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► **To cite this version:**

Séverine Rosat, Yves Rogister. Excitation of the Slichter mode by collision with a meteoroid or pressure variations at the surface and core boundaries. *Physics of the Earth and Planetary Interiors*, 2012, 190, pp.25-33. 10.1016/J.PEPI.2011.10.007 . hal-00673280

HAL Id: hal-00673280

<https://hal.science/hal-00673280v1>

Submitted on 24 Feb 2012

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Excitation of the Slichter mode by collision with a meteoroid or pressure variations at the surface and core boundaries

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Abstract

We use a normal-mode formalism to compute the response of a spherical, self-gravitating anelastic PREM-like Earth model to various excitation sources at the Slichter mode period. The formalism makes use of the theory of the Earth's free oscillations based upon an eigenfunction expansion methodology. We determine the complete response in the form of Green's function obtained from a generalization of Betti's reciprocity theorem. Surficial (surface load, fluid core pressure), internal (earthquakes, explosions) and external (object impact) sources of excitation are investigated to show that the translational motion of the inner-core would be best excited by a pressure acting at the core boundaries at time-scales shorter than the Slichter eigenperiods.

Keywords: Slichter mode; outer and inner core; extraterrestrial object impact

1. Introduction

The three free translational oscillations of the inner core, the so-called Slichter modes (Slichter, 1961), have been a subject of observational contro-

23 versy since the first detection by Smylie (1992) of a triplet of frequencies that
24 he attributed to the Slichter modes. This detection has been supported by
25 Courtier et al. (2000) and Pagiatakis et al. (2007) but has not been confirmed
26 by other authors (Hinderer et al., 1995; Jensen et al., 1995; Rosat et al., 2006;
27 Guo et al., 2007). Also, it motivated additional theoretical studies (Crossley,
28 1992; Rochester and Peng, 1993; Rieutord, 2002; Rogister, 2003). The search
29 for the Slichter modes was invigorated by the development of worldwide data
30 recorded by superconducting gravimeters (SGs) of the Global Geodynamics
31 Project (Hinderer and Crossley, 2000). Thanks to their long-time stability
32 and low noise level, these relative gravimeters are the most suitable instru-
33 ments to detect the small signals that would be expected from the Slichter
34 modes (Hinderer et al., 1995; Rosat et al., 2003, 2004).

35 The theory is now better understood and computation predicts eigen-
36 periods between 4 and 6 h (Rogister, 2003) for the seismological reference
37 PREM (Dziewonski and Anderson, 1981) Earth model. A more recent study
38 by Grinfeld and Wisdom (2010) states that the period could be much shorter
39 because of the kinetics of phase transformations at the inner-core boundary
40 (ICB).

41 The observation of the Slichter modes is fundamental because, the restor-
42 ing force being Archimedean, their periods are directly related to the density
43 jump at the ICB. This parameter is still poorly known: by analyzing seismic
44 PKiKP/PcP phases, Koper and Pyle (2004) found that it should be smaller
45 than 450 kg/m^3 , later increased to 520 kg/m^3 (Koper and Dombrowskaya,
46 2005), whereas Masters and Gubbins (2003) obtained $820 \pm 180 \text{ kg/m}^3$ from
47 normal modes observation. Tkalčić et al. (2009) have shown that the un-

48 certainties associated with the seismic noise might partially explain such
49 discrepancies for the estimates of the ICB density contrast. Gubbins et al.
50 (2008) have proposed a model with a large overall density jump between the
51 inner and outer cores of 800 kg/m^3 and a sharp density jump of 600 kg/m^3
52 at the ICB itself. Such a model satisfies both the constraints set by powering
53 the geodynamo with a reasonable heat flux from the core, and PKP travel-
54 times and normal mode frequencies. The value of the density jump at ICB
55 for the PREM model is 600 kg/m^3 .

56 This paper aims at evaluating the possible amplitude of the Slichter modes
57 for various types of excitation sources.

58 The seismic excitation has been previously studied by Smith (1976),
59 Crossley (1992) and Rosat (2007). They have shown that the best natu-
60 ral focal mechanism to excite the Slichter mode is a vertical dip-slip source.
61 The largest magnitude event in the past was the 1960 Chile earthquake with
62 a magnitude $M_w = 9.6$ for the main shock. A foreshock occurred with a mag-
63 nitude of 9.5 (Kanamori and Cipar, 1974). The combination of both events
64 leads to a seismic source of magnitude $M_w = 9.8$ which would be enough to
65 excite the Slichter modes to the nanoGal level. However, at such frequencies,
66 the noise levels of SGs are of several nGal, even for the quietest sites (Rosat
67 and Hinderer, 2011). Earthquakes are therefore not the most suitable source
68 to excite the Slichter modes to a level sufficient for the SGs to detect the
69 induced surface gravity effect.

70 Surficial pressure flow acting in the core has been considered by Greff-
71 Lefftz and Legros (2007) as a possible excitation source. In this work, we
72 reconsider the pressure flow acting in the core using a Green function formal-

73 ism for a non-rotating anelastic PREM Earth model. Then, we investigate
 74 the surface load and meteoroid impact as possible sources of excitation of
 75 the Slichter modes.

76 **2. Green function formalism**

77 We consider a spherical non-rotating anelastic Earth model. The dis-
 78 placement \mathbf{u} at a point \mathbf{r} and time t produced by any body force density \mathbf{f}
 79 acting in volume V and surface force density \mathbf{t} acting upon surface S can
 80 be written as a convolution of the impulse response G with the entire past
 81 history of the forces \mathbf{f} and \mathbf{t} (Dahlen and Tromp, 1998):

$$\mathbf{u}(\mathbf{r}, t) = \int_{-\infty}^t \int_V G(\mathbf{r}, \mathbf{r}'; t - t') \mathbf{f}(\mathbf{r}', t') dV' dt' + \int_{-\infty}^t \int_S G(\mathbf{r}, \mathbf{r}'; t - t') \mathbf{t}(\mathbf{r}', t') d\Sigma' dt', \quad (1)$$

82 where \mathbf{r}' is the integrated position vector. This relation is inferred from
 83 Betti's reciprocity relation in seismology (Aki and Richards, 1980). Seismic
 84 Green's tensor G of a non-rotating anelastic Earth is given in terms of the
 85 normal-mode complex frequencies $\nu_k = \omega_k(1 + \frac{i}{2Q_k})$ and eigenfunctions \mathbf{s}_k by

$$G(\mathbf{r}, \mathbf{r}'; t) = \Re \sum_k (i\nu_k)^{-1} \mathbf{s}_k(\mathbf{r}) \mathbf{s}_k(\mathbf{r}') e^{i\nu_k t} H(t), \quad (2)$$

86 where \Re denotes the real part of the complex expression and $H(t)$ is the
 87 Heaviside function.

