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A Variable Selection Method for Analyzing Supersaturated Designs

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<p>Note: The following files were submitted by the author for peer review, but cannot be converted to PDF. You must view these files (e.g. movies) online.</p> <p>LSSP-2010-0077.zip</p>	

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A Variable Selection Method for Analyzing Supersaturated Designs

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Abstract

Supersaturated designs are a large class of factorial designs which can be used for screening out the important factors from a large set of potentially active variables. The huge advantage of these designs is that they reduce the experimental cost drastically, but their critical disadvantage is the confounding involved in the statistical analysis. In this paper, we propose a method for analyzing data using several types of supersaturated designs. Modifications of widely used information criteria are given and applied to the variable selection procedure for the identification of the active factors. The effectiveness of the proposed method is depicted via simulated experiments and comparisons.

Keywords and phrases: Supersaturated designs, Variable selection, Best subset method, Information criteria.

AMS Subject Classification: 62-07, 62K15.

1 Introduction

Supersaturated designs (SSDs) are used in the initial stage of an industrial or scientific experiment for identifying the active effects, and they are very useful when there is a large number of factors under testing while only a limited number of experimental runs is available. The analysis of supersaturated designs relies on the assumption of effect sparsity (Box and Meyer ([4])), i.e. only few of the experimental factors have significant influence on the response. SSDs can be generally described as designs with p factors and n runs where $n \leq p$. The idea of SSDs was initiated by Satterthwaite ([25]). Even though the construction methods for SSDs have been widely studied (Lin ([14], [15]), Nguyen ([20]), Tang and Wu ([27]), Liu and Zhang ([17]), Liu and Dean ([16]),

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4 Ryan and Bulutoglu ([24]), Nguyen and Cheng ([21])), the analysis of SSDs is yet in an early
5 research stage.
6

7 Westfall et al. ([29]) proposed an error control skill in forward selection. Abraham et al. ([1])
8 pointed out that the analysis of SSDs can give uncertain results independently of the method used.
9 They applied stepwise and all-models methods to identify the active factors. A two-stage Bayesian
10 model selection strategy for SSDs, able to keep all possible models under consideration while
11 providing a level of robustness similar to Bayesian analysis incorporating noninformative priors,
12 was proposed by Beattie et al. ([3]). Li and Lin ([12]) introduced a variable selection approach to
13 identify the active effects in SSDs via nonconvex penalized least squares. To find the solution of
14 the penalized least squares an iterative ridge regression is employed. This method is empirically
15 compared to the Bayesian variable selection (Li and Lin ([13])). The forward selection method
16 has been suggested and performed by Lin ([14]), where the data were based on a half fraction
17 of Plackett-Burman design. Wang ([28]) applied the Lin's analysis on the other half fraction of
18 Plackett-Burman design and observed that four of the five active factors found in one half fraction
19 were not found in the other. However, when effect sparsity holds, Type I errors can easily occur.
20 In a recent paper, Holcomb et al. ([8]) proposed the bootstrap method with contrasts and the
21 contrast variance method to analyze data using a broad range of supersaturated designs. The
22 objective of the contrast variance method they proposed is to reduce Type II error rates. Hamada
23 and Wu ([6]) recommended an iterative guided stepwise regression strategy for analyzing the data
24 from these designs that allows entertainment of interactions. However, their strategy provides a
25 restricted search in a rather large model space. Chipman et al. ([5]) proposed a Bayesian variable
26 selection approach for analyzing experiments with complex aliasing and Lu and Wu ([18]) proposed
27 an analysis method based on the idea of staged dimensionality reduction. Phoa et al. ([22]) studied
28 a variable selection method via the Dantzig selector for analyzing supersaturated designs.
29

30 The paper is organized as follows. In Section 2, we present the proposed method and implement
31 its steps using a best subset variable selection approach. Also, we introduce two modifications of
32 the widely used information criteria; Akaike Information Criterion (AIC, Akaike ([2])) and Bayesian
33 Information Criterion (BIC, Schwarz ([26])). Simulation results are listed and discussed in Section
34 3. Some conclusion comments are summarized in the last section.
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40 2 Proposed Method

41 Let us consider the model for a screening experiment

$$42 \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad (1)$$

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4 where \mathbf{y} is the $(n \times 1)$ response vector, \mathbf{X} is the $n \times p$ design matrix, $\boldsymbol{\beta}$ is the $(p \times 1)$ vector of
5 the unknown coefficients, and $\boldsymbol{\varepsilon}$ is the $(n \times 1)$ random error vector with $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)'$ and
6 $\varepsilon_i \sim N(0, \sigma^2)$ for all $i = 1, 2, \dots, n$. The aim of factor screening is to identify those factors which
7 have non-zero effects. The experimenter can assume that there are only up to m active factors
8 from the total set of p factors involved. Usually, the process is more robust when m is at most half
9 the number of runs (effect sparsity ([4])).

