Discussion of ”Understanding the influence of slope on the threshold of coarse grainmotion: Revisiting critical stream power” by C. Parker, N.J. Clifford, and C.R. Thorne Geomorphology, Volume 126, March 2011, Pages 51-65

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The original publication is available at http://www.sciencedirect.com/
http://dx.doi.org/10.1016/j.geomorph.2011.10.033

Discussion of "Understanding the influence of slope on the threshold of coarse grain motion: revisiting critical stream power" by C. Parker, N.J. Clifford, and C.R. Thorne

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B. Camenen[1]

Cemagref, UR HHLY, 3 bis quai Chauveau CP 220, 69336 Lyon cedex 09, France

Abstract

The authors present an interesting review on the critical stream power for inception of motion. They also present a new data set of experimental data. Eventually, they suggest the use of a dimensionless critical stream power, which may be of interest. However, some of the remarks and conclusions appear to be subjective and need to be clarified; and there is no sensitivity analysis nor theoretical development to support their analysis. A more extensive discussion on some aspects of the slope effects on the critical Shields parameter and stream power is also lacking in their paper and, thus, additional material is provided here.

Keywords: threshold of motion; critical Shields parameter; critical stream power; relative water depth; slope

[1] Tel.: +33 (0)4 72 20 86 26 ; Fax: +33 (0)4 78 47 78 75 ; E-mail: benoit.camenen@cemagref.fr

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The original publication is available at http://www.sciencedirect.com/
http://dx.doi.org/10.1016/j.geomorph.2011.10.033

1. Estimation of the bed shear stress

A serious question for the work by Parker et al. (2011) is about the estimation of the mean bed shear stress over the section τ. As presented by Parker et al. (2011), geomorphologists generally used an approximation of the bed shear stress (and stream power ω) by assimilating the hydraulic radius R_h to the mean water depth h. However, to be more rigorous, especially in the of laboratory measurements, one should be careful about the computation of τ and ω:

$$\omega = \tau U = \rho g R_h S U = \frac{\rho g Q S}{W} \frac{R_h}{h} \tag{1}$$

Some differences may thus be observed depending on the geometrical characteristics of the channel. In the case of the laboratory channel used by Parker et al. (2011), one should also use wall corrections such as suggested by Vanoni and Brooks (1957). In Fig. 1, estimations of the bed shear stress is plotted versus the total discharge using the experimental set up of Parker et al. (2011) using either the mean water depth, the hydraulic radius. The Manning-Strickler law was used here to link the discharge with the bed shear stress assuming a constant Strickler coefficient $K = 21/d^{1/6}$ (Strickler, 1923). Even if the wall correction tends to reduce the differences, an overestimation by 10% is made by using the mean water depth to compute the bed shear

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stress. Nevertheless, it would be interesting to have this new data set available in the form of a table in order to be able to reuse these data.

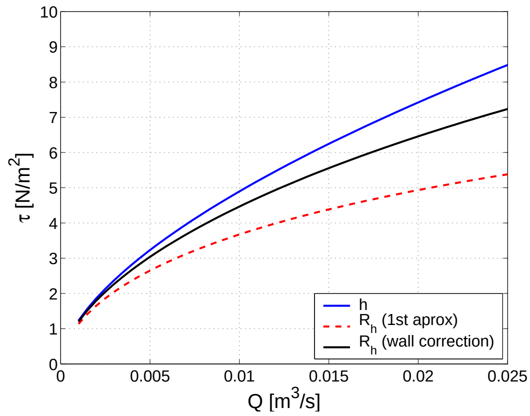

Figure 1: Estimation of the bed shear as a function of the discharge for the channel used by Parker et al. (2011) ($S = 0.01$, $d_{50} = 6$ mm)

Moreover, using section-averaged values may lead to significant errors when computing bedload transport. Indeed, as observed by Camenen et al. (in press), large variations of the bed shear stress may be observed throughout the section; and as bedload transport is non linear, one should compute bedload locally to yield a proper estimation of the total bedload transport across the section. This remark is even more important if a variation of the grain size is found throughout the section, as is often observed (Habersack and Laronne, 2001; Frings and Kleinhans, 2008). It may also explain the large dispersion observed by Petit et al. (2005) for the critical stream power

ω_{cr}.

