



# Adaptive feedforward compensation algorithms for AVC systems in presence of a feedback controller

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is the sum of  $d(t)$  and of the effect of the actuator used for compensation.

The fixed feedback controller  $K$ , is characterized by the stable transfer function:

$$K(q^{-1}) = \frac{B_K(q^{-1})}{A_K(q^{-1})} = \frac{b_0^K + b_1^K q^{-1} + \dots + b_{n_{BK}}^K q^{-n_{BK}}}{1 + a_1^K q^{-1} + \dots + a_{n_{AK}}^K q^{-n_{AK}}}. \quad (3)$$

The "a posteriori" output of the feedforward filter is denoted by:  $\hat{y}_1(t+1) = \hat{y}_1(t+1|\hat{\theta}(t+1))$ .

The "a priori" output of the estimated feedforward filter is given by:

$$\begin{aligned} \hat{y}_1^0(t+1) &= \hat{y}_1(t+1|\hat{\theta}(t)) \\ &= \hat{\theta}^T(t)\phi(t) = [\hat{\theta}_S^T(t), \hat{\theta}_R^T(t)] \begin{bmatrix} \phi_{\hat{y}_1}(t) \\ \phi_{\hat{u}}(t) \end{bmatrix} \end{aligned} \quad (4)$$

where

$$\hat{\theta}^T(t) = [\hat{s}_1(t) \dots \hat{s}_{n_S}(t), \hat{r}_0(t) \dots \hat{r}_{n_R}(t)] = [\hat{\theta}_S^T(t), \hat{\theta}_R^T(t)] \quad (5)$$

$$\begin{aligned} \phi^T(t) &= [-\hat{y}_1(t) \dots -\hat{y}_1(t-n_S+1), \hat{u}(t+1), \hat{u}(t) \dots \hat{u}(t-n_R+1)] \\ &= [\phi_{\hat{y}_1}^T(t), \phi_{\hat{u}}^T(t)] \end{aligned} \quad (6)$$

and  $\hat{y}_1(t)$ ,  $\hat{y}_1(t-1) \dots$  are the "a posteriori" outputs of the feedforward filter generated by:

$$\hat{y}_1(t+1) = \hat{y}_1(t+1|\hat{\theta}(t+1)) = \hat{\theta}^T(t+1)\phi(t) \quad (7)$$

while  $\hat{u}(t+1)$ ,  $\hat{u}(t) \dots$  are the measurements provided by the primary transducer<sup>1</sup>.

The control signal applied to the secondary path is given by

$$\hat{y}(t+1) = \hat{y}_1(t+1) - \frac{B_K}{A_K} \chi^0(t+1) \quad (8)$$

where  $\chi^0(t+1)$  is the measured residual acceleration.

### 3. Development of the Algorithms

The algorithms for adaptive feedforward compensation in presence of feedback controller will be developed under the hypothesis H1, H3 and H4 from [7] and new hypothesis H2:

H2 - Perfect matching condition. There exists a filter  $N(q^{-1})$  of finite dimension such that<sup>2</sup>:

$$\frac{N}{(1-NM)}G = -D \quad (9)$$

and the characteristic polynomials (i) of the "internal" positive coupling loop:

$$P = A_MS - B_MR \quad (10)$$

(ii) of the closed loop (G-K):

$$P_{cl} = A_G A_K + B_G B_K \quad (11)$$

and of the coupled feedforward-feedback loop:

$$P_{fb-ff} = A_MS[A_G A_K + B_G B_K] - B_MR A_K A_G \quad (12)$$

are Hurwitz polynomials.

A first step in the development of the algorithms is to establish a relation between the errors on the estimation of the parameters of the feedforward filter and the measured residual acceleration. This is summarized in the following lemma.

