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1 Discussion of “Understanding the influence of slope on
2 the threshold of coarse grain motion: revisiting critical
3 stream power” by C. Parker, N.J. Clifford, and C.R.
4 Thorne

5 Geomorphology, Volume 126, March 2011, Pages 51-65

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8 **Abstract**

The authors present an interesting review on the critical stream power for inception of motion. They also present a new data set of experimental data. Eventually, they suggest the use of a dimensionless critical stream power, which may be of interest. However, some of the remarks and conclusions appear to be subjective and need to be clarified; and there is no sensitivity analysis nor theoretical development to support their analysis. A more extensive discussion on some aspects of the slope effects on the critical Shields parameter and stream power is also lacking in their paper and, thus, additional material is provided here.

9 *Keywords:* threshold of motion; critical Shields parameter; critical stream
10 power; relative water depth; slope

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11 **1. Estimation of the bed shear stress**

12 A serious question for the work by Parker et al. (2011) is about the
13 estimation of the mean bed shear stress over the section τ . As presented by
14 Parker et al. (2011), geomorphologists generally used an approximation of the
15 bed shear stress (and stream power ω) by assimilating the hydraulic radius R_h
16 to the mean water depth h . However, to be more rigorous, especially in the
17 of laboratory measurements, one should be careful about the computation of
18 τ and ω :

$$\omega = \tau U = \rho g R_h S U = \frac{\rho g Q S}{W} \frac{R_h}{h} \quad (1)$$

19 Some differences may thus be observed depending on the geometrical char-
20 acteristics of the channel. In the case of the laboratory channel used by
21 Parker et al. (2011), one should also use wall corrections such as suggested
22 by Vanoni and Brooks (1957). In Fig. 1, estimations of the bed shear stress
23 is plotted versus the total discharge using the experimental set up of Parker
24 et al. (2011) using either the mean water depth, the hydraulic radius. The
25 Manning-Strickler law was used here to link the discharge with the bed shear
26 stress assuming a constant Strickler coefficient $K = 21/d^{1/6}$ (Strickler, 1923).
27 Even if the wall correction tends to reduce the differences, an overestimation
28 by 10% is made by using the mean water depth to compute the bed shear

29 stress. Nevertheless, it would be interesting to have this new data set avail-
30 able in the form of a table in order to be able to reuse these data.

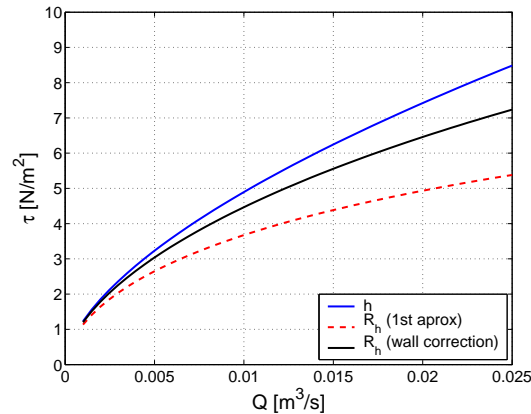


Figure 1: Estimation of the bed shear as a function of the discharge for the channel used by Parker et al. (2011) ($S = 0.01$, $d_{50} = 6$ mm)

31 Moreover, using section-averaged values may lead to significant errors
32 when computing bedload transport. Indeed, as observed by Camenen et al.
33 (in press), large variations of the bed shear stress may be observed throughout
34 the section; and as bedload transport is non linear, one should compute
35 bedload locally to yield a proper estimation of the total bedload transport
36 across the section. This remark is even more important if a variation of the
37 grain size is found throughout the section, as is often observed (Habersack
38 and Laronne, 2001; Frings and Kleinhans, 2008). It may also explain the
39 large dispersion observed by Petit et al. (2005) for the critical stream power

40 ω_{cr} .

41 Having in mind all these remarks, the discussion on the correlation be-
42 tween bedload transport and τ or ω appears to be quite subjective. Many of
43 the remarks in section 2.1 criticizing the application of τ_{cr} could be applied
44 to ω_{cr} as well.

