Two Dimensional Deterministic Model of a Thin Body with Micro High Stiffness Fibers Randomly Distributed

Nait-Ali Azdine[*], **Michaille Gérard**[†], **Pagano Stéphane**[*]

[*]LMGC, UMR-CNRS 5508, Université Montpellier 2, Cc 048, Place E. Bataillon, 34095 Montpellier Cedex 5, France, azdine.nait-ali@univ-montp2.fr,
[†]I3M and MIPA, UMR-CNRS 5149, Université Montpellier 2 and Université de Nîmes, Case courier 051, Place Eugène Bataillon, 34095 Montpellier Cedex 5, France, micha@math.univ-montp2.fr

ABSTRACT

Using ergodic theory and a variational process, we study the macroscopic behavior of a thin body with micro high stiffness fibers randomly distributed according to a stationary point process. The thickness of the body, the stiffness and the size of the cross sections of the fibers depend on a small parameter ε. The variational limit functional energy obtained as ε tends to 0 is deterministic and depends on two variables: one is the deformation of a two dimensional body, which describes the behavior of the medium in the matrix, and the other captures the limit behavior of deformations in the fibers when the thickness, the stiffness and the size section become increasingly thiner.

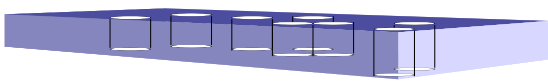

Figure 1: A slice of randomly fibered body of thikness $h(\varepsilon)$

We proposed a deterministic model of a randomly reinforced material with reference configuration (Figure 1), an open $\mathcal{O} = \hat{\mathcal{O}} \times (0,h)$, $\hat{\mathcal{O}} \subset \mathbb{R}^2$ including randomly distributed thin rigid fibers $T_\varepsilon(\omega) = \varepsilon D(\omega) \times (0,h)$ where $D(\omega)$ are disks randomly distributed in $\mathbb{R}^2$ following a stochastic point process $\omega = (\omega_i)_{i\in\mathbb{N}} \subset \mathbb{R}^2$ associated with a suitable probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Our objective as to provide a simplified, but accurate model of the slices of the geomaterial TexSol^{TM} [2, 4, 5]. Let us recall that this soil reinforcement process mixes the soil with a wire and that the obtained material has a better mechanical resistance than the sand without wire. Note that the randomly process of fibers distribution is supposed to be ergodic.

The parameter h, that depends on ε and is assumed to go to zero with ε. Then, we obtain a two dimensional deterministic model which is a first attempt in the scope of non linear elasticity at obtaining a variational equivalent model of a very thin slices of randomly reinforced materials like Texsol^{TM}, with unidirectional orientation of fibers. Indeed, the fibers are considered vertical for slice thin enough.

Thanks to variational convergence method (related to the Γ-convergence [1, 3]) the total energy

$$E_\varepsilon(\omega,u) := \int_{\mathcal{O}\setminus T_\varepsilon} f(\nabla u)\,dx + \frac{1}{\varepsilon^a}\int_{\mathcal{O}\cap T_\varepsilon} g(\nabla u)\,dx - \int_{\mathcal{O}} L_\varepsilon.u\,dx - \int_{\widehat{\mathcal{O}}\cap\varepsilon D} l_\varepsilon.u(\hat{x},h(\varepsilon))\,d\hat{x}$$

converges to the effective total energy E_0.

$$E_0(u,v) := \int_{\hat{\mathcal{O}}} f_0(u)\, d\hat{x} + \theta^{1-p} \int_{\mathcal{O}} g_0(\frac{\partial v}{\partial x_3})\, dx - \int_{\mathcal{O}} L.u\, dx - \int_{\hat{\mathcal{O}}} l.v d\hat{x}$$

Remark this limit energy depending only u and not ∇u. The energy density f_0 is the limit energy in the matrix, g_0 is the limit energy in fibers and θ is the volume fraction of fibers basis in $\hat{Y} := [0,1]^2$. And the constant p is the dimension of space and $a > 0$ a costing depending to the thickness of fibers.

The first functional E_ε is the energy in the matrix, the second the energy in the fibers and L_ε, l_ε are the loading in the matrix and the fibers respectively.
In order to validate this convergence result, we calculate the evolution of a suitable error between the solution of $\inf\limits_{u \in W_\varepsilon^{1,p}(\mathcal{O},\mathbb{R}^3)} E_\varepsilon(\omega, u)$ and $\inf\limits_{(u,v) \in L^p(\mathcal{O},\mathbb{R}^3)\times V_0} E_0(u,v)$ when ε decreases where

$$W_\varepsilon^{1,p}(\mathcal{O},\mathbb{R}^3) := \left\{ u \in W^{1,p}(\mathcal{O},\mathbb{R}^3) : u = 0 \text{ on } \hat{\mathcal{O}} \cap \varepsilon D(\omega) \right\},$$

$$V_0(\mathcal{O},\mathbb{R}^3) := \left\{ v \in L^p(\mathcal{O},\mathbb{R}^3) : \frac{\partial v}{\partial x_3} \in L^p(\mathcal{O},\mathbb{R}^3),\ v(\hat{x},0) = 0 \right\}.$$

We approximate these solutions with the software cast3M® in the particularly case of the random chessboard-like, and, for simplicity we choose $g = f = |.|^2$

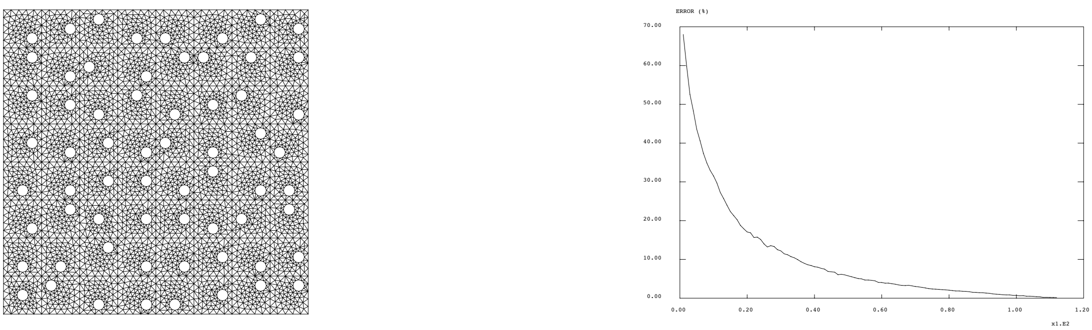

Figure 2: The random "chessboad-like" and error's convergence of result on the matrix

REFERENCE

1. G. Dal Maso. *An introduction to* Γ*-convergence.* Birkäuser, Boston, 1993.

2. M. Frémond. *Non-Smooth Thermo-mechanics.* Springer-Verlag, Berlin Heidelberg New York, 2002.

3. G. Michaille, A. Nait Ali, S. Pagano. *Macroscopic behavior of a randomly fibered medium.* Journal de Mathmatiques Pures et Appliques, vol(42), **3** (2011) 230-252.

4. R. Laniel, P. Alart, S. Pagano. *Consistent thermodynamic modelling of wire-reinforced geomaterials.* European Journal of Mechanics - A/Solids, vol 26, **5** (2007) 854 - 871.

5. R. Laniel, P. Alart, S. Pagano. *Discrete element investigations of wire-reinforced geomaterial in a three-dimensional modeling.* Computational Mechanics, vol(42), **1** (2008) 67-76.