Having in mind all these remarks, the discussion on the correlation between bedload transport and τ or ω appears to be quite subjective. Many of the remarks in section 2.1 criticizing the application of τ_{cr} could be applied to ω_{cr} as well.

2. Influence of grain size, water depth, and slope on the existing models for estimating ω_{cr}

Parker et al. (2011) presented and criticized developments made by Bagnold (1980) and Ferguson (2005) but did not explain where these models need to be improved. Some additional discussion is proposed hereafter. As suggested by Ferguson (2005) revisiting the development by Bagnold (1980), the critical stream power ω_{cr} may be written as follows:

$$\omega_{cr} = \frac{2.3\rho}{\kappa} \left[\theta_{cr}(s-1)gd\right]^{3/2} \log\left(\frac{30h_{cr}}{\exp(1)k_s}\right) \tag{2}$$

where $\kappa = 0.4$ is the Von Karman constant, θ_{cr} is the critical Shields parameter, s is the sediment relative density ($s \approx 2.65$), d is the grain size, h_{cr} is the critical water depth at the inception of motion, and k_s is the roughness height. Hereafter, following Camenen et al. (2006), one can assume $k_s = 2d$ at the inception of motion.

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One assumption taken by Bagnold (1980) is that $\theta_{cr} = 0.04$. Parker et al. (2011) did not discuss this assumption, whereas many authors showed a dependence on the grain size following the work by Shields (1936). For example, Soulsby (1997) suggested a simple fit for the Shields curve:

$$\theta_{cr,0} = \frac{0.30}{1 + 1.2d_*} + 0.055 \left[1 - \exp(-0.02d_*)\right] \tag{3}$$

with $d_* = [g(s-1)/\nu^2]^{1/3}d$. The subscript $_0$ indicates here that the critical Shields parameter is estimated excluding slope effects, i.e. for $S \approx 0$. As shown by several authors (Recking, 2009), the slope appears to strongly affect the critical Shields parameter as well.

Using the extensive data set set compiled by Buffington and Montgomery (1997), Eq. (3) was plotted together with the data both as a function of d_* (Fig. 2). Equation (3) apparently yields a correct estimation of the critical Shields parameter, corresponding roughly to the minimum values measured. As observed by Buffington and Montgomery (1997), most of the scatter is because of methods used to estimate θ_{cr}: extrapolation of bedload transport rate to a zero value, visual observation, development of competence functions that relate shear stress to the largest mobile grain, theoretical calculation, equivalence of bedload and surface grain size, and smallest transport captured by bedload traps.

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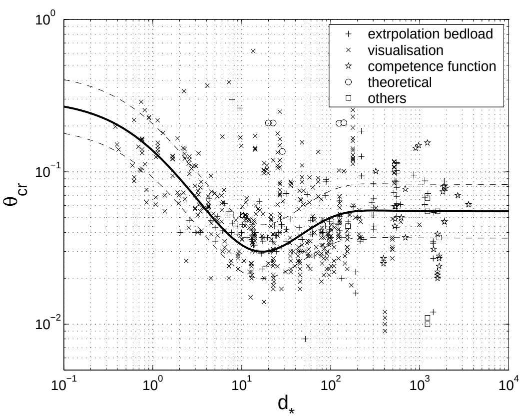

Figure 2: Shields curve with $\theta_{cr,0}$ based on Eq. (3) (solid line; dashed lines correspond to the 50% error bar) as a function of d_*.

As shown in Fig. 3A, the use of Eq. (3) instead of $\theta_{cr} = 0.04$ affects the results given by Eq. (2) as much as the water depth. Indeed, for coarse sediments ($d > 0.5$ mm), $\theta_{cr} \approx 0.055$ following the Shields curve, that means an increase by a factor two for ω_{cr}. As a comparison, one must increase the water depth by a factor of 10 to obtain similar changes.