45 **2. Influence of grain size, water depth, and slope on the existing** 46 **models for estimating ω_{cr}**

47 Parker et al. (2011) presented and criticized developments made by Bag-
48 nold (1980) and Ferguson (2005) but did not explain where these models
49 need to be improved. Some additional discussion is proposed hereafter. As
50 suggested by Ferguson (2005) revisiting the development by Bagnold (1980),
51 the critical stream power ω_{cr} may be written as follows:

$$\omega_{cr} = \frac{2.3\rho}{\kappa} [\theta_{cr}(s-1)gd]^{3/2} \log\left(\frac{30h_{cr}}{\exp(1)k_s}\right) \quad (2)$$

52 where $\kappa = 0.4$ is the Von Karman constant, θ_{cr} is the critical Shields param-
53 eter, s is the sediment relative density ($s \approx 2.65$), d is the grain size, h_{cr} is
54 the critical water depth at the inception of motion, and k_s is the roughness
55 height. Hereafter, following Camenen et al. (2006), one can assume $k_s = 2d$
56 at the inception of motion.

57 One assumption taken by Bagnold (1980) is that $\theta_{cr} = 0.04$. Parker
58 et al. (2011) did not discuss this assumption, whereas many authors showed
59 a dependence on the grain size following the work by Shields (1936). For
60 example, Soulsby (1997) suggested a simple fit for the Shields curve:

$$\theta_{cr,0} = \frac{0.30}{1 + 1.2d_*} + 0.055 [1 - \exp(-0.02d_*)] \quad (3)$$

61 with $d_* = [g(s - 1)/\nu^2]^{1/3}d$. The subscript $_0$ indicates here that the critical
62 Shields parameter is estimated excluding slope effects, i.e. for $S \approx 0$. As
63 shown by several authors (Recking, 2009), the slope appears to strongly
64 affect the critical Shields parameter as well.

65 Using the extensive data set set compiled by Buffington and Montgomery
66 (1997), Eq. (3) was plotted together with the data both as a function of d_*
67 (Fig. 2). Equation (3) apparently yields a correct estimation of the critical
68 Shields parameter, corresponding roughly to the minimum values measured.
69 As observed by Buffington and Montgomery (1997), most of the scatter is
70 because of methods used to estimate θ_{cr} : extrapolation of bedload transport
71 rate to a zero value, visual observation, development of competence functions
72 that relate shear stress to the largest mobile grain, theoretical calculation,
73 equivalence of bedload and surface grain size, and smallest transport captured
74 by bedload traps.

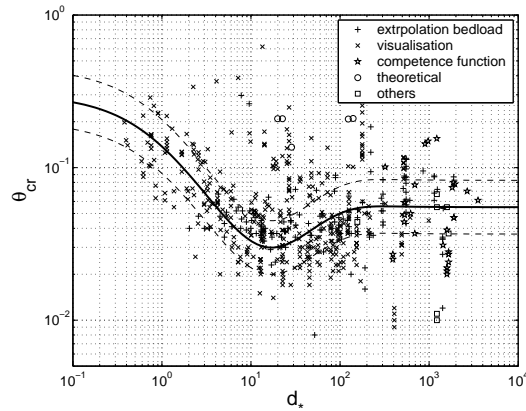


Figure 2: Shields curve with $\theta_{cr,0}$ based on Eq. (3) (solid line; dashed lines correspond to the 50% error bar) as a function of d_* .

75 As shown in Fig. 3A, the use of Eq. (3) instead of $\theta_{cr} = 0.04$ affects
 76 the results given by Eq. (2) as much as the water depth. Indeed, for coarse
 77 sediments ($d > 0.5$ mm), $\theta_{cr} \approx 0.055$ following the Shields curve, that means
 78 an increase by a factor two for ω_{cr} . As a comparison, one must increase the
 79 water depth by a factor of 10 to obtain similar changes.

80 For a uniform flow, one can use the relationship between the Shields
 81 parameter and the local slope of the bed S :

$$\theta_{cr} = \frac{R_{h,cr} S}{(s-1)d} \quad (4)$$

82 where $R_{h,cr}$ is the critical hydraulic radius at the inception of motion. $R_{h,cr} \approx$
 83 h_{cr} if the width of the river $W \gg h_{cr}$. However, for laboratory data and small

