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Gang Xiao

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# Tilting mirror strips in a linear Fresnel reflector

Gang Xiao (University of Nice, France)

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## Abstract

When a linear Fresnel reflector solar concentrator is installed in a site with high latitude, important losses of optic efficiency will occur due to the low angle of the Sun, especially during the winter season, and especially if the concentrator is installed with north-south orientation.

In this work, we make detailed computations on the method of tilting the rotation angle of the mirror strips, to an angle about half of the latitude of the site. According to the computation, this method provides the best mirror yield for zones of latitude between 25 and 45 degrees, compared to other configurations of the linear Fresnel reflector.

Technical restrictions and difficulties of this method are also discussed.

## Introduction

Linear Fresnel reflector (LFR) technology is one of the most promising technologies for solar thermal power generation. A LFR solar concentrator uses a multiple of independently rotating mirror strips to reflect solar radiations onto a receiver tube located at the common focal line of the mirror strips.

In the basic setup of the LFR solar concentrator, both the rotating axes of the mirrors and the receiver are in horizontal positions, and they are parallel to each other. Optically this is the most obvious choice, but the efficiency of horizontally rotating mirrors suffer from the cosine factor due to the north-south inclination of the Sun, especially when the concentrator is installed in geographic zones with high latitude.

The loss of mirror efficiency due to the cosine factor is accentuated by an important end loss (the reflected light goes beyond the end of the receiver and gets lost, due to the inclination of the Sun in the direction of the axis of the receiver), and an important seasonal variation of the energy output. The seasonal variation of the output is due to the fact that the Sun is more inclined towards the horizon during winter, so that both the cosine factor loss and the end loss are more important. As a result, the output of the installation is significantly during the winter than during the summer. The seasonal variation greatly decreases the real value of the output of a solar power plant, because for high-latitude regions, winter is the season where electricity demand is the highest.

We name the above three problems *declination penalties*.

The effect of the declination penalties can be felt right from latitude  $20^\circ$  or so, and becomes more important as the latitude increases. For geographic zones with latitude over  $40^\circ$ , the penalties are so serious that the installation of a horizontally mounted concentrator is not recommended [3, p. 14].

This is a big trouble particularly for European countries, as most European countries are located at latitudes above  $40^\circ$ . It is proposed that large scale solar fields be installed in north African countries, then long distance power transmission lines be drawn to transport the generated power to Europe [5].

For this proposition, one first remarks that north African regions near Europe are of latitude near  $35^\circ$ , so that the declination penalties are already significant. Then besides the cost and loss of long power transmissions, the political and social risks of such a strategy constitute a serious problem.

Here we propose a variation of the LFR technology that combines a horizontal receiver in the north-south direction, with mirror strips whose rotation axes are tilted towards the direction of the Sun. The tilting angle of these rotation axes are roughly half of the latitude of the site. (Figure 1)

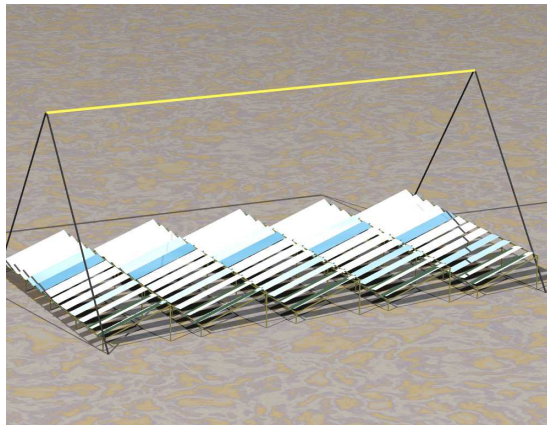


Figure 1: LFR with horizontal receiver and tilted mirrors

The objective of this article is to discuss the geometric aspects of this approach, and compare it with the existing installation modes of LFR. The conclusion drawn at the end of the article shows a clear advantage of the new approach with respect to the existing ones, within the range of latitude from  $25^\circ$  to  $45^\circ$ . This is the majority of geographical zones for which solar power is of big importance. With a mirror yield systematically above 80% within the whole range of latitude, the performance is excellent.

In particular, a LFR with tilted mirrors installed on a site of latitude  $45^\circ$  (central France) performs significantly better than a traditional flat LFR installed at latitude  $35^\circ$  (north Africa).

We start the article by fixing the computational hypotheses and deriving the geometric formulae in Section 1. Then using these formulae, performance

(mirror yield and seasonal variation factor) of existing installation modes are obtained in Section 2. These values will be used to compare with the new mode.

In Section 3, the new tilted mirror mode is described and its performance is computed.

The two principal geometric issues of the new mode is studied in Section 4. Namely, the coincidence of the focal line of the mirrors with the receiver, and the seasonal variation of the normal focal distance.