10 In the present framework we propose an analysis method for testing the factorial effects in a
11 supersaturated design with a block structure. Our method is based on the procedure of subset
12 selection used with certain criteria which combine statistical measures with penalties for increasing
13 the number of predictors in the model. Reviews on best subset variable selection method can be
14 found in Hocking ([7]), Miller ([4]) and Rao and Wu ([23]). Basically, the criteria used in the
15 best subset variable selection procedure are classified into four categories: (1) Prediction criteria;
16 (2) Information-based criteria; (3) Data-reuse and Data-driven procedures; (4) Bayesian variable
17 selection.

18 We use here only the information-based criteria which are related to likelihood or divergence
19 measures. Each of these criteria can be considered as an approximately unbiased estimator of
20 the expected overall discrepancy, a nonnegative quantity which measures the distance between the
21 underlying model and the fitted approximated (or candidate) model. These criteria usually consist
22 of two terms where the first represents a biased estimator of the expected overall discrepancy and
23 the second is the appropriate correction (or penalty) term that makes the estimator asymptotically
24 unbiased. The most popular criteria of this class include the Akaike Information Criterion (AIC)
25 and the Bayesian Information Criterion (BIC). AIC was proposed by Akaike ([2]), and it selects
26 the model that minimizes $AIC = -2l + 2p$, where l is the loglikelihood of the model and p is the
27 number of parameters in the model. BIC was proposed by Schwarz ([26]) and has a similar form
28 to AIC except that the log-likelihood is penalized by $p \log(n)$ instead of $2p$, selecting the model
29 that minimizes $BIC = -2l + p \log(n)$, where n is the number of observations.

30 Phoa et al. ([22]) suggested a modified AIC criterion for model comparison, which is equivalent
31 to $mAIC = -2l + 2p^2$. They posed a quadratic penalty of model complexity p than the linear
32 penalty in AIC; thus, $mAIC$ chooses more parsimonious model than AIC. Inspired from this work,
33 we propose two more modifications of the above criteria; the modified AIC and the modified BIC
34 which are given from the relationships $modAIC = \frac{-2l}{p} + \frac{2p^{3/2}}{\log n^{3/2}}$ and $modBIC = \frac{-2l}{p} + \frac{p \log n^2}{\log p}$,
35 respectively. In our criteria, the conventional AIC and BIC are standardized by dividing the
36 number of active factors and include it in the first term. Thus, the effect of the number of factors
37 is minimized. Also, a penalized multiplier including the model complexity and the number of
38 observations is added in both modified criteria. Several trials and simulated experiments showed
39

that these multipliers give the best results compared to other multipliers and the original versions of the criteria. The efficiency of these modifications is established via a comparative simulation study which is presented in Section 3.

Our method is applicable to the s -block two-level $E(s^2)$ -optimal supersaturated designs with n runs and $p = s(n - 1)$ factors. Such designs can be provided by the construction methods of Tang and Wu ([27]) and Koukouvinos et al. ([10],[11]).

According to the Tang and Wu method ([27]), one can construct a supersaturated design \mathbf{X} with n rows and $s(n - 1)$ columns by juxtaposing s equivalent Hadamard matrices $\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_s$ of order n , with the requirement that none of the $s(n - 1)$ columns be fully aliased with another (two columns \mathbf{c}_i and \mathbf{c}_j are said to be fully aliased if $\mathbf{c}_i = \pm \mathbf{c}_j$). Indeed, these designs can be represented by the design matrix $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_s]$, where \mathbf{X}_i is a balanced $n \times (n - 1)$ matrix with elements ± 1 , $i = 1, 2, \dots, s$. The criterion of minimization of $r_{max} = \max_{i < j} |s_{ij}/n|$, where $s_{ij}/n = r_{ij}$ is the correlation of two columns $\mathbf{c}_i, \mathbf{c}_j$, is taken into account in the construction of these SSDs. Naturally, the matrix $\mathbf{X}_i, i = 1, 2, \dots, s$ has its columns pairwise orthogonal. An important feature of these designs is that any two columns which have non zero correlation appear in different blocks. So, the correlations of all pairs of columns within each block of the design matrix is equal to zero. In this work, the proposed method takes advantage of this feature, having as goal to overcome problems arising if a column of the design is correlated with another column and both correspond to active factors.

