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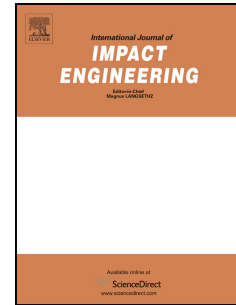
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An approach to robust optimization of impact problems using random samples and meta modelling

David Lönn^{*,a}, Ørjan Fyllingen^b, Larsgunnar Nilsson^a

^a*Division of Solid Mechanics, Linköping University, SE-58183 Linköping, Sweden*

^b*Structural Impact Laboratory (SIMLab), Department of Structural Engineering, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway*

Abstract

Conventionally optimized structures may show a tendency to be sensitive to variations, for instance in geometry and loading conditions. To avoid this, research has been carried out in the field of robust optimization where variations are taken into account in the optimization process. The overall objective is to create solutions that are optimal both in the sense of mean performance and minimum variability. This work presents an alternative approach to robust optimization, where the robustness of each design is assessed through multiple sampling of the stochastic variables at each design point. Meta models for the robust optimization are created for both the mean value and the standard deviation of the response. Furthermore, the method is demonstrated on an analytical example and an example of an aluminium extrusion with quadratic cross section subjected to axial crushing. It works well for the chosen examples and it is concluded that the method is especially well suited for problems with a large number of random variables,

*Corresponding author. Tel.: +46-13-282475; fax: +46-13-282717.
Email address: david.lonn@liu.se (David Lönn)

since the computational cost is essentially independent of the number of random variables. In addition, the presented approach makes it possible to take into consideration variations that cannot be described with a variable. This is demonstrated in this work by random geometrical perturbations described with the use of Gaussian random fields.

Key words: Robust optimization, Geometric imperfections, Meta model, Robustness

1. Introduction

To study the axial buckling of a straight profile is challenging. Small perturbations of the design or loading conditions may create large variations in structural responses, as seen in for instance Fyllingen et al. [1]. Structures that strongly change their behaviour when subjected to small variations are not robust, and this property is rarely sought. Traditional deterministic optimization methods applied to structures subjected to impact loading may also create non-robust designs. This problem can be resolved by accounting for variations in the design variables and environmental conditions when performing the optimization, i.e. performing a robust optimization. This rather fast growing research area is summarised by e.g. Beyer et al. [2].

The concept of robust design is to find an optimal design, which is not only optimal in the sense of mean response value, but also has a minimal variation of the response when subjected to stochastic variations. However, there are several ways to account for variations, i.e. different ways of including the variations in the optimization formulation. Several papers have included uncertainties in the constraints by changing the constraints from being al-

ways fulfilled to being fulfilled to some probability chosen. These methods with probabilistic inequality constraints are denoted Reliability-Based Design Optimization approaches (RBDO), see e.g. Gu et al. [3]. According to for instance Beyer et al. [2], there is no consensus in the literature as to whether RBDO should be regarded as robust optimization or not, since the formulation does not imply a minimisation of the response variations, but rather an inclusion of a safety margin in the constraints. In order to explicitly minimise the variations of a response, this entity must either be present in the objective function, e.g. as in Doltsinis et al. [4], or be given an upper limit in a constraint so that the variations in the response is minimised in order to satisfy the constraint.

The grand challenge lies in evaluating the responses and their variations for computationally costly applications. Responses from impact loading conditions generally require long computing times and it is common to use meta models for approximating the responses. The meta models are built from carefully selected response evaluations, where the chosen sets of variable values are denoted the Design Of Experiments (DOE). Using the meta models, it is then possible to get an approximation of the non-evaluated designs. Several papers extend the usage of the constructed meta models to making approximative evaluations of response dispersion data, typically Monte Carlo analyses performed on the meta models, e.g. Lee et al. [5], Sinha et al. [6] and Ait Brik et al. [7]. In order for this methodology to be accurate, the meta model must also be accurate. For a complete assessment of the dispersions, one must also consider the uncertainties in the meta model itself, e.g. as in the work of Martin et al. [8].

Inspired by the work of Vining et al. [9] and [10], several other recent papers use the dual response surface approach in which two response surfaces are created, one for the mean and one for the variance or standard deviation of a response. In contrast to meta model-based Monte Carlo analyses, e.g. Kovach et al. [11] and Shin et al. [12] use replicates of the same design in order to obtain estimates of the variances for the different responses.

Our work presents an alternative approach to robust optimization. Two meta models are used, one for the mean and one for the standard deviation of the response. Random (uncontrollable) variables are not included in the variable space. Instead, in each design point, an assessment of the mean and standard deviation of a response is made based on a predetermined set of random samples for the stochastic variables, i.e. the random variables and the design variables that are non-deterministic. The mean and the standard deviation of the response are approximated over the design variable space using Artificial Neural Network (ANN) meta models. Standard optimization strategies implemented in the software LS-OPT are then applied to the robust optimization formulation in order to find an optimal robust design. The novelty in this work is the combination of several factors; the robust optimization formulation, using the same random samples throughout the optimization, the calculation of robustness in each design, the use of random fields and the optimization procedure.

Two examples of the proposed robust optimization approach are provided, one analytical design example and one Finite Element (FE) example, where the objective is to find the robust optimal size and position of a buckling trigger on a square aluminium tube subjected to impact loading. For

the latter, a similar problem setup has been presented by e.g. Missoum [13] and [14], but the solution techniques are different, as he maximises the probability that the mean response is above a certain value, whereas in this work, the variance in each design is also studied and used as an objective in the optimization. Finally, it is concluded that the method proposed here is especially well suited for problems with a large number of random variables.

2. Objective

The objective of this work is twofold. First, an alternative approach to robust optimization is presented. The method is not necessarily restricted to usage in a structural impact context, but the chosen structural example serves as a good demonstrator. Advantages and restrictions of the proposed approach are presented in the discussion, as well as possible fields of application. Second, the approach is applied to the robust optimization problem of finding the optimal size and placement of a buckling trigger on an axially crushed aluminium profile. In this example, it is shown why a small trigger should be placed at one end of the axially crushed beam.

3. Theory

The following sections briefly present the theories that constitute the basis for this work.

3.1. Robust optimization

A robust optimization is an optimization where dispersions of the variables and responses are taken into account. Thus, the optimization problem

can be formulated as a multiobjective problem with the minimum of the dispersion as an additional objective.

$$\min_x [\mu(f(x)), \sigma(f(x))] \quad (1)$$

with appropriate constraints. A common approach is to weight the two objectives linearly, i.e.

$$\min_x \bar{f}(x) = \alpha \frac{\mu(f(x))}{\mu_0(f(x_0))} + (1 - \alpha) \frac{\sigma(f(x))}{\sigma_0(f(x_0))} \quad (2)$$

where the new objective function \bar{f} is a linear combination of the mean, μ , and the standard deviation, σ , of a stochastic response $f(x)$. By performing a normalisation, introducing μ_0 and σ_0 denoting the mean and standard deviation of the initial designs response, the trade-off situation becomes independent of the size of the two terms in the objective function. The robustness of the optimal solution design will then only depend on the choice of the parameter α . This is not required when the mean value and the standard deviation are of comparable magnitude.