Optically, a simple tilting the mirror strips will not work, for then the receiver is not coplanar with the rotation axes of the mirror strips, so that the reflected light can only meet the receiver at isolated points. Two solutions exist for this problem: one can either put the mirror strips in positions so that each rotation axis is in a same plane with the receiver, or leaving the mirror strips parallel to each other while (slightly) twist them to correct the paths of the reflected light. As the first solution leads to extra cost for the structure as well as more shading losses, we fix our attention on the second solution.

The required amount of the twist of a mirror varies with the position of the Sun. detailed calculations are carried out to show that the scope of the variation of the required amount of the twist is not far away from the tolerance of the context, so that it can be compensated by simple means with little cost overhead.

The second geometric issue treated in Section 4 is that once the rotation axis of a mirror is not parallel with the receiver, the normal focal distance varies with the inclination angle of the Sun. Our calculations show that the scope of this variation of focal distance is so important that if flat mirrors or curved mirrors with static curvature are used, the concentration ratio of the concentrator would be too low for high-temperature applications such as power generation. Therefore curved mirrors with dynamic curvature are preferred, and we refer to another article [7] for how to realize such mirrors with good precision and low cost.

Some minor issues related to the tilted mirror approach are studied in Section 5, before the conclusion in the last section.

## 1 Computing the mirror yield

First of all, we fix the following notations and definitions.

$\delta$  The Sun's declination angle, that is, the angle between the rays of the Sun and the plane of the Earth's equator.

$h$  The hour angle of the Sun that equals 0 at noon.

$\gamma$  The cosine factor of a mirror.

$\varphi$  The latitude of the site.

$\alpha$  The east-west rotation angle of a mirror.

$\beta$  The north-south tilting angle of a mirror.

$\lambda$  The length of a mirror strip.

$\sigma$  The ratio of seasonal variation.

For the movement of the Sun,  $\delta$  oscillates between  $-23.44^\circ$  and  $23.44^\circ$  in a yearly basis, that is, putting into radians

$$\delta = \delta(\tau) = 0.409 \cos(\tau) ,$$

where  $\tau$  is the yearly time variable. Each interval of  $2\pi$  for  $\tau$  represents one year. For any geometric function  $f(\delta)$ , its average over the variation of the Sun's inclination angle can thus be obtained by taking the integral on  $\tau$  over one period:

$$\bar{f} = \frac{1}{2\pi} \int_0^{2\pi} f(\delta) d\tau$$

To compute daily averages of geometric functions depending on  $h$ , we will use unweighted integrals for  $h$  going from  $-\pi/3$  to  $\pi/3$ , corresponding to a daily operation from 8h am to 4h pm. The real situation is more complicated and varies from one setup to another, as the yield of the concentrator gradually drops with increasing  $|h|$ . So a flat integral over-estimates the influence of values of  $h$  close to  $\pm\pi/3$ , while it ignores values of  $h$  beyond  $\pm\pi/3$ . However for most real concentrators these two errors more or less cancel each other, so the result is a reasonably good estimation.

Therefore for a geometric function  $f(\delta, h)$  depending on both  $\delta$  and  $h$ , the average is given by the following double integral.

$$\bar{f} = \frac{3}{4\pi^2} \int_0^{2\pi} \left( \int_{-\pi/3}^{\pi/3} f(\delta, h) dh \right) d\tau . \quad (1)$$

The formula for the cosine factor of a moving sunlight on a rotating mirror is fairly complicated. Here we deduce it using matrix algebra, based on the following fact.

**Lemma 1.** *Let  $P$  be a plane in 3D space generated by two orthonormal vectors  $v_1$  and  $v_2$ , and let  $v$  be a vector in the space with  $\|v\| = 1$ . Let  $a$  be the angle between  $v$  and  $P$ . Then*

$$\cos(a) = |\det(v_1, v_2, v)| ,$$

where  $M = (v_1, v_2, v)$  is the  $3 \times 3$  matrix formed by the coordinate components of the 3 vectors under any orthonormal basis of the space.

*Proof.* It is well known that the determinant of a  $3 \times 3$  matrix is the volume of the parallelepiped generated by its column vectors. Now by the choice of  $v_1, v_2$  and  $v$ , this volume equals the cosine factor.  $\square$

Let  $\alpha$  be the east-west rotation angle of the mirror, and  $\beta$  the north-south tilting angle of the mirror. Choose the coordinate system of the space such that  $x$  is the polar direction of the Earth,  $z$  is the direction perpendicular to  $x$  and pointing to the direction of the Sun, and  $y$  is the direction perpendicular both to  $x$  and to  $z$ .

