period to maturity risk-free rate and  $D_{t+1}$  the dividends distributed during the  
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51 period  $[t, t+1]$ .  
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58 Considering now the fundamental value under the one-period ahead REH, the future dividends  
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60  $D_{t+1}$  are replaced by the expected dividends  $E_t(D_{t+1})$ , where  $E_t(\cdot)$  is the expectation conditional to

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3 the information available at time  $t$ , the discount rate being defined as the sum of the risk-free rate  $i_{ot}$   
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5 and the constant risk premium  $\Phi_o$ . The fundamental rational value is then given by the forward  
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7 resolution of the following relation:  
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$$10 \quad F_{t+1} = F_t (1 + i_{ot} + \Phi_o) - E_t(D_{t+1}) \quad (6)$$

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13 The generating process of  $F_t$  is based on rational expectations that are revised at each date  
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15 according to new information, and this is a less restrictive hypothesis than the REH at time  $t$  for any  
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17 future horizons often considered in the literature. The estimation of  $F_t$  according to (6) requires not  
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19 only the expected dividends time series but still an initial value  $F_o$  at the beginning of the period and  
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21 the value of the constant risk premium  $\Phi_o$ . To let the fundamental value explain the price as far as  
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23 possible, these parameters are chosen to obtain the minimum sum of squared log-differences between  
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25 prices and the fundamental values over the period of analysis. The fundamental value is estimated for  
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27 the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States)  
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29 using monthly data over the period 1969-2005. Dividend series, which are computed using *Price*  
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31 *Indexes* and *Gross Indexes*, and stock prices are obtained from the Morgan Stanley Capital  
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33 International database.<sup>5</sup> The monthly free-risk discount rate is given by the one month Monetary  
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35 Market Rates (MMR), and the industrial production series (CSA) are obtained from the International  
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37 Monetary Fund's International Financial Statistics. All data are expressed in local currencies. The  
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39 Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) stationarity tests show that the log- G7  
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41 stock prices are I(1). Furthermore, the G7 stock return<sup>6</sup> distributions are found to be asymmetric and  
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43 leptokurtic. This rejection of normality may indicate nonlinearity characterizing the dynamics of stock  
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<sup>5</sup> The gross index takes into account the dividend investment while the price index excludes it. All indexes are closing prices.

<sup>6</sup> The stock return is defined as the stock price logarithmic first difference plus the dividends yield.

Following Driffill and Sola (1998) and Berdin and Hyde (2005), we use a STAR model to estimate the dividend expectations. Given that dividends are I(1), the one period ahead expected dividends  $E_t(D_{t+1})$  are deduced from a rate of growth equation:<sup>7</sup>

$$\Delta ID_t = (\alpha_0 + \alpha_1 \Delta ID_{t-1} + \dots + \alpha_p \Delta ID_{t-p}) + (\beta_0 + \beta_1 \Delta ID_{t-1} + \dots + \beta_p \Delta ID_{t-p}) \times \Omega(\Delta ID_{t-d}, \gamma, c) + \nu_t \quad (7)$$

where  $ID$  refers to the log-value of dividends. As a result of (7), the expected dividends are given by  $E_t(D_{t+1}) = D_t \exp[\Delta ID_t - \nu_t]$ . This model implies two regimes for the dividends associated with the extreme values of the transition function ( $\Omega(\cdot) = 0$  and  $\Omega(\cdot) = 1$ ), but allows for a “continuum” of intermediate regimes when  $0 \leq \Omega(\cdot) \leq 1$ .

We carried out preliminary linearity tests (LM tests) introduced by Lukkonen *et al.* (1988) to test the null hypothesis of linearity against its alternative of nonlinearity. The implementation of these tests is described in Appendix 1 (§ A-1.1). The results show that the dividend dynamics are nonlinear for all countries and highlight two significant regimes that characterize the dividend dynamics of the MSCI indexes (see Appendix 1, § A-1.2). This may be due to the coexistence of heterogeneous dividend policies and to changes in management strategies which can induce persistence and discontinuity in dividend dynamics.<sup>8</sup> Interestingly, the LSTAR model is retained for Germany and the USA while an ESTAR model is estimated for the five other countries. The estimation results point to evidence of different regimes characterizing dividend dynamics. The estimated transition speed ( $\hat{\gamma}$ ) is quite small for most indexes, indicating that the transition between these regimes is slow, due to the smooth character of the dividend series. When applying the misspecification tests proposed by Eitrheim and Teräsvirta (1996) to check the specification of the selected STAR model, we find that

<sup>7</sup>  $(\alpha_0, \alpha_1, \dots, \alpha_p)$  and  $(\beta_0, \beta_1, \dots, \beta_p)$  are respectively the autoregressive coefficients in the first and second regime,  $d$  is the lag parameter defining the transition variable ( $d \geq 1$ ),  $\gamma$  is the transition speed between the regimes, and  $c$  is the threshold parameter.  $\Omega(\cdot)$  is the transition function which is continuous and bounded between 0 and 1.  $\Omega(\cdot)$  is either logistic ( $\Omega(\Delta ID_{t-d}, \gamma, c) = (1 + \exp\{-\gamma(\Delta ID_{t-d} - c)\})^{-1}, \gamma > 0$ ) or exponential. ( $\Omega(\Delta ID_{t-d}, \gamma, c) = 1 - \exp\{-\gamma(\Delta ID_{t-d} - c)^2\}, \gamma > 0$ ). It implies respectively a Logistic STAR (LSTAR) model or an Exponential STAR (ESTAR) model.

<sup>8</sup> For more explanations about nonlinearity characterizing dividend dynamics, see Jawadi (2009).

residual sets have white noise properties, suggesting that representing  $E_t(D_{t+1})$  by a STAR model is in line with the REH.

After replacing  $E_t(D_{t+1})$  in the equation (6) by the estimated values of the appropriate STAR model (7), the initial values  $F_0$  and  $\Phi_0$  were chosen in the interval  $[P_0-50\%, P_0+50\%]$  and  $[0\%, 8\%]$  respectively in order to minimize  $Q = \sum_{t=1}^T (p_t - f_t)^2$ , where  $p_t$  and  $f_t$  are the log- values of price and fundamental value respectively, while  $T$  indicates the number of observations. Estimates for  $F_0$  and  $\Phi_0$  given in table 2.

**Table 2 - Initial fundamental values and risk premia estimates**

	Germany	Canada	USA	France	UK	Italy	Japan
$\hat{F}_0$	73.11	80.32	85.12	72.57	86.13	57.25	129.15
$P_0$	100	100	100	103.67	100	80.51	100
$\hat{\Phi}_0$	3.8%	4.8%	5.4%	3.95%	4.29%	6.01%	6.58%

Note:  $P_0$  and  $\hat{F}_0$  are the initial values of observed price and of the fundamental value respectively, while  $\hat{\Phi}_0$  is the risk premium estimate.

We note that, apart from Japan, all the price indexes were overvalued at the beginning of the period. Otherwise, the risk premium values seem realistic since the G7 premia average is about 5% per year, which is coherent with the values obtained in the literature (among others, see Mehra and Prescott (1985), Siegel (1992), Cochrane (1997) and Pastor and Stambaugh (2000)). Figures presented in Appendix 2 show that the fundamental values are smooth in comparison with market prices for all seven countries, and this property is in accordance with the results proposed by Manzan (2003) and Boswijk *et al.* (2007).<sup>9</sup> This feature means stock prices are often last away from their fundamentals for a long time, as underlined by Black *et al.* (2003) and Manzan (2003).

<sup>9</sup> The smooth character of fundamental values is implied by the DDM, not by the STAR model used to determine the expected dividend. Indeed, according to the DDM, the fundamental value is the sum of discounted future dividends, this sum leading to formally removing the short term movements in dividend and interest rate.



### 3.2 - Modeling stock price deviations with a STECM

In a frictionless market and in the absence of arbitrage costs in particular, stock price adjustment is symmetrical, continuous and characterized by a constant speed of adjustment (see § 2.1).

A linear error correcting model (LECM) is therefore appropriate:

$$\Delta z_t = k + \rho z_{t-1} + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \varepsilon_t \quad (8)$$

where  $\rho$  characterizes the intensity of the stock price mean-reversion mechanism while  $\varepsilon_t$  is a white noise. However, when the stock market is not frictionless, the LECM cannot describe stock price adjustment. In particular, arbitrage costs induce discontinuities in arbitrages and imply a nonlinear mean reversion phenomenon with a time-varying speed. Moreover, as shown in § 2.1, when arbitrage costs are heterogeneous, the relevant model is a STECM. Introduced by Granger and Teräsvirta (1993), Van Dijk and Franses (2000) and Van Dijk *et al.* (2002), the STECM defines an adjustment process that depends on the sign (LSTECM) or size (ESTECCM) of the deviation. Let  $z_t = p_t - f_t$  be the relative deviation, where  $p_t$  and  $f_t$  are the log-values of price and the fundamentals, respectively.

The general expression of a STECM is given as:

$$\Delta z_t = k + \rho_1 z_{t-1} \times [1 - \Omega(\gamma, z_{t-d}, c)] + \rho_2 z_{t-1} \times \Omega(\gamma, z_{t-d}, c) + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \mu_t \quad (9)$$

where  $\rho_1$  and  $\rho_2$  are the adjustment coefficients in the first and second regime respectively,  $z_{t-1}$  is the lagged error-correction term,  $z_{t-d}$  is the transition variable,  $\phi_i$  are the AR parameters,  $\Omega(\cdot)$  is the transition function and  $\mu_t \rightarrow N(0, \sigma_\mu^2)$  is an error term.

It is worth noting that relationship (9) corresponds to Anderson's model (see (3) and (4)) if the transition function  $\Omega(\cdot)$  is an exponential function and if the conditions  $k = \rho_1 = c = 0$  and  $\phi_i = 0 \forall i = 2, \dots, p$  hold.<sup>10</sup> For  $\Omega(\cdot) = 0$  or  $\Omega(\cdot) = 1$ , the STECM (9) leads to the LECM (8). For the values of  $\Omega(\cdot)$  ranging between 0 and 1, the adjustment is gradual rather than abrupt and its speed depends on

<sup>10</sup> For more details about these conditions, see equations (10) and (11).