For a uniform flow, one can use the relationship between the Shields parameter and the local slope of the bed S:

$$\theta_{cr} = \frac{R_{h,cr} S}{(s-1)d} \tag{4}$$

where $R_{h,cr}$ is the critical hydraulic radius at the inception of motion. $R_{h,cr} \approx h_{cr}$ if the width of the river $W \gg h_{cr}$. However, for laboratory data and small

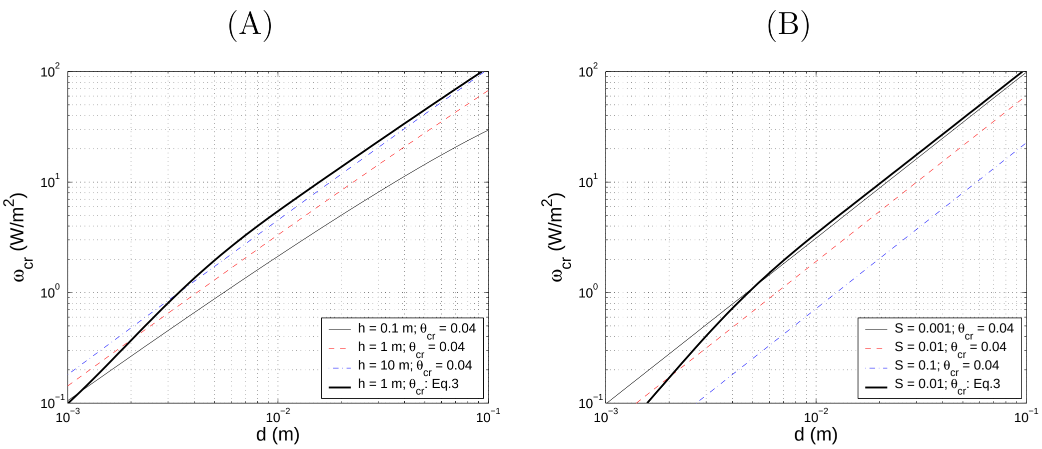

Figure 3: Critical stream power based on the development by Bagnold (1980) (Eq. (2)) as a function of the grain size d, with the water depth influence emphasized (A) or the bed slope influence emphasized (B).

rivers, this assumption may be too strong. Using Eq. (4), the use of Eq. (3) instead of $\theta_{cr} = 0.04$ also affects the results given by Eq. (2) by a factor of 2 (Fig. 3B).

Ferguson (2005) showed that the choice of the velocity profile in the development of Eq. (2) does not affect the results. The formula based on the Manning-Strickler resistance law is not discussed here, but similar conclusions were obtained using this second formula. In Fig. 3, Eq. (2) indicates an increasing critical stream power with the critical water depth and a decreasing critical stream power with slope. As observed by Parker et al. (2011), this is not confirmed by the data and Petit et al. (2005)'s work. However, it does not mean the model is weak but that some of the hypotheses need to be improved (constant θ_{cr}, $R_{h,cr}/d$ inversely proportional to S). Some improvements of the existing models are presented in the following sections.

3. Influence of the slope on the critical Shields parameter

Results presented by Parker et al. (2011) are not particularly new. Many authors showed a dependency between the critical Shields parameter and the slope S, especially for mountain rivers where the slope is not negligible (Recking, 2009). Lamb et al. (2008) showed that there is a trend of increasing critical Shields stress with channel slope despite a large data scatter.

In order to get a better estimation of θ_{cr}, one should improve the relationship between the relative flow depth R_h/d and the slope S as suggested by Ferguson (2005) or Lamb et al. (2008). In the same way as some relationships have been found between the equilibrium slope of a river and the median grain size, such an idea appears to be physically realistic. Indeed, it corresponds to the discussion by Lamb et al. (2008) or Recking (2009) on the computation of the friction coefficient depending on the relative flow depth R_h/d. And some attempts to develop analytical models were proposed (Armanini and Gregoretti, 2005). Based on Eq. (4), if we assumed that $\theta_{cr} \approx \theta_{cr,0}$ for S close to zero, one can obtain an asymptote for $R_{h,cr}/d$:

$$\left(\frac{R_h}{d}\right)_{cr,0} = \frac{(s-1)\theta_{cr,0}}{S} \quad \text{if} \quad S \to 0 \tag{5}$$

Using the data set by Recking (2009), the relative flow depth at incipient motion was plotted as a function of the bed slope S in Fig. 4. Eq. (5) yields a correct tendency but does not fit well to the data. An empirical fit is suggested (see Fig. 4):

$$\left(\frac{R_h}{d}\right)_{cr} = \frac{(s-1)\theta_{cr,0}}{S}\left(0.5 + 6S^{0.75}\right) \tag{6}$$