Assuming that the mirror is installed on a ground that is the  $xy$ -plane, and that there is no north-south tilting, the east-west rotation of the mirror is a transformation represented by the matrix

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} .$$

Next, rotating the ground to latitude  $\varphi$  and tilting to angle  $\beta$  is a rotation of the ground in the  $xz$ -plane, with rotation angle  $\varphi - \beta$ . It is represented by the matrix

$$M_2 = \begin{pmatrix} \cos(\varphi - \beta) & 0 & -\sin(\varphi - \beta) \\ 0 & 1 & 0 \\ \sin(\varphi - \beta) & 0 & \cos(\varphi - \beta) \end{pmatrix} .$$

Finally, the self-rotation of the Earth is represented by

$$M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(h) & -\sin(h) \\ 0 & \sin(h) & \cos(h) \end{pmatrix} .$$

Now the unit vector pointing to the direction of the Sun is  $v = \begin{pmatrix} \sin(\delta) \\ 0 \\ \cos(\delta) \end{pmatrix}$ ,

and the position of the mirror can be obtained by first putting it in the  $xy$ -plane, then rotating east-west to angle  $\alpha$ , then tilting to angle  $\varphi - \beta$ , and finally following the self-rotation of the Earth. This series of rotation is represented by the multiplied matrix

$$M = M_3 M_2 M_1 .$$

Let  $w_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ,  $w_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$  be the standard orthonormal basis of the  $xy$ -plane. As  $M$  is an orthogonal matrix, the transformed vectors  $v_1 = Mw_1$  and  $v_2 = Mw_2$  form an orthonormal basis of the mirror plane. According to Lemma 1, the cosine factor of the sunlight to the mirror is

$$\gamma = \gamma(\delta, h, \alpha, \beta) = |\det(v_1, v_2, v)| . \quad (2)$$

Note that both  $\alpha$  and  $\beta$  are functions of  $\delta$  and  $h$ . Once these functions are determined, the average cosine factor of a mirror strip can be obtained via (1):

$$\bar{\gamma} = \frac{3}{4\pi^2} \int_0^{2\pi} \left( \int_{-\pi/3}^{\pi/3} |\det(v_1, v_2, v)| dh \right) d\tau , \quad (3)$$

and the average cosine factor of a LFR concentrator is the average of  $\bar{\gamma}$  over all its mirror strips.

The computation will be done on a hypothetical LFR solar concentrator having 14 mirror strips having a width of  $1.4m$  each, with a gap of  $0.6m$  between adjacent strips. The receiver is placed at a height of  $16m$  above the mirror strips. These dimensions differ slightly from what is commonly practised actually, but we will show in other occasions [6] that with the mirrors of dynamic curvature and a receiver with optimised secondary reflectors, the above parameter ratios offer a better performance.

The average distance between a mirror strip and the receiver is 1.1 times the height of the receiver.

For the evaluation of end loss, we assume that the length of the LFR row is 15 times the height of the receiver, that is,  $240m$ . Although longer LFR rows will reduce end loss, increasing the length-to-height ratio will bring about less than optimal choices in other factors such as installation flexibility, reducing the performance-to-cost ratio. If the row is lengthened or the height is lowered to reduce end loss, the adverse effects should be put into the account of end loss. Therefore 15 times should be a faire point for appreciating the effect of end loss.

On the other hand, low-cost methods to partially recover end loss do exist. In general these methods recover a slight end loss much better than an end loss long away from the end of the receiver. For this reason, we subtract a constant value 0.01 from all the end loss estimations.

Let  $\theta_a = \theta_a(\delta, h)$  be the angle between the sunlight and the normal plane of the receiver axis. The end loss ratio is given by the function

$$E(\delta, h) = \frac{1.1}{15} |\tan(\theta_a)| - 0.01 ,$$

and by (1), the average end loss is

$$\bar{E} = \frac{3.3}{60\pi^2} \int_0^{2\pi} \left( \int_{-\pi/3}^{\pi/3} |\tan(\theta_a)| dh \right) d\tau - 0.01 . \quad (4)$$

In order to compare the seasonal variation of efficiency among different methods, we define the *seasonal variation ratio* of a concentrator,  $\sigma$ , to be the ratio between the yields of unit mirror surface at winter and summer solstice noon. Of course, the actual seasonal variation of the output is worse than  $\sigma$ , due to other inevitable factors such as a shorter day during winter or the thermal losses that remain constant.

## 2 Existing LFR modes

Basically, there are currently 3 common modes to install a LFR solar concentrator.

1. Flat east-west axis: The mirror field is horizontally placed above the ground, and the axes of the mirror and of the receiver are in the east-west direction.
2. Flat north-south axis: The mirror field is horizontally placed above the ground, and the axes of the mirror and of the receiver are in the north-south direction.
3. Inclined east-west axis: the axes are in the east-west direction, and the whole mirror field is inclined towards the sun, in the direction of the width, with an inclination angle that equals the latitude of the site.