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3 the size or the sign of the deviation: the larger the deviation, the stronger the tendency to move back to  
4 zero. This implies that even though  $\rho_1 \geq 0$ ,  $\rho_2$  and  $(\rho_1 + \rho_2)$  should be strictly negative and the linear  
5 adjustment term  $\rho$  must belong to the interval  $[\rho_1, \rho_1 + \rho_2]$  in order to comply with a nonlinear mean-  
6 reversion process in stock prices (see Michael *et al.* (1997) among others). In the central regime, when  
7 the deviations are small,  $z_t$  is close to a unit root process approaching a random walk ( $z_t \rightarrow I(1)$ ), and  
8 may also demonstrate explosive behavior (when  $\rho_1 \geq 1$ ). In this regime, the deviations are persistent  
9 and stock prices can remain distant from their fundamentals for a long time. On the other hand, in the  
10 outer regimes, when deviations are large enough to pay for arbitrage costs, the process would be  
11 mean-reverting with a convergence speed that depends on the deviation size, and  $z_t$  may approach a  
12 white noise. Furthermore, for each date, the adjustment process is described by a combination of the  
13 two adjustment patterns weighted by the transition function  $\Omega_t$ , and scaled by the coefficients  $\rho_1$  and  
14  $\rho_2$ . The greater the value of  $\rho_2$  relative to  $\rho_1$ , the larger stock price deviations will be. Note that such  
15 behavior can escape from the conventional linear cointegration framework in the sense that  $H_0: \rho = 0$   
16 (i.e. LECM) may not be rejected even though stock prices are nonlinearly mean-reverting (i.e.  $(\rho_1 +$   
17  $\rho_2) < 0$  in the STECM). Conventional cointegration tests thus appear to be relatively ineffective in the  
18 presence of market frictions (see Taylor *et al.* (2001)). In fact, what appears important is testing the  
19 linear adjustment hypothesis against its alternative of nonlinearity and testing the cointegration  
20 hypothesis in a nonlinear framework by using nonlinear cointegration tests.  
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55 Before moving on to the nonlinear adjustment tests, in line with Peel and Taylor (2000), we  
56 now consider three hypotheses leading to a restricted specification of the STECM which have not  
57 previously been considered for stock markets:  
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$$H_0^a : k' = c = 0,$$

$$H_0^b : \rho_1 + \rho_2 = -1 / H_0^a, \quad (10)$$

$$H_0^c : \rho_1 = 0 \text{ s.t. } H_0^a \text{ and } H_0^b$$

Under these conditions, the equation (10) may be simplified to:

$$\Delta z_t = -z_{t-1} \times \Omega(\gamma, z_{t-d}) + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \mu_t \quad (11)$$

The equation (11) is in conformity with the theoretical relation prevailing when arbitrage costs are heterogeneous (see above, (3) and (4)): stock price deviations are characterized by two regimes, namely, a random walk in the central regime (when arbitrage costs are larger than expected arbitrage gains) and a white noise in the outer regimes (when arbitrage costs are smaller than expected arbitrage gains). Furthermore, as it is shown below, these hypotheses enable us to compute two indicators proposed by Peel and Taylor (2000), the first giving the magnitude of under- and overvaluation of stock prices per date, and the second a measure per date of the speed of convergence between stock prices and fundamentals. In practice, both the unconstrained STECM (9) and the constrained STECM (11) have been estimated independently in order to test the restrictive hypotheses  $(H_0^a, H_0^b, H_0^c)$  by using a likelihood ratio test.

### 3.3 – Empirical relevance of the STECM specification

To be defined, the STECM specification requires both the form of the transition function  $\Omega(\cdot)$  and of the basic linear model (LECM) from which regimes can be deduced. Concerning the linear model, in order to capture the interdependence or contagion between stock markets, we introduce the current and lagged US stock price deviations in the LECM as an exogenous variable in the adjustment process of the other G7 countries. The German (respectively French) deviations are also introduced for France (respectively Germany) to capture the interdependences or contagion between these two markets. In the same way, the Japanese deviations are introduced in the US stock price adjustment equation. Moreover, change in the domestic risk-free interest rate is retained as an exogenous variable in the model to capture a possible liquidity effect while rate of growth in the domestic industrial production is also introduced to capture a possible influence of the economic activity. Formally, the equation (8) has been extended for each country as follows:

$$\begin{aligned} \Delta z_t^l = & k + \rho z_{t-1} + \sum_{i=1}^p \phi_i \Delta z_{t-i}^l + \sum_{j=0}^p \alpha_j \Delta z_{t-j}^{USA} + \sum_{j=0}^p \alpha_j' \Delta z_{t-j}^F + \sum_{j=0}^p \alpha_j'' \Delta z_{t-j}^G + \\ & + \sum_{j=0}^p \beta_j \Delta z_{t-j}^J + \sum_{j=0}^p \theta_j \Delta i_{0,t-j}^l + \sum_{j=0}^p \theta_j' \Delta q_{t-j}^l + \varepsilon_t^l \end{aligned} \quad (12)$$

where  $z_t^{USA}$ ,  $z_t^F$ ,  $z_t^G$  and  $z_t^J$  are the US, French, German and Japanese stock price deviations respectively.  $l = USA, France (F), Germany (G), United Kingdom (UK), Canada (Ca), Italy (I)$  and Japan (J). For  $l = USA$ ,  $\alpha_j = \alpha_j' = \alpha_j'' = 0, \forall j$ . For  $l = F$ ,  $\alpha_j' = \beta_j = 0, \forall j$ . For  $l = G$ ,  $\beta_j = \alpha_j'' = 0, \forall j$ . For  $l = UK = Ca = I = J$ ,  $\alpha_j' = \alpha_j'' = \beta_j = 0, \forall j$ .  $i_0$  is the risk-free interest rate and  $q_t$  is the log-index of industrial production.

Many specifications have been tested to determine the number of lags, using the AIC, BIC, Ljung-Box Statistics and the autocorrelation functions. As a result, we retain  $p = 1$  for Germany, the USA, France, Italy and Japan;  $p = 2$  for the UK and  $p = 3$  for Canada. The LECMs (12) are estimated by the OLS and the results are given in Appendix 3. Since contemporary values of residuals  $\varepsilon_t^l$  for all seven countries are found to be insignificantly correlated, it was not necessary to estimate the seven equations as a system.<sup>11</sup> Our results show that most of the AR parameters are statistically significant at 5% or 10%. The adjustment coefficient  $\hat{\rho}$  is negative and statistically significant, suggesting a mean reversion phenomenon in stock prices for all countries, apart from the US and Italy. For these last two countries, the rejection only concerns the strong hypothesis of a linear mean reversion and then leaves entire the possibility of a nonlinear influence of the error correction term. In addition, an interdependence or contagion effect is identified at the 5% level, since the US market has a strong positive effect on all the other stock markets. A mutual contagion effect is also shown respectively between German and French and between US and Japanese stock markets. Otherwise, as expected with the liquidity effect hypothesis, changes in short-term interest rate have a negative effect on

<sup>11</sup> We nevertheless applied an SUR system estimate: estimates were insignificantly different from those obtained with the OLS. This result confirms that the seven equations can be estimated independently.

changes in the deviations for all countries, while changes to industrial production have a positive delayed effect for Canada, the USA, the UK and Japan.

Because the linear modeling and usual unit root tests are less powerful when data are generated by a nonlinear process, we used “mixing tests”, which are relevant for time series that are stationary but generated by a nonlinear process characterizing different types of behavior. In particular, several mixing tests were developed to investigate nonlinearity and nonstationarity hypotheses for time series.<sup>12</sup> Among these tests, the KPSS test by Kwiatkowski et al. (1992) and the R/S test by Lo (1991) were applied to check the mixing and nonlinear cointegration hypotheses. Both tests consider the null hypothesis of “mixing” or short-range dependence against the alternative of “non-mixing.” For the KPSS test, Schwert (1989) suggests two values for the truncation parameter:

$$l_4 = \text{int} \left[ 4 \left( \frac{T}{100} \right)^{\frac{1}{4}} \right] \text{ and } l_{12} = \text{int} \left[ 12 \left( \frac{T}{100} \right)^{\frac{1}{4}} \right]$$

where  $T$  is the number of observations and  $\text{int}[\cdot]$  denotes the integer part. In performing the R/S test,  $q$  is determined as follows (Andrews, 1991):

$$q_t = [K_T], \text{ where } K_T = \left( \frac{3T}{2} \right)^{\frac{1}{3}} \left( \frac{2\hat{\rho}}{1-\hat{\lambda}^2} \right)^{\frac{2}{3}}, \quad [K_T] = \text{int}(K_T) \text{ and } \hat{\lambda} \text{ is the first-order}$$

autocorrelation coefficient. The results of both tests are reported in Table 3.<sup>13</sup> The mixing hypothesis is accepted at 5% for Germany, the UK and Italy according to the KPSS and at 10% for France, while it is retained for all countries according to the R/S test. This suggests the existence of a cointegration relationship between stock prices indexes and their fundamentals and the existence of a nonlinear mean reversion.

<sup>12</sup> For more details about mixing tests and conditions, see Dufrenot and Mignon (2002).

<sup>13</sup> Using similar tests, Hasanov (2009) tests the weak efficiency form for the Australian and New Zealand stock markets

**Table 3 - Nonlinear cointegration using mixing tests**

countries	KPSS test		R/S test
	$I_4$	$I_{12}$	Andrews (q)
Germany	0.35	0.14	1.1
Canada	0.57	0.25	1.3
USA	1.02	0.42	1.4
France	0.50	0.22	1.2
UK	0.27	0.12	1.1
Italy	0.22	0.10	1.0
Japan	0.92	0.38	1.5

Note: statistics are described in the text above the table.

In order to check for the nature of the nonlinear dependence, we will now turn to the relevance of the nonlinear stock price adjustment hypothesis. We applied the LM linearity tests where the transition variable is supposed to be the lagged deviation  $z_{t-d}$  for  $1 \leq d \leq 12$  months.<sup>14</sup> Besides the standard LM linearity tests, we apply linearity tests that are robust to heteroscedasticity (Van Dijk *et al.* (2002)). According to these tests, the rejection of linearity implies that nonlinearity is relevant, hence suggesting the rejection of the one regime hypothesis.