88 Tromp and Mitrovica (1999) have generalized Betti's reciprocity relation
 89 to a representation theorem suited for surface-load problems, so that the
 90 displacement \mathbf{u} due to a surface load σ located at \mathbf{r}' is given by

$$\mathbf{u}(\mathbf{r}, t) = \int_{-\infty}^t \int_S \sigma(\mathbf{r}', t') \mathbf{\Gamma}(\mathbf{r}, \mathbf{r}'; t - t') d\Sigma' dt', \quad (3)$$

91 where $\mathbf{\Gamma}$ is the surface-load Green's vector defined by

$$\mathbf{\Gamma}(\mathbf{r}, \mathbf{r}'; t) = -[G(\mathbf{r}, \mathbf{r}'; t) \cdot \nabla' \Phi(\mathbf{r}') + \mathbf{g}(\mathbf{r}, \mathbf{r}'; t)], \quad (4)$$

92 ∇' is the gradient with respect to \mathbf{r}' , Φ is the unperturbed gravitational
93 potential, and \mathbf{g} is

$$\mathbf{g}(\mathbf{r}, \mathbf{r}'; t) = \Re \sum_k (i\nu_k)^{-1} \phi_k(\mathbf{r}) \mathbf{s}_k(\mathbf{r}') e^{i\nu_k t} H(t), \quad (5)$$

94 ϕ_k denoting the perturbation of the gravitational potential associated with
95 the normal mode $\{\mathbf{s}_k, \phi_k\}$. Green's tensor $\{G, \mathbf{g}\}$ represents the complete
96 point-source response.

97 A spheroidal mode of harmonic degree l and order m and radial overtone
98 number n can be decomposed into three components in spherical coordinates:

$${}_n \mathbf{s}_l^m(\mathbf{r}) = {}_n U_l^m(r) Y_l^m(\theta, \phi) \hat{\mathbf{r}} + k^{-1} {}_n V_l^m(r) \frac{\partial Y_l^m}{\partial \theta} \hat{\theta} + k^{-1} {}_n V_l^m(r) \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} \hat{\phi}, \quad (6)$$

99 where $Y_l^m(\theta, \phi)$ are the real spherical harmonics of degree l and order m
100 (Dahlen and Tromp, 1998), $k = \sqrt{l(l+1)}$ and $\hat{\mathbf{r}}$, $\hat{\theta}$ and $\hat{\phi}$ are the usual
101 unit vectors of the spherical coordinates. The associated perturbation of the
102 gravitational potential has the form

$${}_n \phi_l^m(\mathbf{r}) = {}_n P_l^m(r) Y_l^m(\theta, \phi). \quad (7)$$

103 The eigenfunctions ${}_n U_l^m(r)$, ${}_n V_l^m(r)$ and ${}_n P_l^m(r)$ are functions of the radius
104 only. Because the model is non-rotating and spherically symmetric, the $2l+1$
105 eigenfrequencies for each fixed l and n are degenerate into a single eigenfre-
106 quency that we can therefore denote by ${}_n \nu_l$. The summation over k in Eqs
107 (2) and (5) is actually a triple summation over l , m and n . Since the eigen-
108 frequencies do not depend on m , the summation over m can be performed

109 using the addition theorem for surface spherical harmonics, with the result
 110 [Dahlen and Tromp (1998), Eqs 10.28 and 10.34]:

$$\begin{aligned}
 G(\mathbf{r}, \mathbf{r}'; t) = \Re \sum_n \sum_l \frac{2l+1}{4\pi} \frac{e^{i_n \nu_l t}}{i_n \nu_l} & \\
 \{ {}_n U_l(r) {}_n U_l(r') \hat{\mathbf{r}} \hat{\mathbf{r}}' P_{l0} & \\
 + k^{-1} [{}_n U_l(r) {}_n V_l(r') \hat{\mathbf{r}} \hat{\Theta}' - {}_n V_l(r) {}_n U_l(r') \hat{\Theta} \hat{\mathbf{r}}'] P_{l1} & \\
 + \frac{1}{2} k^{-2} [{}_n V_l(r) {}_n V_l(r') \hat{\Theta} \hat{\Theta}'] (k^2 P_{l0} - P_{l2}) & \\
 + k^{-2} [{}_n V_l(r) {}_n V_l(r') \hat{\Phi} \hat{\Phi}'] (\sin \Theta)^{-1} P_{l1} \}, & \quad (8)
 \end{aligned}$$

111 where $\hat{\Phi} = \hat{\mathbf{r}} \times \hat{\Theta}$ and Θ is the angular distance between the receiver at $\hat{\mathbf{r}}$
 112 and the point source at $\hat{\mathbf{r}}'$:

$$\cos \Theta = \hat{\mathbf{r}} \cdot \hat{\mathbf{r}}' = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi'). \quad (9)$$

113 The Slichter mode is the spheroidal mode of harmonic degree one and
 114 radial overtone number one. For a non-rotating spherical model, the three
 115 Slichter frequencies are degenerate into a single eigenfrequency. As

$$P_{10}(\cos \Theta) = \cos \Theta \quad (10)$$

116 and

$$P_{11}(\cos \Theta) = \sin \Theta, \quad (11)$$

117 the term for which $l = 1$ and $n = 1$ in Eq. (8) writes:

$$\begin{aligned}
 {}_1 G_1(\mathbf{r}, \mathbf{r}'; t) = \frac{3}{4\pi} \Re \{ \frac{e^{i\nu}}{i\nu} [U(r) U(r') \hat{\mathbf{r}} \hat{\mathbf{r}}' \cos \Theta & \\
 + \frac{1}{\sqrt{2}} (U(r) V(r') \hat{\mathbf{r}} \hat{\Theta}' - V(r) U(r') \hat{\Theta} \hat{\mathbf{r}}') \sin \Theta & \\
 + \frac{1}{2} V(r) V(r') \hat{\Theta} \hat{\Theta}' \cos \Theta + \frac{1}{2} V(r) V(r') \hat{\Phi} \hat{\Phi}'] \} & \quad (12)
 \end{aligned}$$

118 being understood that $\nu = {}_1\nu_1$, $U = {}_1U_1$, $V = {}_1V_1$ and $P = {}_1P_1$. For PREM,
119 the Slichter eigenperiod is 5.42 h (Rogister, 2003) and the eigenfunctions U ,
120 V and P are plotted in Fig. A.1.

121 The damping rate depends on the dissipation processes involved. A sum-
122 mary of plausible dissipation processes is given by Greff-Lefftz and Legros
123 (2007); Guo et al. (2007); Rosat et al. (2007). The role of the outer core vis-
124 cosity has been studied by Smylie and McMillan (2000) and Rieutord (2002),
125 the effect of a mushy zone at the ICB, by Peng (1997), the influence of the
126 magnetic field, by Buffett and Goertz (1995) and the anelastic dissipation
127 for the core modes, by Crossley et al. (1992). Such studies have revealed
128 that it is unlikely that the damping factor of the Slichter mode be less than
129 2000, corresponding to a damping time of 144 days. In this case, the induced
130 surface gravity perturbation should be more easily detectable by SGs. We
131 assume, in the following, a quality factor of 2000.