Koukouvinos et al. ([10]) construct $E(s^2)$ -optimal and minimax-optimal, two level cyclic structured supersaturated designs through a metaheuristic approach guided via multi-objective simulated annealing. Although the blocks constituting these designs have not necessarily an orthogonal structure, these SSDs appear optimal with respect to the criterion of minimization of r_{max} . In this way the value of the appeared correlation between the columns is as small as possible. Also, Koukouvinos et al. ([11]) propose a hybrid simulated annealing genetic algorithm for generating $E(s^2)$ -optimal cyclic structured supersaturated designs.

The procedure we suggest is briefly described by the following steps.

1. For $i = 1, 2, \dots, s$ apply the best subset variable selection method, combined with one of the information criteria described previously, using as design matrix the $n \times (n - 1)$ matrix \mathbf{X}_i , and the $n \times 1$ response vector \mathbf{y} .
2. Define S_i be the set of all active variables derived when \mathbf{y} regress with $\mathbf{X}_i, i = 1, 2, \dots, s$.
3. Let $S = \bigcup_{i=1}^s S_i$ is the union of all factors derived in each block of the design and consider the set of screened active variables to be S .

This method is applied to both type of designs as well as to SSDs without a block structure ([20]) with very good results presented in the next section.

3 Simulation Study

In this section a comparative simulation study is provided in order to test the performance of the proposed method. The SCAD method ([12],[9]) is also included in the simulation experiments, so as the best subset variable selection method is compared to an existing analysis method of supersaturated designs. Since the proposed method is run into blocks, SCAD is also run into blocks as it is proposed in [9]. The necessary parameters for the SCAD method are selected according to the suggestion in the original paper they appear. Specifically, we choose $a = 3.7$ and a generalized cross-validation is used to estimate the thresholding parameter λ .

1000 data sets are generated from the linear model (1) with randomly selected coefficients and an $\varepsilon_i \sim N(0,1)$ for all $i = 1, 2, \dots, n$ random error is added to each corresponding observation y_i , while only main effects models are considered. The true active variables are also selected randomly from the set of $1, \dots, p$ potentially active factors, with respect to the already determined number of active variables in each block of the design matrix. Following the principle of effect sparsity, all possible numbers of active factors are chosen. Thus, $0, 1, 2, 3, \dots, \frac{n}{2}$, active factors are tested for each of the six $n \times p$ design matrices we study. We run all the possible combinations of $0, 1, 2, 3, \dots, \frac{n}{2}$ active factors per s blocks in order to examine all the possible cases. We have observed symmetry between the numbers of active factors in the several blocks; hence, we limited the results listed only for the cases where $t_i < t_j, i \neq j, i, j = 1, \dots, s$ and t_i denote the number of active factors in the i -th block. The coefficients of the non-active variables, in the true model, are generated from $N(0, 0.05)$ and the absolute values of the active factors are randomly selected in the range $[0.7, 2.0]$, considering both negative and positive signs assigned to the coefficients of the active factors. In total, for obtaining Tables I-V the number of the used linear models is equal to the number of the considered cases appeared in the corresponding tables.

In these simulation experiments our aim is to control two types of error rates; the cost of declaring an inert effect to be active (Type I error), and the cost of declaring an active effect to be inactive (Type II error).

Various types of supersaturated designs are used in the simulations as examples of the performance of the method when it is applied in designs with different structure. In Tables I-V are presented the results obtained by one supersaturated design constructed by the method presented in Tang and Wu ([27]), two supersaturated designs constructed by Koukouvinos et al. ([10]), one supersaturated design constructed by Koukouvinos et al. ([11]) and one supersaturated design

constructed by Nguyen ([20]), respectively. In the first columns of each table we present the number of true active factors $t_i, i = 1, \dots, s$, per each of the s blocks used in the simulated models. In the next columns we present the Type I and Type II error rates occurred by the application of the best subset variable selection and the block SCAD ([9]) method to the simulated models. The procedure of best subset variable selection terminates using the five information-based criteria described in Section 2.