**Table 4 – LM<sub>3</sub> linearity test**

Delay	Germany	Canada	USA	France	UK	Italy	Japan
$p$	1	3	1	1	2	1	1
$\hat{d}$	10	2	6	2	1	6	10
$p$ -value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: Table 4 gives the p-values.  $p$  indicates the number of lags in the change of the deviation and  $\hat{d}$  the optimal number of lags in the transition variable  $z_{t-d}$ . For the nature of the LM<sub>3</sub> test, see footnote (14).

<sup>14</sup> In line with Teräsvirta (1994) and recently Van Dijk *et al.* (2002), we applied several LM tests (LM<sub>1</sub>, LM<sub>2</sub>, LM<sub>3</sub>, LM<sub>3</sub><sup>s</sup> and LM<sub>4</sub>) for all possible values of  $d$ :  $1 \leq d \leq 12$ . The optimal value of the delay parameter  $\hat{d}$  is such that linearity is rejected the most strongly. Thus,  $\hat{d}$  should maximize the LM statistics and minimize the p-values of the linearity tests. In practice, all tests unanimously support nonlinearity, so we focus only on the results of the LM<sub>3</sub> test that is available to test linearity against both exponential and logistic STECMs.

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3 The results presented in table 4 show that the  $LM_3$  test strongly rejects the linearity  
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5 hypothesis at 5% for the seven stock markets, and this conclusion is in accordance with that of  
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7 Manzan (2003) and Boswijk *et al.* (2007).<sup>15</sup> Although the optimal value of  $d$  varies across the different  
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9 countries ( $d = 10$  for Germany and Japan,  $d = 2$  for Canada and France,  $d = 6$  for the USA and Italy,  
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11 and  $d = 1$  for the UK), the validity of the STECM to describe stock price adjustment suggests that the  
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13 expected effects of heterogeneous arbitrage costs are not rejected.<sup>16</sup>  
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19 The last step in the STECM specification is the choice of transition function  $\Omega(\cdot)$ . Even though  
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21 several previous studies retained *a priori* an exponential function which is in line with the arbitrage  
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23 cost hypothesis (i.e. Michael *et al.* (1997), Manzan (2003) and Boswijk *et al.* (2007)), we tested the  
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25 ESTECM against the LSTECM on the basis of tests developed by Teräsvirta (1994) and Escribano  
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27 and Jordā (1999). Table 5 gives the results for the unrestricted STECM.  
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<sup>15</sup> These authors only apply the standard LM linearity tests.

<sup>16</sup> We briefly describe the STECM methodology and LM tests. More details can be found in Van Dijk *et al.* (2002) and Jawadi (2006).

Table 5 - Selecting the transition function  $\Omega(\cdot)$ 

Countries	Delay parameter	<i>p-values</i> (Teräsvirta tests)			<i>p-values</i> (Escribano and Jordä tests)		Conclusion
		$H_{03}$	$H_{02}$	$H_{01}$	$H_{0L}$	$H_{0E}$	
	$\hat{d}$						<i>Model</i>
<b>Germany</b>	10	0.09	0.01	0.00	0.00	0.001	ESTECCM
<b>Canada</b>	2	0.01	0.00	0.00	0.008	0.00	ESTECCM
<b>The USA</b>	6	0.0009	0.00	0.001	0.003	0.00	ESTECCM
<b>France</b>	2	0.15	0.008	0.04	0.002	0.00	ESTECCM
<b>The UK</b>	1	0.00	0.00	0.01	0.00	0.00	ESTECCM or LSTECCM
<b>Italy</b>	6	0.21	0.002	0.54	0.007	0.00	ESTECCM
<b>Japan</b>	10	0.24	0.004	0.001	0.00	0.00	ESTECCM or LSTECCM

Note: Teräsvirta tests and Escribano & Jordä tests allow to specify the transition function while testing whether it is exponential ( $\Omega(z_{t-d}, \gamma) = 1 - \exp\{-\gamma (z_{t-d})^2\}$ ) or logistic ( $\Omega(z_{t-d}, \gamma) = (1 + \exp\{-\gamma (z_{t-d})\})^{-1}$ ).  $H_{01}$ ,  $H_{02}$  and  $H_{03}$  are the null hypotheses in Teräsvirta tests which are based on the Fisher statistic.  $H_{0L}$  and  $H_{0E}$  are the null hypotheses tested by Escribano and Jordä (1999) and correspond to the auxiliary regression of the linearity tests (LM<sub>3</sub> and LM<sub>4</sub>). More details about these tests and about the  $H_{01}$ ,  $H_{02}$  and  $H_{03}$  null hypotheses can be found in Van Dijk *et al.* (2002) and Jawadi (2006).

According to table 5, the ESTECCM can be retained to describe the stock price adjustment for most of the countries since the  $H_{02}$  hypothesis is rejected more strongly than the  $H_{01}$  and  $H_{03}$  hypotheses. These results are as one would expect when arbitrage costs are heterogeneous. Although both models may be retained for the UK and Japan, while estimating these two models, the information criteria appear to conclude in favor of the ESTECCM. As a result, the ESTECCM is therefore retained for all the G7-MSCI indexes.



### 3.4 – ESTECM estimations for the G7 stock prices

The non-restricted ESTECM (9) and the restricted ESTECM (11) are estimated by the NLS method, both models being augmented with exogenous variables as indicated in (12). We tested the  $H_0^a, H_0^b, H_0^c$  restrictions (10) using the likelihood ratio  $LR=2 [L(\theta_1)-L(\theta_0)]$ , where  $L(\theta_0)$  and  $L(\theta_1)$  are respectively the log-likelihood of the restricted and non-restricted STECM. The LR ratio follows a  $\chi^2(q)$  distribution where  $q$  is the number of constraints. The results reported in Table 6 show that, for the seven MSCI indexes, the  $H_0^a, H_0^b$  and  $H_0^c$  restrictions are statistically accepted at 5%. According to this restricted specification of the ESTECM, arbitrage costs are implicitly captured at each date. Indeed, since the calculated value of the endogenous variable at time  $t$  is a weighted average of the values corresponding to the outer and central regimes, the first regime (white noise) will appear to be dominant when the costs are smaller than the expected gains, while the second regime will appear to be dominant (random walk) when arbitrage costs are higher than expected gains. This property of the model is far more interesting than it appears at first sight since arbitrage costs are time-varying (for example, transaction costs tended to decrease during last years of the period).

**Table 6 - Testing  $H_0^a, H_0^b$  and  $H_0^c$  restrictions with the Likelihood Ratio**

Countries	Germany	Canada	USA	France	UK	Italy	Japan
LR <sup>a</sup>	0.8	0.79	0.85	0.58	0.12	0.79	0.28
LR <sup>b</sup>	0.89	0.93	0.98	0.82	0.09	0.77	0.11
LR <sup>c</sup>	0.93	0.74	0.97	0.90	0.08	0.67	0.80

Note: the table gives the *p-values* issued from the LR test.

The ESTECM estimates under  $H_0^a, H_0^b$  and  $H_0^c$  are reported in Table 7. The AR parameters are statistically significant at 5%. There is strong evidence of contagion or interdependence between the stock markets. In particular, the current and lagged US stock price deviations significantly affect the stock price adjustment of the other countries. There is also significant interdependence between the French and German and between the US and Japanese stock markets. Furthermore, interest rate

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3 variations negatively affect the stock market deviations, while changes in industrial production have a  
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5 significant positive influence for Japan (at 5%) and the USA (at 10%) only. The transition speed  $\gamma$  is  
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7 statistically significant at 5% (10% for the UK). The values of  $\gamma$  are relatively low, hence confirming  
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9 the hypothesis of a smooth transition. This implies that stock prices are nonlinearly mean-reverting  
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11 with an adjustment speed that depends on the size of deviations from the fundamentals at each date.  
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13 For small deviations, stock prices can remain a long time distant from their fundamentals, but, for  
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15 large deviations, when deviations exceed the arbitrage costs, arbitrage becomes active and the prices  
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17 quickly revert back to fundamentals.  
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Table 7 - Restricted ESTECM estimates

	Germany	Canada	USA	France	UK	Italy	Japan
p	1	3	1	1	2	1	1
$\hat{d}$	10	2	6	2	1	6	10
$\hat{\gamma}$	0.62 (3.8)*	0.10 (4.4)*	0.57 (3.6)*	8.53 (3.29)*	0.64 (1.63)**	9.94 (2.7)*	7.65 (2.18)*
$\hat{\phi}_1$	-0.06 (-1.75)**	-0.08 (-1.63)**	-0.03 (-1.69)**	0.06 (2.1)*	-0.02 (-0.44)	0.14 (2.9)*	-0.02 (-1.63)**
$\hat{\phi}_2$	-	-0.02 (-1.1)	-	-	-0.46 (-9.7)*	-	-
$\hat{\phi}_3$	-	0.17 (5.4)*	-	-	-	-	-
$\hat{\alpha}_0$	0.16 (3.07)*	0.68 (16.1)*	-	0.44 (7.9)*	1.08 (21.7)*	0.98 (13.1)*	0.06 (1.2)
$\hat{\alpha}_1$	0.12 (2.4)*	0.16 (2.9)*	-	-	-0.05 (-0.9)	0.38 (5.08)*	0.35 (6.07)*
$\hat{\alpha}_2$	-	-	-	-	0.37 (6.2)*	0.42 (5.8)*	-
$\hat{\alpha}'_0$	0.19 (3.6)*	-	-	-	-	-	-
$\hat{\alpha}''_0$	-	-	-	0.9 (20.4)*	-	-	-
$\hat{\beta}_0$	-	-	0.18 (3.9)*	-	-	-	-
$\hat{\theta}_0$	-0.007 (-1.73)**	-0.01 (-4.2)*	-0.03 (-6.06)	-0.02 (-5.8)*	-0.005 (-1.8)**	-0.06 (-10.3)*	-0.01 (-2.3)*
$\hat{\theta}'_0$	-	-	-	-	-	-	0.34 (1.98)*
$\hat{\theta}'_1$	-	-	0.41 (1.8)**	-	-	-	-
$\hat{\gamma} \times \sigma_z^2$	0.07	0.006	0.08	1.2	0.04	1.3	1.1
ADF (p)	-13.9* (p = 0)	-14.3* (p = 0)	-14.8* (p = 0)	-14.6* (p = 0)	-20.3* (p = 0)	-14.6* (p = 0)	-14.07* (p = 0)
DW	1.97	2.04	2.02	2.03	2.01	2.0	2.02
Q(4)	0.12	0.6	2.07	1.5	0.95	4.6	2.2
Q(12)	5.31	29.2	9.34	13.07	14.2	15.5	6.7
ARCH (q)	5.06* (q = 1)	10.8* (q = 1)	14.3* (q = 1)	0.55* (q = 1)	17.7* (q = 1)	7.9* (q = 1)	18.8* (q = 2)
Nb. of iterations	18	47	30	45	27	25	28