The formulation above has been selected for this paper. However, true expressions for mean and standard deviation of responses are generally not known, and these entities must be replaced by approximations that are valid over the design domain. The robust optimization formulation changes to

$$\min_x \tilde{f}(x) = \alpha \frac{\tilde{\mu}(f(x))}{\mu_0(f(x_0))} + (1 - \alpha) \frac{\tilde{\sigma}(f(x))}{\sigma_0(f(x_0))} \quad (3)$$

where $\tilde{\mu}$ and $\tilde{\sigma}$ are meta model approximations of the true responses.

Since \tilde{f} now is a smooth function, any standard optimization algorithm can be used for this problem. This work uses the Leapfrog Optimizer for

Constrained optimization (LFOPC) in conjunction with two different types of neural networks as meta models, as implemented in LS-OPT, see Stander et al. [19].

3.2. Meta modelling

Meta models are constructed approximations of the responses over the design space. The approximations are built up from evaluations of response values for a carefully selected set of designs, denoted the Design Of Experiments (DOE). Meta models are most commonly used when each evaluation of the response is computationally costly and when a global or local approximation may increase the efficiency, e.g., in traditional optimization or sensitivity analysis. It is important to note that the robust optimization approach presented here is independent of the choice of meta model. However, since every design point in this method requires several costly evaluations, it is wise to choose a meta model where the results from every design evaluation are saved and reused, and where, consequently, the meta model is refined for each iteration in the optimization procedure.

In this work, a meta model based on a Neural Network which meets the above criteria is used. Moreover, according to e.g. Redhe [17], Neural Networks are also capable of capturing local changes of the response, such as a bifurcation in the buckling mode where the energy absorption changes rapidly with a small change in the design. This is not the case when a polynomial meta model is used over the entire region of interest.

3.3. Artificial Neural Network

A Neural Network, or more precisely, an Artificial Neural Network (ANN), may be used to approximate complex relations between input and output data, and thus serve as a meta model. An ANN consists of neurons, i.e. small computing devices, that are connected. The output y_k from neuron k is calculated as

$$y_k(\mathbf{x}) = f \left(\sum_{i=0}^d w_{ki}x_i \right) \Big|_{x_0=1} = f(a) \quad (4)$$

where f is the activation function and w_{ki} is the weight of the corresponding input signal x_i . The latter is either an input variable value or an output value from a previous neuron in the network. The term w_{k0} corresponds to the bias parameter and it may be included in the summation by adding an input signal $x_0 = 1$.

The nature of the connection topology between the neurons, the weights and the type of activation functions, together determine the type of ANN used. The two most common approaches for function approximation are the multilayer feedforward (FF) network and the radial basis function (RBF) network. In a multilayer FF network, no information travels backward in the network, i.e. the output of each layer serves as an input to the next. Furthermore, the activation functions in the hidden layers are usually sigmoidal functions

$$f(a) = \frac{1}{1 + e^{-a}} \quad (5)$$

and the input and output layers are usually linear, i.e. $f(a) = a$.

The network is called an RBF network using Gaussian basis functions when the following mapping is used

$$y_k(\mathbf{x}) = \sum_{i=1}^d w_{ki} \phi_i(\mathbf{x}) + w_{k0} \quad (6)$$

$$\phi_i(\mathbf{x}) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{c}_i\|_2^2}{2\sigma_{c_i}^2}\right)$$

where $\|\dots\|_2^2$ denotes the square of the Euclidian distance, and where \mathbf{c}_i and σ_{c_i} are the center and width of the i :th Gaussian basis function, respectively.

Both multilayer FF and RBF networks are used in this work. More information regarding ANN in the context of function approximation is found in e.g. Bishop [18].

3.4. Random Fields

The finite element model of the axially crushed aluminium profile, which will be used here to demonstrate the method of robust optimization, is based on the model developed by Fyllingen et al. [1]. By stochastic simulations, Fyllingen et al. investigated the influence of assumed random geometrical imperfections on the behaviour of a slender aluminium profile subjected to axial crushing. As in that model, the geometrical imperfections, in this work only the perturbations of the position of the wall sides, will be modelled by the use of Gaussian random fields.

The algorithms which were used for implementation of the Gaussian zero mean homogenous random fields were adopted from Shinozuka and Deodatis [15] and Stefanou and Papadrakakis [16]. It was chosen to use the following autocorrelation function R_{ff} , which yields quite smooth realizations

$$R_{ff}(x_1, x_2) = s^2 \exp \left[- \left(\frac{x_1}{b_1} \right)^2 - \left(\frac{x_2}{b_2} \right)^2 \right] \quad (7)$$

where s denotes the standard deviation for the stochastic field and b_1 and b_2 are parameters, which are proportional to the correlation distance of the stochastic field along the x_1 and x_2 axis, respectively.

4. Work procedure

The work procedure is presented in Figure 1. To describe the variations in a model, it is assumed that m different stochastic variables are required. Random samples of these m variables are generated n times in the beginning of the optimization, creating n different sets with different values of the stochastic variables. The same random variations are used throughout the optimization, and the reason for this will be explained later.

The next step is to choose an initial DOE, after which the iteration loop for the optimization begins with evaluating every design n times, once for every set of random variations. From the n evaluations of a design, it is possible to evaluate the mean and the standard deviation of the response, which is done for all designs. Next, two meta models are created to approximate the mean and the standard deviation over the region of interest. A standard optimization approach is then applied to the robust optimization formulation, Equation (3), where the iterations stop when convergence has been reached.