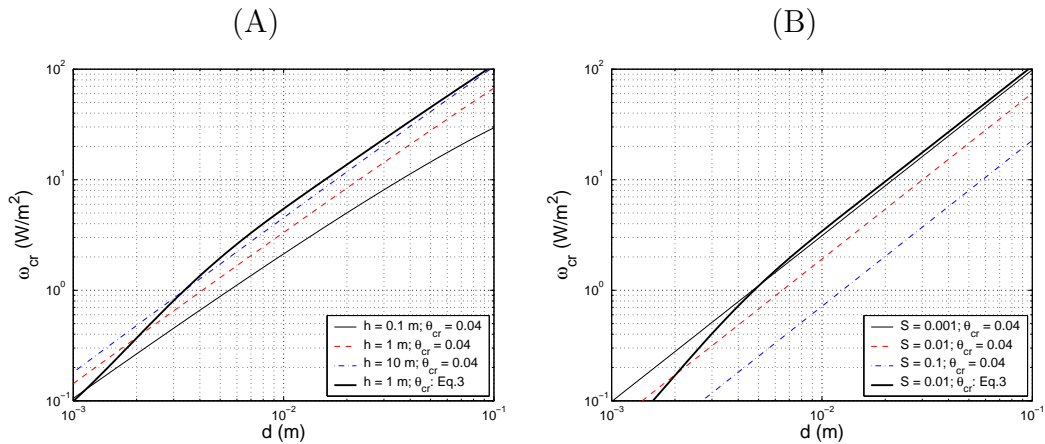


Figure 3: Critical stream power based on the development by Bagnold (1980) (Eq. (2)) as a function of the grain size d , with the water depth influence emphasized (A) or the bed slope influence emphasized (B).

84 rivers, this assumption may be too strong. Using Eq. (4), the use of Eq. (3)
 85 instead of $\theta_{cr} = 0.04$ also affects the results given by Eq. (2) by a factor of
 86 2 (Fig. 3B).

87 Ferguson (2005) showed that the choice of the velocity profile in the de-
 88 velopment of Eq. (2) does not affect the results. The formula based on the
 89 Manning-Strickler resistance law is not discussed here, but similar conclu-
 90 sions were obtained using this second formula. In Fig. 3, Eq. (2) indicates
 91 an increasing critical stream power with the critical water depth and a de-
 92 creasing critical stream power with slope. As observed by Parker et al. (2011),
 93 this is not confirmed by the data and Petit et al. (2005)'s work. However,

94 it does not mean the model is weak but that some of the hypotheses need
95 to be improved (constant θ_{cr} , $R_{h,cr}/d$ inversely proportional to S). Some
96 improvements of the existing models are presented in the following sections.

97 **3. Influence of the slope on the critical Shields parameter**

98 Results presented by Parker et al. (2011) are not particularly new. Many
99 authors showed a dependency between the critical Shields parameter and
100 the slope S , especially for mountain rivers where the slope is not negligible
101 (Recking, 2009). Lamb et al. (2008) showed that there is a trend of increasing
102 critical Shields stress with channel slope despite a large data scatter.

103 In order to get a better estimation of θ_{cr} , one should improve the rela-
104 tionship between the relative flow depth R_h/d and the slope S as suggested
105 by Ferguson (2005) or Lamb et al. (2008). In the same way as some rela-
106 tionships have been found between the equilibrium slope of a river and the
107 median grain size, such an idea appears to be physically realistic. Indeed,
108 it corresponds to the discussion by Lamb et al. (2008) or Recking (2009)
109 on the computation of the friction coefficient depending on the relative flow
110 depth R_h/d . And some attempts to develop analytical models were pro-
111 posed (Armanini and Gregoretti, 2005). Based on Eq. (4), if we assumed

112 that $\theta_{cr} \approx \theta_{cr,0}$ for S close to zero, one can obtain an asymptote for $R_{h,cr}/d$:

$$\left(\frac{R_h}{d}\right)_{cr,0} = \frac{(s-1)\theta_{cr,0}}{S} \quad \text{if } S \rightarrow 0 \quad (5)$$

113 Using the data set by Recking (2009), the relative flow depth at incipient
114 motion was plotted as a function of the bed slope S in Fig. 4. Eq. (5)
115 yields a correct tendency but does not fit well to the data. An empirical fit
116 is suggested (see Fig. 4):

$$\left(\frac{R_h}{d}\right)_{cr} = \frac{(s-1)\theta_{cr,0}}{S} (0.5 + 6S^{0.75}) \quad (6)$$