132 Using the Green function formalism, we can compute the excitation of
133 the Slichter mode by any body or surface forces.

134 **3. Excitation by fluid core pressure**

135 Observational evidence for motions in the core comes from core-sensitive
136 seismic modes, which have periods smaller than one hour, the free core nu-
137 tation, which is a rotational mode of nearly-diurnal period, and variations of
138 the magnetic field that can be related to motions in the core with timescales
139 larger than one year. Therefore, the dynamics of the fluid core at the Slichter
140 frequencies lacks observational evidence. Theoretical results suggest that, at
141 timescales smaller than one day and outside the seismic band, plausible mo-

142 tions are to be searched for in the turbulent convection or in the spectrum
143 of the core.

144 An account of small-scale turbulence driven by convection is given by
145 Loper (2007). The timescale may be less than one day but, because of
146 the small characteristic length-scales, turbulence is unlikely to excite the
147 translation of the whole inner core.

148 Valette (1989a,b) has shown that the inertia-gravity spectrum of an in-
149 viscid liquid core is continuous and set bounds on it. The bounds depend on
150 both the speed of rotation and squared Brunt-Väisälä frequency. Rogister
151 and Valette (2009) and Rogister (2010) have suggested that the rotational
152 modes might be influenced by the continuous spectrum in which they are
153 embedded. In particular, the nearly-diurnal free inner core nutation and
154 long-period inner core wobble might be double or even multiple and have
155 energy in the liquid core. Pending on the value of the squared Brunt-Väisälä
156 frequency in the outer core, the Slichter modes could also be embedded in
157 the continuous spectrum. Similarly to what has been found for the two ro-
158 tational modes of the inner core, significant motion and pressure variation
159 in the liquid core could accompany the Slichter modes. Although a Slichter
160 mode with its associated motion in the liquid core should then be considered
161 as a single normal mode, we can for simplicity assume that the pressure vari-
162 ations in the liquid core excite the translational motions of the inner core.
163 This is somewhat the opposite of what Buffett (2010) did to investigate the
164 attenuation of the free inner core nutation: he assumed that the tilt of the
165 inner core generates shear layers in the outer core where Ohmic and viscous
166 dissipation occur.

167 As Greff-Lefftz and Legros (2007), we will assume that the pressure at
 168 the CMB takes the following analytical form:

$$P^c(\theta, \phi, t) = P_0^c(\theta, \phi)e^{-\left(\frac{t-T_0}{\tau}\right)^2}, \quad (13)$$

169 where T_0 is the starting time of application of the pressure, τ is the time
 170 duration of the pressure source and $P_0^c(\theta, \phi)$ includes three terms of harmonic
 171 degree 1:

$$P_0^c(\theta, \phi) = P_{10}^c \cos \theta + (P_{11}^c \cos \phi + \tilde{P}_{11}^c \sin \phi) \sin \theta. \quad (14)$$

172 According to Okubo and Endo (1986) and Greff-Lefftz and Legros (2007),
 173 the total force exerted at the core boundaries must vanish for the centre of
 174 mass to be kept fixed. This translates into the so-called Consistency Relation
 175 (Farrell, 1972) and imposes a relation between the pressures P^c and P^{ic} at
 176 the CMB and ICB, respectively:

$$P^{ic} = \frac{r_c^2}{r_{ic}^2} P^c. \quad (15)$$

177 Greff-Lefftz and Legros (2007) analytically solved the equations of the
 178 elasto-gravitational deformation (Alterman et al. , 1959) for a simple Earth
 179 model made up of three homogeneous incompressible layers and investigated
 180 the excitation of the Slichter mode by a pressure acting at the outer core
 181 boundaries. In this section, we consider the same simple excitation sources
 182 to test the Green function approach for PREM, which is a more realistic
 183 Earth model. We compute the displacement \mathbf{u} by means of Eqs (1) and (12).
 184 As we are mainly interested in the surface gravity effect, we only need the
 185 radial component of the displacement:

$$u_r(r, \theta, \phi, t) = \frac{3}{4\pi} \Re \left\{ \frac{U(r)}{i\nu} \int_{-\infty}^t e^{i\nu(t-t')} e^{-\left[\frac{t'-T_0}{\tau}\right]^2} dt' \right.$$

$$\begin{aligned}
& [U(r_c) \int_{\text{CMB}} \cos \Theta (P_{10}^c \cos \theta' + (P_{11}^c \cos \phi' + \tilde{P}_{11}^c \sin \phi') \sin \theta') d\Sigma' \\
& - U(r_{ic}) \int_{\text{ICB}} \cos \Theta (P_{10}^{ic} \cos \theta' + (P_{11}^{ic} \cos \phi' + \tilde{P}_{11}^{ic} \sin \phi') \sin \theta') d\Sigma'] \} \quad (16)
\end{aligned}$$

186 At the CMB, $d\Sigma' = r_c^2 \sin \theta' d\theta' d\phi'$ and, at the ICB, $d\Sigma' = r_{ic}^2 \sin \theta' d\theta' d\phi'$.

187 Taking Eqs (9) and (15) into account, the integration over θ' and ϕ' gives:

$$\begin{aligned}
u_r(r, \theta, \phi, t) = \Re \left\{ \frac{r_c^2 U(r)}{i\nu} [U(r_c) - U(r_{ic})] \int_{-\infty}^t e^{i\nu(t-t')} e^{-[\frac{t'-T_0}{\tau}]^2} dt' \right. \\
\left. (P_{10}^c \cos \theta + P_{11}^c \sin \theta \cos \phi + \tilde{P}_{11}^c \sin \theta \sin \phi) \right\} \quad (17)
\end{aligned}$$

188 The integral

$$I(t) = \int_{-\infty}^t e^{i\nu(t-t')} e^{-[\frac{t'-T_0}{\tau}]^2} dt' \quad (18)$$

189 is calculated in the Appendix. It gives

$$I(t) = \frac{\sqrt{\pi}}{2} \tau e^{i\nu(t-T_0)} e^{-\nu^2 \tau^2 / 4} [1 + \operatorname{erf}(\frac{t-T_0}{\tau} + i\frac{\nu\tau}{2})], \quad (19)$$

190 where erf denotes the error function. The radial displacement then becomes:

$$\begin{aligned}
u_r(r, \theta, \phi, t) = -\frac{r_c^2 U(r)}{\omega(1 + \frac{1}{4Q^2})} [U(r_c) - U(r_{ic})] \left[\frac{\Re\{I(t)\}}{2Q} - \Im\{I(t)\} \right] \\
(P_{10}^c \cos \theta + P_{11}^c \sin \theta \cos \phi + \tilde{P}_{11}^c \sin \theta \sin \phi), \quad (20)
\end{aligned}$$

191 where $\Re\{ \}$ and $\Im\{ \}$ respectively denote the real and imaginary parts of the
192 expression between brackets.