Note: The values under the estimates are the t-value ratios. Q(4) and Q(12) are the Ljung-Box statistics. (\*) and (\*\*) indicate respectively the significativity at 5% and 10%. ADF and ARCH are the statistics of the ADF and ARCH tests. The estimated model is the equation (9) under hypotheses  $H_0^a$ ,  $H_0^b$  and  $H_0^c$  and augmented by exogenous variables as in equation (12):

$$\Delta z_t^l = -z_{t-1} \times \Omega(\gamma, z_{t-d}) + \sum_{i=1}^p \phi_i \Delta z_{t-i}^l + \sum_{j=0}^p \alpha_j \Delta z_{t-j}^{USA} + \sum_{j=0}^p \alpha'_j \Delta z_{t-j}^F + \sum_{j=0}^p \alpha''_j \Delta z_{t-j}^G + \sum_{j=0}^p \beta_j \Delta z_{t-j}^J + \sum_{j=0}^p \theta_j \Delta i_{0,t-j}^l + \sum_{j=0}^p \theta'_j \Delta q_{t-j}^l + \mu_t^l$$

Figures in Appendix 4 plot for the seven countries the estimated values of the transition functions (vertical axis) against the lagged values of the stock price deviations (horizontal axis). It can be seen that observations are distributed around the equilibrium value on the left and the right side, hence confirming the choice of the exponential function and the relevance of the regimes. Moreover, the shapes of the functions are sharper for France, Italy and Japan (i.e. the functions increase quickly with deviations), suggesting that the transition occurs faster in these countries compared to the others. Furthermore, for these countries, the transition functions sometimes reach the unity, indicating strong evidence of significant persistence.<sup>17</sup> Finally, to check the validity of the ESTECM estimations under  $H_0^a$ ,  $H_0^b$  and  $H_0^c$ , three misspecification tests are applied: a test of residual autocorrelation, a test of parameter stability and a test of omitted linearity (Appendix 5). First, the results show that the residuals are independent for the seven stock markets. Second, the hypothesis of parameter stability is accepted at 5% except for the UK. Third, applying the robust linearity tests to the residuals for different values of  $d$ ,  $1 \leq d \leq 12$ , we find that the nonlinearity is well captured by the ESTECM, except for the UK. Overall, these results confirm our ESTECM specification, although they suggest that there is a missing exogenous variable in the UK equation.

### 3.5 - Gauging under- and overvaluation phases and mean reversion strengths

To gauge the degree of the under- and overvaluation of stock prices and the mean reversion strength over time, we estimate the two indicators  $\Pi(z_t)$  and  $\Psi(z_t)$  proposed by Peel and Taylor (2000) for the foreign exchange market, but which has not yet been applied to stock markets. The first indicator is defined as follows:

$$\Pi(z_t) = 100 \times \Omega(z_t) \times \text{sign}(z_t), \quad \text{sign}(z_t) \equiv \frac{z_t}{|z_t|}, \quad -100 \leq \Pi(z_t) \leq 100 \quad (13)$$

The use of  $\Pi(z_t)$  is based on the property that the transition function  $\Omega(\cdot)$  measures the magnitude of the deviation from equilibrium since it implies a low degree of mean reversion for small

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<sup>17</sup> Overall, our results are in line with Shen *et al.* (2007) who used a nonlinear cointegration test to examine long-run symmetric equilibrium relationships between the Chinese Shanghai and Shenzhen stock markets. Their findings also show evidence of nonlinear mean reversion with time-varying adjustment speed.

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3 deviations and a high degree of mean reversion for large deviations. This is why, substituting  $z_t$  to  
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5  $z_{t-d}$  in the exponential function  $\Omega(\cdot)$ , and affecting the sign of  $z_t$  to the latter, allows us to determine  
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7 the magnitude of the under- or overvaluation at any date. The condition  $\Pi(z_t) \rightarrow 0$  means that the  
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9 stock price approaches the fundamental value, while  $\Pi(z_t) > 0$  (respectively  $\Pi(z_t) < 0$ ) implies that the  
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11 stock price is overvalued (respectively undervalued).  
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18 The indicator of the mean reversion strength proposed by Peel and Taylor depends directly on  
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20 the importance of the autoregressive component in the STECM, and it can be shown that it just equals  
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22 one minus the transition function:  
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$$\Psi(z_t) = 1 - \Omega(z_{t-d}), \quad 0 \leq \Psi(z_t) \leq 1 \quad (14)$$

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28 When  $\Psi(z_t)$  moves toward 1, the speed of adjustment decreases and  $z_t$  converges toward a random  
29  
30 walk. Conversely, when  $\Psi(z_t)$  moves toward 0, the speed of adjustment increases and  $z_t$  converges  
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32 toward a white noise.  
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38 Calculating these two indicators for the stock markets leads to interesting new results. The  
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40 values of  $\Pi(z_t)$  per date for the G7 stock markets are reported by the figures given in Appendix 6.  
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42 One can observe long durations and high magnitude of under - and overvaluation of the MSCI stock  
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44 indexes over the period. The values per date of  $\Psi(z_t)$  are reported in figures given in Appendix 7.<sup>18</sup>  
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46 The convergence speeds appear to be strongly time-varying, sometimes smooth and sometimes abrupt.  
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48 The adjustment speeds often appear to be greater when the stock price deviations are large. The  
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50 adjustment speeds tend to be higher during crises (i.e. 1973, 1979, 1987). For the US, our results  
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52 confirm those of Boswijk *et al* (2007), showing that the S&P500 index was poorly mean-reverting  
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54 during the period 1990-95. Overall, the dynamics of  $\Pi(z_t)$  and  $\Psi(z_t)$  show that, at each date, stock  
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<sup>18</sup> Note that the average adjustment delay from prices to fundamentals is about 5 months for the seven countries. This average is given by the sum of the optimal values of  $d$  for the G7-MSCI indexes divided by 7.

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3 price adjustment is highly dependant on the country considered. However, during the last years of the  
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5 period, it can be seen that, for almost all the countries, stock prices are near the fundamental value,  
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7 hence suggesting low expected profits. Although the fall in transaction costs has been a recognized  
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9 fact during the last years of the period, it is not surprising to observe that the speed of adjustment tends  
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11 to be slow or decreasing since increased risk of arbitrage and decreased expected profits may largely  
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13 compensate the decrease in transaction costs. This may also be considered as the emergence of a new  
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15 fundamentalist predominance phase, leading the stock markets back to the fundamentals.  
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#### 20 21 **4 - Concluding remarks** 22 23 24 25

26 This paper analyses the G7 stock price adjustments toward fundamentals represented by the  
27 dividend discount model. Adjustments are viewed as an “on/off” threshold error-correction model  
28 which works only when stock price deviations exceed a threshold determined by the investors’  
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30 arbitrage costs defined as the sum of transaction costs plus a risky arbitrage premium. We found  
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32 strong evidence of nonlinear mean-reversion in the G7 stock price indexes since the adjustment speeds  
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34 increase with the magnitudes of the deviations from fundamentals. The ESTECM proposed takes into  
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36 account the interdependences or contagion effects between stock markets and shows that stock price  
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38 deviations follow a process close to a random walk in the central regime where prices are close to  
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40 fundamentals (i.e. arbitrage costs are higher than expected gains) while deviations approach a white  
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42 noise process in the outer regimes (i.e. arbitrage costs are lower than expected gains). In accordance  
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44 with the expected effects due to heterogeneous arbitrage costs, our results show that the transition  
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46 from one regime to the other generally follows a smooth, although sometimes abrupt, pattern. Finally,  
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48 the dynamics of stock price adjustment are highly dependent on the date and the country under  
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50 consideration, and this result seems intuitive since both arbitrage costs and expected gains are  
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52 magnitudes characterized by rather high volatility.  
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## Appendix 1 – Estimating the expected dividends

### A1.1 – Preliminary tests of linearity: methodology

Below, we briefly describe linearity tests. Details of these tests may be found in Van Dijk *et al.* (2002). Formally, in equation (7), since  $\Omega(\cdot) = 0$  when  $\gamma = 0$ , the linearity hypothesis can be expressed as  $H_0 : \gamma = 0$ , and the linearity test consists of testing  $H_0$  against  $H_1 : \gamma > 0$ . In order to apply this test, Luukkonen *et al.* (1988) developed Lagrange Multiplier-type tests with a standard  $\chi^2$  distribution under  $H_0$ . However, in practice, the implementation of these tests yields some nuisance problems since the null hypothesis can be defined differently.<sup>19</sup> To solve this problem, the authors proposed replacing the transition function by its Taylor approximation. For an exponential function, they show that the appropriate LM linearity test associated with the first-order of the auxiliary regression is based on the statistic  $LM_2 = \frac{T(SCR_0 - SCR_2)}{SCR_0}$ , where  $SCR_2$  is the residual squared sum of nonlinear model and  $SCR_0$  is that of the linear model under  $H_0$ . The  $LM_2$  statistic is asymptotically distributed as a  $\chi^2(2(p + 1))$ . More recently, Escribano and Jorda (1999) showed that the first-order approximation of the exponential function is insufficient, and recommend using the second order Taylor approximation. Consequently, they developed another LM test. The statistic of this test is noted  $LM_4$  and is also distributed as  $\chi^2(4(p + 1))$ .<sup>20</sup> In practice, all the tests (five LM tests) are carried out to test the null of linearity for dividends against the alternative of nonlinearity. The tests are applied for several values of the delay parameter ( $d$ ). The optimal value of  $d$  that defines the transition variable is the one where linearity is most strongly rejected.