The fit suggested that the critical Shields parameter for S close to zero is approximately twice smaller than given by the Shields curve (Eq. (3)). This may be explained as the Shields curve was proposed based on experimental results where $S > 0$ and assuming that θ_{cr} is independent of S. In Fig. 3, Eqs. 5 and 6 were plotted using the mean value of the experimental data for $\theta_{cr,0}$ (using Eq. (3)), i.e. $\theta_{cr,0} = 0.051$. The empirical fit suggested by Parker et al. (2011) for θ_{cr} (Eq. 14 in their paper) may be rewritten using Eq. (4):

$$\left(\frac{R_h}{d}\right)_{cr} = 0.19(s-1)S^{-0.72} \tag{7}$$

This third relationship is also plotted in Fig. 4 but tends to overestimated the results.

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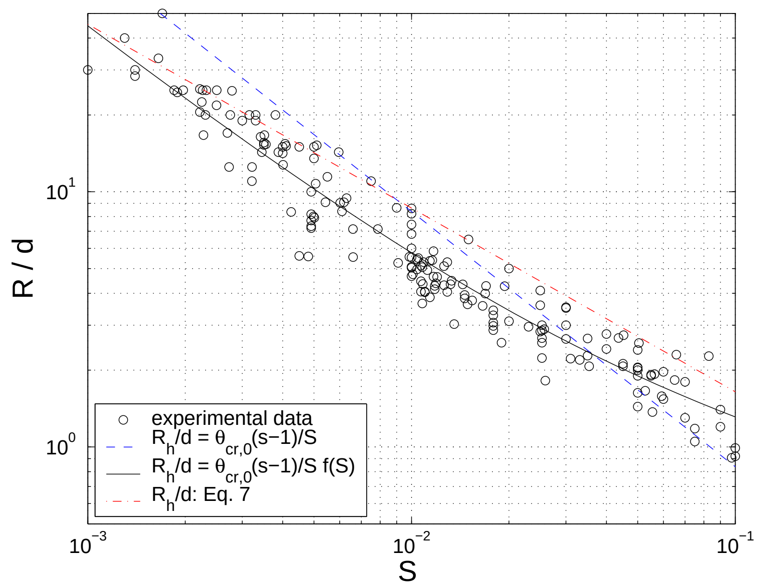

Figure 4: Relative flow depth as a function of the bed slope ($f(S) = 0.5 + 6S^{0.75}$).

The approach developed by Shields assumed no direct effect of the slope $S \approx 0$. There exists a critical angle ϕ_s (angle of repose of the sediment) at which slope the sediment will avalanche in a zero flow. For an arbitrary (positive) slope S, gravity provides a component of force on the grain, which decreases the threshold such as (Ikeda, 1982):

$$\frac{\theta_{cr,S}}{\theta_{cr,0}} = \frac{\sin(\phi_s - \arctan S)}{\sin(\phi_s)} = \cos(\arctan S)\left[1 - \frac{S}{\tan(\phi_s)}\right] \qquad (8)$$

Finally, by combining Eq. (4) with Eqs. (3), (6), and (8), an equation for the critical Shields parameter may be written as follows:

$$\theta_{cr} = \left(\frac{R_h}{d}\right)_{cr} \frac{S}{(s-1)} \frac{\theta_{cr,S}}{\theta_{cr,0}} = \theta_{cr,0} \frac{\sin(\phi_s - \arctan S)}{\sin(\phi_s)} \left(0.5 + 6S^{0.75}\right) \qquad (9)$$

Equation (9) (with $\theta_{cr,0} = 0.051$ corresponding to an average value, $s = 2.65$, and $\tan\phi_s = 1.2$) is plotted in Fig. 5 together with the experimental data collected by Recking (2009) and shows a correct behaviour compared to the data thanks to the empirical fit ($f(S) = 0.5 + 6S^{0.75}$).