193 The degree- l gravity variation measured by a gravimeter at the surface of
194 the Earth r_s is the sum of three terms: the free-air gravity variation owing
195 to the displacement of the ground in the surrounding unperturbed gravity
196 field g_0

$$g_{\text{free}} = (-4\pi G \bar{\rho} + \frac{2}{r_s} g_0) U(r_s), \quad (21)$$

197 the inertial acceleration of the ground

$$g_{\text{in}} = -\omega_l^2 U(r_s) \quad (22)$$

198 and the perturbation of the gravitational attraction

$$g_{\text{pot}} = 4\pi G \bar{\rho} U(r_s) + \frac{2}{r_s} P(r_s). \quad (23)$$

199 In these expressions, $\bar{\rho}$ is the mean density of the Earth.

200 The degree-1 gravity variation measured by a gravimeter is therefore

$$\begin{aligned} \Delta g(\theta, \phi, t) = & \frac{r_c^2}{\omega(1 + \frac{1}{4Q^2})} [U(r_c) - U(r_{ic})] \\ & [P_{10}^c \cos \theta + P_{11}^c \sin \theta \cos \phi + \tilde{P}_{11}^c \sin \theta \sin \phi] \\ & [\frac{\Re\{I(t)\}}{2Q} - \Im\{I(t)\}] [-\omega^2 U(r_s) + \frac{2}{r_s} g_0 U(r_s) + \frac{2}{r_s} P(r_s)] \end{aligned} \quad (24)$$

201 We consider a zonal pressure $P_{10}^c = 150$ Pa and compute the induced
 202 geocentre motion, inner-core translation and surface gravity perturbation for
 203 both $\tau = 1.5$ h and 15 h (Fig. A.2). As the centre of mass is fixed, the
 204 geocentre motion, which is the displacement of the figure centre with respect
 205 to the centre of mass, corresponds to the surface displacement (Greff-Lefftz
 206 and Legros, 1997).

207 For P_{10}^c varying from 0 to 1000 Pa and τ ranging from 0.1 and 10 h, we
 208 compute a 2D-map (Fig. A.3) of the surface gravity perturbation when the
 209 exciting pressure has vanished (i.e. $t > T_0 + \tau$).

210 Our results for the PREM model show excitation amplitudes larger by
 211 70 % than the amplitudes computed by Greff-Lefftz and Legros (2007), who
 212 used a very simple model made up of three incompressible homogeneous
 213 layers with a solid inner core, a liquid outer core and a rigid mantle. If

214 we consider such a 3-layer model with the average densities of the PREM
215 inner core, outer core and mantle, we obtain a period of 3.09 h for the
216 Slichter mode and excitation amplitudes in close agreement with the values
217 of Greff-Lefftz and Legros (2007). They do not provide numerical details for
218 the structure of their model but mention that its Slichter period is 3.08 h,
219 which is almost equal to the Slichter period of our 3-layer model. Therefore,
220 the Green function formalism we have adopted gives the same result as the
221 analytical solution for the degree-1 deformation of a simple 3-layer model
222 obtained by Greff-Lefftz and Legros (2007).

223 The difference between the excitation amplitudes we obtain for PREM
224 and the amplitudes computed by Greff-Lefftz and Legros (2007) comes from
225 the different Earth models. The elasticity of the mantle and inner core, the
226 compressibility of the outer core and the density jump at the ICB all come
227 into play. Greff-Lefftz and Legros (2007) mention that the elasticity of the
228 mantle perturbs the solutions by 30 %, without any further specifications.
229 We have checked that, by making the outer core of our simple 3-layer model
230 compressible, with a P-wave velocity of 11083 m/s, the excitation amplitude
231 of the Slichter mode is decreased by 40 % with respect to the incompressible
232 model. The influence of the compressibility and stratification of the core on
233 the Slichter mode was also investigated by Rogister (2003).

234 The perturbation of the surface gravity field is the largest when τ is
235 smaller than half the Slichter eigenperiod. When τ is one fourth of the
236 Slichter period, a 10 Pa pressure acting at the CMB, which by Eq. (15)
237 imposes a 81 Pa pressure at the ICB, is enough to induce a 10 nGal (0.1
238 nm/s²) surface gravity perturbation, which should be detectable by SGs.

239 **4. Excitation by a surface load**

240 A degree- l surficial mass distribution σ^s at the surface r_s exerts forces
 241 over the Earth in two ways. First, at the interface between the Earth and
 242 the load σ^s , the static contact forces give rise to a degree- l pressure

$$P^s = g_0 \sigma^s. \quad (25)$$

243 Second, the gravitational attraction of the load σ^s over the entire Earth is
 244 described by a degree- l potential

$$\phi = \frac{4\pi G}{2l+1} \sigma^s r_s \begin{cases} \left(\frac{r}{r_s}\right)^l \text{ if } r \leq r_s \\ \left(\frac{r_s}{r}\right)^l \text{ if } r > r_s \end{cases} \quad (26)$$

245 Atmospheric pressure models are sampled at 3 h at best. So instead of
 246 using actual data, we write the surface density load in the analytical form:

$$\sigma^s(\theta, \phi, t) = \sigma_0^s(\theta, \phi) e^{-\left[\frac{t-T_0}{\tau}\right]^2}, \quad (27)$$

247 which is the same as the expression used for the fluid core pressure in Section
 248 3. The degree-one load σ_0^s , like P_0^c , contains three terms:

$$\sigma_0^s(\theta, \phi) = \sigma_{10} \cos \theta + (\sigma_{11} \cos \phi + \tilde{\sigma}_{11} \sin \phi) \sin \theta. \quad (28)$$

249 After integration of Eq. (3) over the whole surface, the radial displace-
 250 ment is given by:

$$u_r(r, \theta, \phi, t) = \frac{r_s^2 U(r)}{\omega(1 + \frac{1}{4Q^2})} [U(r_s)g_0 + P(r_s)] \left[\frac{\Re\{I(t)\}}{2Q} - \Im\{I(t)\} \right] \\ [\sigma_{10} \cos \theta + \sigma_{11} \sin \theta \cos \phi + \tilde{\sigma}_{11} \sin \theta \sin \phi], \quad (29)$$

251 and the perturbation of the surface gravity is

$$\begin{aligned}
\Delta g(t) = & \frac{r_s^2}{\omega(1 + \frac{1}{4Q^2})} [U(r_s)g_0 + P(r_s)] [\frac{\Re\{I(t)\}}{2Q} - \Im\{I(t)\}] \\
& [\sigma_{10} \cos \theta + \sigma_{11} \sin \theta \cos \phi + \tilde{\sigma}_{11} \sin \theta \sin \phi] \\
& [-\omega^2 U(r_s) + \frac{2}{r_s} g_0 U(r_s) + \frac{2}{r_s} P(r_s)] \tag{30}
\end{aligned}$$

252 We use a zonal surface load pressure of 1000 Pa (the surface mass density
253 is then $\sigma_{10} = P_{10}/g_0$) and we compute the induced geocentre motion, the
254 inner-core translation and the surface gravity perturbation for two excitation
255 time-scales (1.5 h and 15 h) (Fig. A.4). When applying a surface load of
256 1000 Pa during $2\tau = 3$ h, the induced surface gravity perturbation has an
257 amplitude of 5 nGal (0.05 nm/s²) corresponding to an inner-core translation
258 of 15 mm and a geocentre motion in the opposite direction with an amplitude
259 of 0.015 mm. When the excitation time-scale (15 h) is larger than the Slichter
260 period, the excitation amplitude is smaller.