Among the three, the flat east-west axis mode is of substantially similar cost as the flat north-south axis mode but offers lower performance under all latitudes. So it is excluded from our comparison for lack of interest.

We also exclude the possibility of installing a LFR with a non-horizontal receiver. A LFR row is practical only when it has a sufficient length; and then the receiver is too long to be tilted for many practical reasons.

**Flat north-south axis mode.** (Figure 2) We have  $\beta = 0$ , and  $v_1$  is in the direction of the axis of the receiver, so  $\theta_a = \arcsin(\langle v_1, v \rangle)$ . Strictly speaking,  $\arcsin(\langle v_1, v \rangle)$  is the angle of the incoming sunlight with the axis, but as the mirrors are parallel to the receiver, this is also the angle of the reflected light.

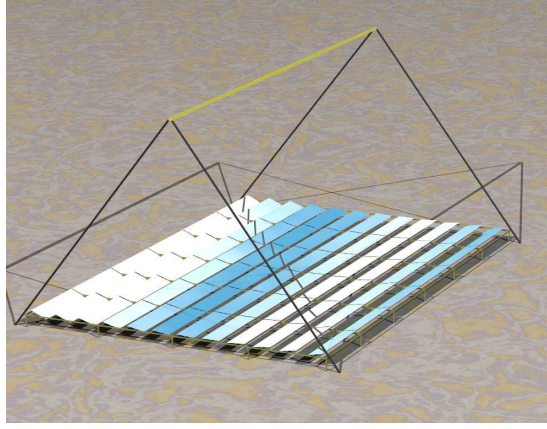


Figure 2: Flat north-south axis LFR

To compute  $\alpha$ , let  $r$  be the angle between the vertical plane above the mirror axis and the plane containing the mirror axis and the receiver center. The series of values of  $r$  for the different mirror strips are

$$\arctan(-13/16) \sim -39.1^\circ, \arctan(-11/16) \sim -34.5^\circ, \dots, 39.1^\circ.$$

Let  $v' = v - \langle v_1, v \rangle v_1$  be the orthogonal projection of  $v'$  in the normal plane of the axis of the rotation axis of the mirror, and let  $u = M_3 M_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$  be the



unit vector pointing to the zenith.  $\alpha$  is determined by the angle  $\alpha'$  between  $u$  and  $v'$ , which is given by the formula

$$\alpha' = \arccos \left( \frac{\langle u, v' \rangle}{||v'||} \right) .$$

Now the mirror has to reflect the sunlight onto the receiver, so it should be rotated to the angle in the middle of  $r$  and  $\alpha'$ , that is,

$$\alpha = \frac{1}{2} (r + \alpha') = \frac{1}{2} \left( r + \arccos \left( \frac{\langle u, v' \rangle}{\sqrt{\langle v', v' \rangle}} \right) \right) . \quad (5)$$

This value of  $\alpha$  can be used to compute the average cosine factor  $\bar{\gamma}$  of a mirror strip. Average will then be taken for all the mirror strips to get the global  $\bar{\gamma}$  of the concentrator.

Shading by adjacent mirror strips can be easily computed to be 1.2% for our setup, and is independent of the latitude.

Table 1 summarizes the mirror yield data for various latitudes.

latitude $\varphi$	$\bar{\gamma}$	$1 - \bar{E}$	1-shading	Mirror yield	$\sigma$
25°	0.83	0.98	0.99	0.8	0.66
30°	0.8	0.97	0.99	0.77	0.6
35°	0.77	0.96	0.99	0.73	0.53
40°	0.74	0.96	0.99	0.7	0.47
45°	0.7	0.95	0.99	0.65	0.39
50°	0.66	0.94	0.99	0.61	0.32

Table 1: Flat north-south axis performances

**Inclined east-west axis mode.** (Figure 3) For this case the mirror behavior is independent on the latitude, and  $\alpha = 0$ ,  $\theta_a = h$ . There is no shading by adjacent mirror strips.

In order that the reflected light be on the receiver, the rotation angle  $\beta - \varphi$  must be in the middle of  $r$  and  $\delta$ , or  $\beta - \varphi = \frac{1}{2} (r + \delta)$ . And the formulae in Section 1 give  $\bar{\gamma} = 0.8$ ,  $1 - \bar{E} = 0.96$ , so the overall mirror yield is 0.77. We have clearly  $\sigma = 1$ .

latitude $\varphi$	$\bar{\gamma}$	$1 - \bar{E}$	1-shading	Mirror yield	$\sigma$
any	0.8	0.96	1	0.77	1

Table 2: Inclined east-west axis performances

It would cost too much to incline a wide LFR. Only concentrators with limited width can be inclined with reasonable overcost for the structure, but then if mirrors with static curvature are used, they will be too narrow and the overcost is important.