- **A two-block orthogonal SSD**

The first idea was to apply the proposed method to a two-block orthogonal design in order to take advantage of this special structure. We choose the $E(s^2)$ -optimal SSD for $p = 22$ factors in $n = 12$ runs with $r_{max} = 0.67$ constructed in Tang and Wu ([27]) for this purpose and the results are presented in the following table.

t		AIC		BIC		m AIC		mod AIC		mod BIC		SCAD	
t_1	t_2	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II
0	0	0.1639	-	0.1652	-	0.0781	-	0.0792	-	0.0835	-	0.1324	-
0	1	0.2279	0.0980	0.2310	0.0960	0.0615	0.2270	0.1224	0.1800	0.1345	0.1740	0.1604	0.1290
0	2	0.3033	0.0630	0.3070	0.0615	0.0696	0.3535	0.1620	0.1215	0.1875	0.1060	0.1829	0.1175
1	1	0.3245	0.0660	0.3274	0.0635	0.0559	0.2850	0.1698	0.1350	0.1983	0.1190	0.1962	0.1540
0	3	0.3583	0.0950	0.3613	0.0933	0.0835	0.4940	0.1819	0.1907	0.2081	0.1590	0.2132	0.1867
1	2	0.3890	0.0307	0.3911	0.0293	0.0549	0.2150	0.1798	0.0773	0.2164	0.0637	0.2071	0.1093
0	4	0.3931	0.0602	0.3947	0.0587	0.1057	0.5383	0.1998	0.1838	0.2189	0.1358	0.2277	0.1417
1	3	0.3801	0.0305	0.3815	0.0302	0.0670	0.3342	0.1785	0.1038	0.2011	0.0800	0.2147	0.1525
2	2	0.4095	0.0073	0.4116	0.0073	0.0436	0.2190	0.1802	0.0382	0.2112	0.0267	0.2165	0.0685
0	5	0.4725	0.0564	0.4747	0.0562	0.1160	0.6096	0.2262	0.2222	0.2678	0.1668	0.2675	0.2182
1	4	0.4836	0.0874	0.4849	0.0870	0.0686	0.4396	0.2085	0.1976	0.2594	0.1682	0.2126	0.2412
2	3	0.4999	0.1044	0.5020	0.1036	0.1181	0.6462	0.2515	0.2648	0.3025	0.3142	0.3245	0.4296
0	6	0.4538	0.0467	0.4549	0.0450	0.1251	0.6695	0.2432	0.2922	0.2592	0.2087	0.2527	0.3732
1	5	0.5336	0.0712	0.5339	0.0712	0.0810	0.5508	0.2123	0.2392	0.2607	0.1950	0.1773	0.3293
2	4	0.5208	0.1060	0.5216	0.1053	0.0750	0.5372	0.2236	0.3017	0.2738	0.2517	0.2326	0.3812
3	3	0.4079	0.0437	0.4095	0.0433	0.0504	0.4680	0.1840	0.1603	0.2243	0.1232	0.2330	0.1527

Table I: Methods performance for random model coefficients using 1000 simulations in Tang-Wu design for $p = 22$ factors and $n = 12$ runs, considering up to 6 active factors per block (11 factors in total per block).

From Table I, it is shown that the best subset variable selection methods using AIC and BIC behave similarly overestimating the non-active variables (high Type I error rates), while the application of m AIC results to extreme values of Type II. The criteria mod AIC and mod BIC improve the results of the corresponding original versions and compete the results obtained from block SCAD.