261 We also plot the surface gravity perturbation associated to the Slichter
262 mode excited by a surface load for different excitation time-scales and various
263 zonal pressure amplitudes in Fig. A.5. The conclusion is similar to the one
264 for an internal pressure flow, except that the surface gravity variations are
265 about 300 times smaller.

266 In this section, we have estimated the effect of a surface load on the
267 Slichter mode. Another source that, intuitively, could make the inner core
268 oscillate is a shock at the surface. Hence, in the next section, we study the
269 excitation by an object impact on the Earth's surface.

270 **5. Excitation by an object impact**

271 Stellar objects, such as asteroids or comets, are dragged by the Earth's
272 atmosphere and reach the Earth's surface at relatively modest velocities,
273 typically a few tens of km/s. The released energy is comparable to nuclear
274 explosions (according to Table 6 of Collins et al. (2005), from 3.2 to $3.9 \cdot 10^8$
275 megatons of TNT, where $1 \text{ Mt} = 4.2 \cdot 10^{15} \text{ J}$). The collisions between the
276 Earth and the largest meteoroids, with diameters from hundreds of meters
277 to several kilometers, blast out the impacting objects, create wide craters,
278 generally twenty times larger than the diameter of the meteoroids, and melt
279 terrestrial rocks. Fortunately, such collisions are rare events: statistically,
280 a 100 to 200 m meteoroid hits the Earth every 1000 years, a 500 to 800 m
281 meteoroid every 30000 years, and a 5 km meteoroid every 40 millions years.

282 We believe it is reasonable to assume that both the mass and linear mo-
283 mentum of the impacting object are negligible with respect to the Earth's
284 mass and linear momentum, so the orbit of the Earth is not disturbed. Be-
285 sides, the rotation period and tilt of the rotation axis of the Earth could
286 be changed by the impact if the angular momentum of the object is large
287 enough. We, however, consider impactors for which the angular momen-
288 tum is at least one hundred times smaller than the Earth's; the change of
289 the Earth's angular momentum is therefore negligible. The major known
290 meteoroid impacts had such characteristics (Table A.1).

291 Consequently, we reduce the extraterrestrial impact issue to the com-
292 putation of the equivalent seismic magnitude corresponding to the released
293 energy. The computation proposed here is based on the equations and drastic
294 simplifications used by Collins et al. (2005), which are summarized below.

295 When an object enters the Earth's atmosphere, it loses its kinetic energy
 296 through deceleration and ablation. The rate of change of the velocity v is
 297 given by the drag equation (Collins et al., 2005; Melosh, 1989):

$$\frac{dv}{dt} = -\frac{3\rho z C_D}{4\rho_i L_0} v^2, \quad (31)$$

298 where z is the altitude, C_D is the drag coefficient, taken equal to 2, and ρ_i
 299 and L_0 are, respectively, the impactor density and diameter. By assuming
 300 an exponential atmosphere,

$$\rho(z) = \rho_0 e^{-z/H}, \quad (32)$$

301 where $H = 8$ km is the scale height and $\rho_0 = 1$ kg/m³, the velocity of the
 302 impactor as a function of altitude is given by:

$$v(z) = v_0 \exp\left(-\frac{3\rho(z)C_D H}{4\rho_i L_0 \sin \alpha}\right), \quad (33)$$

303 where α is the entry angle and v_0 , the velocity at the top of the atmosphere.
 304 On its trajectory down to the ground, the impactor goes through the in-
 305 creasing atmospheric pressure and, possibly, breaks up. Collins et al. (2005)
 306 have established an empirical strength-density relation to estimate the yield
 307 strength Y_i (in Pa)

$$\log_{10} Y_i = 2.107 + 0.0624\sqrt{\rho_i} \quad (34)$$

308 and give an approximate expression for the altitude of breakup z^* :

$$z^* \approx -H \left[\ln\left(\frac{Y_i}{\rho_0 v_i^2}\right) + 1.308 - 0.314 I_f - 1.303 \sqrt{1 - I_f} \right], \quad (35)$$

309 where

$$I_f = 4.07 \frac{C_D H Y_i}{\rho_i L_0 v_i^2 \sin \alpha}, \quad (36)$$

310 and v_i is the impactor velocity at the surface.

311 Equation (35) holds provided that $I_f < 1$. Otherwise, and more rarely,
 312 the object does not break up and the velocity at the impact is given by
 313 Equation (33).

314 Following Collins et al. (2005), we use the approximative pancake model
 315 (Chyba et al., 1993; Melosh, 1981) to describe the disintegration of the me-
 316 teoroid. Let us denote by z_b the airburst altitude, which is the altitude of
 317 complete dispersion of the fragments. According to the simplifying assump-
 318 tions of the pancake model, z_b is given by

$$z_b = z^* - 2H \ln\left(1 + \frac{\ell}{2H} \sqrt{f_p^2 - 1}\right), \quad (37)$$

319 where ℓ is the dispersion length scale:

$$\ell = L_0 \sin \alpha \sqrt{\frac{\rho_i}{C_D \rho(z^*)}} \quad (38)$$

320 and the pancake factor f_p is between 2 and 10. We shall adopt Collins et al.
 321 (2005)'s value of 7. If $z_b > 0$, the airburst occurs in the atmosphere, there is
 322 neither impact nor associated seismic event. If $z_b \leq 0$, the fragments are not
 323 dispersed when they collide with the ground and the impact velocity is:

$$v_{zr} = v(z^*) \exp \left\{ \left(-\frac{3}{4} \frac{C_D \rho(z^*)}{\rho_i L_0^3 \sin \alpha} \frac{H^3 L_0^2}{3\ell^2} \left(32 + \left(\frac{\ell}{H} \right)^2 e^{z^*/H} \right) \right. \right. \\ \left. \left. + 6e^{2z^*/H} - 16e^{3z^*/2H} - 3\left(\frac{\ell}{H} \right)^2 \right) \right\} \quad (39)$$

324 The remaining kinetic energy at the moment of impact is

$$E_{cr} = \frac{\pi}{12} \rho_i L_0^3 v_{zr}^2. \quad (40)$$

325 After the impact, a fraction k_s of E_{cr} is radiated as seismic waves. Exper-
 326 imental data (Schultz and Gault, 1975) provide $k_s \in [10^{-5}, 10^{-3}]$. We will

327 take $k_s = 10^{-4}$. The seismic moment being given by

$$M_0 = 2 \frac{\mu}{\Delta\sigma_S} k_s E_{cr}, \quad (41)$$

328 where the stress release $\Delta\sigma_S \approx 3$ MPa and the rigidity $\mu = 30$ GPa, the
 329 seismic magnitude is then:

$$M_w = \frac{2}{3} \log_{10}(M_0) - 10.73 \quad (42)$$

330 with M_0 in dyn.cm (1 dyn.cm=10⁻⁷ N/m).