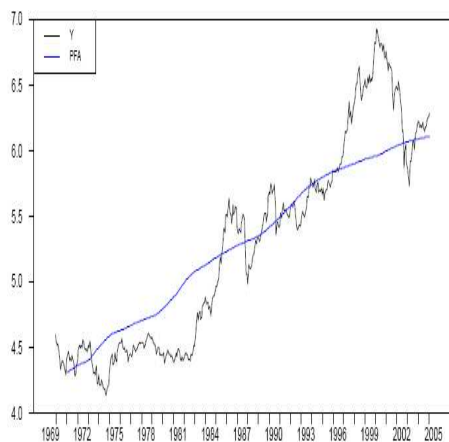
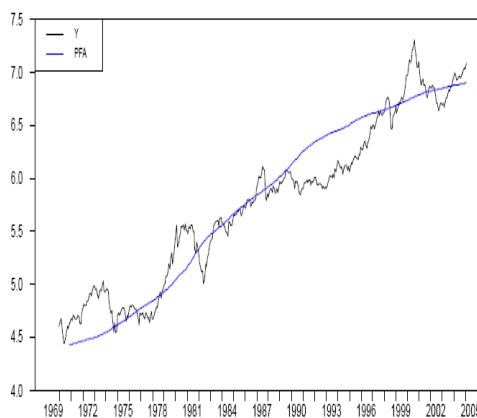
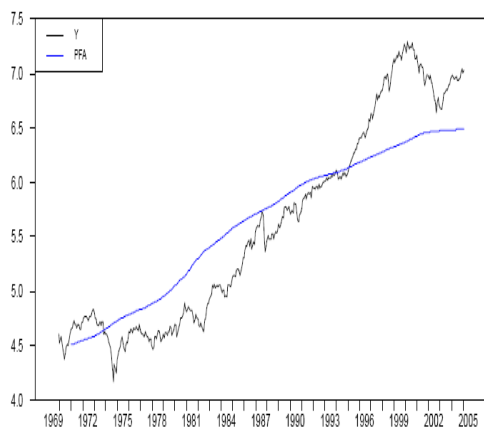
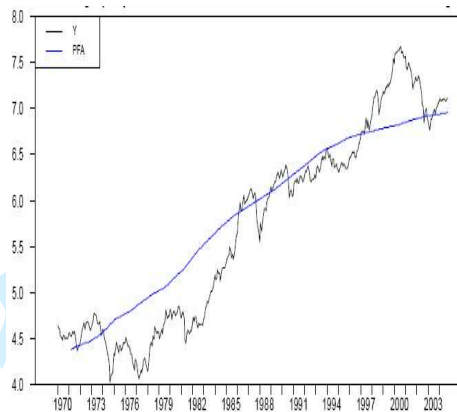
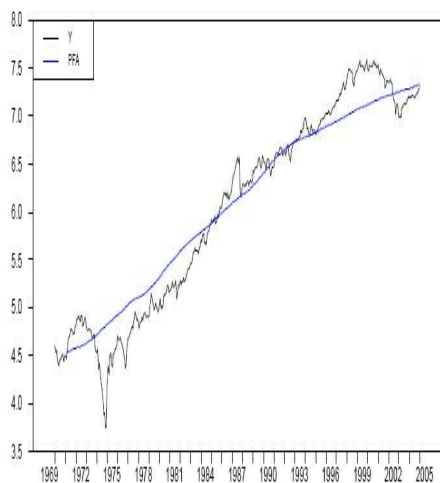
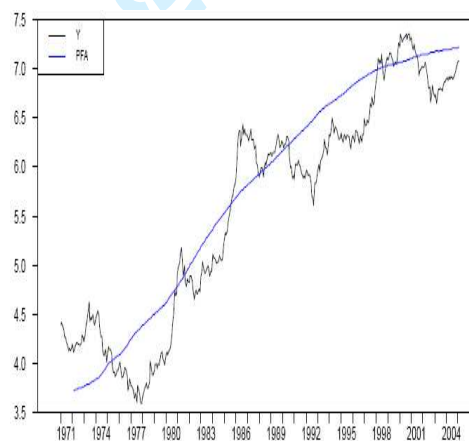
<sup>19</sup> For more details, see Luukkonen *et al.* (1988).

<sup>20</sup> For more details about these linearity tests, see Van Dijk *et al.* (2002), Jawadi and Koubbaa (2006) among others.

## A1.2 - STAR estimations of dividends (equation (7))

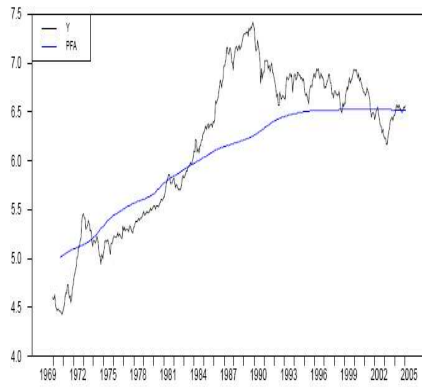
	Germany	Canada	USA	France	UK	Italy	Japan
$\alpha_0$	0.06 (0.9)	-0.26 <sup>a</sup> (-2.8)	0.28 <sup>a</sup> (2.2)	-0.006 (-0.1)	-0.008 (-0.27)	0.01 (0.1)	5.9 <sup>a</sup> (12.7)
$\alpha_1$	0.08 (1.1)	0.9 <sup>a</sup> (2.1)	-0.17 <sup>a</sup> (-10.8)	-0.73 <sup>a</sup> (-7.9)	-0.62 <sup>a</sup> (-4.6)	0.08 <sup>a</sup> (2.3)	-1.02 <sup>a</sup> (-5.7)
$\alpha_2$	-0.002 (-1.04)	-0.27 (-0.4)	-3.5 <sup>a</sup> (-6.3)	-0.71 <sup>a</sup> (-7.5)	-0.78 <sup>a</sup> (-7.3)	-0.06 <sup>a</sup> (-2.3)	-2.3 <sup>a</sup> (-6.8)
$\alpha_3$	0.2 <sup>b</sup> (1.9)	0.75 (1.1)	-2.0 <sup>a</sup> (-3.6)	-0.82 <sup>a</sup> (-9.7)	-0.49 <sup>a</sup> (-4.3)	0.03 (0.7)	-1.62 <sup>a</sup> (-13.9)
$\alpha_4$	0.01 <sup>a</sup> (2.1)	-0.17 (-0.28)	-1.7 <sup>a</sup> (-3.2)	-0.64 <sup>a</sup> (-5.6)	-0.52 <sup>a</sup> (-4.7)	0.01 (0.5)	-1.6 <sup>a</sup> (-15.1)
$\alpha_5$	-0.004 (-1.2)	1.5 <sup>a</sup> (2.2)	-0.33 (-0.7)	-0.68 <sup>a</sup> (-6.9)	-0.13 (-1.2)	-0.1 <sup>a</sup> (-3.3)	0.41 (1.3)
$\alpha_6$	0.08 <sup>a</sup> (2.2)	3.1 <sup>a</sup> (4.0)	0.13 (0.2)	-0.95 <sup>a</sup> (-8.1)	0.57 <sup>a</sup> (3.8)	0.25 <sup>a</sup> (8.6)	-0.69 <sup>a</sup> (-5.6)
$\alpha_7$	-0.04 (-0.7)	-3.09 <sup>a</sup> (-4.1)	0.39 (0.8)	-0.86 <sup>a</sup> (-7.3)	0.44 <sup>a</sup> (2.8)	-0.03 (-0.5)	-0.66 <sup>a</sup> (-3.7)
$\alpha_8$	0.07 (1.6)	1.02 <sup>a</sup> (3.5)	1.07 <sup>a</sup> (2.2)	-0.84 <sup>a</sup> (-6.7)	0.34 <sup>a</sup> (2.1)	0.04 <sup>b</sup> (1.8)	1.7 <sup>a</sup> (6.0)
$\alpha_9$	0.11 <sup>b</sup> (1.7)	0.15 <sup>b</sup> (1.7)	1.2 <sup>a</sup> (2.7)	1.02 <sup>b</sup> (1.6)	0.004 (0.03)	-0.11 (-1.0)	-
$\alpha_{10}$	0.09 (1.3)	-	1.6 <sup>a</sup> (2.9)	-2.2 <sup>a</sup> (-10.7)	-0.09 (-0.7)	0.02 (0.4)	-
$\alpha_{11}$	0.06 <sup>b</sup> (1.9)	-	0.59 (1.5)	-0.35 <sup>a</sup> (-2.1)	-0.32 <sup>a</sup> (-3.2)	0.06 (0.6)	-
$\alpha_{12}$	0.05 <sup>a</sup> (7.5)	-	-	0.27 <sup>a</sup> (2.8)	-0.16 <sup>b</sup> (-1.9)	-2.1 <sup>a</sup> (-2.0)	-
$\beta_0$	1.9 <sup>a</sup> (5.5)	0.3 <sup>a</sup> (3.1)	-0.26 <sup>a</sup> (-2.0)	5.1 <sup>a</sup> (2.2)	2.36 <sup>a</sup> (5.5)	4.1 <sup>a</sup> (7.1)	-5.8 <sup>a</sup> (-12.6)
$\beta_1$	0.42 <sup>a</sup> (4.2)	-2.0 <sup>a</sup> (-4.8)	0.7 <sup>a</sup> (3.8)	3.3 <sup>a</sup> (2.5)	-1.06 <sup>a</sup> (-5.2)	0.01 <sup>a</sup> (0.1)	0.96 <sup>a</sup> (4.3)
$\beta_2$	-0.31 <sup>a</sup> (-3.7)	-0.99 <sup>b</sup> (-1.7)	2.5 <sup>a</sup> (-4.4)	-1.9 <sup>a</sup> (-0.9)	-1.4 (-0.6)	0.04 (0.2)	2.4 <sup>a</sup> (6.8)
$\beta_3$	0.1 (1.1)	-1.3 <sup>b</sup> (-1.9)	1.3 <sup>a</sup> (2.3)	4.6 <sup>a</sup> (2.4)	-1.7 <sup>a</sup> (-5.5)	-2.3 <sup>a</sup> (-5.3)	1.6 <sup>a</sup> (13.8)
$\beta_4$	-0.43 <sup>a</sup> (-5.2)	-0.3 (-0.5)	1.0 <sup>b</sup> (1.7)	-4.1 <sup>a</sup> (-1.6)	-1.6 <sup>a</sup> (-4.4)	-1.1 <sup>a</sup> (-5.8)	1.7 <sup>a</sup> (15.1)
$\beta_5$	-0.12 (-1.3)	-2.0 <sup>a</sup> (-2.7)	-0.34 (-0.7)	0.21 (0.22)	-2.03 <sup>a</sup> (-5.1)	0.06 (0.2)	-0.4 (-1.2)
$\beta_6$	-0.25 <sup>a</sup> (-2.4)	-3.7 <sup>a</sup> (-4.6)	-0.59 (-1.1)	1.5 <sup>b</sup> (1.8)	-2.4 <sup>a</sup> (-7.4)	0.1 (0.4)	1.4 <sup>a</sup> (10.2)
$\beta_7$	-0.09 (-1.0)	-3.4 <sup>a</sup> (-4.4)	-0.84 (-1.5)	1.8 <sup>a</sup> (2.1)	-2.5 <sup>a</sup> (-7.9)	-3.8 <sup>a</sup> (-8.7)	0.86 <sup>a</sup> (3.8)
$\beta_8$	-0.33 <sup>a</sup> (2.6)	-1.3 <sup>a</sup> (-4.2)	-1.7 <sup>a</sup> (-3.3)	1.1 (1.3)	-2.1 <sup>a</sup> (-8.1)	-0.03 (-0.1)	-1.6 <sup>a</sup> (-5.8)
$\beta_9$	0.46 <sup>a</sup> (4.9)	-0.08 (-0.3)	-1.8 <sup>a</sup> (-3.6)	-0.7 (-0.8)	-1.6 <sup>a</sup> (-6.6)	7.3 <sup>a</sup> (3.8)	-
$\beta_{10}$	-0.26 (-0.9)	-	-1.9 <sup>a</sup> (-3.3)	2.1 <sup>a</sup> (2.4)	-1.9 <sup>a</sup> (-7.1)	-1.2 <sup>a</sup> (-12.8)	-
$\beta_{11}$	-0.09 (-0.3)	-	-0.8 <sup>a</sup> (-2.0)	0.3 (0.4)	-1.1 <sup>a</sup> (-6.4)	-0.17 <sup>a</sup> (-1.0)	-
$\beta_{12}$	0.29 <sup>a</sup> (2.0)	-	-	-0.5 <sup>b</sup> (-1.7)	-0.08 <sup>b</sup> (-1.8)	0.32 <sup>b</sup> (1.8)	-
$\gamma$	5.3 <sup>a</sup> (2.8)	1.43 <sup>a</sup> (6.9)	0.24 <sup>a</sup> (2.5)	5.2 <sup>a</sup> (2.8)	0.17 <sup>a</sup> (4.9)	0.16 <sup>a</sup> (3.8)	66.4 <sup>a</sup> (5.8)
$c$	0.78 <sup>a</sup> (14.8)	-0.34 <sup>a</sup> (-23.1)	-0.27 <sup>a</sup> (-6.9)	0.05 <sup>a</sup> (4.7)	-0.31 <sup>a</sup> (-2.0)	0.45 <sup>b</sup> (1.8)	0.04 <sup>a</sup> (22.1)
$R^2$	0.78	0.87	0.85	0.81	0.91	0.91	0.92
N	32	26	71	53	40	50	51

