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Recent Investigations on BAW Resonators at Cryogenic Temperatures

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Abstract— This work presents the results of investigations on different aspects of cryogenic operation of BAW resonators. For the quartz crystal resonators the losses mechanisms, the temperature sensitivity, the amplitude-frequency effect are described for liquid helium temperatures. The corresponding problems are discussed. To overcome some of these problems, the possible solution of operation at higher modes is considered. Some of these higher modes exhibit outstanding quality factors. The highest (for BAW resonators) quality factor value and quality factor-frequency product are measured for the 11th and 13th overtones respectively. In addition, two LGT resonators have been characterized in a wide temperature range. Finally, some preliminary results on utilization of cryogenic quartz resonators as a part of a frequency stabilization closed loop system are given.

I. INTRODUCTION

The quality factor (Q -factor) of resonators and the frequency stability of frequency sources are closely related. Thus, the time and frequency community is always eager for new resonators with higher Q values of any physical nature, i.e. electromagnetic, acoustic, mechanical, etc. Bulk acoustic wave (BAW) quartz resonators at room temperatures have rather high $Q \cdot f$ product of about $15 \cdot 10^{13}$ for the frequency band of 1-100 MHz. The properties of these devices at room temperatures are widely investigated and their limitations are known [1]–[6]. Indeed, the Q -factor is not the only parameter that counts for the frequency stability or even for resonator applicability in frequency sources. Others may be frequency-temperature characteristics, nonlinear phenomena, vibration sensitivity, required environment, etc. Contrary to that, though it is well-known that quartz resonators have much higher Q -factor at cryogenic temperatures, their properties and limitations at these temperatures have not yet been deeply investigated. This work is dedicated to this problem. First, the working conditions and measurement methods for BAW resonators are discussed. Second, three vibration modes, A, B and C are considered at liquid helium temperatures for quartz SC-cut [7] BVA (Boîtier à Vieillessement Amélioré) [8] resonators optimized for working at their 3rd or 5th overtones at 5 or 10 MHz at usual temperatures. An important point here is the temperature sensitivity of this devices. Also, this section concerns amplitude-frequency and self-heating effects and associated limitations. In addition, some results for two higher overtones are given. The next section gives the results of investigation on resonators of a new synthetic material

known as Lanthanum Gallium Tantalate (LGT). Finally, the frequency stability results of one possible reference system with a cryogenic quartz resonator are considered.

II. EXPERIMENTAL SET-UP

Since the first investigations on cryogenic crystal resonators, the cryocooling technology has made a considerable progress. Thus, for example, cryostats can be substituted with cryogenerators. This substitution simplifies the cryogenic system operation and does not superposes any limitations on experiment time duration. But cryogenerators themselves introduce additional environmental disturbances, such as temperature fluctuations and vibrations mainly due to the helium pulsation at the frequency of 1.7 Hz.

A. Cryogenic System

For all the presented measurements a two-stage pulse-tube cryocooler is used (F-70H from Sumitomo Heavy Industries). The cryocooler is equipped with two shields: a vacuum chamber and an anti-radiation shield to prevent the heat exchange by means of air conduction and radiation respectively. The vacuum level inside the chamber is about 10^{-7} mBar. Both primary and secondary pumps are cut off from the chamber during measurements. Nominally, the cryocooler can absorb up to 1 W at the second stage at the temperature of 4 K. The temperature of the second stage can go down to almost 3 K if no additional thermal losses are present. The thermal losses are mainly introduced with the connecting cables needed for measurement purposes and environmental control. The resonators are inserted inside a copper block attached to the second pulse-tube stage [6].

B. Environmental Control

As it is said above, two main environmental disturbances are temperature fluctuations and vibrations. The temperature fluctuations of the cryocooler second stage due to helium pulses are about 0.3 K. The power spectral density of temperature fluctuations of the second stage in the steady state regime consists of f^{-2} region for frequencies less than about 10^{-2} Hz and f^0 region for higher frequencies. In addition, a clear spurious frequency of 1.7 Hz and its harmonics are present due to the temperature pulsation. To improve the thermal conditions passive and active instability suppression

methods are used. First, a thermal filter is installed between the second stage and a device under test (DUT). The filter reduces the temperature pulsation down to 15 mK, but increases the absolute temperature difference between these parts. Second, the active closed loop temperature regulation is implemented with a 332 Temperature Controller from Lake Shore, a CER-NOX sensor and 20 Ohms heating resistor. The sensor and the actuator are installed in a copper block under the filter. The control system eliminates the f^{-2} region in the temperature fluctuation spectrum and reduces pulses down to 5 mK.