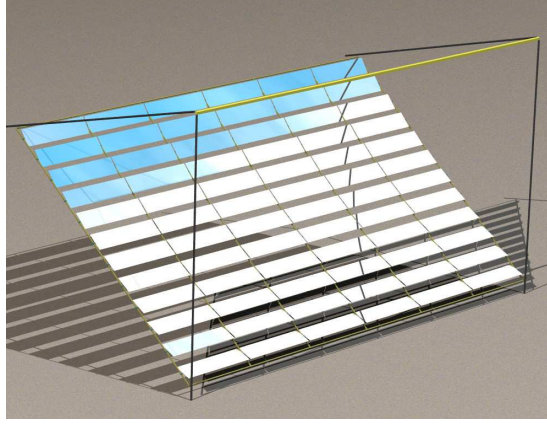


Figure 3: Inclined east-west LFR

Using mirrors of dynamic curvature, modules with only 3 to 5 mirror strips can be constructed, avoiding the overcost of narrow mirrors. For example, a module with 3 mirror strips of  $1.8m$  wide each is a typical choice. However, overcosts due to the inclining structure and performance drop due to a smaller receiver still exist. We hope to deal with this setup more in detail in a separate occasion later.

### 3 The new mode: north-south tilted mirror

We start from the flat traditional north-south axis mode. But instead of horizontally rotating mirrors, we incline the rotation axis of the mirrors to an angle facing the Sun, that equals 0.5 times the latitude of the site. As shown in Figure 1. Note that the receiver remains horizontal, therefore the flow of the heat transfer fluid is exactly as usual.

Here we first compute the performances of this setup, pushing the feasibility and optimization studies to later sections.

We have  $\beta = 0.5\varphi$ , and  $\alpha$  is given by (5) as in the case of flat north-south axis. As for  $\theta_a$ , the tilting of the mirrors makes approximately

$$\theta_a = \arcsin(\langle v_1, v \rangle) - \beta .$$

The integral (4) then gives  $\overline{E} = 0.01$ , which is about the minimal possible value of end loss, and is valid for any latitude. This is because that the “half-tilting” of the mirrors reflect the incoming light to angles that are as close as possible to be perpendicular to the receiver.

The computation of  $\overline{\gamma}$  goes exactly the same way as for the flat north-south axis mode with latitude equal to  $\frac{1}{2}\varphi$ . Shading by adjacent mirror strips is also sensibly the same as for the flat north-south axis mode. The resulting overall mirror yield results are shown in Table 3.

latitude $\varphi$	$\bar{\gamma}$	$1 - \bar{E}$	1-shading	Mirror yield	$\sigma$
25°	0.88	0.99	0.99	0.86	0.82
30°	0.87	0.99	0.99	0.85	0.79
35°	0.86	0.99	0.99	0.84	0.76
40°	0.85	0.99	0.99	0.83	0.73
45°	0.84	0.99	0.99	0.82	0.7
50°	0.83	0.99	0.99	0.81	0.66

Table 3: North-south tilted mirror performances

## 4 Optical adaptation of the mirror strips

It must be noted that, if the mirrors are tilted and if the rotation axes of the mirrors are still kept in parallel positions with each other, most of these rotation axes no longer share a same plane with the receiver axis. On the other hand, if the mirrors are constructed “as usual”, its surface is a cylindrical surface whose generatrices are parallel to the rotation axis. Therefore the focus of the reflected light is a line that is in the same plane with the rotation axis, that can only meet the receiver at an isolated point. That is, the concentration would fail.

One solution to this problem is to place each mirror strip in a way that the rotation axis of each mirror is coplanar with the receiver axis, as shown in Figure 4.

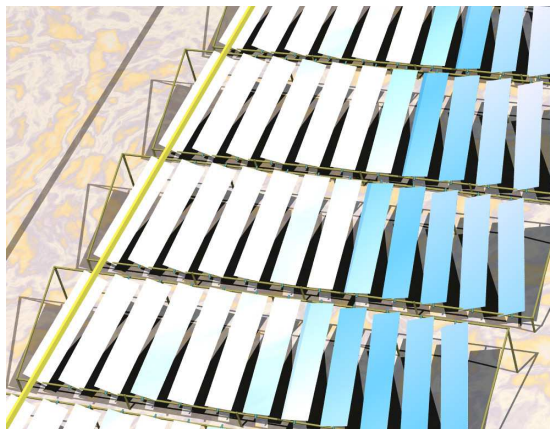


Figure 4: LFR with tilted mirrors each coplanar with the receiver

The disadvantages of this approach are obvious. As the gap between adjacent mirror strips is no longer uniform, optimization on mirror shading and land use is less obvious. And more importantly, the non-parallel mirror strips make the support structure and rotation mechanism more complicated to design and manufacture, so a higher cost is inevitable.