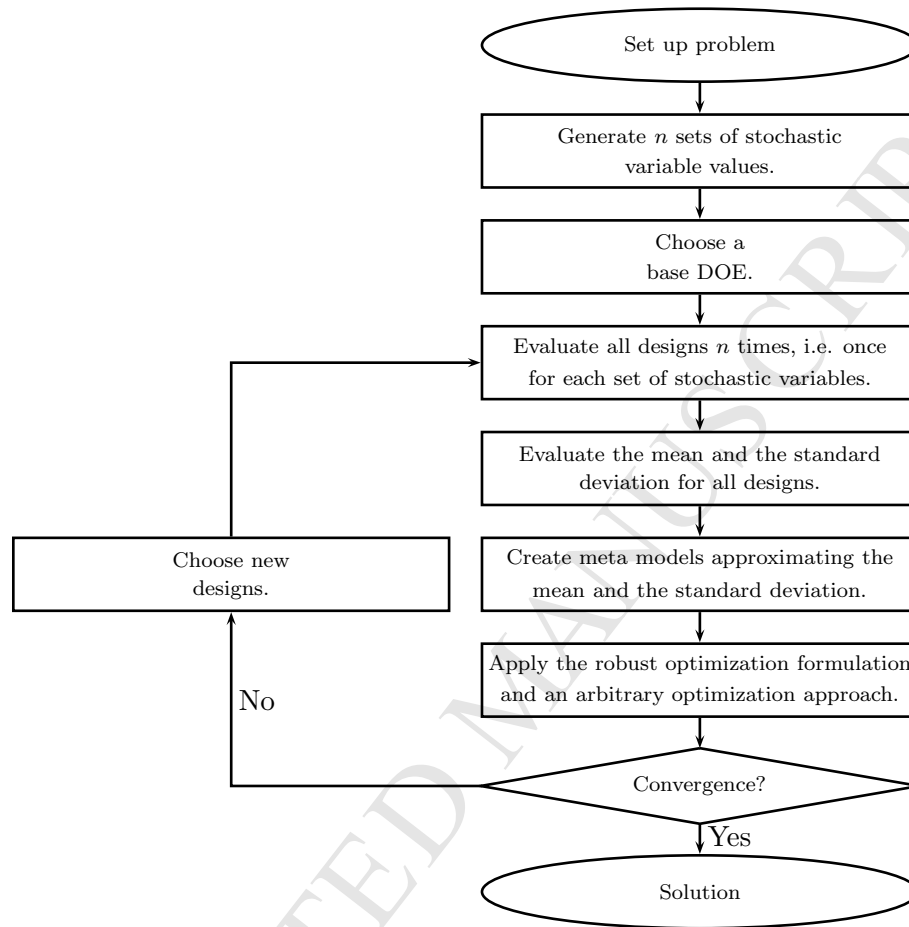


Figure 1: Flow chart for the methodology used.

5. Analytical example

In order to demonstrate the proposed approach, a one-dimensional analytical example is used. The problem is formulated according to Equation (3) as follows

$$\begin{aligned} \min_x \quad & \tilde{f}(x) = \alpha \cdot \tilde{\mu}(f(x)) + (1 - \alpha) \cdot \tilde{\sigma}(f(x)) \\ \text{s.t.} \quad & 0 \leq x \leq 1 \end{aligned}$$

where

$$\begin{aligned} f(x) &= g(x) + \varepsilon(x) \\ g(x) &= (x - 0.6)^2 \cdot (2 - \sin(3.5\pi(x - 0.55))) \\ \varepsilon(x) &= k(x) \cdot \text{"random field, } N(0, 1)\text{"} \\ k(x) &= 0.1(1.4 + \sin(2\pi(x - 0.4))) \\ \alpha &= 0.3 \end{aligned} \tag{8}$$

A robust optimization of the stochastic response $f(x)$ is to be performed, where x is the only design variable. The objective is to minimise a linear combination of the mean performance and the variability. The response function $f(x)$ is a sum of a mean function $g(x)$ and some random error $\varepsilon(x)$, which causes the dispersion. The random error is represented by a one-dimensional Gaussian random field that is normally distributed with zero mean and variance one, $N(0, 1)$. The random fields that are generated are further multiplied with a weight function $k(x)$, creating the error function $\varepsilon(x)$, which consequently has different dispersion levels for different x -values. The random error of the function $f(x)$ will therefore also depend on x . The functions $g(x)$ and $k(x)$ are shown in Figure 2.

50 samples of the random field are generated ($n = 50$), creating the 50 response curves seen in Figure 3. It may be noted that the generated response functions $f(x)$ are normally distributed around $g(x)$, and that the dispersion is larger where $k(x)$ is large. The stochastic variations are applied directly to the response function in this analytical example, thereby

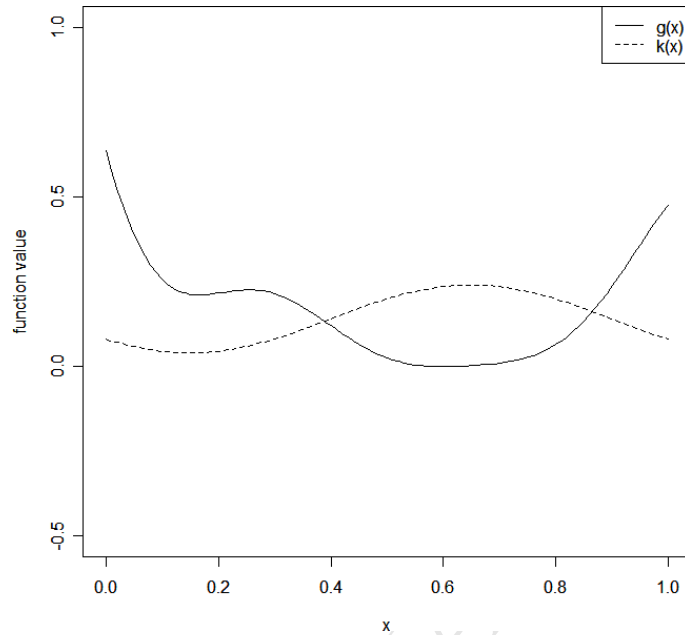


Figure 2: Values of $g(x)$ and $k(x)$ for the analytical function.

simulating variations induced by random variables.

In order to find a robust design with the robust optimization formulation in Equation (8), the response is evaluated 50 times for certain values of x , i.e. the value of the 50 different curves are extracted for given values of x . Basic statistics yield approximations of the mean value and the standard deviation for those values of x , and the mean value and standard deviation can be approximated over the entire design space by the use of meta models, $\tilde{\mu}$ and $\tilde{\sigma}$. Standard optimization procedures may then be applied to find the optimum in a robustness sense.

Moreover, the same 50 curves of $f(x)$ are used throughout the optimization for all design evaluations of the response, i.e. the stochastic samples

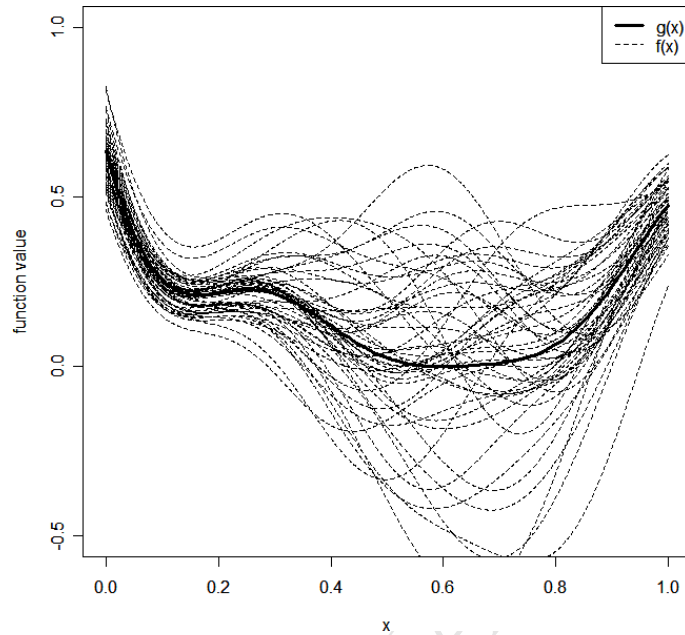
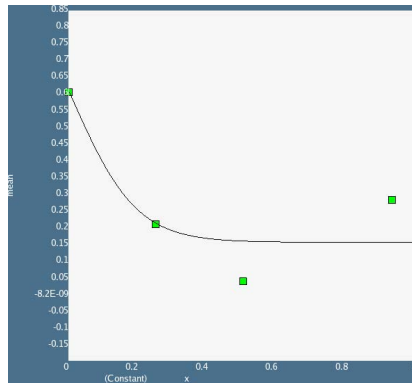


Figure 3: 50 randomly generated analytical curves $f(x)$.