117 The fit suggested that the critical Shields parameter for S close to zero is
118 approximately twice smaller than given by the Shields curve (Eq. (3)). This
119 may be explained as the Shields curve was proposed based on experimental
120 results where $S > 0$ and assuming that θ_{cr} is independent of S . In Fig. 3,
121 Eqs. 5 and 6 were plotted using the mean value of the experimental data for
122 $\theta_{cr,0}$ (using Eq. (3)), i.e. $\theta_{cr,0} = 0.051$. The empirical fit suggested by Parker
123 et al. (2011) for θ_{cr} (Eq. 14 in their paper) may be rewritten using Eq. (4):

$$\left(\frac{R_h}{d}\right)_{cr} = 0.19(s-1)S^{-0.72} \quad (7)$$

124 This third relationship is also plotted in Fig. 4 but tends to overestimated
125 the results.

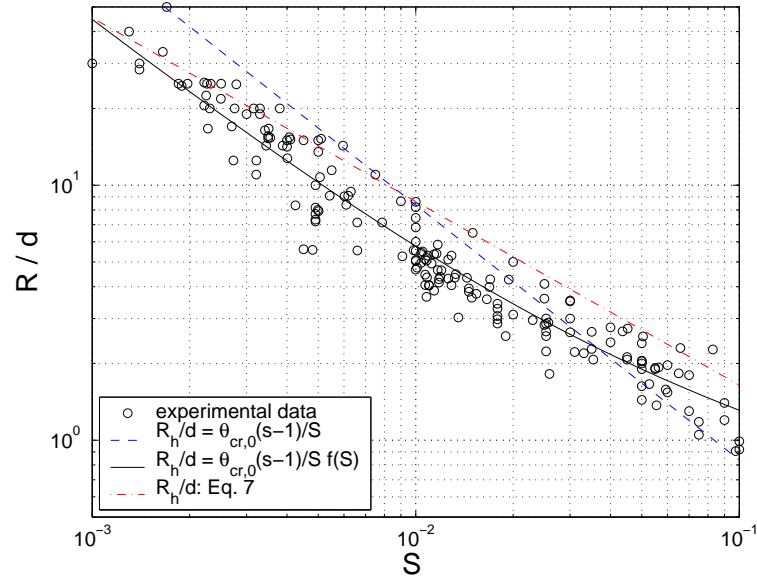


Figure 4: Relative flow depth as a function of the bed slope ($f(S) = 0.5 + 6S^{0.75}$).

126 The approach developed by Shields assumed no direct effect of the slope
 127 $S \approx 0$. There exists a critical angle ϕ_s (angle of repose of the sediment)
 128 at which slope the sediment will avalanche in a zero flow. For an arbitrary
 129 (positive) slope S , gravity provides a component of force on the grain, which
 130 decreases the threshold such as (Ikeda, 1982):

$$\frac{\theta_{cr,S}}{\theta_{cr,0}} = \frac{\sin(\phi_s - \arctan S)}{\sin(\phi_s)} = \cos(\arctan S) \left[1 - \frac{S}{\tan(\phi_s)} \right] \quad (8)$$

131 Finally, by combining Eq. (4) with Eqs. (3), (6), and (8), an equation
 132 for the critical Shields parameter may be written as follows:

$$\theta_{cr} = \left(\frac{R_h}{d} \right)_{cr} \frac{S}{(s-1)} \frac{\theta_{cr,S}}{\theta_{cr,0}} = \theta_{cr,0} \frac{\sin(\phi_s - \arctan S)}{\sin(\phi_s)} (0.5 + 6S^{0.75}) \quad (9)$$

133 Equation (9) (with $\theta_{cr,0} = 0.051$ corresponding to an average value, $s =$
 134 2.65, and $\tan \phi_s = 1.2$) is plotted in Fig. 5 together with the experimental
 135 data collected by Recking (2009) and shows a correct behaviour compared
 136 to the data thanks to the empirical fit ($f(S) = 0.5 + 6S^{0.75}$).