**Lemma 3.1:** Under hypotheses H1, H2, H3 and H4, for the system described in Section 2, using a feedforward compensator  $\hat{N}$  with constant parameters and a feedback controller  $K$ , one has:

$$v(t+1) = \frac{A_M A_G A_K G}{P_{fb-ff}} [\theta - \hat{\theta}]^T \phi(t) \quad (13)$$

where

$$\theta^T = [s_1, \dots, s_{n_S}, r_0, r_1, \dots, r_{n_R}] = [\theta_S^T, \theta_R^T] \quad (14)$$

is the vector of parameters of the optimal filter  $N$  assuring perfect matching

$$\hat{\theta}^T = [\hat{s}_1 \dots \hat{s}_{n_S}, \hat{r}_0 \dots \hat{r}_{n_R}] = [\hat{\theta}_S^T, \hat{\theta}_R^T] \quad (15)$$

is the vector of constant estimated parameters of  $\hat{N}$

$$\begin{aligned} \phi^T(t) &= [-\hat{y}_1(t) \dots -\hat{y}_1(t-n_S+1), \hat{u}(t+1), \hat{u}(t) \dots \hat{u}(t-n_R+1)] \\ &= [\phi_{\hat{y}_1}^T(t), \phi_{\hat{u}}^T(t)] \end{aligned} \quad (16)$$

and  $\hat{u}(t+1)$  is given by:<sup>3</sup>

$$\hat{u}(t+1) = d(t+1) + \frac{B_M^*}{A_M} \hat{y}(t). \quad (17)$$

The proof is given in the Appendix.

**Corollary 1:** For  $B_K = 0$  (absence of the feedback controller), the error equation for pure feedforward compensation given in [7], is obtained.

**Corollary 2:** For  $B_M = 0$  (absence of the mechanical coupling), the error equation is given by:

$$v(t+1) = \frac{B_G A_K}{P_{cl} S} [\theta - \hat{\theta}]^T \phi(t) = \frac{G_{cl}}{S} [\theta - \hat{\theta}]^T \phi(t) \quad (18)$$

where:  $G_{cl}$  is the closed loop transfer function  $(G, K)$  defined by:  $G_{cl} = \frac{B_G A_K}{P_{cl}}$ .

Filtering the vector  $\phi(t)$  through an asymptotically stable filter  $L(q^{-1}) = \frac{B_L}{A_L}$ , equation (13) for  $\hat{\theta} = \text{constant}$  becomes:

$$v(t+1) = \frac{A_M A_G A_K G}{P_{fb-ff} L} [\theta - \hat{\theta}]^T \phi_f(t) \quad (19)$$

$$\phi_f(t) = L(q^{-1})\phi(t). \quad (20)$$

Equation (19) will be used to develop the adaptation algorithms neglecting the non-commutativity of the operators when  $\hat{\theta}$  is time varying (however an exact algorithm can be derived in such cases - see [8]).

<sup>1</sup>  $\hat{u}(t+1)$  is available before adaptation of parameters starts at  $t+1$

<sup>2</sup> In many cases, the argument  $q^{-1}$  or  $z^{-1}$  will be dropped out

<sup>3</sup>  $B(q^{-1}) = q^{-1} B^*(q^{-1})$

Replacing the fixed estimated parameters by the current estimated parameters, equation (19) becomes the equation of the a-posteriori residual (adaptation) error  $v(t+1)$  (which is computed):

$$v(t+1/\hat{\theta}(t+1)) = \frac{A_M A_G A_K}{P_{fb-ff} L} G[\theta - \hat{\theta}(t+1)]^T \phi_f(t). \quad (21)$$

Equation (21) has the standard form for an a-posteriori adaptation error ([8]), which immediately suggests to use the following parameter adaptation algorithm (same as in [7]):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\psi(t)v(t+1); \quad (22)$$

$$v(t+1) = \frac{v^0(t+1)}{1 + \psi^T(t)F(t)\psi(t)}; \quad (23)$$

$$F(t+1) = \frac{1}{\lambda_1(t)} \left[ F(t) - \frac{F(t)\psi(t)\psi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \psi^T(t)F(t)\psi(t)} \right] \quad (24)$$

$$1 \geq \lambda_1(t) > 0; 0 \leq \lambda_2(t) < 2; F(0) > 0 \quad (25)$$

$$\psi(t) = \phi_f(t) \quad (26)$$

where  $\lambda_1(t)$  and  $\lambda_2(t)$  allow to obtain various profiles for the matrix adaptation gain  $F(t)$  (see section 4 and [8]).