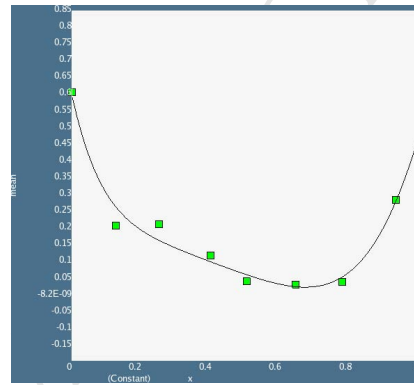
are only generated once. The reason for this is that the chosen optimization algorithm might behave in a non-robust manner if new perturbations are used for every design. If the same perturbations are used all the time, the error with which the mean and the variance are estimated for different designs will not depend on the specific samples that have been generated for that specific evaluation. This issue becomes more relevant when fewer samples are used for the estimation of the stochastic values. An alternative interpretation of this approach may be that the optimization depends on a broader spectrum of deterministic simulations that are selected in order to represent a stochastic behaviour of the model.

5.1. Solution

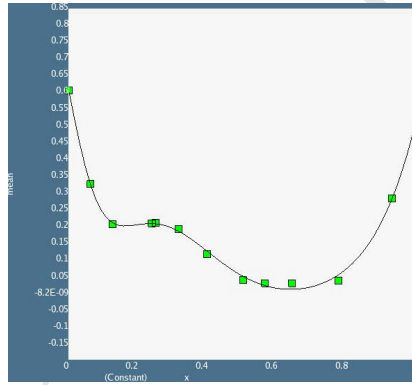
The optimization problem is solved using RBF networks as meta models. The initial DOE is retrieved using a space filling algorithm with the number of designs set to the default value 4. Successive designs are chosen by the internal optimization algorithm.



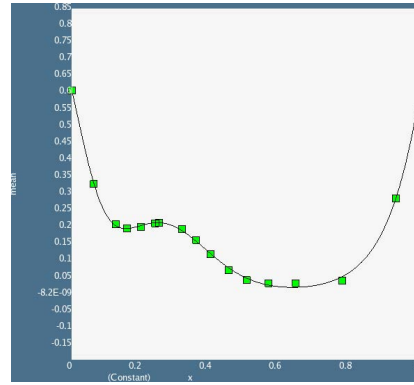
(a) Mean value approximation, iteration 1.



(b) Mean value approximation, iteration 2.



(c) Mean value approximation, iteration 3.

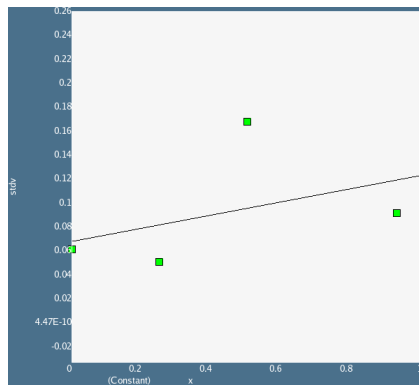


(d) Mean value approximation, iteration 4.

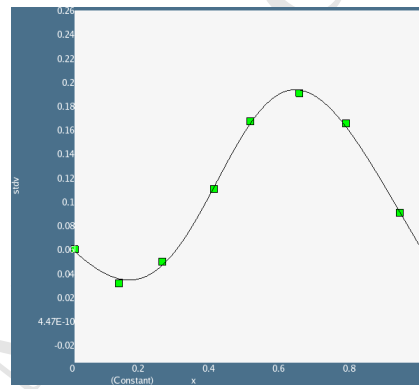
Figure 4: Mean approximations in analytical problem.

As seen in Figures 4 and 5, the meta models better approximate the given

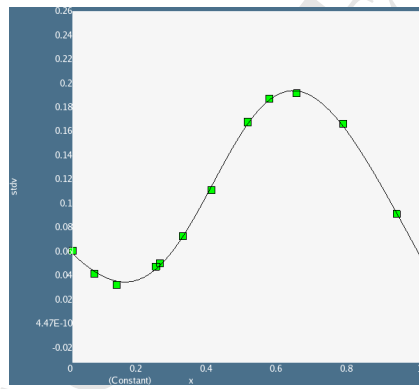
analytical curves with an increased number of evaluations. However, the meta models will not converge to the true analytical curves as the mean and standard deviation values calculated in each design x are only approximations made from 50 randomly generated samples. A comparison between the given analytic expressions and the approximations based on the 50 samples is given in Figure 6.



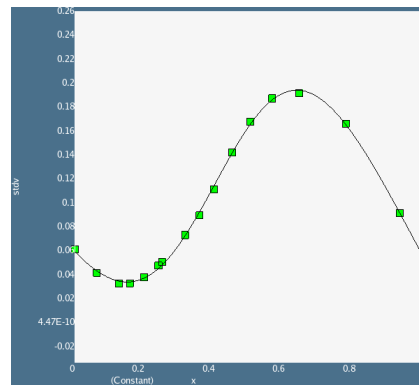
(a) Standard deviation approximation, iteration 1.



(b) Standard deviation approximation, iteration 2.



(c) Standard deviation approximation, iteration 3.



(d) Standard deviation approximation, iteration 4.

Figure 5: Mean approximations in analytical problem.

From Figure 6, it can also be noted that the accuracies of the approximations are higher where the dispersions are low, i.e. $k(x)$ has a low value. This is in accordance to basic statistical theory, stating that the number of samples required to estimate a mean value or a standard deviation accurately, increases with higher dispersion in the samples. For instance, the confidence interval of the mean value I_m is given by

$$I_m = [\bar{x} - t_{\beta/2} \cdot d, \bar{x} + t_{\beta/2} \cdot d] \quad (9)$$

where d is the estimated standard deviation and $t_{\beta/2}$ is a number given by the Student's t-distribution, which depends on the confidence level β and the number of samples chosen, see e.g. Casella et al. [20]. For example, a 95% confidence interval has $\beta = 0.05$ and if the number of samples are 20, $t_{\beta/2} = 2.09$. Nevertheless, the confidence interval will be larger with a larger dispersion (larger d) and consequently the approximated mean value will be more uncertain. The behaviour of the confidence interval for the standard deviation is similar.