A better solution is to leave the mirror strips in the traditional parallel

positions, while slightly twist the mirrors to “correct” the focus line onto the receiver (Figure 5).

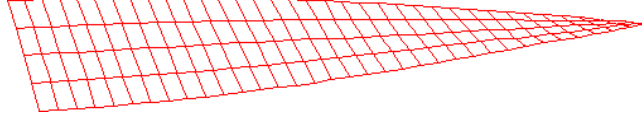


Figure 5: A twisted mirror strip (with a heavily exaggerated twisting)

Strictly speaking, the amount of twisting that is required to move the focus line onto the receiver depends on the position of the Sun, more precisely on the north-south angle of the Sun, because the variation of this angle changes the focal distance. Now we compute the scope of this variation of the twisting.

For this, we need only to do a worst case study. So let  $\varphi = 50^\circ$ ,  $\beta = 0.5\varphi = 25^\circ$ , and take the outer-most mirror strip with a horizontal distance of  $13m$  between the receiver and the rotation axis of the mirror. this is the mirror strip that requires the maximal amount of twisting. Let the receiver be  $17m$  above the lower end of the mirror strips, and take the length of the mirror strip to be  $\lambda = 4m$ . With  $25^\circ$  tilting, the higher end of the mirror strip is  $15.3m$  below the receiver.

Take the coordinate system of the space so that the  $x$  axis points horizontally to the east, the  $z$  axis is the direction of the rotation axis of the mirror strip, and so the  $y$  axis is pointing to the sky with a tilting degree of  $25^\circ$ . Put the origin of the coordinate system on the lower end of the rotation axis of the mirror strip. Under this coordinate system, the receiver line is defined by the system of equations

$$\begin{cases} x = 13 \\ y = 17/\cos(25^\circ) - \tan(25^\circ)z \sim 18.757 - 0.4663z \end{cases}$$

Let  $p_1 = (0, 0, 0)$  and  $p_2 = (0, 0, 4)$  be the two points at the two extremities of the (rotation axis) of the mirror strip. For a fixed sunlight hitting one of the points  $p_i$ , the reflected lights by the mirror under all rotation angles form a quadratic cone  $C_i$ , whose equation is  $z = c_i\sqrt{x^2 + y^2} + d_i$ , where the coefficient  $c_i$  depends on the angle of the incoming sunlight in the north-south direction. It is clear that  $c_1 = c_2$ , and  $d_1 = 0$ ,  $d_2 = 4$ .

Each cone  $C_i$  meets the receiver line at one point  $q_i$ . The orthogonal projection  $L_i$  of the line  $\overline{p_i q_i}$  on the  $xy$  plane has an angle  $a_i$  with the  $x$  axis. The difference  $a_2 - a_1$  is the error that should be corrected by twisting the mirror strip, because if the mirror is non-twisted, the two reflected lights are parallel.

Now if the mirror rotates an angle  $t$ , the orthogonal projection of the reflected light on the  $xy$  plane takes a rotation of angle  $2t$ . Therefore to correct the error angle  $a_2 - a_1$ , the mirror must be twisted to the angle  $a = (a_2 - a_1)/2$  from one end to the other. This is the required amount of twisting.

When the Sun rotates in the east-west direction with respect to the mirror, the cones  $C_1$  and  $C_2$  remains the same. So the twisting angle  $a$  does not change.

On the other hand, the north-south movement of the Sun changes all the data. So we have to compute different values of  $a$  and compare them.

First, take the equinox noon Sun (the middle case). The cones' angular radius is  $90^\circ - 25^\circ = 65^\circ$ , so  $c_1 = c_2 = \tan(25^\circ) \sim 0.4663$ . We have  $q_1 = (13, 14.52, 9.088)$  and  $q_2 = (13, 12.91, 12.543)$ ,  $a_1 = 41.84^\circ$ ,  $a_2 = 45.2^\circ$ , hence

$$a = 1.68^\circ = 29.35 \text{ mrad} .$$

Next let the Sun be at winter solstice noon (the lowest position). We have  $c_1 = c_2 = \tan(25^\circ + 23.44^\circ) \sim 1.128$ ,

$$a = 2.067^\circ = 36.1 \text{ mrad} .$$

At summer solstice morning, the Sun is at the highest position with an angle of about  $-10^\circ$  with respect to the normal plane of the mirror rotation axis. This gives  $c_1 = c_2 = \tan(-10^\circ) \sim -0.1763$ , and

$$a = 1.33^\circ = 23.2 \text{ mrad} .$$

Therefore, the maximal variation of the amount of twisting is about 13 mrad. Now if the mirror strip is adjusted to a fixed twist in the middle between the maximum and the minimum, and if the tracking is adjusted to the middle of the mirror strip, the maximal angular deviation of the mirror rotation is 3.25 mrad, hence the maximal deviation of the reflected light is 6.5 mrad. This is about 3 times more than what is normally tolerated.