- **A two-block SSD**

Next, we apply the proposed method in a two-block supersaturated design provided in [10]. This design is the $E(s^2)$ -optimal SSD for $p = 18$ factors in $n = 10$ runs with $r_{max} = 0.600$. The results of the simulations using the above design are listed in Table II.

t		AIC		BIC		m AIC		mod AIC		mod BIC		SCAD	
t_1	t_2	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II
0	0	0.3668	-	0.4317	-	0.0912	-	0.0997	-	0.1479	-	0.1271	-
0	1	0.4706	0.1370	0.5211	0.1300	0.0741	0.2880	0.1360	0.2090	0.2134	0.2130	0.1259	0.2140
0	2	0.5691	0.1225	0.6025	0.1210	0.0839	0.4405	0.1916	0.2155	0.3064	0.1800	0.1124	0.2185
1	1	0.5671	0.3385	0.6163	0.3350	0.0861	0.5875	0.2028	0.4730	0.2938	0.4250	0.1509	0.5215
0	3	0.4771	0.1540	0.5145	0.1483	0.0897	0.5990	0.2003	0.3097	0.2726	0.2377	0.1835	0.3427
1	2	0.5231	0.1863	0.5544	0.1773	0.0917	0.5493	0.2246	0.3583	0.3114	0.2820	0.1905	0.4243
0	4	0.5876	0.1330	0.6120	0.1270	0.1139	0.6005	0.2457	0.3113	0.3620	0.2115	0.1487	0.3945
1	3	0.5692	0.1403	0.5999	0.1320	0.0821	0.4808	0.2390	0.2858	0.3405	0.2218	0.1746	0.4060
2	2	0.6841	0.1452	0.7059	0.1260	0.1024	0.5517	0.2886	0.4035	0.4314	0.3225	0.1269	0.5357
0	5	0.6876	0.1090	0.6978	0.1062	0.1492	0.6284	0.2866	0.3402	0.4577	0.2194	0.1045	0.4628
1	4	0.7327	0.1784	0.7492	0.1698	0.1452	0.6238	0.3385	0.4838	0.4782	0.3914	0.1336	0.5344
2	3	0.6419	0.3150	0.6636	0.2898	0.1597	0.7452	0.3432	0.5186	0.4375	0.5334	0.2445	0.6676

Table II: Methods performance for random model coefficients using 1000 simulations in Koukouvinos et al. design for $p = 18$ factors and $n = 10$ runs, considering up to 5 active factors per block (9 factors in total per block).

The previous observations stand for the results of Table II too; when best subset terminates using AIC and BIC, the values of Type I are very high, while the use of m AIC results to extreme Type II error rates. In contrary, the modified versions mod AIC and mod BIC behave better than AIC and BIC, since they reduce the values of Type I and also, give lower Type II rates than SCAD.

• **A larger SSD**

In cases that the number of experimental factors is large, best subset approaches could not be applied because of their computer burden. So, it is of interest to test if a further division within the blocks could be an alternative approach in such cases. We use the supersaturated design for $p = 66$ factors and $n = 12$ runs ([10]) and we divide each one of the two original blocks into three blocks; so, we have six blocks in total. For simplicity, we keep the same notation for the number of active factors as in previous tables, but note that we now refer in a subdivision of the two-block design. Active factors are assigned randomly in each of the six sub-blocks.

t		AIC		BIC		m AIC		mod AIC		mod BIC		SCAD	
t_1	t_2	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II
0	0	0.2118	-	0.2301	-	0.0497	-	0.0838	-	0.1360	-	0.0861	-
0	1	0.3627	0.1820	0.3788	0.1820	0.0814	0.2140	0.1236	0.1840	0.2363	0.1800	0.1381	0.1830
0	2	0.4252	0.2250	0.4394	0.2285	0.1068	0.4565	0.1249	0.2595	0.2723	0.2350	0.1676	0.2880
1	1	0.4698	0.0860	0.4790	0.0900	0.1100	0.1575	0.1496	0.1270	0.3057	0.1115	0.1573	0.1095
0	3	0.5090	0.2220	0.5171	0.2220	0.1327	0.4367	0.1750	0.3360	0.3288	0.3090	0.1839	0.3807
1	2	0.4794	0.2743	0.4888	0.2687	0.1305	0.4853	0.1690	0.3897	0.3228	0.3570	0.1903	0.4350
0	4	0.5622	0.0747	0.5707	0.0752	0.1395	0.2288	0.1794	0.1358	0.3377	0.1155	0.1899	0.2208
1	3	0.5842	0.1318	0.5896	0.1345	0.1462	0.3320	0.1868	0.1683	0.3548	0.1452	0.1880	0.2402
2	2	0.5831	0.0210	0.5874	0.0205	0.1352	0.1817	0.1622	0.1678	0.3603	0.0532	0.1781	0.1920
0	5	0.6351	0.1642	0.6402	0.1618	0.1504	0.4744	0.2140	0.3032	0.3703	0.2662	0.1991	0.4236
1	4	0.6881	0.1066	0.6925	0.1054	0.1393	0.3046	0.1736	0.2208	0.3893	0.2008	0.1857	0.2656
2	3	0.6748	0.0740	0.6792	0.0728	0.1413	0.3278	0.2283	0.2514	0.3931	0.2110	0.2118	0.3432
0	6	0.6545	0.1822	0.6577	0.1793	0.1570	0.5843	0.2034	0.3870	0.4071	0.3350	0.2033	0.4458
1	5	0.6989	0.0952	0.7020	0.0962	0.1391	0.3933	0.2263	0.1690	0.3983	0.1387	0.2006	0.3938
2	4	0.7018	0.0820	0.7047	0.0812	0.1434	0.4357	0.2096	0.3227	0.4092	0.2388	0.2109	0.3810
3	3	0.6438	0.1197	0.6480	0.1190	0.1493	0.4912	0.2216	0.2955	0.4001	0.2263	0.2311	0.3705