A solution to this problem could be to let the number of samples vary from design to design. The width of the confidence interval could be used as a stopping criterion, indicating when enough samples have been used for that design. However, an implementation of this is not within the scope of this work. It is also uncertain how that approach would work in conjunction with an optimization algorithm, see the discussion in the last paragraph of the previous section. Instead, we make sure that the number of samples used is sufficient. It is also possible to check the robustness of the final design by performing more evaluations, i.e. use more than n samples for that design.

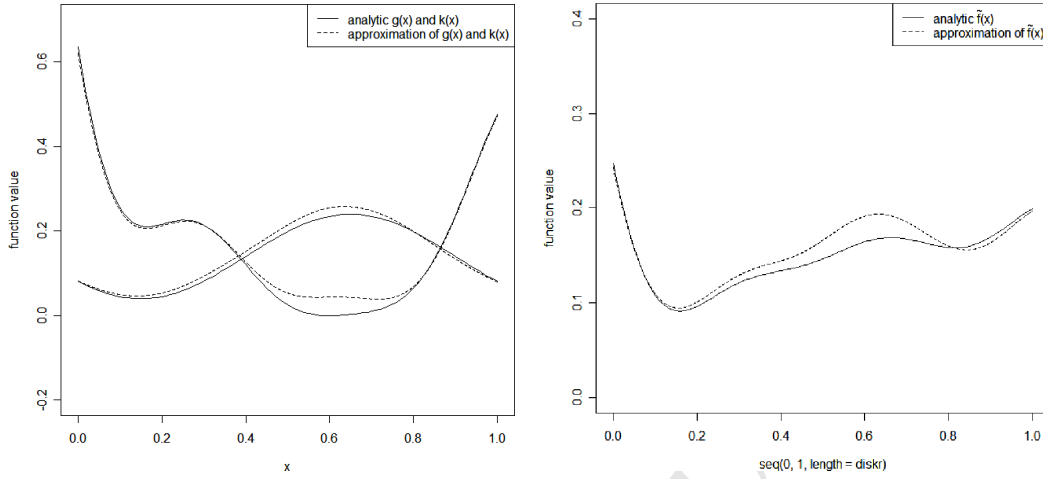
(a) Comparisons of $g(x)$ and $k(x)$.(b) Comparison of $\tilde{f}(x)$ for $\alpha = 0.3$.

Figure 6: Comparison of the true given analytic expressions and the approximations based on the 50 samples.

The optimal analytical solution to this problem is $x \approx 0.1596$, whereas the optimization algorithm with 50 samples gives $x \approx 0.1525$ within four iterations. The convergence criteria for the optimization was set to 0.001 for change both in design and objective.

6. Square aluminium tube example

Next, the robust optimization approach is applied to the problem of a thin walled aluminium extrusion with a quadratic cross section subjected to axial crushing. The example is essentially picked from Fyllingen et al. [1] and Jensen et al. [21]. In this work, the problem is used to exemplify the robust optimization approach, while the predictability of the FE model is not emphasized.

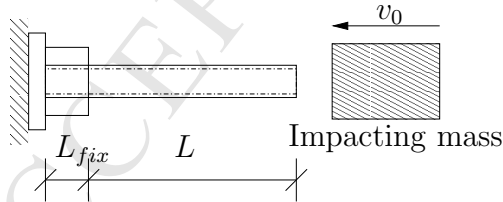
The objective is to demonstrate the presented method on the problem

of finding a robust optimal placement of a buckling trigger. The purpose of the trigger is to improve the predictability of the buckling as a result of axial loading, i.e. increase the robustness, while maintaining good energy absorption properties. From an engineer's point of view, a robust and efficient crash absorbing component buckles in a progressive mode rather than in a global mode every time.

6.1. Model description

The test set-up is given in Table 1. A mass of 600 kg impacts the profile at an initial velocity of 20 m/s. The profile is fixed to a reaction wall. The length of the fixation (L_{fix}) is 100 mm and the free length (L) of the profile is 1500 mm. The profile is made of the aluminium alloy 6060 T6, with a cross section of 80 mm \times 80 mm, and its nominal thickness is 2.5 mm.

Table 1: Test set-up.

<i>Test set-up</i>	<i>Parameters</i>
 <p>The diagram illustrates the test setup. On the left, a vertical hatched line represents a reaction wall. A horizontal profile is fixed to this wall. The fixed portion is labeled L_{fix} and the free portion is labeled L. To the right of the free end, a rectangular block labeled 'Impacting mass' is shown moving towards the profile with an initial velocity v_0 indicated by an arrow pointing left.</p>	Wall thickness
	2.5 mm
	Cross section
	80 mm \times 80 mm
	Free length
1500 mm	
Impact velocity and mass	20 m/s and 600 kg

Both the local and global geometric imperfections are applied in the same

way as in Fyllingen et al. [1]. The local imperfections are represented by four independent fields, one field for each of the extrusion walls. The field values are generated at the same locations as the FE nodes, and the intersection of the different fields at the corners are found by use of linear extrapolation of the intersecting fields. The x_1 direction is directed perpendicular to the longitudinal direction and the x_2 direction is directed parallel to the longitudinal direction.

The global geometric imperfections are represented by two independent stochastic fields generated on two perpendicular planes. The field on one wall is generated in the same way as for the local geometric imperfections, and then the parallel wall is given the same geometric imperfections in the same direction. The perpendicular walls are given a sideways geometric imperfection equal to the geometric imperfections at the corners. Examples of the local and global geometric imperfections can be seen in Figure 7.

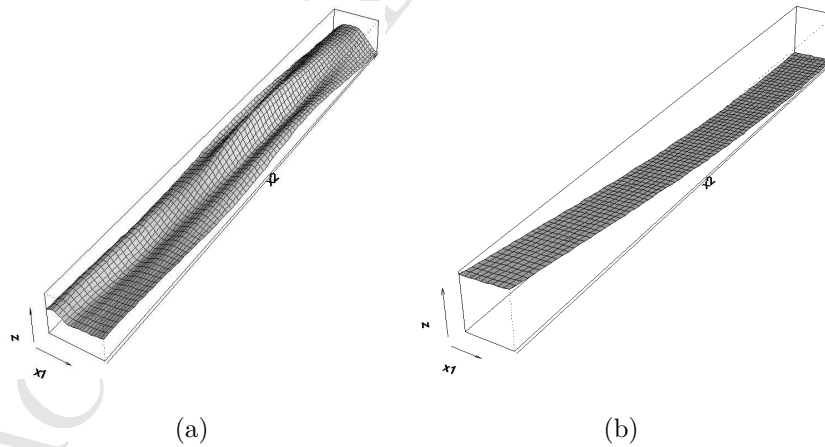


Figure 7: Examples of the geometric imperfections that are added to the model. (a) Local imperfections; (b) Global imperfections.

Measurements of initial geometrical imperfections on aluminium extrusions of AA6060 T6 with a cross section of 80 mm \times 80 mm and a nominal thickness of 2.5 mm have been performed by Clausen and Remseth [22]. It was not possible to estimate the parameters of the spectral density functions using these measurements. However, after some trial and error, a similar shape and size in the local and global imperfections in the model were obtained. The chosen parameters for the spectral density function for the local and global geometrical imperfections are given in Table 2.