Notes: The values in brackets are the t-value ratio of the estimators. (a) and (b) designate respectively the significance at 5% and 10%. Canada: 1969:12-2005:02, France: 1970:01-2004:10, Germany: 1969:12-2005:02, Italy: 1971:01-2005:02, Japan: 1969:12-2005:02, the UK: 1969:12-2005:01 and the USA: 1969:12-2005:02.

Appendix 2 - Stock prices and fundamental values <sup>21</sup>*Germany**Canada**USA**France**UK**Italy*

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*Japan*



**Note:** **Y** and **PFA** are respectively the observed price and its estimated fundamental value in logarithm.

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## Appendix 3 - Stock price deviations: LECM estimations

	Germany	Canada	USA	France	UK	Italy	Japan
p	1	3	1	1	2	1	1
$\hat{\rho}$	-0.015 (-2.31)*	-0.011 (-1.68)**	0.06 (0.9)*	-0.0001 (-1.74)**	-0.025 (-2.95)*	-0.005 (-0.63)	-0.012 (-2.05)*
$\hat{\phi}_1$	-0.013 (-1.63)**	0.017 (0.4)	-0.04 (-1.8)**	0.029 (1.71)**	-0.007 (-1.15)	-0.016 (-1.81)**	0.012 (1.83)**
$\hat{\phi}_2$	-	-0.019 (-1.74)**	-	-	-0.14 (-2.92)*	-	-
$\hat{\phi}_3$	-	0.102 (3.18)*	-	-	-	-	-
$\hat{\alpha}_0$	0.293 (5.14)*	0.83 (22.6)	-	0.4 (7.05)*	0.79 (16.2)*	0.52 (7.06)*	0.43 (7.91)*
$\hat{\alpha}_1$	0.131 (2.35)*	0.09 (1.65)**	-	-	0.09 (1.65)**	0.16 (2.0)	0.2 (3.37)*
$\hat{\alpha}_2$	-	-	-	-	0.13 (2.09)*	0.14 (1.96)*	-
$\hat{\alpha}_3$	-	-	-	-	0.15 (3.06)*	-	-
$\hat{\alpha}_0$	0.49 (11.7)*	-	-	-	-	-	-
$\hat{\alpha}_0''$	-	-	-	0.51 (11.67)*	-	-	-
$\hat{\beta}_0$	-	-	0.15 (4.4)*	-	-	-	-
$\hat{\theta}_0$	-0.0007 (1.65)**	-0.011 (-3.8)*	-0.008 (2.57)	-0.011 (-2.4)*	-0.022 (-5.42)*	-0.011 (-2.16)*	-0.001 (-1.99)*
$\hat{\theta}_0$	-	-	-	-	-	-	0.29 (1.64)**
$\hat{\theta}_1$	-	0.22 (1.7)**	0.29 (1.69)**	-	-	-	-
$\hat{\theta}_2$	-	-	-	-	0.25 (1.76)**	-	-
R <sup>2</sup>	0.49	0.60	0.44	0.53	0.46	0.17	0.21
$\sigma_L$	0.04	0.03	0.03	0.04	0.04	0.06	0.04
Q(4)	0.09	0.46	2.37	1.77	1.25	3.18	1.84
Q(12)	3.56	31.01	10.06	13.1	14.9	17.56	5.8
J-B	31.95*	23.58*	7.66**	27.54*	372.2*	20.3*	24.55*

Note: Values under estimates are the Student t-values. R<sup>2</sup> is the determination coefficient, J-B is statistic of Jarque-Berra test and  $\sigma_L$  is standard deviation of linear model. Q(4) and Q(12) are Ljung-Box statistics. (\*) and (\*\*) designate respectively the significativity at 5% and 10%.

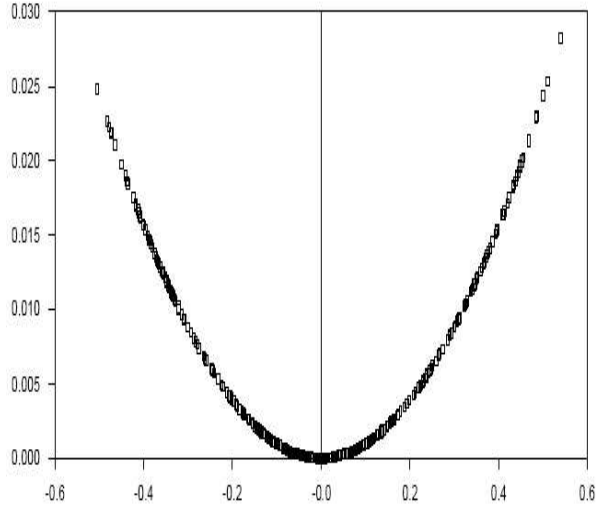
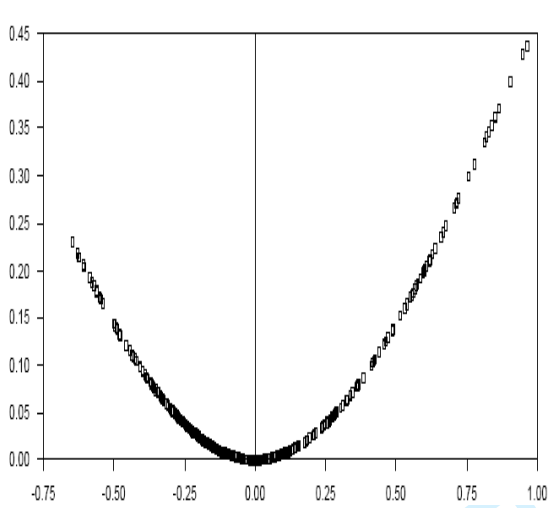