Table III: Methods performance for random model coefficients using 1000 simulations in Koukouvinos et al. design for $p = 66$ factors and $n = 12$ runs, considering up to 6 active factors per block (11 factors in total in each of the six blocks). Each one of the two original blocks is sub-divided in three blocks.

Let us note that the further split of the design in six blocks is less time consuming than the division in two blocks. The modified versions *modAIC* and *modBIC* improve the results of the original versions. *modBIC* yields lower Type II error rates than SCAD, but higher values of Type I. On the other hand, *modAIC* outperforms SCAD run into blocks. The above results show that when the number of experimental factors enlarges a further division into sub-blocks could be an alternative approach of the proposed best subset method using *modAIC*.

- **A three-block SSD**

We also use the $E(s^2)$ -optimal SSD for $p = 21$ factors in $n = 8$ runs with $r_{max} = 0.500$ ([11]) divided in three blocks in this simulation study. The obtained results are shown in Table IV, where t_1, t_2, t_3 denote the number of active factors in the first, second and third block respectively. Again *modAIC* and *modBIC* perform better than AIC and BIC giving lower Type I rates and preserving Type II rates in satisfactory levels. It is shown also, that *modAIC* gives better results than SCAD run into three blocks.

t			AIC		BIC		<i>mAIC</i>		<i>modAIC</i>		<i>modBIC</i>		SCAD	
t_1	t_2	t_3	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II
0	0	0	0.3672	-	0.4120	-	0.0820	-	0.0963	-	0.2160	-	0.1053	-
1	0	0	0.4501	0.4290	0.4894	0.3950	0.0916	0.6040	0.1943	0.4580	0.2882	0.3640	0.2182	0.5020
2	0	0	0.6033	0.2470	0.6310	0.2370	0.1345	0.5570	0.2104	0.3450	0.3962	0.1725	0.2588	0.4575
1	1	0	0.5761	0.2955	0.6127	0.2720	0.1162	0.5185	0.2017	0.4995	0.3808	0.2880	0.2593	0.5270
3	0	0	0.5723	0.3157	0.5975	0.2927	0.1553	0.6587	0.2183	0.4827	0.3846	0.2603	0.2981	0.5970
2	1	0	0.6593	0.3243	0.6792	0.2963	0.1676	0.6433	0.2954	0.6203	0.4681	0.3447	0.2828	0.6143
1	1	1	0.6889	0.3237	0.7139	0.3077	0.1196	0.4070	0.2958	0.4810	0.4851	0.3083	0.3298	0.4090
4	0	0	0.6421	0.3115	0.6615	0.2953	0.1886	0.7083	0.2728	0.5058	0.4378	0.2643	0.3305	0.6138
3	1	0	0.6121	0.2432	0.6341	0.2350	0.1509	0.5433	0.2505	0.3927	0.4136	0.2013	0.2954	0.4970
2	2	0	0.6209	0.0850	0.6491	0.0818	0.1476	0.2630	0.2016	0.1535	0.4049	0.1108	0.2395	0.3200
2	1	1	0.6521	0.0153	0.6760	0.0115	0.0851	0.1820	0.2249	0.1143	0.4302	0.1362	0.2276	0.2330

Table IV: Methods performance for random model coefficients using 1000 simulations in Koukouvinos et al. design for $p = 21$ factors and $n = 8$ runs, considering up to 4 active factors per block (7 factors in total per block).