Table 2: Parameters of the autocorrelation function for the local and global imperfection.

Imperfection	\mathbf{b}_1 [mm]	\mathbf{b}_2 [mm]	\mathbf{s} [mm]
Local	40	400	0.07
Global	1500	1500	2.0

By introducing geometrical triggers, the performance of the profile may be improved. It was chosen to use a geometrical trigger which consists of a product of trigonometric waves:

$$f(u, v) = A \sin\left(\pi \frac{u - u_1}{w}\right) \sin\left(\pi \frac{v - v_1}{a}\right) \quad (10)$$

One half sine wavelength was used both in the width (u-direction) and in the axial direction (v-direction), and all four walls of the profile were triggered. The trigger points outwards on two of the parallel walls, while it points inwards on the remaining two parallel walls, see Figure 8(b). The amplitude A was chosen to be 10 mm.

In Figure 8(a), a schematic view of the profile with the geometrical trigger

is shown. The position of the trigger is given as b , while the size of one half-wavelength in the axial direction is denoted a . Optimal values of these two variables, in the robust optimal sense, are sought in the robust optimization.

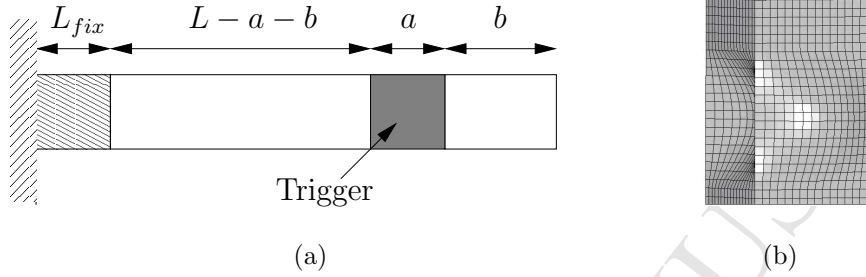


Figure 8: (a) Schematic view of the profile; (b) Geometric trigger.

It was chosen to use an FE mesh with Belytschko-Tsay shell elements, five integration points through the thickness and 16 square shaped elements across the width of the profile. At the clamped part of the member, which has a length of $L_{fix} = 100$ mm, all degrees of freedom were constrained, except for the longitudinal translation, in order to allow deformation in the axial direction. The lower end of the specimen was fixed by constraining all six degrees of freedom. To account for contact between the impactor and the profile, a single surface contact algorithm was used, while the self contact was modelled using an automatic surface to surface contact algorithm. The friction coefficients were set to 1.05 and 0.61 for the self contact and contact between the impactor and profile, respectively.

The material was assumed to be elastic-viscoplastic with an anisotropic yield criterion and isotropic hardening, see material model 103 in LS-DYNA, Hallquist [23]. However, the parameters of the yield criterion were chosen such that it was equivalent to the von Mises yield criterion. The extended

Voce isotropic hardening rule was used, see Equation 11, with the following coefficients: $\sigma_0 = 148$ MPa, $Q_1 = 48$ MPa, $C_1 = 10492$, $Q_2 = 49$ MPa and $C_2 = 22$.

$$\sigma = \sigma_0 + \sum_{i=1}^2 Q_i (1 - \exp(-C_i \varepsilon^p)) \quad (11)$$

The density was set to 2700 kg/m^3 , Young's modulus to 70 GPa and Poisson's ratio to 0.33 . The impactor was modelled as a $400 \text{ mm} \times 400 \text{ mm} \times 2 \text{ mm}$ plate of cubic solid elements and rigid body material.

6.2. Response

The response chosen for this model was a measure of the energy absorption of the beam. As the energy absorption strongly depends on the different buckling modes, i.e. global, mixed mode or progressive buckling, it is a suitable response to use in the optimization process, cf. Fyllingen et al. [1]. Figure 9 shows the three different buckling modes obtained experimentally and one simulation result where unlike in the experiments, a trigger has been introduced in the model.

By integrating the force that acts on the impactor as it travels a distance of 60% of the free length of the beam, the absorbed energy response is retrieved, see Figure 10.

6.3. Problem formulation

Again, the robust optimization problem is formulated according to the principle in Equation (3), more precisely

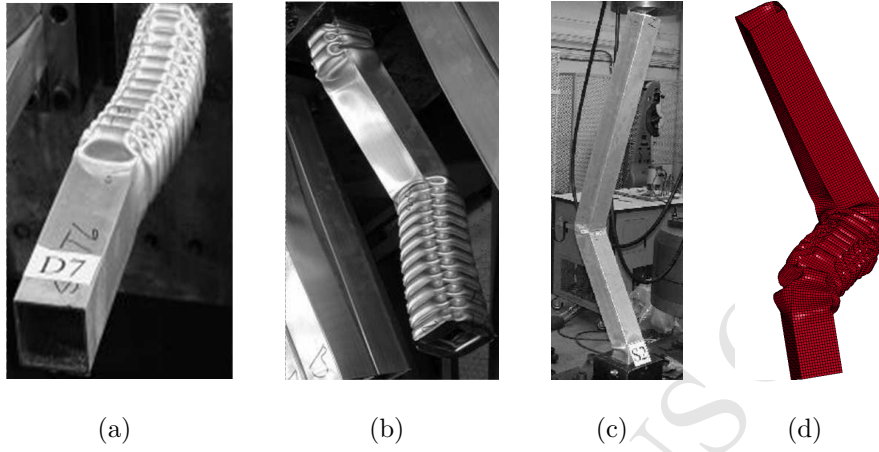


Figure 9: The three different possible experimental buckling modes and one example of a simulation deformation mode. (a) Progressive buckling mode; (b) Mixed buckling mode; (c) Global buckling mode; (d) Mixed buckling mode (simulation).

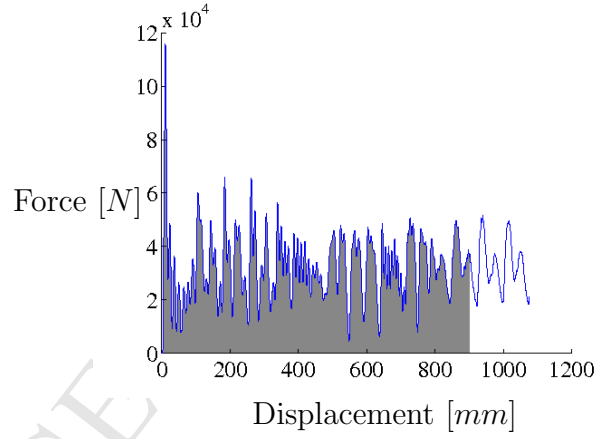


Figure 10: The definition of the energy absorption response as the shaded region in the graph.

$$\begin{aligned}
 \max_{a,b} \quad & \tilde{f}(a,b) = \alpha \frac{\tilde{\mu}(f(a,b))}{\mu_0(f(a_0,b_0))} - (1-\alpha) \frac{\tilde{\sigma}(f(a,b))}{\sigma_0(f(a_0,b_0))} \\
 \text{s.t.} \quad & 10 \leq a \leq 200 \\
 & 10 \leq b \leq L - (10 + a)
 \end{aligned} \tag{12}$$

In this robust optimization, the objective is to maximise the mean and minimise the variability of the energy absorption $f(a, b)$. Size and position (a and b) of the trigger are the two deterministic design variables and the geometric perturbations described by the random fields are the only random variables. Constraints for the design variables are given in mm.