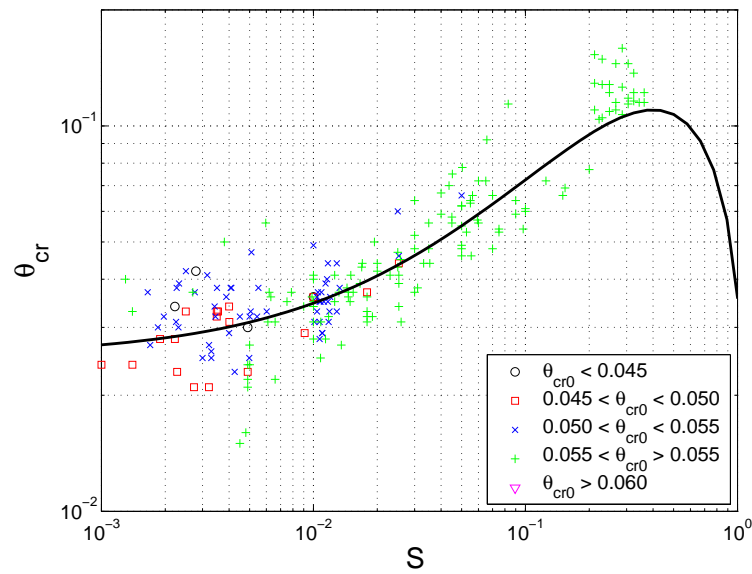


Figure 5: Critical Shields parameter as a function of the bed slope (The solid line correspond to Eq. (9) with $\theta_{cr,0} = 0.051$, $s = 2.65$, and $\tan \phi_s = 1.2$).

137 4. Influence of slope and grain size on critical stream power

138 Plotting both critical Shields parameter and critical stream power against
 139 the slope, Parker et al. (2011) showed that there is some correlation between
 140 θ_{cr} and S , whereas no correlation is found between ω_{cr} and S . Indeed a larger

141 dispersion is observed for the second case. How is it possible to conclude
142 that ω_{cr} is independent of S , and so, more convenient for modelling bedload
143 transport?

144 Following the previous development and assuming $k_s = 2d$ (Camenen
145 et al., 2006), the critical stream power may be written using Eqs. (2) and
146 (9):

$$\omega_{cr} = \frac{2.3\rho}{\kappa} \left[\left(\frac{R_h}{d} \right)_{cr} \frac{\theta_{cr,S}}{\theta_{cr,0}} Sgd \right]^{3/2} \log \left[\frac{15}{\exp(1)} \left(\frac{R_h}{d} \right)_{cr} \right] \quad (10)$$

147 Equations (9) and (10) yield satisfactory results compared to the exper-
148 imental data set collected by Recking (2009). Most of the data are prop-
149 erly predicted within a factor of 1.5 allowed ($P1.5$, or criterion $1/1.5 \leq$
150 $X_{predicted}/X_{measured} \leq 1.5$ satisfied). Some larger dispersion is however ob-
151 served for the prediction of the critical critical stream power, indicating that
152 its prediction is not as easy as the critical Shields parameter. The model
153 suggested by Parker et al. (2011) ($\omega_{cr}^* = \omega_{cr}/(g\rho(s-1)\sqrt{(s-1)gd^3}) = 0.1$)
154 yields relatively poor results compared to Eq. (10) in the same order as Eq.
155 (9) yields poorer results than Eq. (3). The Ferguson model (Eq. 11 in Parker
156 et al.) yields actually better results than the Parker et al. model ($\omega_{cr}^* = 0.1$)
157 although a much larger dispersion is observed.

Table 1: Statistical results for the proposed models compared to the experimental data set collected by Recking (2009) ($f = X_{predicted}/X_{measured}$).

Parameter	Equation	$P1.2$	$P1.5$	mean(f)	std(f)
θ_{cr}	Eq. 3	20%	46%	+0.09	0.21
	Eq. 9	61%	93%	+0.02	0.09
ω_{cr}	Eq. 10	35%	69%	+0.05	0.17
	Eq. Parker et al.	16%	35%	+0.18	0.21
	Eq. Fergusson	21%	46%	-0.17	0.61

158 In Fig. 6A, Eq. (10) made dimensionless as suggested by Parker et al.
159 (2011) ($s = 2.65$, and $\tan \phi_s = 1.2$ and for four different grain sizes) is
160 plotted together with the experimental data collected by Recking (2009).
161 The critical dimensionless stream power appears to be sensitive to both grain
162 size and slope. The influence of the grain occurs through the critical Shields
163 parameter $\theta_{cr,0}$ only; it explains why curves for $d = 1$ mm and $d = 3$ mm are
164 merged. The proposed model presents a coherent behaviour although it does
165 not show an increasing ω_{cr}^* value for coarse sediments as observed in Fig. 6B.
166 The empirical fit by Parker et al. (2011), which corresponds to $\omega_{cr}^* = 0.1$,
167 tends to overestimate the results and a significant dispersion is observed.