Three choices for the filter  $L$  will be considered, leading to three different algorithms:

Algorithm I:  $L = G$

Algorithm II:  $L = \hat{G}$

Algorithm III:  $L = \frac{\hat{A}_M \hat{A}_G A_K}{\hat{P}_{fb-ff}} \hat{G}$

where:

$$\hat{P}_{fb-ff} = \hat{A}_M \hat{S} [\hat{A}_G A_K + \hat{B}_G B_K] - \hat{B}_M \hat{R} A_K \hat{A}_G \quad (27)$$

is an estimation of the characteristic polynomial of the coupled feedforward-feedback loop computed on the basis of available estimates of the parameters of the filter  $\hat{N}$  and estimated models  $\hat{G} = \frac{\hat{B}_G}{\hat{A}_G}$  and  $\hat{M} = \frac{\hat{B}_M}{\hat{A}_M}$ . For the Algorithm III several options for updating  $\hat{P}_{fb-ff}$  can be considered:

- Run Algorithm II for a certain time to get estimates of  $\hat{R}$  and  $\hat{S}$  and compute  $\hat{P}_{fb-ff}$
- Update  $\hat{P}_{fb-ff}$  at each sampling instant or from time to time using Algorithm III (after a short initialization horizon using Algorithm II)

### 3.1. Analysis of the Algorithms

For Algorithms I, II and III the equation for the a-posteriori adaptation error has the form:<sup>4</sup>

$$v(t+1) = H(q^{-1})[\theta - \hat{\theta}(t+1)]^T \psi(t) \quad (28)$$

where:

$$H(q^{-1}) = \frac{A_M A_G A_K}{P_{fb-ff} L} G, \quad \psi = \phi_f. \quad (29)$$

Neglecting the non-commutativity of time varying operators, one has the following result:

**Lemma 3.2:** Assuming that eq. (28) represents the evolution of the a posteriori adaptation error and that the parameter adaptation algorithm (22) through (26) is used, one has:

$$\lim_{t \rightarrow \infty} v(t+1) = 0 \quad (30)$$

$$\lim_{t \rightarrow \infty} \frac{[v^0(t+1)]^2}{1 + \psi^T(t)F(t)\psi(t)} = 0 \quad (31)$$

$$\|\psi(t)\| \text{ is bounded} \quad (32)$$

$$\lim_{t \rightarrow \infty} v^0(t+1) = 0 \quad (33)$$

for any initial conditions  $\hat{\theta}(0), v^0(0), F(0)$ , provided that:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda_2}{2}, \max_t [\lambda_2(t)] \leq \lambda_2 < 2 \quad (34)$$

is a strictly positive real (SPR) transfer function.

The proof is similar to that given in [7] for  $B_K = 0$  and  $A_K = 1$  (absence of the feedback controller) and it is omitted.

## 4. Experimental results

The same AVC system as in [7] has been used.

### 4.1. Design of the feedback controller

The objective of the feedback controller  $K$  is to reduce the disturbance effect on the residual acceleration  $\chi(t)$  where the secondary path  $G$  has enough gain, without using the disturbance correlated measurement  $\hat{u}(t)$ .