Another type of environmental disturbances is vibration. The permanent pulsation of the helium inside the pulse-tube causes DUT vibrations with a nominal amplitude of $A = 10\mu\text{m}$ and frequency $f = 1.7$ Hz. The influence of the vibration on resonator parameters and noise can be reduced simply by choosing a proper angle of resonator orientation relatively to the main (vertical) axis of the vibration. No other vibration compensation system is installed for the presented measurements.

C. Measurement Techniques

The resonator characteristics are measured with a 10Hz-500MHz network analyzer HP 4195A equipped with its impedance test adapter. This method is preferable to traditional π -network method for this kind of measurements because the resonator parameters, in particular its equivalent motional resistance, change significantly with temperature. This fact results in an impedance mismatch in the π -network. The measurement capability provides a frequency resolution of 1 mHz, a phase resolution of 0.01 degree and a magnitude resolution of 0.01 dB. The minimum power of the signal source is -50 dBm. A distinctive feature of the network analyzer method is the need for compensation of long connecting cables. Indeed, the resonators are connected to the analyzer with 1 m coaxial cables passing a temperature range from 4 K to the ambient temperature. To compensate cable effects, three calibration references - the open circuit, the short cut and a 50 Ohms resistor - are put close to DUTs at the end of identical cables. Measurements are made with -40 dBm source power for 3rd and 5th overtones and -50 dBm for higher overtones.

The network analyzer is used to trace a so-called GB-plot, i.e. real and imaginary parts of the device admittance $Y(j\omega) = G(\omega) + jB(\omega)$ versus frequency $f = 2\pi\omega$ near a resonance of interest. These data are acquired by means of the GPIB connection and Labview software. After that the parameters of the traditional series $R - L - C$ lumped resonator model are optimized to fit the measured data.

III. SC-CUT QUARTZ RESONATORS AT CRYOGENIC TEMPERATURES

Quartz BVA resonators are the subject of investigation of the present section, because it has been already shown that electrodeless resonators exhibit higher quality factors than electrode-deposited resonators [5]. More precisely the plano-convex SC-cut resonators optimized at room temperatures are

chosen here as the most modern devices (active part diameter: 11 mm, thickness: 0.54 mm, curvature radius: 300 mm).

A. Resonator Losses at Cryogenic Temperatures

The main property of the devices of interest is the total losses that are inversely proportional to the device quality factor Q . The total losses (actually measured results) are a sum of all losses presented in the device, i.e. the material losses, mounting losses, electrical losses, etc. Concerning material part, this type of losses are represented in perfect material with three physical mechanisms [9]–[13]: acoustic wave-phonon dissipation (usually dominant intrinsic loss mechanism in insulating resonators), thermoelastic dissipation (pronounced in plates or rods when the wavelength of flexural vibrations is larger than the thickness of the rod or the plate), phonon-electron dissipation (usually negligible in insulators). Since the first type of the listed material losses is dominant in the devices of interest, it is considered further. The losses due to interaction of the acoustic wave and thermal phonons can be described with two distinct approaches. The first one, introduced by Akheiser [12], considers the interaction of a macroscopic strain wave in an insulator crystal with thermal phonons. Dependency of the phonon frequency on the strain leads to transport of the phonons between macroscopic regions of a crystal restoring the thermal equilibrium. This process is accompanied by acoustic wave energy loss. This approach is valid when the phonon life time is much smaller than a period of the acoustic wave, i.e. $\omega\tau \ll 1$, where τ is a thermal phonon life time and ω is an acoustic wave angular frequency. According to Akheiser approach the material $Q \cdot f$ product at high temperature is found as follows:

$$f \cdot Q = \frac{\rho V_a^2 (1 + (\omega\tau)^2)}{C_v T \gamma^2 \tau}, \quad (1)$$