### Appendix 4 - Estimating the transition functions $\Omega(\cdot)$

*ESTECSM are estimated under  $H_0^a, H_0^b$  and  $H_0^c$*

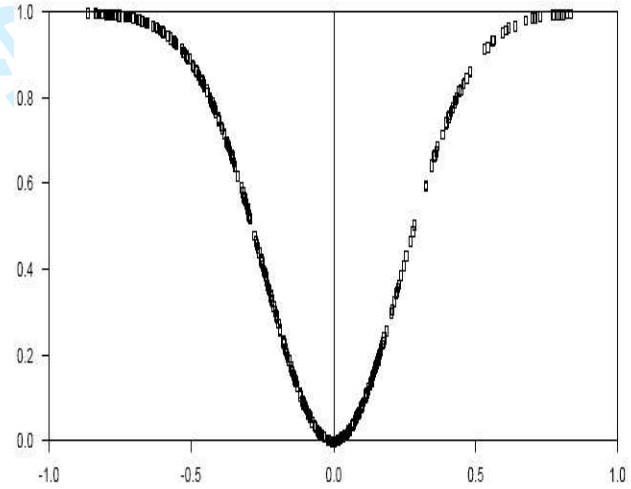
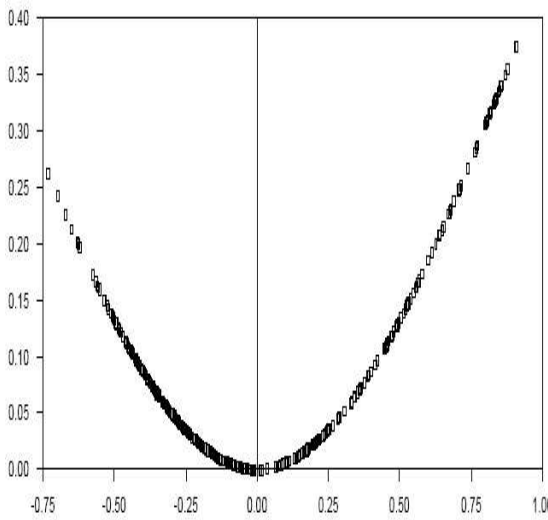
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**Canada**



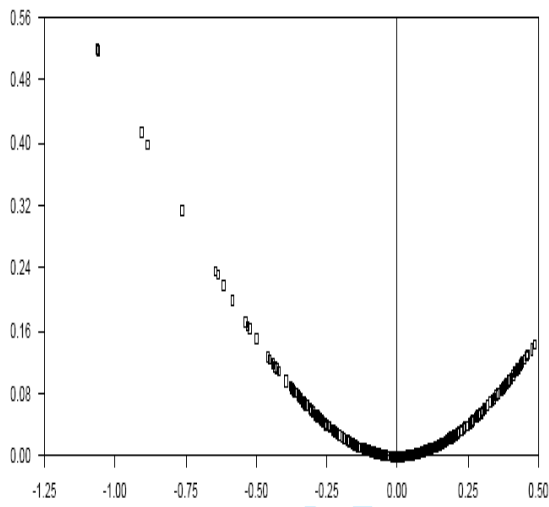
**USA**

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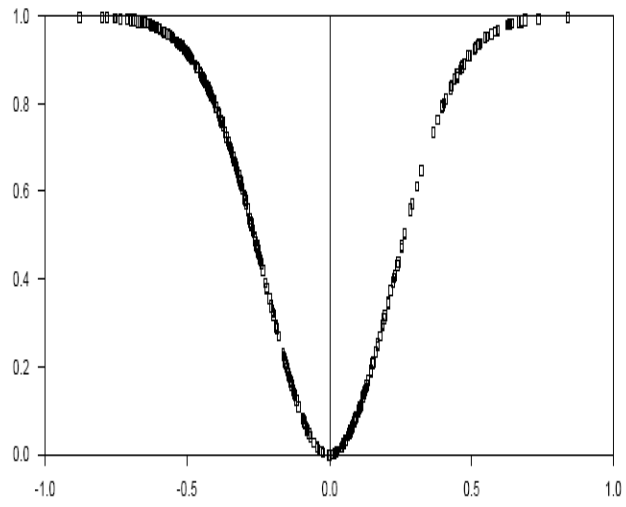


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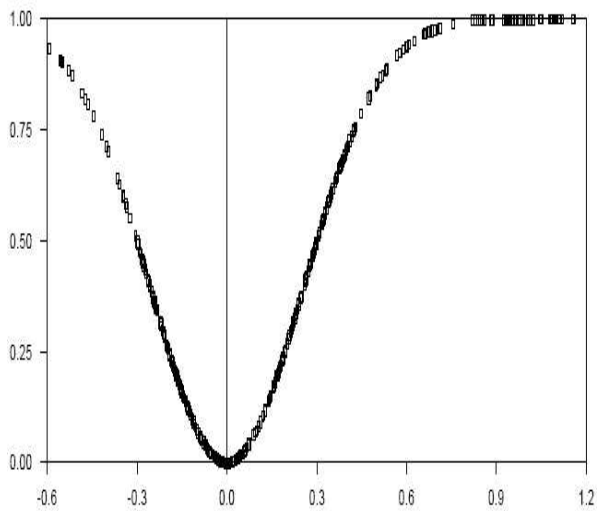
**UK**



**Italy**



**Japan**



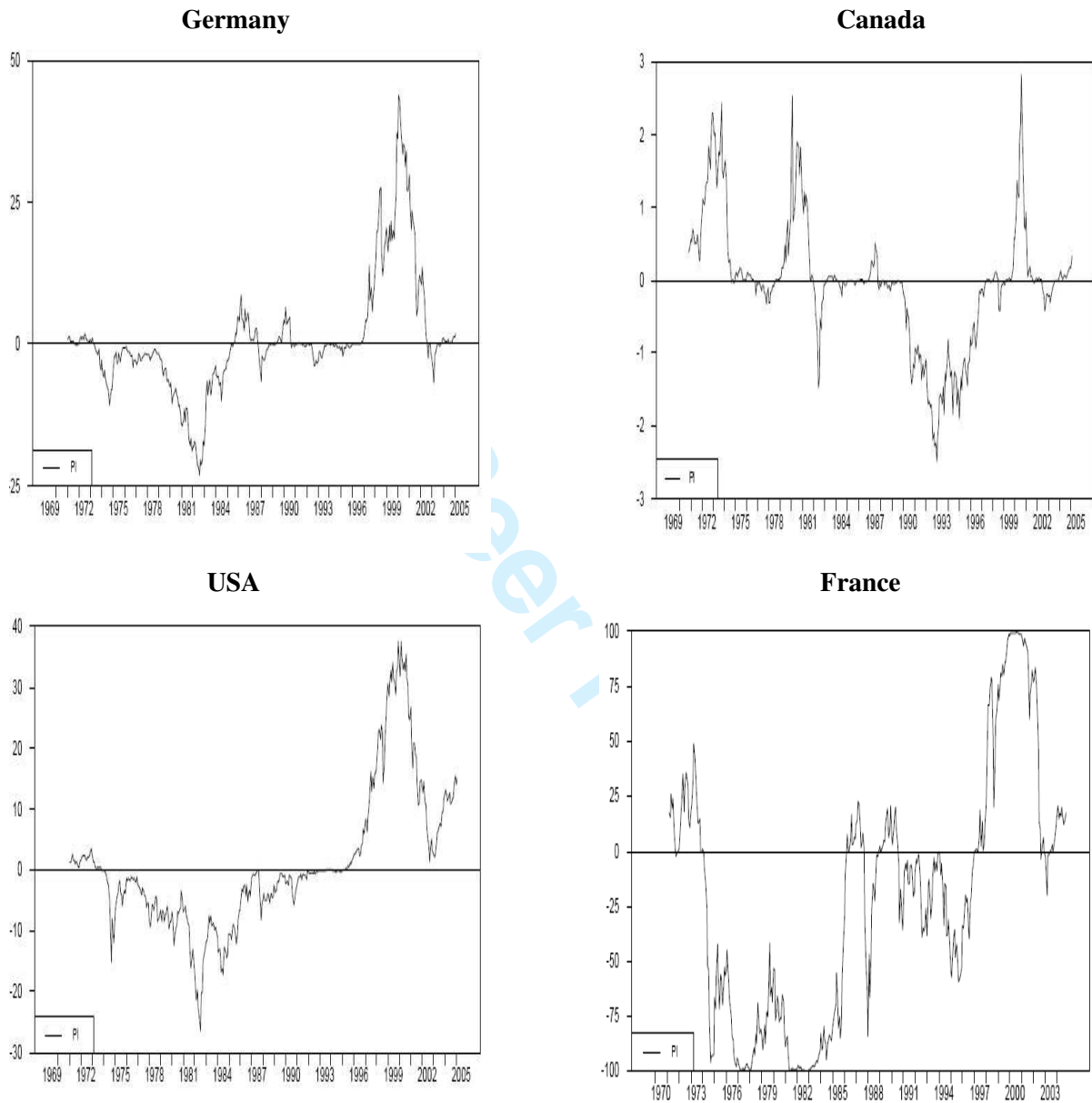
## Appendix 5 - Misspecification tests

*ESTEEM estimations under  $H_0^a$ ,  $H_0^b$  and  $H_0^c$*

Tests of no error autocorrelation (p-values of $LM_{SI}$ )							
q / serie	Germany	Canada	USA	France	UK	Italy	Japan
q = 1	0.35	0.11	0.17	0.24	0.20	0.55	0.13
q = 2	0.62	0.13	0.24	0.44	0.22	0.46	0.28
q = 3	0.80	0.12	0.42	0.51	0.43	0.53	0.31
q = 4	0.90	0.23	0.53	0.63	0.57	0.33	0.29
q = 8	0.69	0.20	0.73	0.28	0.39	0.16	0.23
q = 12	0.89	0.35	0.75	0.27	0.10	0.17	0.40
Test of parameter stability (p-values of $LM_{c,i}$ , $\forall i = 1, 2, 3$ )							
$LM_{c,1}$	0.48	0.22	0.18	0.17	0.02	0.34	0.23
$LM_{c,2}$	0.67	0.23	0.44	0.10	0.01	0.55	0.38
$LM_{c,3}$	0.88	0.55	0.68	0.30	0.03	0.75	0.63
Test of no remaining nonlinearity (p-values of $LM_{AMR}$ )							
$d = 1$	0.84	0.63	0.97	0.19	0.11	0.11	0.11
$d = 2$	0.92	0.49	0.94	0.27	0.01	0.59	0.07
$d = 3$	0.94	0.57	0.87	0.46	0.13	0.11	0.06
$d = 4$	0.95	0.64	0.79	0.62	0.05	0.15	0.16
$d = 5$	0.98	0.54	0.92	0.74	0.11	0.18	0.39
$d = 6$	0.98	0.47	0.92	0.63	0.14	0.13	0.07
$d = 7$	0.92	0.45	0.80	0.40	0.29	0.48	0.15
$d = 8$	0.92	0.29	0.93	0.37	0.04	0.16	0.23
$d = 9$	0.87	0.53	0.86	0.39	0.11	0.87	0.30
$d = 10$	0.68	0.43	0.80	0.52	0.03	0.30	0.13
$d = 11$	0.80	0.41	0.69	0.64	0.03	0.57	0.52
$d = 12$	0.66	0.32	0.66	0.68	0.07	0.74	0.29