Depending on the relative priorities between the energy absorption and its variations, it is presumed that different solutions may be obtained by changing the value of α . This was tested with the RBF network meta model in order to study the trade-off between maximising the energy absorption and minimising the variability. $\alpha = 0.8$ was tested for both meta models.

15 samples were used to estimate the mean value and the standard deviation of the energy absorption, i.e. 15 different samples of the random fields were generated in the beginning of the optimization ($n = 15$). The same geometrical perturbation fields were used throughout the optimization, as in the analytical example. A convergence study was conducted on the least robust design point found in the optimization, the design where the standard deviation in the response was the largest, as this design governs the number of samples that are required. This design was evaluated for 100 samples and it was concluded that the values of the mean and standard deviation of the response had reached sufficiently accurate values already after 15 samples, thereby showing that 15 samples were sufficient for this analysis.

6.4. Solution

The optimization problem was solved testing two different available ANN meta models: FF networks and RBF networks. For the RBF networks, different values of the weighting parameter α were also tried. The initial values

of the variables a and b were set to 105 mm and 697.5 mm, respectively, representing the midpoint of the design space, i.e. a medium size trigger placed at the middle of the free length. The initial DOE was retrieved using a space filling algorithm with the number of designs set to 5, and the successive designs were chosen by the optimization algorithm. The tolerance for termination was chosen to 0.001 for both change in design and objective.

Table 3: Solutions of the optimization problem for optimal placement of the trigger.

Type of ANN	α -value	Iterations	Optimum (a,b)
FF	0.8	15	(10.0, 1480)
RBF	0	4	(10.0, 10.0)
RBF	0.8	4	(10.0, 10.0)
RBF	1	2	(10.0, 263)

The number of iterations and the optimal solutions are presented in Table 3 and the optimization evolutions are seen in Figure 11. The optimal solutions indicates a small trigger at one of the ends of the beam. From studying the design evaluations more closely, it can be concluded that there are only small differences in energy absorption properties between placing the small trigger in one end or the other. To which end the optimization converges depends on the first steps of the optimization loop. Choosing another starting point for the optimization would also affect to which of the two solutions the optimization would converge.

In all the optimal designs suggested, all 15 samples buckled in a progressive mode, which were improvements compared to the initial design where

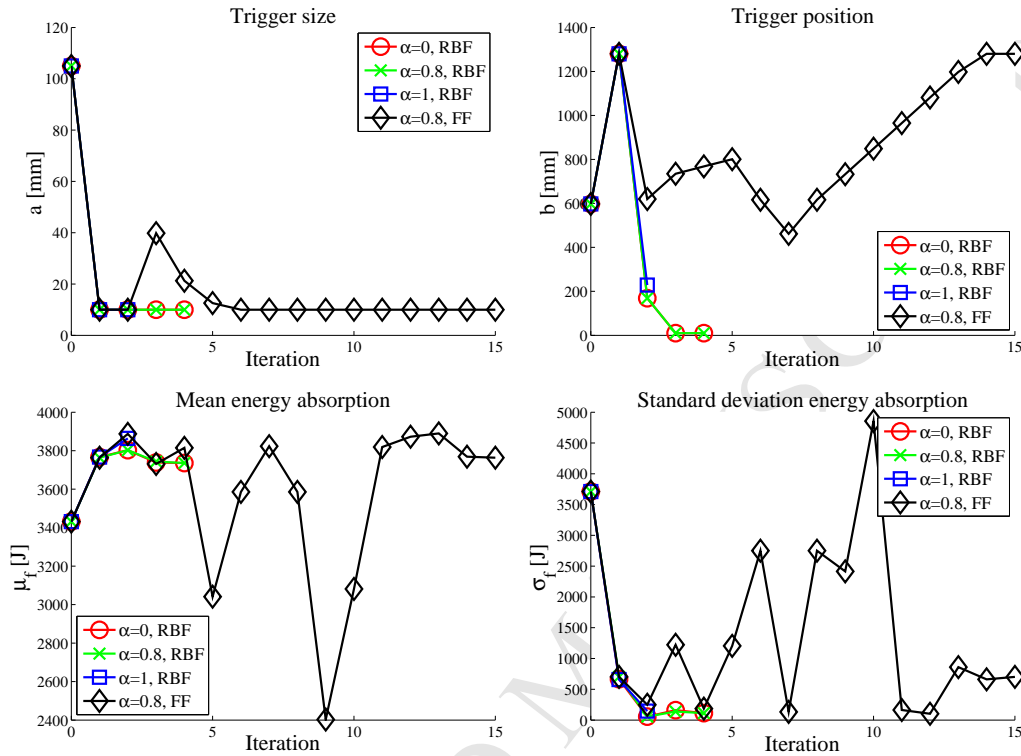


Figure 11: Evolution of variables and responses for the square aluminium tube robust optimization example. The response values shown are actual design point evaluations.

11 samples buckled in a progressive mode and 4 in a mixed mode. In the case where $\alpha = 1$, the optimization stops after only 2 iterations. This is due to the fact that the change in the predicted objective value happens to be less than the given tolerance of 0.001. If this tolerance had been changed, the optimum design would probably had changed too, most certainly to one of the other suggested optima in Table 3.

The results further show that in this example, there is not really a trade-off situation present between maximising the mean energy absorption and

minimising the standard deviation of the energy absorption. An increase in the mean response is because of fewer profiles buckling in the mixed mode and more in the progressive mode. The standard deviation of the response simultaneously decreases, as most of the profiles initially buckle in a progressive mode. The absence of the tradeoff situation is established in Table 3, where it is seen that the solutions are independent of the parameter α . However, if a majority of the initial designs had buckled in a mixed mode instead, which for instance could be achieved by using a longer beam, there would have been a trade-off situation present.

Examples of final RBF meta models are shown in Figure 12. As this example problem involves only two variables, it is possible to illustrate how the stochastic values of the response vary over the design space. It may, however, be noted that in design regions where few evaluations are made, the meta models should not be trusted.