For  $\varphi = 40^\circ$ , this maximal deviation drops to 4.5 mrad for a mirror length of  $5m$ .

As this deviation depends only on the north-south angle of the Sun, it is seasonal. It is therefore possible to periodically readjust the twisting manually (4 or 6 times a year), or to incorporate a temperature-sensitive compensating device into the twist adjustment mechanism (more twisting at low temperature). These can bring the deviation of the twisting to less than 1/3 of the original amount, therefore within the acceptable range of 2 mrad.

The variation of the normal focal distance of the a mirror strip is another geometric parameter that has to be dealt with. By definition, the normal focal distance from the point  $p_i$  on the mirror is the distance between the orthogonal projections of  $p_i$  and  $q_i$  on the  $xy$  plane.

The variation of the normal focal distance is most important for the innermost strips, so we assume  $x = 1$ . The geometric analysis above used to compute mirror twist is also valid for this case.

The first case is the variation of the normal focal distance from one end to the other of the mirror strip. This is the direct consequence of the change of the distance from the mirror strip to the receiver, from one end to the other, due to the tilting of the mirror. The extent of the variation depends on the length  $\lambda$  of the mirror strip: the longer the strip, the more is the difference of the distances. If the  $\lambda$  is limited in a way that the difference of the distances is no more than 10 – 15%, the same will be for the variation of the normal focal distance.

This variation is of limited scope. For the case of mirror strips with fixed curvature, it is much less than the variation of focal length brought about by the rotation of the mirror.

With our mirror of dynamic curvature, simple mechanisms can be incorporated into the curvature generating structure so that the curvature varies slightly and continuously from one end to the other. However, the discussion of such mechanisms is beyond the scope of this article. We will return to this question in another occasion.

The more important case is the seasonal variation of the normal focal distance, that is, the variation created by the north-south movement of the Sun. Table 4 gives the scope of this variation, computed using the above geometric analysis, in the form of the ratio maximum:minimum. In this table,  $\rho_1$  (resp.  $\rho_2$ ) is the ratio over the lowest point  $p_1$  (resp. the highest point  $p_2$ ).

$\varphi$	$\beta$	$\lambda$	$x$	$\rho_1$	$\rho_2$
30°	15°	7	1	1.31	1.31
30°	15°	7	13	1.24	1.23
35°	17.5°	6	1	1.39	1.39
40°	20°	5	1	1.47	1.47
40°	20°	5	13	1.36	1.34
45°	22.5°	5	1	1.57	1.57
50°	25°	4	1	1.68	1.68
50°	25°	4	13	1.5	1.47

Table 4: Seasonal variation ratio of normal focal distance

The scope of this seasonal variation is important. Putting together the focal variation due to the rotation of the mirror, the optimal curvature of a mirror will varie in a range of up to 1 : 2. This range is too important for a mirror of fixed curvature to offer a satisfactory performance. On the other hand, mirrors of dynamic curvature can offer this scope of variation of the curvature, with the error of approximation not exceeding  $\pm 1\%$  of the width of the mirror for the reflected light [7]. For our choice of sizes, this means an error less than  $\pm 1$  mrad, which is very satisfactory.

## 5 Other issues

In a north-south tilted mirror mode LFR row, it is clear that the length of the receiver is longer than the added lengths of the mirrors in one row. In order to avoid shading by north-south adjacent mirror banks even in the worst case (winter solstice noon), sufficient space must be left between the adjacent banks.

This north-south gap is easy to compute if the effect of the mirror rotation is not taken into account. In this case the higher end of the mirror bank is  $\lambda \sin(\beta) = \lambda \sin(0.5\varphi)$  above the lower end of the mirror bank behind it. At

winter solstice noon, the sunlight comes with a north-south angle equal to

$$a = \varphi + 23.44^\circ ,$$

so the gap should be

$$\mu = \lambda \sin(0.5\varphi) \tan(a) .$$

And the length  $R$  of the portion of the receiver corresponding to one bank of mirrors is equal to the distance on the ground between the lower end of the preceeding mirror bank to the lower end of the next mirror bank, which equals

$$R = \lambda \cos(0.5\varphi) + \mu = \lambda (\cos(0.5\varphi) + \sin(0.5\varphi) \tan(a)) .$$