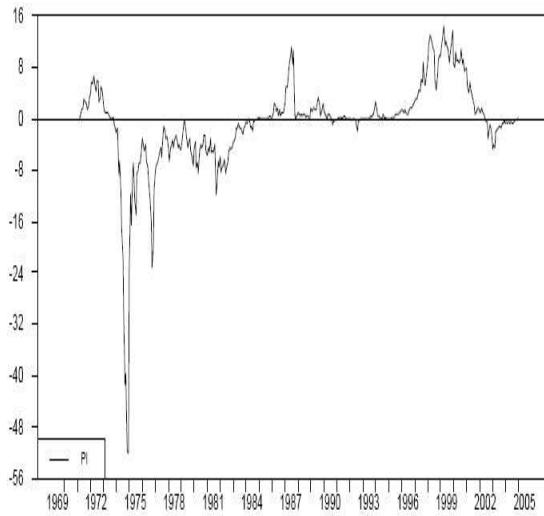
## Appendix 6 - Under- and overvaluation of stock price $\Pi(z_t)$

*ESTECM are estimated under  $H_0^a$ ,  $H_0^b$  and  $H_0^c$*

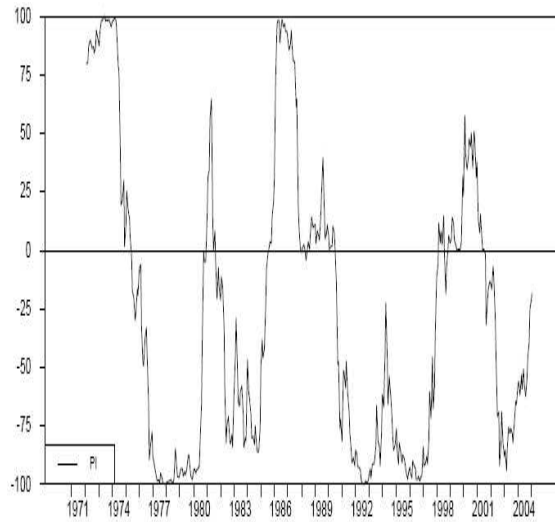


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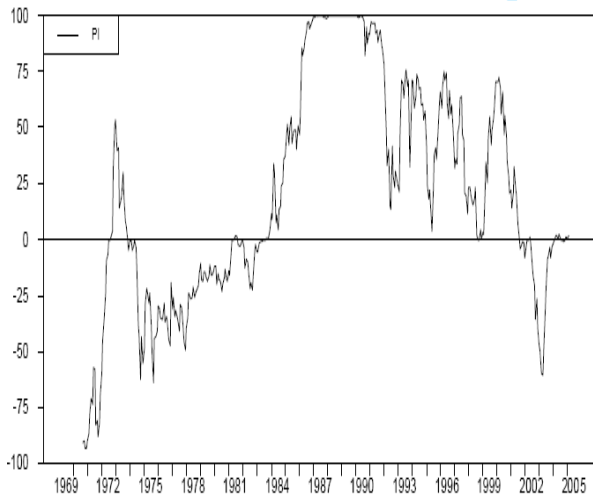
**UK**



**Italy**



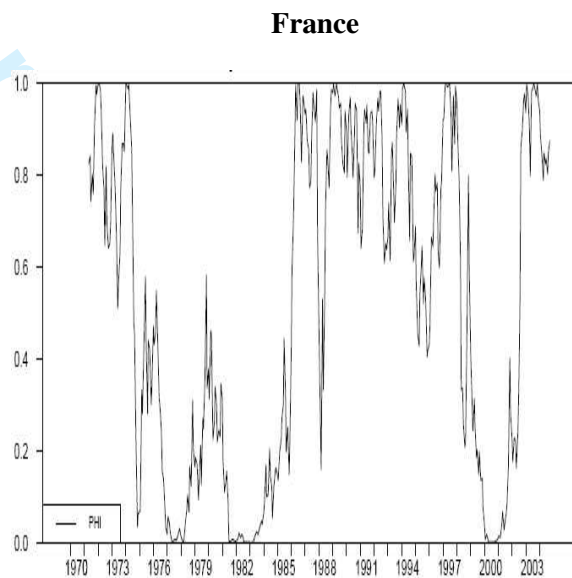
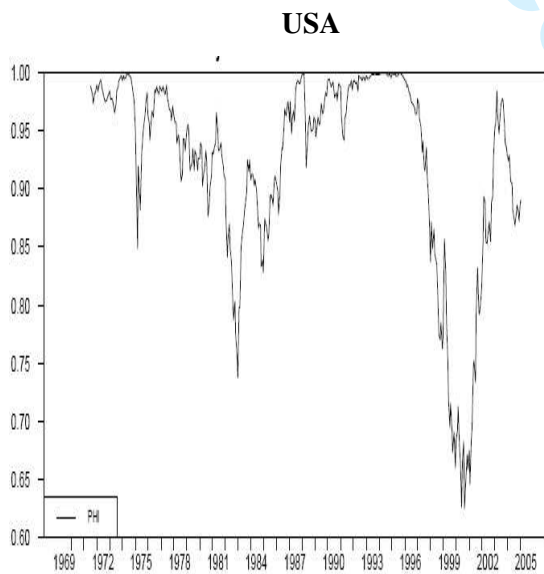
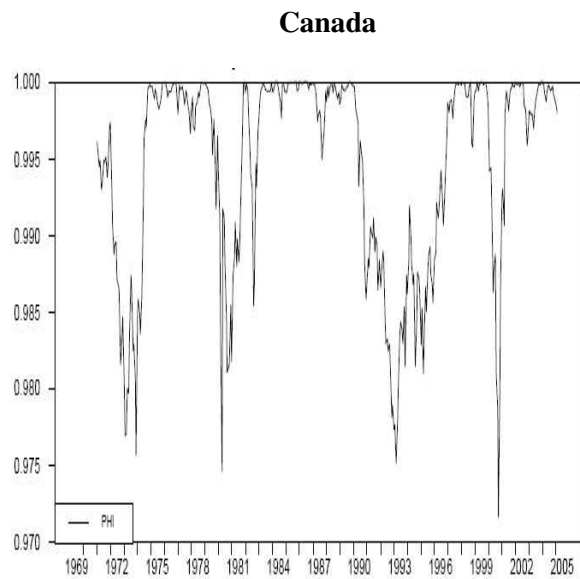
**Japan**



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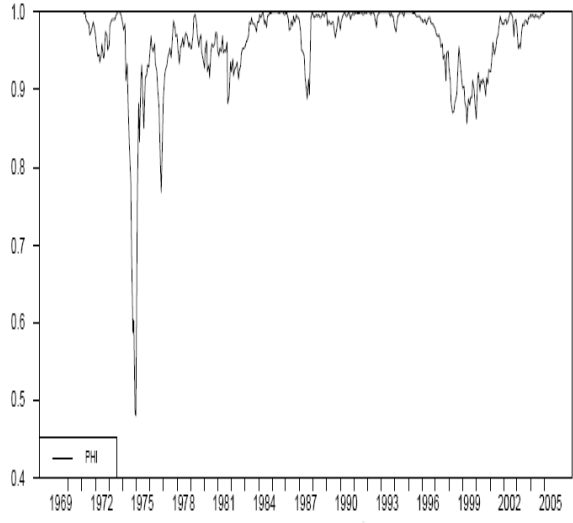
## Appendix 7 - Stock price adjustment speeds $\Psi(z_t)$

*ESTECM are estimated under  $H_0^a, H_0^b$  and  $H_0^c$*

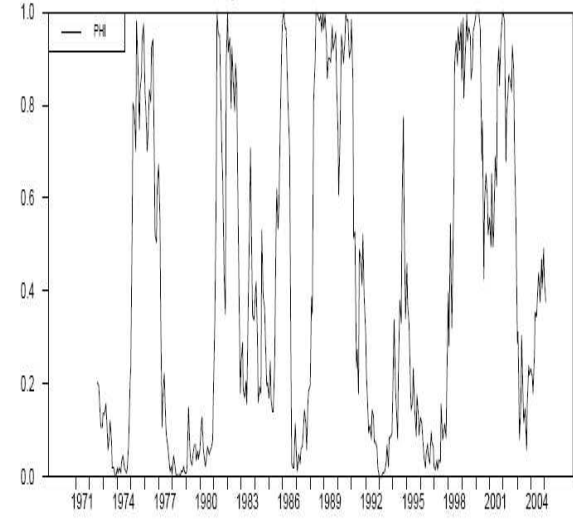


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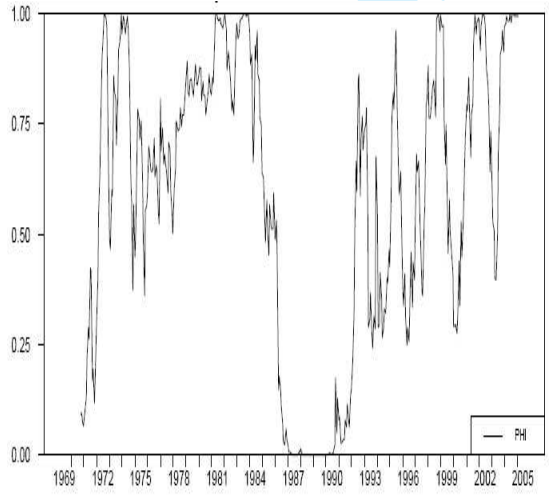
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