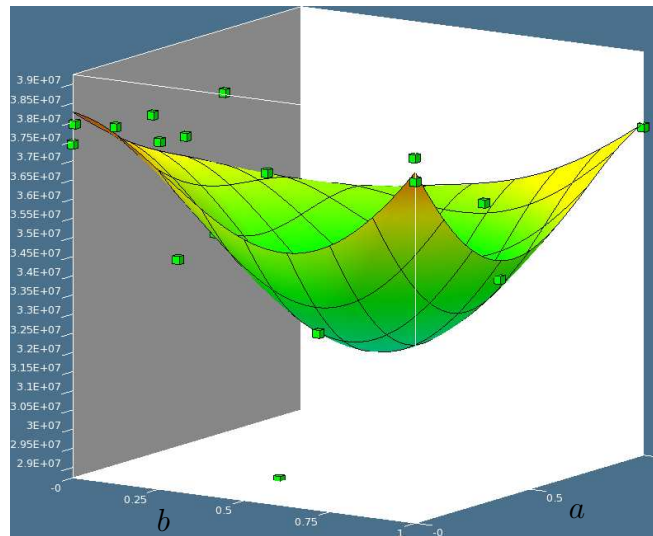
6.5. Error analysis

To check the validity of the meta models, the root mean square (RMS) error was studied

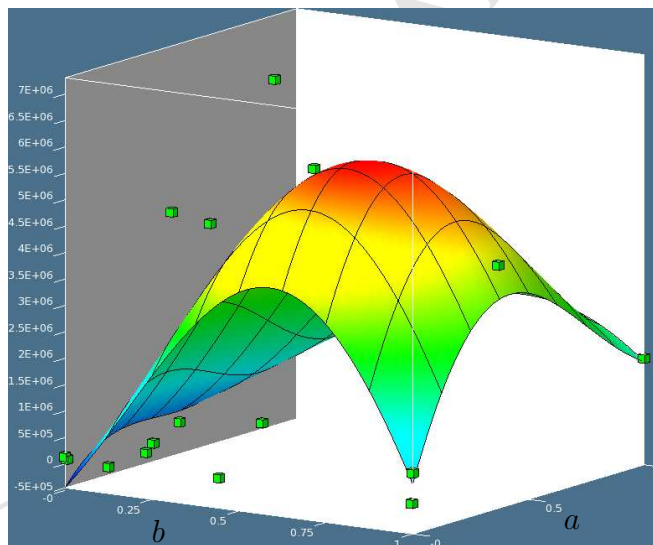
$$RMS = \sqrt{\sum_{i=1}^P \frac{(\hat{y}_i - y_i)^2}{P}} \quad (13)$$

where \hat{y}_i is the predicted response by the meta model, y_i is the actual response from a design point evaluation and P is the number of design points.

The RMS error summarises the overall error of the model. An error measurement of this kind is appropriate as this work aims at improving a global fit sequentially. One downside of the RMS error is that rapid changes in



(a) Meta model for mean.



(b) Meta model for standard deviation.

Figure 12: Final meta models for mean and standard deviation of the energy absorption with a normalised design variable space. These figures show the final meta models for the optimization conducted with the RBF network and $\alpha = 0.8$. The variable space is normalised so that both design variables vary between 0 and 1.

the response may cause large residuals at certain points affecting the measure, as the meta model is unable to reflect these steep changes satisfactorily. Large local approximation errors is the prize you pay for trying to maintain a global approximation. These errors may be avoided if the region of interest is decreased sequentially in the optimization, as in for instance sequential response surface methods (SRSM), e.g. Stander et al. [19].

The RMS error for the meta models for the optimizations above are presented in Tables 4 and 5, where the relatively large errors are due to large residuals in design points located where there is a transition between progressive and mixed buckling modes. This error is pronounced in the standard deviation meta models, as the local residuals have almost the same magnitudes as the response value range of the meta model.

7. Discussion

The main advantage of the proposed approach to robust optimization is that it essentially is computationally independent of the number of random variables used. This makes the method well suited for problems with a large number of random variables and relatively few design variables, as each design point is more computationally expensive to evaluate. Normally, random variables are included in the variable space, but here the dimension of the variable space is reduced as it only includes the design variables. This may make up for the increase in computational cost due to the many evaluations that are made on the same design. The choices of random variables for the method are of course not restricted to variables that require random fields.

The approach presented is an extension of classic optimization strate-

Table 4: RMS errors for the mean value meta models for the different optimizations performed.

Iteration	$\alpha = 0$, RBF	$\alpha = 0.8$, RBF	$\alpha = 1$, RBF	$\alpha = 0.8$, FF
1	4.44 %	4.44 %	4.44 %	0.000549 %
2	1.51 %	1.52 %	1.51 %	0.445 %
3	7.19 %	4.92 %		1.15 %
4	6.51 %	4.48 %		1.63 %
5				5.48 %
6				5.57 %
7				9.79 %
8				11.9 %
9				14.0 %
10				14.4 %
11				14.2 %
12				14.4 %
13				13.8 %
14				13.5 %
15				13.2 %

gies and require no additional knowledge from the field of optimization. As mentioned, arbitrary standard optimization strategies are applicable and the presented approach is also easy to use and implement in any optimization environment. However, as the number of samples to be used must be set by the user, an evaluation of the required number of samples, in order to get

Table 5: RMS errors for the standard deviation meta models for the different optimizations performed.

Iteration	$\alpha = 0$, RBF	$\alpha = 0.8$, RBF	$\alpha = 1$, RBF	$\alpha = 0.8$, FF
1	27.0 %	27.0 %	27.0 %	69.4 %
2	24.5 %	24.5 %	24.5 %	16.0 %
3	60.9 %	66.8 %		9.23 %
4	69.9 %	67.9 %		14.7 %
5				50.2 %
6				48.8 %
7				56.5 %
8				77.7 %
9				76.5 %
10				78.5 %
11				82.6 %
12				84.0 %
13				86.4 %
14				88.7 %
15				89.6 %

good approximations of the mean value and the standard deviation, must be performed. It is advisable to study the least robust design, as this design governs the numbers of samples needed.

Before applying random variations to a simulation model, the real variations must be known a priori, preferably from real measurements. If random

fields are used, e.g. for geometry imperfections, appropriate range parameters ("wave lengths") and amplitudes must be set. However, the simulation result's dependency on the quality of the input dispersion data is similar to that found in all other sensitivity or robustness studies. Furthermore, the presented approach makes it possible to apply variations as they are seen in measurements, exemplified by the geometry perturbations in the FE example above. It is not possible to set these kind of variations as variables in a traditional way.

Both the examples of robust optimization presented assume that the design variables are deterministic. For non-deterministic design variables, these dispersions should definitely be taken into account in the random sampling phase by generating random perturbations to the variables.

Furthermore, when searching for a global optimum, it is customary to try different starting points for the optimization algorithm, since only local convergence is guaranteed. This is not done here, but it should be considered in a more comprehensive analysis.

Through a robust optimization procedure, it has also been shown here that the best way of using a single trigger to improve the robustness of an axial buckling process, is by making it small and placing it at one end of the axially crushed beam.

8. Conclusions

An approach to robust optimization has been presented and tested on both an analytical example and on the problem of an axially crushed aluminium profile. The method works well for the examples presented and the

main advantage of the method is that it is well suited for problems with a large number of random variables. Moreover, the approach is applicable when different impact deformation modes occur.

9. Acknowledgements

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