where $f = 2\pi\omega$, V_a is the wave velocity, h the Planck constant, γ the Grüneisen parameter, C_v the volumetric heat capacity, T the absolute temperature, ρ the density. The second approach, proposed by Landau and Rumer [13], considers the acoustic wave as a parallel beam of interacting acoustic phonons. The interaction process involves acoustic quanta and individual thermal phonons. It is shown that the acoustic phonon attenuation is a result of three phonon interaction and is proportional to the angular frequency ω in the most studied case (also assuming that $\omega\tau \gg 1$ which is relevant at low temperature) [10]. So, the product of the quality factor and the resonance frequency is given by

$$f \cdot Q = \frac{15\rho V_a^5 h^3}{\pi^5 \gamma^2 \tau K^4 T^4} \omega, \quad (2)$$

where K is the Boltzmann constant. According to this approach the material quality factor does not depend on the angular frequency ω .

It is clear that if the acoustic wave-thermal phonon attenuation is dominant for some range of temperatures, the dependence of the losses on temperature can show which regime is valid for the considered material in given conditions. To

do so, for the first experiments, SC-cut BVA quartz resonators optimized for working on their C-mode (about 5 MHz at room temperature) are characterized between 3.8 and 20 K. The results for 5th overtones of three different modes are shown in Fig. 1.

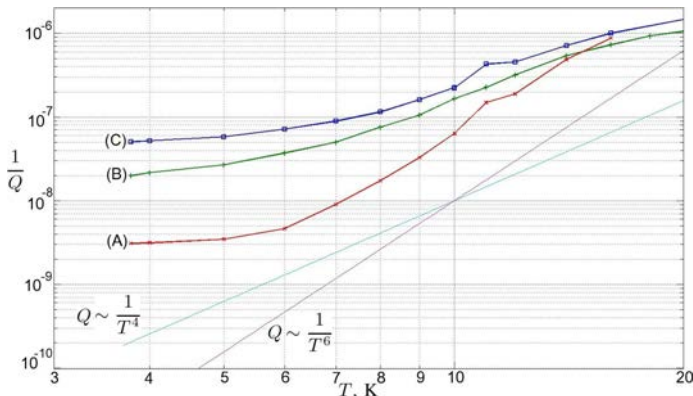


Fig. 1. Measured device losses for 5th overtones of different modes: A-mode (quasi-longitudinal), B-mode (fast quasi-shear), C-mode (slow quasi-shear).

The dependence of the losses on temperature (Fig. 1) clearly shows three regions: a plateau for temperatures less than 6 K (device losses are dominant, such as acoustic diffraction at the resonator surface or damping at the edge of the resonant disk), a linear region in the log-log scale (attributed to acoustic wave - phonon losses) and the second plateau for temperatures about 20 K (region of the maximum absorption [4]). The medium region clearly shows the dependency of the type $Q^{-1} = aT^n + \text{constant}$, where n is close to 4 for quasi-shear modes and to 6 for a quasi-longitudinal mode. Though for the latter mode, the dependence does not strictly confirm the slope predicted by the Landau-Rumer theory, the corresponding losses regime is actually dominant in quartz material at the liquid helium temperature. The increase of the power n has been already observed [10], [11].

The highest measured Q -values among 5th overtones of three modes are $325 \cdot 10^6$ for the A-mode, $50.5 \cdot 10^6$ for the B-mode, $1.99 \cdot 10^6$ for the C-mode. This sequency corresponds to the list of modes sorted out from the best trapped mode to the less trapped one according to the Stevens and Tiersten model [14]. Among all modes of 3rd and 5th overtones the best result is achieved for the 5th overtone of the A mode with $Q \cdot f$ product of about $5.1 \cdot 10^{15}$ which is the highest result in comparison with previous investigations [6]. But already starting at this stage, the problems of practical utilization arise. Since at cryogenic temperatures the resonators have very low values of equivalent motional resistance (e.g. about 3 Ohm for the 5A mode), minimization of the active load is required in order to preserve the high quality factor. This considerably complicates the choice of signal sources and amplifiers.