168 Using Eqs. (1) and (4), some simple manipulations yield to the following

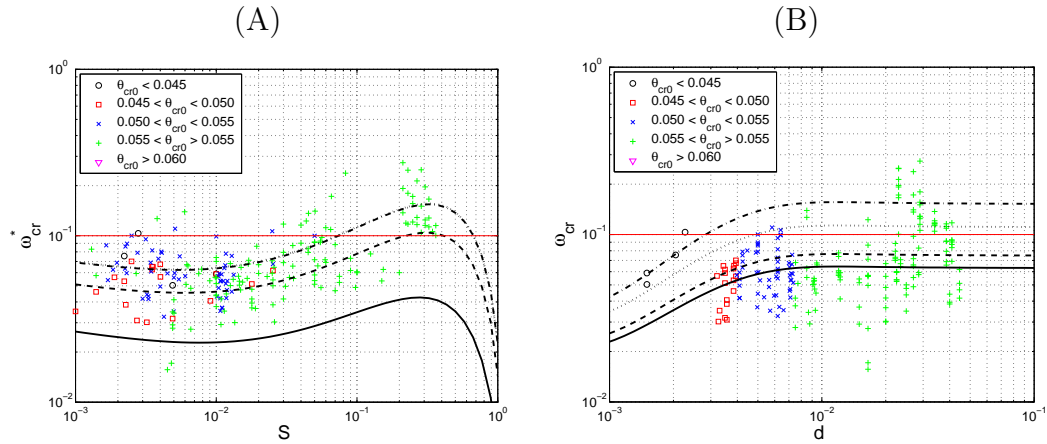


Figure 6: Dimensionless critical stream power as a function of the bed slope (A) or of the grain size (B) (the four lines correspond to Eq. (10) with $s = 2.65$, $\tan \phi_s = 1.2$, and four different grain sizes $d = 0.1, 0.3, 1$, and 3 mm, or four different slopes $S = 0.01, 0.03, 0.1$, and 0.3 , respectively for the solid, dashed, pointed, and dashed-pointed lines).

169 relationship for the dimensionless critical stream power:

$$\omega_{cr}^* = \frac{R_h S U}{(s-1)\sqrt{(s-1)gd^3}} = \frac{\theta_{cr} F_r}{\sqrt{s-1}} \sqrt{\frac{R_h}{d}} \quad (11)$$

170 with $F_r = U/\sqrt{gR_h}$ a Froude number. Thus, it appears that the only differ-
 171 ences between these two dimensionless parameters lie in the Froude number
 172 and the relative flow depth. The Froude number may be rewritten such as
 173 $F_r = \sqrt{S/C_d}$ with $C_d = [\kappa/(1 + \ln(k_s/(30R_h)))]^2$ a dimensionless logarithmic
 174 drag coefficient. As shown by Rickenmann (2011), many empirical equations
 175 were suggested to relate the drag coefficient to the relative flow depth R_h/d ,

176 or to the slope S considering a potential relationship between R_h/d and S
177 close to the inception of movement (see Eq. 6). As a results, θ_{cr} and ω_{cr}^* may
178 be related through the relative flow depth only (or though the slope only).
179 However, both Froude number and relative flow depth do affect the velocity
180 profile and roughness height, especially in mountain rivers (Camenen et al.,
181 2006; Rickenmann, 2011). And the problem is getting even more complex
182 for poorly sorted sediment mixtures (Rickenmann, 2011).

183 5. Conclusions

184 If the review and the proposed formula for ω_{cr} by Parker et al. (2011) is of
185 interest, the discussion is generally poor. All the comments to discredit the
186 use of the critical Shields parameter in favour of the critical stream power
187 have no real interest and are often wrong or subjective. Eventually, Parker
188 et al. (2011) suggested an empirical equation for ω_{cr} with no real scientific
189 support: “Using a dimensionless critical stream power criterion to identify
190 the threshold of motion is both conceptually and practically attractive.” One
191 should remember that the Shields parameter is also dimensionless and so
192 could be conceptually and practically attractive. Their equation fits “ex-
193 tremely well to a selection of data,” but this selection is highly questionable.

194 Using the data set selected by Recking (2009), their equation yields not as
195 good results as expected.

196 Some new formulas for θ_{cr} and ω_{cr} are suggested here based on a semi
197 empirical fit for the critical relative flow depth $(R_h/d)_{cr}$. They yield good
198 results compared to an extensive data set collected by Recking (2009) and
199 confirm the interests and limits of the formula proposed by Parker et al.
200 (2011). A simple relationship between θ_{cr} and ω_{cr}^* (Eq. (11)) shows that the
201 relative flow depth $(R_h/d)_{cr}$ (which can be written as a function of the slope
202 S) is the main term that differentiates the two parameters.

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