- **A design without block structure**

Moreover, supersaturated designs without a block structure are used in the simulations in order to illustrate the method's conduct when it is applied to a broader class of such designs. In the following table, the results obtained from Nguyen's ([20]) $ave(s^2)$ -optimal SSD for $p = 14$ factors in $n = 8$ runs with $r_{max} = 0.500$ are listed. The following results show that the proposed method performs well when it terminates using *modAIC* and *modBIC*. Moreover, best subset using *modAIC* gives better results than block SCAD.

t		AIC		BIC		m AIC		mod AIC		mod BIC		SCAD	
t_L	t_R	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II	Type I	Type II
0	0	0.2436	-	0.2478	-	0.1237	-	0.1260	-	0.1483	-	0.1550	-
0	1	0.3229	0.1240	0.3321	0.1210	0.0971	0.2740	0.1728	0.2190	0.2235	0.1980	0.1694	0.2220
0	2	0.3983	0.1000	0.4067	0.0945	0.0962	0.4750	0.2249	0.1950	0.3022	0.1515	0.2339	0.2435
1	1	0.3908	0.1685	0.3998	0.1580	0.1274	0.5710	0.2302	0.3405	0.2886	0.3675	0.2564	0.3750
0	3	0.4511	0.1567	0.4590	0.1487	0.1076	0.6183	0.2576	0.2950	0.3503	0.2170	0.2825	0.3900
1	2	0.4631	0.2107	0.4735	0.2040	0.0740	0.4903	0.2545	0.3220	0.3590	0.2640	0.2836	0.3983
0	4	0.5264	0.1245	0.5322	0.1210	0.1308	0.6775	0.3038	0.3040	0.4211	0.1918	0.2951	0.4895
1	3	0.4824	0.1457	0.4897	0.1442	0.0657	0.4140	0.2495	0.2275	0.3701	0.1840	0.2643	0.2920
2	2	0.5137	0.2205	0.5220	0.2147	0.1179	0.6228	0.3064	0.4022	0.4144	0.3020	0.3151	0.4150

Table VI: Methods performance for random model coefficients using 1000 simulations in Nguyen design for $p = 14$ factors and $n = 8$ runs, considering up to 4 active factors per block (7 factors in total per block).

Some general conclusion remarks are discussed in the next section.

4 Conclusion

In this paper, we have introduced a method for analyzing data in supersaturated designs by dividing them into blocks. We approach the problem of variable selection combining several information criteria, with the block structure of the SSDs given by Tang and Wu ([27]) and Koukouvinos et al. ([10], [11]). The proposed method enables us to make the best subset variable selection procedure applicable even in high order designs. However, when there is a large number of experimental runs, best subset approaches cannot be applied because of their time complexity. In these cases a further division of the large designs into more blocks allows the application of best subset methods. Empirical performance of our method based on simulations is tested and compared to SCAD run into blocks ([9]). Simulation results show that the proposed method gives good results and can compete existing analysis methods when it is applied to supersaturated designs with a block orthogonal structure. Moreover, it outperforms when it is applied to supersaturated designs with a block but not orthogonal structure or even to designs without any block structure by simply dividing them into blocks.

As with any method for analyzing supersaturated designs, high risk is expected for this method as well. Both Type I and Type II error rates are important and should be kept as low as possible. A method with a low Type I error rate has the ability to exclude unnecessary factors, so it can be helpful in reducing the cost of additional experiments based on the selected factors. On the other hand, there are experiments, where the exclusion of one or more active factors and the weakness of the method to identify them could have dramatic results. In these cases, the cost of declaring an active effect to be inactive would be substantial and the application of any of the other methods with lower Type II error rates is suggested. Best subset approach using AIC and BIC gives very high Type I and low Type II error rates. The proposed method using m AIC gives the lowest Type I error rates but extreme values of Type II. The combination of our method with the proposed in

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4 this paper modified information criteria achieves lower error rates than the conventional versions,
5 and especially *modAIC* gives better results than block SCAD in most of the cases.
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9 referees for their useful comments and suggestions.
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Letter to the Referees concerning our ~~Manuscript~~ ~~10077~~ of the submission
 A Variable Selection Method for Analyzing Supersaturated Designs
 by C. Koukouvinos, K. Mylona and A. Skountzou
 October, 2010

This letter describes the ~~SRP10077~~ submission entitled A Variable Selection Method for Analyzing Supersaturated Designs and associated referee comments received September 21, 2010. Taking into consideration the point to point nature of the referee's comments, the point response is as follows.