B. Temperature Sensitivity

Another important characteristic of a resonator is its temperature sensitivity. Indeed, to minimize the influence of

the environment on the frequency source, the sensitivity of the resonance element has to be minimized. Fig. 2 shows the fractional frequency temperature sensitivity for the three modes considered in the previous subsection.

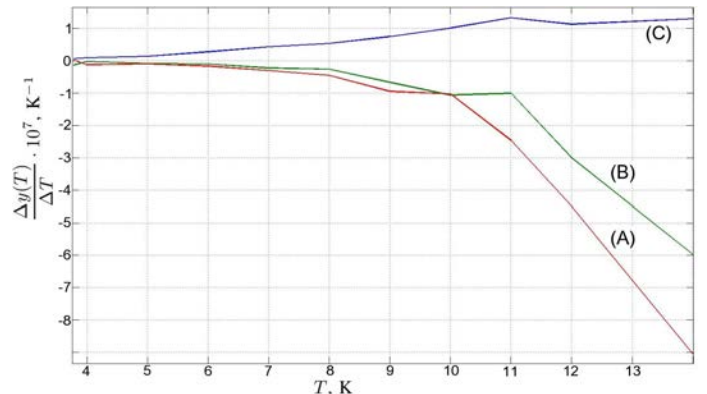


Fig. 2. Fractional frequency temperature sensitivity: A, B and C-modes.

As it is clearly seen from Fig. 2 the investigated modes does not have any turn-over point. The absence of a turn-over point poses serious problems in terms of crystal temperature stabilization. In general, the smaller sensitivity corresponds to lower temperatures for these devices. At 4 K the sensitivities are $2 \cdot 10^{-9} \text{ K}^{-1}$ for the A-mode, $3 \cdot 10^{-9} \text{ K}^{-1}$ for the B-mode and $1 \cdot 10^{-8} \text{ K}^{-1}$ for the C-mode. These results suggest that to obtain a desired fractional frequency stability of 10^{-13} , in best case, the temperature has to be controlled within 0.5 mK.

C. Amplitude-Frequency Effect

BAW resonators are nonlinear devices in nature. Resonator properties considerably depend on the power level of the excitation signal. This dependence becomes even more pronounced with the increase of the quality factors, since the resonance frequency shift is proportional to the product of the dissipated power and Q . To characterize this effect in cryogenic temperatures, the impedance of the resonator under test is measured with different levels of an excitation signal for the 5th overtone of the A-mode (that with the highest Q). The results are shown in Fig. 3.

Fig. 3 suggests that already for the dissipation power of $P = 0.22 \mu\text{W}$ (Curve (2)) the impedance curve distortion exists. Furthermore, for $P = 1.6 \mu\text{W}$ the resonator self-heating effect becomes apparent. Also, it can be seen that for an increasing power P , the equivalent motional resistance also increases. These effects can even lead to temperature-amplitude oscillations. These oscillations are characterized by two states of low and high dissipation powers corresponding also to higher and lower values of motional resistance. In addition, the higher driving signal levels degrades resonator noise characteristics [15].

The strong amplitude-frequency and self-heating effects arise another problem concerning effective utilization of cryogenic quartz crystal resonators in actual applications. Indeed, in order to avoid this effect the driving signal has to be kept