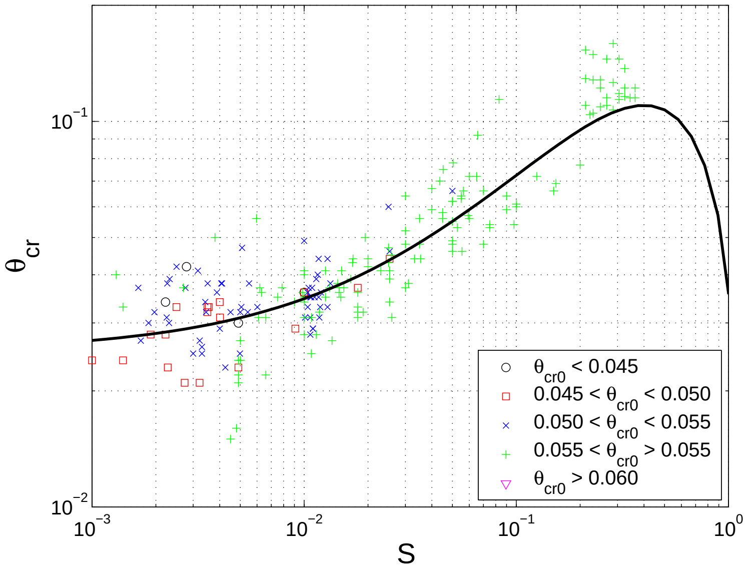

Figure 5: Critical Shields parameter as a function of the bed slope (The solid line correspond to Eq. (9) with $\theta_{cr,0} = 0.051$, $s = 2.65$, and $\tan\phi_s = 1.2$).

4. Influence of slope and grain size on critical stream power

Plotting both critical Shields parameter and critical stream power against the slope, Parker et al. (2011) showed that there is some correlation between θ_{cr} and S, whereas no correlation is found between ω_{cr} and S. Indeed a larger dispersion is observed for the second case. How is it possible to conclude that ω_{cr} is independent of S, and so, more convenient for modelling bedload transport?

Following the previous development and assuming $k_s = 2d$ (Camenen et al., 2006), the critical stream power may be written using Eqs. (2) and (9):

$$\omega_{cr} = \frac{2.3\rho}{\kappa}\left[\left(\frac{R_h}{d}\right)_{cr}\frac{\theta_{cr,S}}{\theta_{cr,0}}Sgd\right]^{3/2}\log\left[\frac{15}{\exp(1)}\left(\frac{R_h}{d}\right)_{cr}\right] \qquad (10)$$

Equations (9) and (10) yield satisfactory results compared to the experimental data set collected by Recking (2009). Most of the data are properly predicted within a factor of 1.5 allowed ($P1.5$, or criterion $1/1.5 \leq X_{predicted}/X_{measured} \leq 1.5$ satisfied). Some larger dispersion is however observed for the prediction of the critical critical stream power, indicating that its prediction is not as easy as the critical Shields parameter. The model suggested by Parker et al. (2011) ($\omega^*_{cr} = \omega_{cr}/(g\rho(s-1)\sqrt{(s-1)gd^3}) = 0.1$) yields relatively poor results compared to Eq. (10) in the same order as Eq. (9) yields poorer results than Eq. (3). The Ferguson model (Eq. 11 in Parker et al.) yields actually better results than the Parker et al. model ($\omega^*_{cr} = 0.1$) although a much larger dispersion is observed.

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Table 1: Statistical results for the proposed models compared to the experimental data set collected by Recking (2009) ($f = X_{predicted}/X_{measured}$).

Parameter	Equation	$P1.2$	$P1.5$	mean(f)	std(f)
θ_{cr}	Eq. 3	20%	46%	+0.09	0.21
	Eq. 9	61%	93%	+0.02	0.09
ω_{cr}	Eq. 10	35%	69%	+0.05	0.17
	Eq. Parker et al.	16%	35%	+0.18	0.21
	Eq. Fergusson	21%	46%	−0.17	0.61

In Fig. 6A, Eq. (10) made dimensionless as suggested by Parker et al. (2011) ($s = 2.65$, and $\tan\phi_s = 1.2$ and for four different grain sizes) is plotted together with the experimental data collected by Recking (2009). The critical dimensionless stream power appears to be sensitive to both grain size and slope. The influence of the grain occurs through the critical Shields parameter $\theta_{cr,0}$ only; it explains why curves for $d = 1$ mm and $d = 3$ mm are merged. The proposed model presents a coherent behaviour although it does not show an increasing ω^*_{cr} value for coarse sediments as observed in Fig. 6B. The empirical fit by Parker et al. (2011), which corresponds to $\omega^*_{cr} = 0.1$, tends to overestimate the results and a significant dispersion is observed.