The gap-to-mirror ratio  $\mu : \lambda$  and receiver-to-mirror ratio  $R : \lambda$  for various latitudes are summarized in Table 5. The table also includes the maximal ratio between the mirror length  $\lambda$  and the height  $H$  of the receiver over the lower end of the mirrors, in order that the variation of the focal length be limited to 15%. This last ratio equals

$$\lambda : H = 0.15 / \sin(0.5\varphi) .$$

$\varphi$	$\beta$	$\mu : \lambda$	$R : \lambda$	$\max(\lambda : H)$
$25^\circ$	$12.5^\circ$	0.244	1.22	0.69
$30^\circ$	$15^\circ$	0.35	1.31	0.58
$35^\circ$	$17.5^\circ$	0.49	1.44	0.5
$40^\circ$	$20^\circ$	0.68	1.62	0.44
$45^\circ$	$22.5^\circ$	0.97	1.89	0.39
$50^\circ$	$25^\circ$	1.42	2.32	0.355

Table 5: Length ratios for north-east tilted mirror mode

It should be noted that the values of the north-south gap given in Table 5 are only the maxima but not the optima that should be much less than the maxima. For example for  $\varphi = 45^\circ$ , reducing  $R : \lambda$  to 1.5 from 1.89 reduces the overall mirror yield by less than 2% due to north-south shading, although the seasonal variation factor suffers a bit more than that. If the reductions in cost and thermal losses brought about by a shorter receiver outweigh the performance reduction, the shorter value is better.

We remark also that the north-south gap between mirror banks is not completely wasted: it offers a comfortable maintenance space for the mirror field.

The tracking of the mirror strips is another issue. Now that the mirror strips are no longer parallel to the receiver, the rotations of the various mirror strips are not synchronized due to the north-south movement of the Sun. The amount of desynchronization is limited to within a degree between adjacent mirror. But it is too important to be ignored, and measures must be taken to deal with it.

This can be done either by driving different mirror strips by independent motors, or if one motor is used to drive several adjacent mirror strips, by adding

a compensation mechanism in the mechanical linkage of the mirror driving system. The first method is probably preferred, as even under the flat mirror mode, the economic advantage of using one motor to drive adjacent mirror strips is not obvious. With the precision requirement of the application, what is saved in the motors may very well be paid back for the mechanical linkage. With the desynchronization of the rotations, it is even less obvious that a compensating linkage is economically competitive.

As it is not the intention of this article to make a thorough cost study, we simply make some remarks on the cost. Per unit mirror surface, the tilted mirror mode costs slightly more than the flat mode, and the overcost grows with the tilting angle  $\beta$ . This overcost is due to several factors: a longer receiver, extra cost for the structure, the limited length of the mirror strips, an extra mechanism to twist the mirrors. But the basic sizing remains the same as the flat mode, so that the overcost is less important than for the inclined east-west mode.

The final point is the tilting angle  $\beta$  of the mirror strips. Of course,  $0.5\varphi$  is not the only possible tilting angle. In general, tilting the mirrors to an angle  $\beta < 0.5\varphi$  reduces the overcost, but the mirror yield is reduced too. Inversely, tilting the mirrors to an angle  $\beta > 0.5\varphi$  increases the mirror yield and the overcost at the same time, until some point  $< \varphi$  where the yield starts to decline. Hence an optimization can be done with a best balance between the cost and the yield. But any departure of  $\beta$  from  $0.5\varphi$  will increase endloss, therefore the optimal value of  $\beta$  will not be very far from  $0.5\varphi$  in either direction.

## 6 Conclusion

From the above discussions, we obtain Table 6 which compares the mirror yields of the three modes, with the flat north-south axis mode as reference.

All the three modes are supposed to use curved mirror strips with dynamic curvature. With mirrors with static curvature, the inclined east-west mode suffers from a severe overcost, while the north-south tilted mirror mode has a very poor concentration ratio, so the comparison does not make sense.

Latitude	Flat	Inclined	Tilted
25°	1	0.96	1.08
30°	1	1	1.1
35°	1	1.05	1.15
40°	1	1.1	1.19
45°	1	1.18	1.26
50°	1	1.26	1.33

Table 6: Mirror yield and seasonal variation comparison

A correct comparison of the advantages or disadvantages of the modes must also include the relative differences of the costs. But cost analysis is beyond the



scope of this article, so we simply make some remarks.

Among the three modes, the inclined east-west axis mode has the highest cost per mirror surface, as is noted before. But at the same time, its output is the most valuable too, especially in case of a high latitude, due to its excellent behavior during the winter. On the other hand, the flat north-south axis mode is the most economic one, and in the same time its output is the least valuable due to the huge seasonal variation. The north-south tilted mirror mode is between the two in both aspects.

Hence depending on the point of view of each, the differences in the cost aspect and in the seasonal aspect among the modes cancels each other more or less. In the end, the differences for the mirror yields reflect quite well the order of preference among the modes.

This means the following conclusion.

The flat north-south axis mode is the best choice for latitudes under  $25^\circ$ .

The north-south tilted mirror mode is the best choice for latitudes between  $25^\circ$  and  $45^\circ$ .

The inclined east-west axis mode is the best choice for latitudes above  $50^\circ$ .

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