Response to Editor

The referee is still asking about the modification of AIC and BIC. On what basis was this modification made? We would like to know the evidence, but give the evidence. It seems to me that, in general, increasing Type I error then Type II error will decrease. So what is the choice of modification? Is the proposed modification divided by p ? The referee states that the new method is better than the original, but a careful discussion.

- Inspired from the work of Phoa et al. [22] we propose the ~~modification~~ and BIC with ~~penalties~~ of the model complexity. The use of a ~~new~~ penalty leads to more provident models, modify the original versions of the criteria in order to improve application with the proposed further comments are added in the text in order to clarify which is achieved mainly from the simulation. Moreover, we rerun the simulations more carefully and the ~~conventional version~~ gives very low Type I (thus, low Type I rates), while the modified versions, modAIC and modBIC perform well. In many cases, they outperform SCAD run into blocks.

It is not clear to me which methods are applied in the design of the design. If the modified SCAD is applied to the design then comparing different criteria as well as different methods.

- All the methods are applied to the ~~whole~~ design. This is clarified in Section 3 of the previous manuscript, but not in the current version.

When you made the subdivisions into many blocks, the number of factors per block is smaller than the number of variables. A separate analysis of variance should be done for a main effect. Perhaps you should be looking at the design here. Please add into the captions of the tables the number of factors of the design.

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• A larger design with 66 factors is included in our simulation. The two blocks are subdivided into three blocks; thus, we have 6 blocks in each block. The design number is the total number of the active factors per block are added into the captions of the tables.

The referee points out that it would be better to report the distribution around zero rather than equal to zero.

• We rerun the simulation and study the coefficient distribution from $N(0, 0.05)$.

Page 5. You talk about "up to $n/2$ factors" but the tables show

• The tables show the results up to $n/2$ factors, where n denotes the number of the factors.

Page 5 end. You say "Since we do not have the work to more than two blocks."

• This phrase is corrected.

Did you use a version of SCAD obtained by you or previous days? If not, own, did you use the CV step and parameter values as in [12] and [9]. Please clarify.

• Further comments on the application of SCAD are added. The parameter values as in [9].

Page 9, line 3. It would be clearer to state why the number of runs of the problem with the information is increased (not the number of runs) be run on a large number of variables? You do not seem to have any example with a large number of factors.

• Right it was a typo. The correct phrase is the number of runs is enlarged. As it is mentioned before, we present the results of a design with 66 factors in 12 runs in a design of the subdi-

Response file

First of all, what from the simulation (Tables I to VI)? Does the "fact" that the proposed method is indeed superior at all of these minor points is about the simulated effects: should not be assigned zero, instead it should draw from a distribution, such as $N(0, 0.5)$; while they should all be positive.]

- We rerun the simulation study generating the coefficients of $N(0, 0.5)$ and selecting the coefficients of the active factors $[-2.0, 2.0]$, assigning both negative and positive signs to the coefficients. The proposed method is compared to SCAD. We believe the new simulation results indicate the effectiveness of the proposed method, since the two modified criteria behave better than the conventional ones when they compete with SCAD. Especially, when the proposed method is used, modAIC, it gives better results than SCAD in most of the cases.

Second, The proposed criteria, modAIC and modBIC, need motivation for why so or properties. The authors indicated that at the top of p.4) "Several simulation studies showed that this is not convincing at all. Why this is a good way to "standardize", by the way.

- Motivated from the work of A.P. [22] we propose the two modified AIC and BIC with different penalties of the model complexity. Some further comments are added in the text in the choice, which is verified mainly by simulation. The original criterion penalty is a set of parameters, so we tried to modify the origin of the criterion in order to improve the results of their application to the proposed method.