### 4.2. Broadband disturbance rejection

The adaptive feedforward filter structure for most of the experiments has been  $n_R = 9$ ,  $n_S = 10$  (total of 20 parameters) and this complexity does not allow to verify the "perfect matching condition" (which requires more than 40 parameters). A pseudo-random binary sequence (PRBS) excitation on the global primary path will be considered as the disturbance. For the adaptive operation the Algorithms II and III have been used with decreasing adaptation gain ( $\lambda_1(t) = 1$ ,  $\lambda_2(t) = 1$ ) combined with a constant trace adaptation gain.

The experiments have been carried on by first applying the disturbance in open loop during 50s and after that closing the loop with the hybrid adaptive feedforward-feedback algorithms. Time domain results obtained in open loop and with hybrid control (using Algorithm III) on the AVC system are shown in figure 2. The initial trace of the matrix adaptation gain was 10 and the constant trace has been fixed at 0.2.

Table 1 summarizes the global attenuation results for various configurations. Clearly, hybrid adaptive feedforward-feedback scheme brings a significant improvement in performance with respect to adaptive feedforward compensation alone. Comparing with the results of [7], (Table 2) one can conclude that in terms of performance and complexity it is more interesting to add a linear feedback than augmenting the number of parameters of the adaptive feedforward filter beyond a certain value.

<sup>4</sup>the argument  $\hat{\theta}(t+1)$  has been dropped out

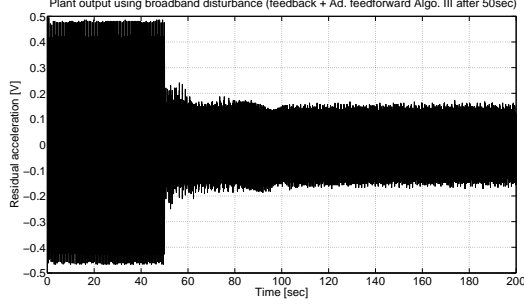


Figure 2: Real time results obtained with feedback controller and adaptive feed-forward Algorithm III.

	No feedback no feedforward	Feedback only	Adaptive feedforward	Feedback & Ad. feedforward
Variance	0.0354	0.0067	0.0054	0.0033
Normalized var.	1	0.1892	0.1525	0.0932
Atten. (dB)	0dB	-14.40dB	-16.23dB	-20.53dB

Table 1: Global attenuation for various configurations.

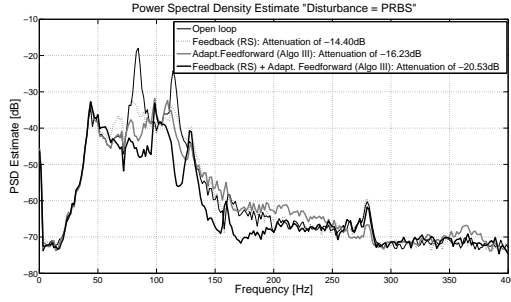


Figure 3: Power spectral densities of the residual acceleration (Disturbance = PRBS).

Figure 3 shows a comparison of the power spectral densities for adaptive feedforward alone with 20 parameters (Alg. III), feedback controller alone, and the hybrid "feedback-adaptive feedforward" scheme with 20 parameters (Alg. III).

## 5. Conclusions

The theoretical analysis presented in this note, has pointed out the interaction between the feedback and the stability conditions for adaptive feedforward compensation. Experimental results on an AVC system featuring an internal "positive" coupling have illustrated the improvement in the performance provided by the hybrid approach.