331 We compute the magnitude M_w for the different meteoroid impacts of
 332 Table A.1 and, in the same table, report the surface gravity perturbation
 333 associated with the translational excitation of the inner core. Note that the
 334 source is represented in terms of moment tensor by three orthogonal force
 335 couples (spherically symmetric explosion) and not as a vertical force. Indeed,
 336 the shock pressure would reach hundreds of gigapascals and the impact en-
 337 ergy would vaporize the rocks and cause a spherically symmetric explosion,
 338 as observed from the spherical shape of known craters.

339 For the biggest meteoroid, the surface excitation amplitude of the Slichter
 340 mode is 0.0067 nm/s², which is less than the detection threshold of 1 nGal
 341 (= 0.01 nm/s²). To determine what kind of impact would be necessary to
 342 excite sufficiently the Slichter mode so that it is detectable in surface gravity
 343 data, we compute the magnitude M_w and gravity perturbation Δg for various
 344 ranges of density ρ_i , velocity v_0 , diameter L_0 of the object and for seismic
 345 efficiency k_s varying between 10⁻⁵ and 10⁻². The resulting maps are plotted
 346 in Fig.A.6. The shaded areas correspond to M_w larger than 9.7, which is the
 347 magnitude required for the surface gravity effect to reach the nGal detection
 348 threshold for a surficial explosive moment source (Fig. A.7).

349 We see from Fig.A.6 that to produce a seismic event of magnitude larger
350 than 9.7, the size, density or velocity of the impacting object should have
351 unrealistic huge values. However, the value of the seismic efficiency k_s has
352 a direct impact on the equivalent seismic magnitude. For instance, with a
353 seismic efficiency of 10^{-3} instead of 10^{-4} , a meteoroid similar to the one
354 which produced the Chicxulub crater in Mexico would be able to induce a
355 seismic event of such a magnitude. Of course, the consequences would have
356 been devastating.

357 We conclude that the surficial seismic events, including extra-terrestrial
358 object impacts and explosions, are not efficient to make the inner-core oscil-
359 late at the Slichter frequency with an amplitude large enough to be observed
360 at the surface. The reason is the same as for earthquakes, i.e. the excitation
361 amplitude is directly linked to the seismic magnitude and the radial eigen-
362 functions of the Slichter mode are constant and close to zero in the mantle
363 (Crossley, 1992; Rogister, 2003; Rosat, 2007).

364 **6. Conclusions and perspectives**

365 We have investigated the excitation of the translational free motion of the
366 inner core by a pressure due to a flow in the outer core and acting at both the
367 ICB and CMB, by a surface load, which can be associated to atmospheric or
368 oceanic loading for instance, and by the collision between the Earth and a
369 stellar object. Our conclusion is that the Slichter mode would be best excited
370 by a pressure acting at the core boundaries at time-scales shorter than half
371 the Slichter eigenperiod.

372 For the pressure source at the ICB and CMB and the loading source at the

373 surface, we have considered Gaussian functions of time. More complicated
374 sources should be considered, in particular stochastic forces produced by
375 some turbulent flow in the core or at the surface. The stochastic excitation,
376 be it oceanic and atmospheric, of normal modes has been studied for instance
377 by Tanimoto and Um (1999), Tanimoto (1999, 2007) and Webb (2007, 2008).
378 However, the time scale for the Slichter mode is larger than for the other
379 seismic normal modes, whose eigenperiod is shorter than 1 hour, so we should
380 consider a theory different from the Kolmogorov theory of turbulence.

381 We have considered an analytic expression for the surface pressure as a
382 source for the excitation of the Slichter mode. A more realistic approach
383 should be based on actual atmospheric data from space correlation of world-
384 wide barometers or from weather institutes (ECMWF, NCEP...), provided
385 the data are available at a time resolution higher than the Slichter period.

386 **Appendix A. Evaluation of the time integral $I(t)$ (Eq. 18)**

387 We evaluate the following integral:

$$I(t) = \int_{-\infty}^t e^{i\nu(t-t')} e^{-[\frac{t'-T_0}{\tau}]^2} dt'$$

388 We put $b = \frac{t-T_0}{\tau}$ and introduce the variable $x = \frac{t'-T_0}{\tau}$. $I(t)$ becomes:

$$I(t) = \tau \int_{-\infty}^b e^{i\nu\tau(b-x)} e^{-x^2} dx = \tau \int_{-\infty}^b e^{i\nu\tau b} e^{-x(x+i\nu\tau)} dx$$

389 We perform the change of variable $y = x + i\frac{\nu\tau}{2}$:

$$\begin{aligned} I(t) &= \tau e^{i\nu\tau b} \int_{-\infty}^{b+i\frac{\nu\tau}{2}} e^{-(y-i\frac{\nu\tau}{2})(y+i\frac{\nu\tau}{2})} dy \\ &= \tau e^{i\nu\tau b} e^{-\frac{\nu^2\tau^2}{4}} \int_{-\infty}^{b+i\frac{\nu\tau}{2}} e^{-y^2} dy \end{aligned}$$

390 The integral $\frac{2}{\sqrt{\pi}} \int_z^{+\infty} e^{-y^2} dy$ is the complementary error function $\text{erfc}(z) =$
 391 $1 - \text{erf}(z)$, where

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-y^2} dy. \quad (\text{A.1})$$

392 So the time integral $I(t)$ is given by:

$$I(t) = \frac{\sqrt{\pi}}{2} \tau e^{i\nu(t-T_0)} e^{-\nu^2\tau^2/4} [1 + \text{erf}(\frac{t-T_0}{\tau} + i\frac{\nu\tau}{2})]$$

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Table A.1: Some meteoroid impacts on the Earth continental crust and oceanic crust. The impact angle is supposed to be 45 degrees and the impact velocity is 20 km/s.