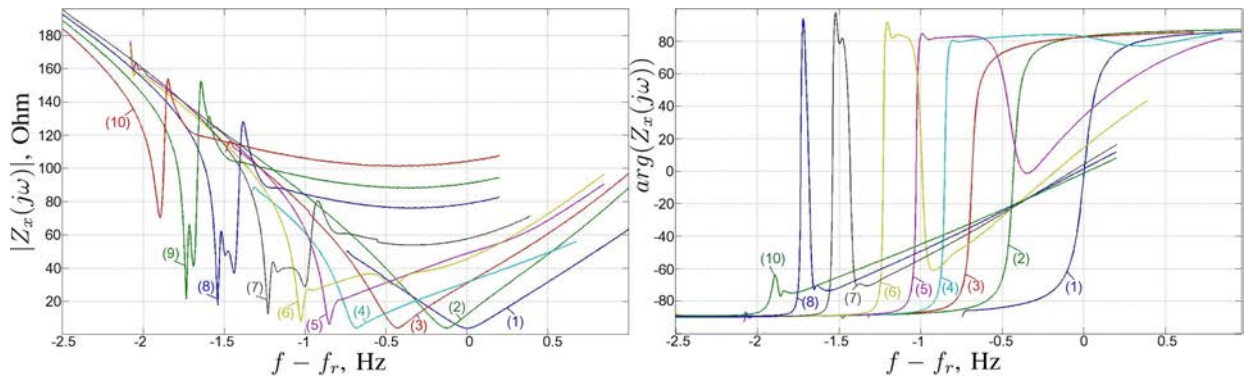


Fig. 3. Impedance of the resonator near the A-mode on 5th overtone for different dissipation powers: (1) $P = 0.09\mu\text{W}$, $I = 0.154\text{mA}$ (2) $P = 0.22\mu\text{W}$, $I = 0.237\text{mA}$, (3) $P = 0.42\mu\text{W}$, $I = 0.324\text{mA}$ (4) $P = 0.64\mu\text{W}$, $I = 0.403\text{mA}$, (5) $P = 1.6\mu\text{W}$, $I = 0.479\text{mA}$ (6) $P = 2.26\mu\text{W}$, $I = 0.538\text{mA}$, (7) $P = 4.87\mu\text{W}$, $I = 0.624\text{mA}$ (8) $P = 9.20\mu\text{W}$, $I = 0.719\text{mA}$, (9) $P = 12.35\mu\text{W}$, $I = 0.757\text{mA}$ (10) $P = 17.20\mu\text{W}$, $I = 0.494\text{mA}$.

very low, but this fact degrades the system signal-to-noise ratio as well as it requires additional amplification stages. This problem is even worse lower values of the resonator equivalent motional resistance.

D. Higher Overtones

In order to deeply investigate the abilities of devices under tests, some higher resonator harmonics have been characterized at cryogenic temperatures. Thus, Fig. 4 demonstrates the presence of different anharmonics of the 11th overtone of the A-mode. The corresponding indexes for identifying the anharmonics are marked on the figure. The best anharmonic mode has a quality factor of $417 \cdot 10^6$ at the frequency of 68823167 Hz with an equivalent resistance of 19 Ohms. The estimated $Q \cdot f$ product for this resonance equals to $2.87 \cdot 10^{16}$ Hz. Thus, the DUT working at this resonance at cryogenic temperatures has the highest reported quality factor ever measured on BAW resonators.

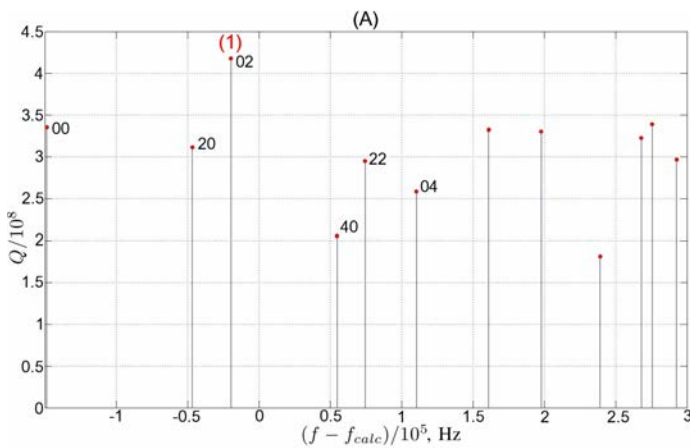


Fig. 4. Different anharmonics of the 11th overtone of the the A mode ($f_{calc} = 68.842620$ MHz).

In the same way the 13th overtone of the A-mode is characterized. Here only three anharmonics are found. The best of them has a Q value of $377 \cdot 10^6$ at 81392723 Hz with

an equivalent resistance of 30 Ohms. Thus, the highest $Q \cdot f$ measured on BAW resonators can reach $3.07 \cdot 10^{16}$ Hz

So, these modes could be perspective for a practical utilization. But the issue of the selection or rejection of the undesired modes should be solved first in autonomous systems such as oscillators. Also, higher Q values result in a stronger amplitude-frequency effect [16].