## 6. Appendix- Proof of lemma 3.1

For a fixed value of the parameter vector  $\hat{\theta}$  characterizing the estimated filter  $\hat{N}(q^{-1})$  of same dimension as the optimal filter  $N(q^{-1})$ , the output of the secondary path can be expressed by (in this case  $\hat{z}(t) = \hat{z}^0(t)$ ,  $\hat{y}(t) = \hat{y}^0(t)$  and  $\chi(t) = \chi^0(t)$ ):

$$\hat{z}(t) = G\hat{y}(t) \quad (35)$$

with:

$$\hat{y}(t) = \hat{y}_1(t) - \frac{B_K}{A_K}\chi(t) = \hat{y}_1(t) + \frac{B_K}{A_K}v(t) \quad (36)$$

where:

$$\hat{y}_1(t+1) = \hat{\theta}^T \phi(t). \quad (37)$$

The key observation is that using equations (63) through (67) from [7] the dummy variable  $y(t+1)$  can be expressed as :

$$y(t+1) = \theta^T \phi(t) - S^*[y(t) - \hat{y}_1(t)] + R[u(t+1) - \hat{u}(t+1)]. \quad (38)$$

Define the dummy error (for a fixed vector  $\hat{\theta}$ )

$$\varepsilon(t+1) = y(t+1) - \hat{y}_1(t+1) - KG\varepsilon(t+1) \quad (39)$$

and the residual error becomes:

$$v(t+1) = -x(t+1) - \hat{z}(t+1) = G\varepsilon(t+1). \quad (40)$$

By taking into account the equations 36 and 40,  $y(t+1)$  becomes:

$$y(t+1) = \theta^T \phi(t) - S^*[y(t) - \hat{y}(t) + \frac{B_K B_G}{A_K A_G} \varepsilon(t)] + R[u(t+1) - \hat{u}(t+1)]. \quad (41)$$

It results from (41) by taking into account the expressions of  $u(t)$  and  $\hat{u}(t)$  given by (67) of [7] and (17) that:

$$y(t+1) = \theta^T \phi(t) - \left[ S^* \left( 1 + \frac{B_K B_G}{A_K A_G} \right) - \frac{R(q^{-1}) B_M^*}{A_M} \right] \varepsilon(t). \quad (42)$$

Using equations (36) and (39), one gets (after passing all terms in  $\varepsilon$  on the left hand side):

$$\varepsilon(t+1) = \frac{A_M A_G A_K}{P_{fb-ff}} [\theta - \hat{\theta}]^T \phi(t). \quad (43)$$

Taking now into account equation (40) one obtains equation (13). End of the proof.

## References

- [1] R. De Callafon. Active noise control method and apparatus including feedforward and feedback controller, March 2010.
- [2] S.J. Elliott and P.A. Nelson. Active noise control. *Noise / News International*, pages 75–98, June 1994.
- [3] S.J. Elliott and T.J. Sutton. Performance of feedforward and feedback systems for active control. *IEEE Transactions on Speech and Audio Processing*, 4(3):214–223, May 1996.
- [4] E. Esmailzadeh, A. Alasty, and A.R. Ohadi. Hybrid active noise control of a one-dimensional acoustic duct. *Journal of vibration and acoustics*, 124(1):10–18, 2002.
- [5] C.A Jacobson, C.R Johnson, D.C Mc Cormick, and W.A Sethares. Stability of active noise control algorithms. *IEEE Signal Processing letters*, 8(3):74–76, 2001.
- [6] M.S. Kuo and D.R. Morgan. Active noise control: A tutorial review. *Proceedings of the IEEE*, 87:943–973, 1999.
- [7] I.D. Landau, M. Alma, and T.B. Airimițoiaie. Adaptive feedforward compensation algorithms for active vibration control with mechanical coupling. *Automatica*, 47(10):2185 – 2196, 2011.
- [8] I.D. Landau, R. Lozano, and M. M'Saad. *Adaptive control*. Springer, London, 1997.
- [9] L.R. Ray, J.A. Solbeck, A.D. Streeter, and R.D. Collier. Hybrid feedforward-feedback active noise reduction for hearing protection and communication. *The Journal of the Acoustical Society of America*, 120(4):2026–2036, 2006.
- [10] J Zeng and R.A de Callafon. Recursive filter estimation for feedforward noise cancellation with acoustic coupling. *Journal of sound and vibration*, 291:1061–1079, 2006.