Location	Date (AD or My BP)	Diameter (m)	Density (kg/m ³)	M_w	Δg (nm/s ²)
Tunguska Fireball Siberia	1908 AD	60	2700 (rock)	No impact	
Ries Crater Germany	15.1 ± 0.1	1500	2700 (rock)	7.4	$3.9 \cdot 10^{-6}$
Rochechouart France	214 ± 8	1500	3350 (stony-iron)	7.5	$4.9 \cdot 10^{-6}$
Chesapeake Bay USA	35.5 ± 0.3	2300	2700 (rock)	7.8	$1.4 \cdot 10^{-5}$
Chicxulub Mexico	64.98 ± 0.05	17500	2700 (rock)	9.6	$6.7 \cdot 10^{-3}$

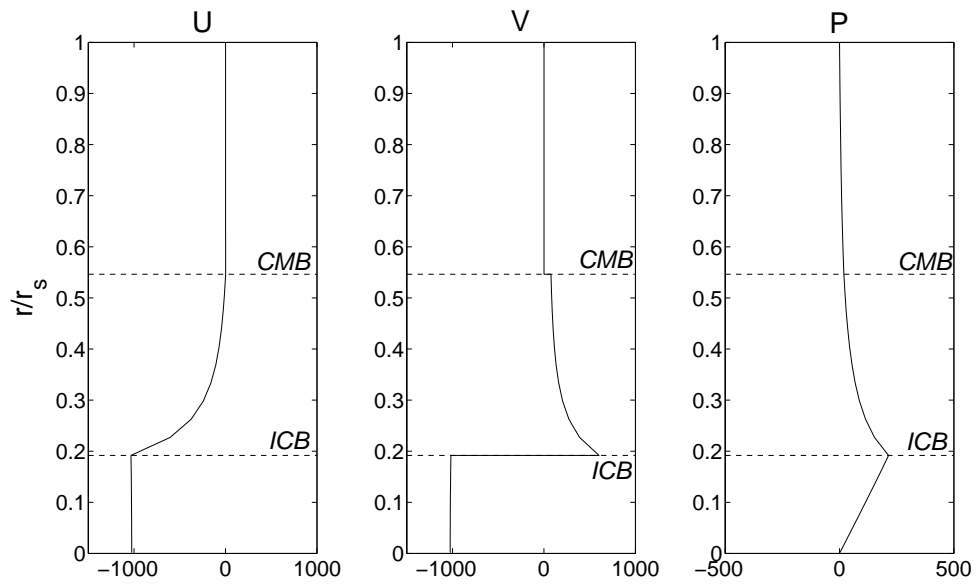


Figure A.1: Eigenfunctions of the Slichter mode ${}_1S_1$ for the PREM model. The vertical axis is the radius normalized by the Earth's surface radius r_s . U and V are the radial dependence of the displacement given by Eq. (6). P is the perturbation of the gravitational potential. The normalization of the eigenfunctions is such that $U(r_s) = 1$ m.

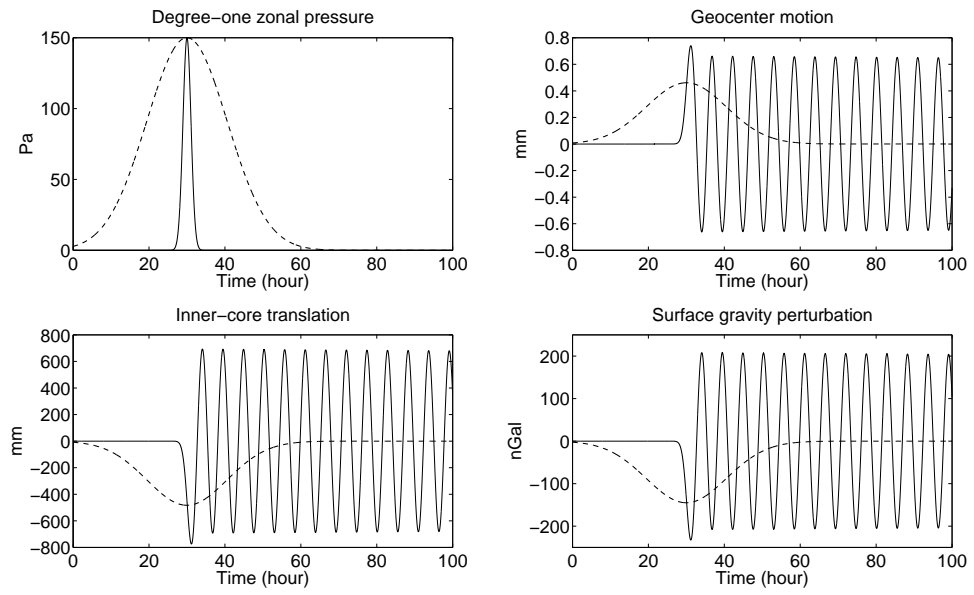


Figure A.2: Effects of the excitation of the Slichter mode by a fluid pressure acting at the CMB for two different excitation time-scales: $\tau = 1.5$ h (solid line) and $\tau = 15$ h (dashed line). This figure is similar to Fig. 3 of Greff-Lefftz and Legros (2007) but we have applied a Green tensor formalism to the PREM model. (a) Degree-one zonal pressure; (b) geocentre motion; (c) inner-core translation; (d) surface gravity perturbation.

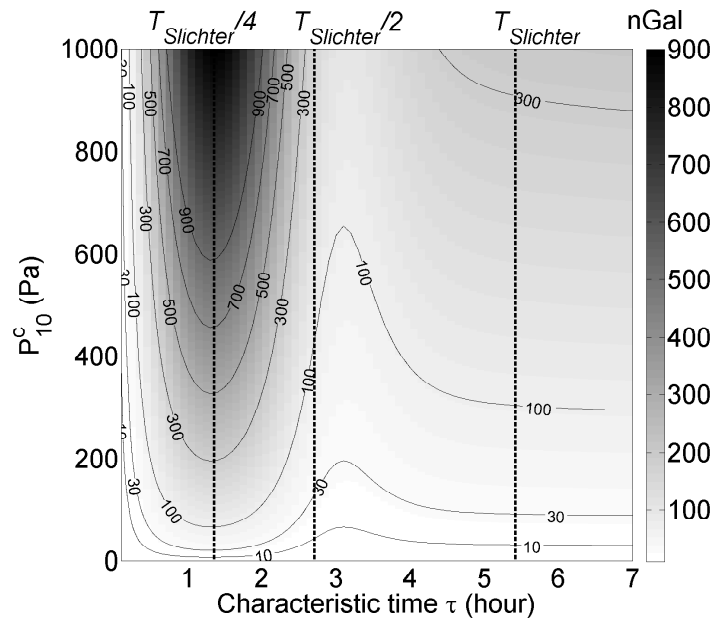


Figure A.3: Surface gravity perturbation induced by the Slichter mode excited by a fluid pressure acting at the CMB for different excitation time-scales and various zonal pressure amplitudes. The vertical dotted lines correspond to one fourth of the Slichter period, one half of the Slichter period and the Slichter period.