Using Eqs. (1) and (4), some simple manipulations yield to the following

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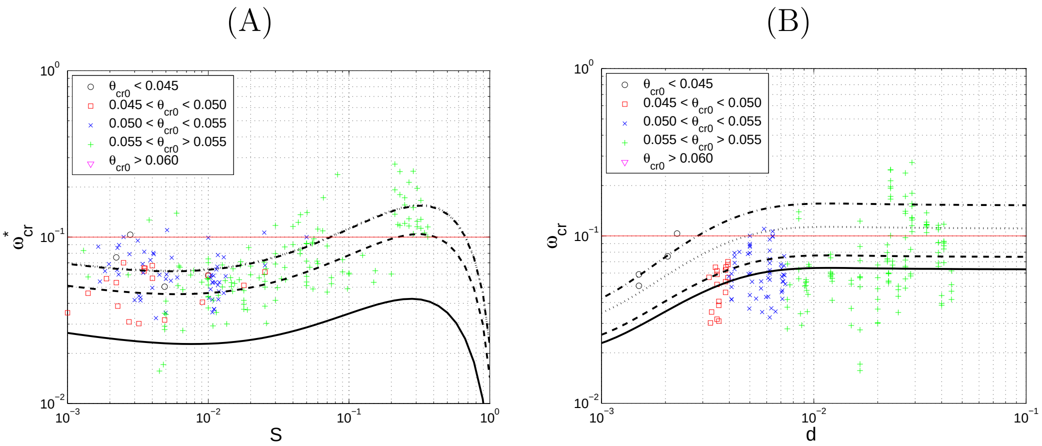

Figure 6: Dimensionless critical stream power as a function of the bed slope (A) or of the grain size (B) (the four lines correspond to Eq. (10) with $s = 2.65$, $\tan\phi_s = 1.2$, and four different grain sizes $d = 0.1$, 0.3, 1, and 3 mm, or four different slopes $S = 0.01$, 0.03, 0.1, and 0.3, respectively for the solid, dashed, pointed, and dashed-pointed lines).

relationship for the dimensionless critical stream power:

$$\omega^*_{cr} = \frac{R_h S U}{(s-1)\sqrt{(s-1)gd^3}} = \frac{\theta_{cr} F_r}{\sqrt{s-1}}\sqrt{\frac{R_h}{d}} \tag{11}$$

with $F_r = U/\sqrt{gR_h}$ a Froude number. Thus, it appears that the only differences between these two dimensionless parameters lie in the Froude number and the relative flow depth. The Froude number may be rewritten such as $Fr = \sqrt{S/C_d}$ with $Cd = [\kappa/(1+\ln(k_s/(30R_h)))]^2$ a dimensionless logarithmic drag coefficient. As shown by Rickenmann (2011), many empirical equations were suggested to relate the drag coefficient to the relative flow depth R_h/d, or to the slope S considering a potential relationship between R_h/d and S close to the inception of movement (see Eq. 6). As a results, θ_{cr} and ω^*_{cr} may be related through the relative flow depth only (or though the slope only). However, both Froude number and relative flow depth do affect the velocity profile and roughness height, especially in mountain rivers (Camenen et al., 2006; Rickenmann, 2011). And the problem is getting even more complex for poorly sorted sediment mixtures (Rickenmann, 2011).

5. Conclusions

If the review and the proposed formula for ω_{cr} by Parker et al. (2011) is of interest, the discussion is generally poor. All the comments to discredit the use of the critical Shields parameter in favour of the critical stream power have no real interest and are often wrong or subjective. Eventually, Parker et al. (2011) suggested an empirical equation for ω_{cr} with no real scientific support: "Using a dimensionless critical stream power criterion to identify the threshold of motion is both conceptually and practically attractive." One should remember that the Shields parameter is also dimensionless and so could be conceptually and practically attractive. Their equation fits "extremely well to a selection of data," but this selection is highly questionable.

Using the data set selected by Recking (2009), their equation yields not as good results as expected.

Some new formulas for θ_{cr} and ω_{cr} are suggested here based on a semi empirical fit for the critical relative flow depth $(R_h/d)_{cr}$. They yield good results compared to an extensive data set collected by Recking (2009) and confirm the interests and limits of the formula proposed by Parker et al. (2011). A simple relationship between θ_{cr} and ω^*_{cr} (Eq. (11)) shows that the relative flow depth $(R_h/d)_{cr}$ (which can be written as a function of the slope S) is the main term that differentiates the two parameters.

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