IV. LGT RESONATORS AT CRYOGENIC TEMPERATURES

The LGX material family is promising for designing BAW resonators with high quality factors. Among this materials the Lanthanum Gallium Tantalate (LGT) and Lanthanum Gallium Silicate (LGS) are the most commonly known. Nevertheless, their properties have not been yet investigated for temperatures lower than 80 K. In this section, investigation results on BVA-type plano-convex Y-cut LGT resonators (active part diameter: 10.2 mm, thickness: 0.65 mm, curvature radius: 100 mm) are presented. The attempts to characterize their LGS counterparts have been abandoned since they show a decrease of quality going from room to cryogenic temperatures.

A. LGT Resonator Losses

The characteristics of two LGT resonators of similar origin have been measured in the wide temperature range (4–80 K). The results for their quality factors are shown in Fig. 5. The maximum Q -factor is just about $7.2 \cdot 10^6$ that is only 5 times higher than its value at room temperature. Unlike in the case of quartz resonators, device losses dependence on temperature seem to be rather uniform. The only physical phenomenon clearly seen is the peak of losses at about 40 K. It can be attributed to relaxation processes due to the presence of impurities. In addition, a very important difference between two resonators is observed. These results suggest that the LGT resonator technology is far from being optimized in terms of different types of losses: device losses cover material losses. On the other hand, further technology development might lead to considerable increase of their quality factors in cryogenic conditions.

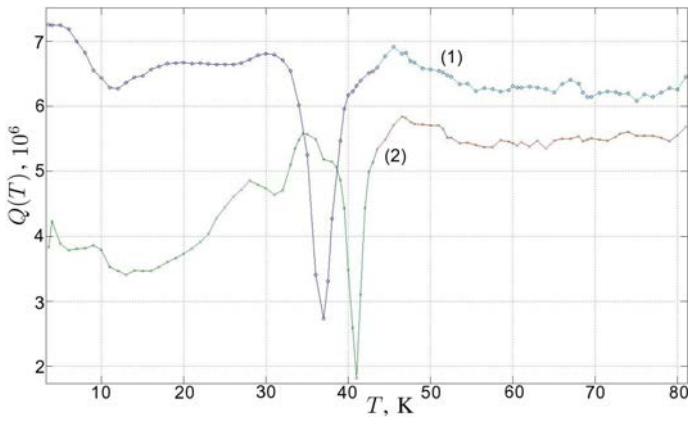


Fig. 5. LGT resonator quality factor Q as a function of temperature T (two resonators).

B. LGT Resonator Temperature Sensitivity

The dependence of the fractional stability on the temperature is also examined for the LGT resonators (see Fig. 6). Both resonators have a turn-over point between 6 and 7 K which is advantageous for creating a high quality temperature stabilization system.

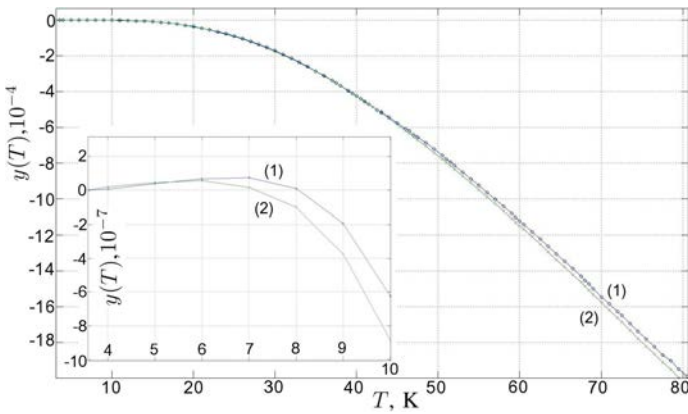


Fig. 6. Fractional frequency y versus temperature T for two LGT resonators.

V. FREQUENCY STABILIZATION SYSTEM BASED ON CRYOGENIC QUARTZ RESONATOR

This section shows the preliminary results on a system based on a quartz resonator working at liquid helium temperatures. For this purpose a frequency stabilization system based on an external high quality resonator is implemented [17]. The general scheme of this system is shown in Fig. 7. The system consists of a voltage-controlled oscillator used as a reference for a frequency synthesizer whose output signal frequency corresponds to the resonant frequency of the BAW device. The resonator is put into a so-called π -network implemented at cryogenic conditions and connected with the long cables with the other components. The signal filtrated by the resonator is compared with the original one (also passed through the cryogenerator to compensate the parasitic effects of cables)

with a double balanced mixer. The resulting signal is filtrated and used to correct the frequency variation of the VCO. In fact, only the resonator π -network and the through connection for the original signal channel are placed inside the cryogenerator.