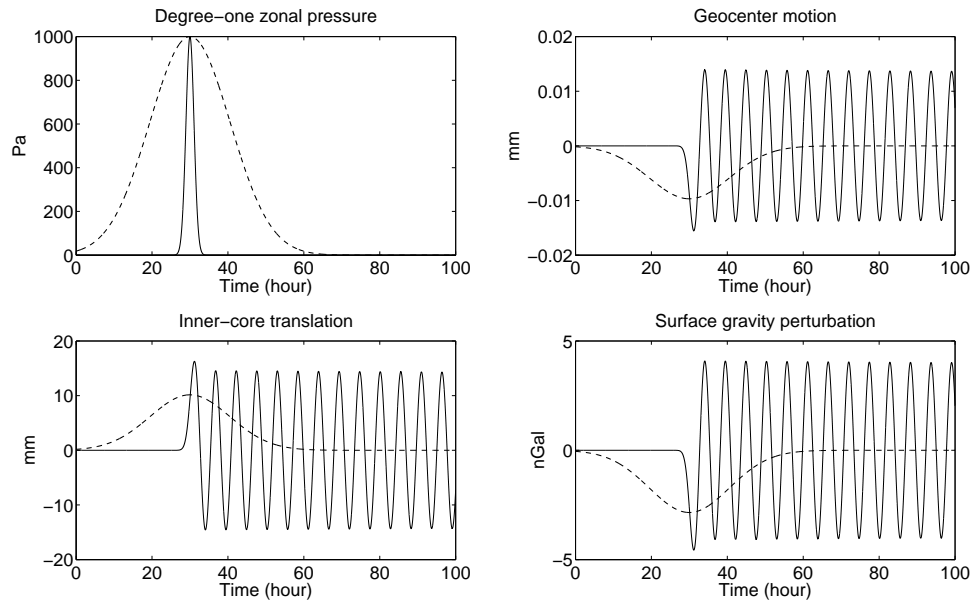


Figure A.4: Resulting effects of the excitation of the Slichter mode by a zonal surface load $\sigma(t) = \sigma_0 e^{-[\frac{t-T_0}{\tau}]^2}$ for two different excitation time-scales: $\tau = 1.5$ h (solid line) and $\tau = 15$ h (dashed line). (a) Degree-one zonal pressure effect; (b) geocentre motion; (c) inner-core translation; (d) surface gravity perturbation.

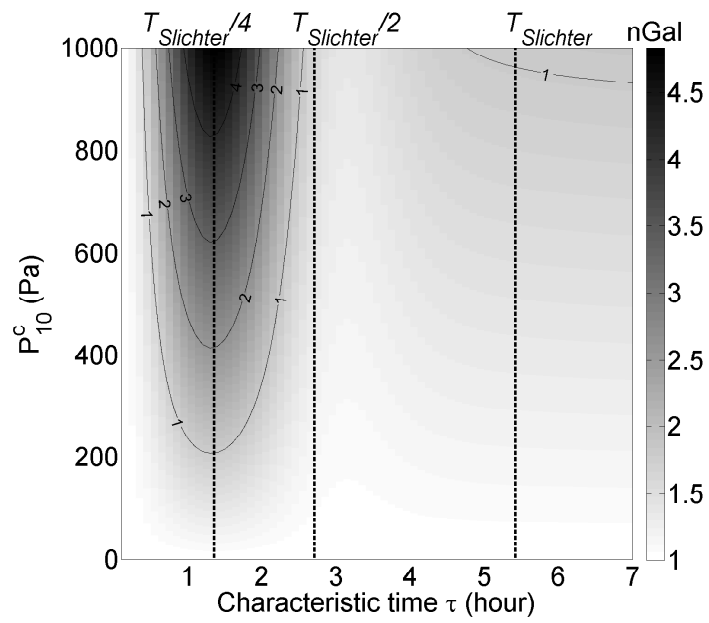


Figure A.5: Surface gravity perturbation induced by the Slichter mode excited by a surface load for different excitation time-scales and various zonal pressure amplitudes.

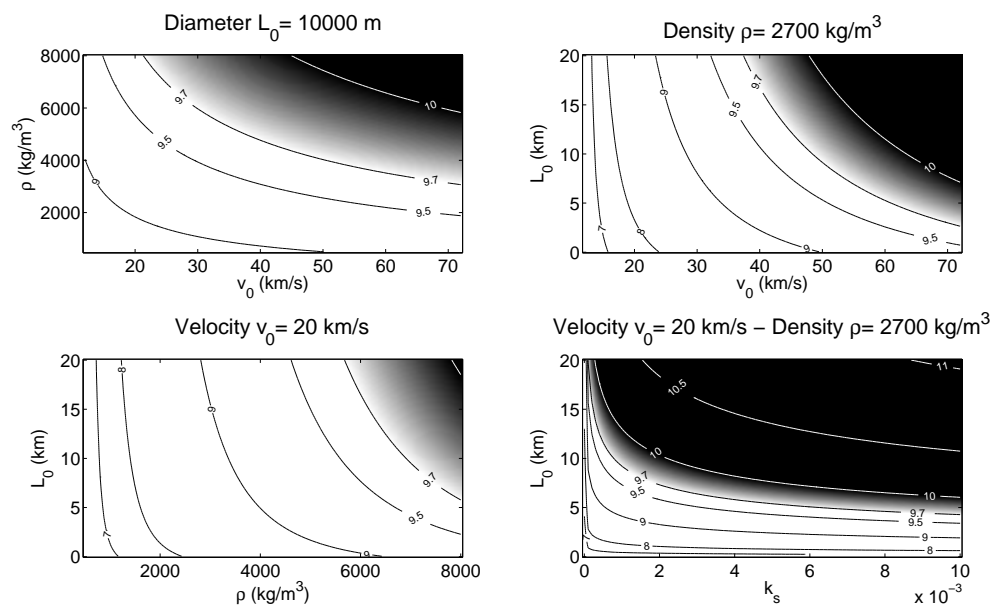


Figure A.6: Seismic magnitude as a function of the impactor parameters and seismic efficiency k_s . The shaded area corresponds to moment magnitudes larger than 9.7, i.e. to induced surface gravity perturbation larger than 1 nGal.

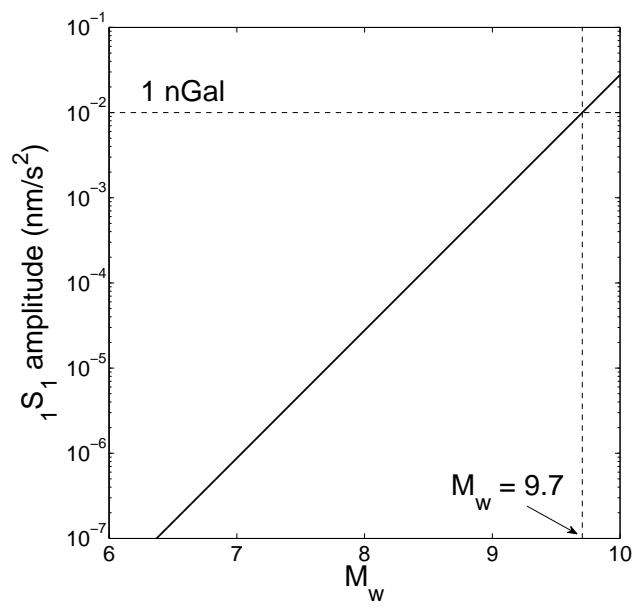


Figure A.7: Surface gravity perturbation induced by the Slichter mode as a function of the moment magnitude of a superficial energy release (explosion or object impact).