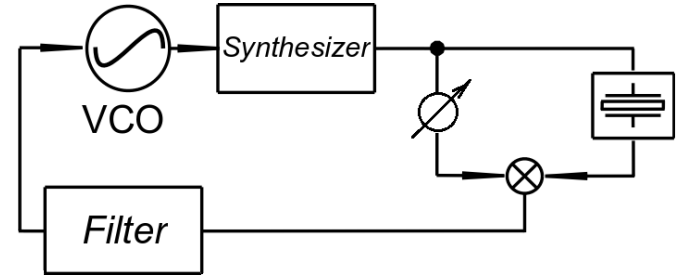


Fig. 7. External frequency discriminator system.

The system is designed for working on the resonator 5th overtone of the A-mode preserving a loaded Q of only $16 \cdot 10^6$. The frequency stability results in terms of Allan deviation are shown in Fig. 8. This figure compares the results measured for the stand-alone voltage-controlled oscillator and the closed loop system after different periods of continuous cryogenic system operation. The stability improvement is clearly seen for the long averaging times τ , though for short τ the stability is degraded due to internal loop noise sources. Comparing two cases of closed loop system, the 12-hours case (measurements are made within first 12 hours of cryogenerator permanent operation) exhibits a τ^{+1} slope of the Allan deviation that corresponds to the linear frequency drift. The 60-hours case (measurements after at least 60 hours of operation) uncover a $\tau^{+1/2}$ behavior that is a result of the VCO f^{-4} or cryogenic resonator f^{-2} phase noises. Both types of noise can be attributed to resonance frequency sensitivity to the temperature random-walk noise [18]. These results suggests the presence of very long time constants in the system (probably due to thermal effects).

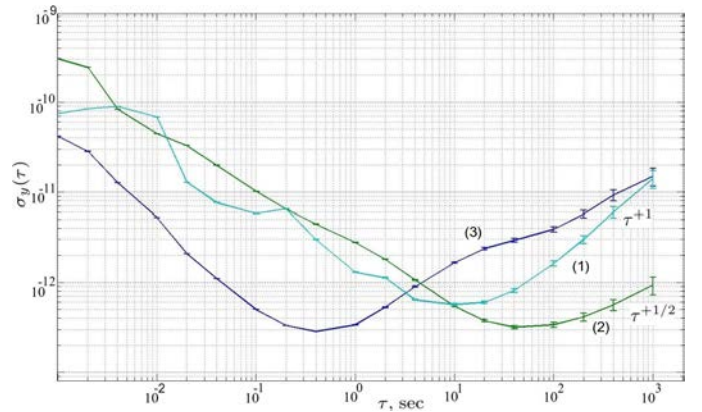


Fig. 8. Allan deviation of the closed-loop frequency stabilization system measured at first 12 hours of cryogenerator operation (1), measured after at least 60 hours of cryogenerator permanent operation (2) and compared with a stand-alone oscillator (3).

Though experimental conditions are not so ideal, tests with a passive reference as well as an oscillator based on the

presented devices are presently in progress. The aim of these tests is to achieve the frequency stability corresponding to the measured values of the quality factor.

VI. CONCLUSION

Though quartz resonators can achieve extraordinary values of quality factors at cryogenic conditions, deeper investigations show the presence of severe problems concerning practical utilization of these devices. The difficulties come from the small values of equivalent motional resistances, the strong amplitude-frequency effect, the self-heating, the absence of a temperature turn-over point. Some of these disadvantages could be overcome for higher overtones, such as 11th or 13th, exhibiting even higher Q values. The LGT resonators are still perspective candidates for cryogenic operation, since the device losses overshadow material losses. The preliminary investigations on the passive frequency stabilization system based on cryogenic BAW resonators show rather moderate results due to a non-optimized environment, internal noise sources and a reduced quality factor.

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