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A Novel Damage Sensitive Feature Based on State-Space Representation

ABSTRACT

Damage detection in civil structure is a challenging task, mainly because of the strong environmental variations and the variable and unknown excitation. There is still a lack of a robust damage detection process. Taking advantage of the development of the nonlinear dynamical systems theory which represents times series in a reconstructed state-space, a novel damage sensitive feature vector is proposed. Statistical modelling using extreme value theory is conducted to classify measurement as damaged or undamaged. The whole approach is tested on two case studies. The first one is a simple 4dof mass/spring numerical model, damaged by stiffness reduction. The second one is a concrete beam subjected to temperature variations to simulate realistic conditions. Damage is introduced buy loading cycles.

From 30% of stiffness reduction, damage is correctly detected with a monotonic trend. In the more realistic case, only few true detection are observed before macro-cracking whereas all points are well classified after. Furthermore, the method is robust against strong temperature variations.

INTRODUCTION

For few years, vibration-based Structural Health Monitoring (SHM) process is presented in terms of statistical pattern recognition [1]. The extraction of Damage Sensitive Feature (DSF), in which high dimensional vibration data are compressed into low dimensional vectors, takes a central place in this process. The main challenge is to preserve information about structure condition. For this purpose, the paradigm of nonlinear dynamical systems offers some promising abilities [2, 3, 4, 5]. The information contained in time-series is extracted using reconstructed state-space representation [6]. Among the numerous candidate features, the Lyapunov Exponents (LE) have been particularly studied [7, 8, 9]. They are related to the long term predictability of dynamical system. To be more precise, two trajectories initially close in the state space will diverge as time evolves with an exponential rate proportional to LE. But two remarks can be formulated on their use as damage sensitive feature. First, the practical calculation of LE is very time consuming as trajectories need to be followed for several thousand of time steps [10]. Then, since damage manifests itself as local irregularities in the signal (due to opening and closing of cracks or loosened assembly) it should be more visible if short evolutions of trajectories are considered.

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This leads to the proposition of a new damage sensitive feature referred to as Jacobian Feature Vector (JFV) [11]. It is formed by the components of the Jacobian matrix of the dynamic estimated in the reconstructed state-space.

Once DSF are extracted from measurement, each new DSF has to be classified as damaged or undamaged (when the aim is limited to damage detection). In civil structures, since only data from the undamaged state are available, damage is detected by comparison with the undamaged database. The statistical classification is carried out in two steps. First, the Mahalanobis distance of the new DSF vector from the undamaged database is calculated to provide a scalar value. Then, exceedances statistical model is used to set classification threshold for control charts. This modelling is related to extreme value theory which is more accurate in determining control limits when dealing with tails of an unknown distribution.

The proposed approach is tested on two case studies. The first one is a 4 dof mass/spring/damper numerical model, and the second one is a concrete beam subjected to temperature variations.

The first section of the paper details the theoretical background of the new DSF and the statistical modelling. Then the case studies are presented before discussing the results in the third section.

THEORETICAL BACKGROUND

State-Space Damage Sensitive Feature

The evolution of any dynamical system can be represented as a trajectory in it state-space where each dimension is a degree of freedom. When it is impossible to measure all dof, as in instrumented civil structures, one can reconstruct qualitatively the state-space based on the measurement of only one scalar time series by using the delayed coordinates method [6].

The scalar time series $(x) = x(1), \ldots, x(N)$ is transformed into a collection of *n*-dimensional vectors:

$$X(k) = \{x(k), x(k+\tau), \ldots, x(k+(n-1)\tau)\}. \tag{1}$$

The first minimum of the autocorrelation function or the first zero of the mutual information function of *(x)* provides an estimate of τ [12]. Then the best embedding dimension, *n*, is estimated with the *false nearest neighbors* method [13]. More details on the reconstruction procedure can be found in [14].

In the reconstructed state-space, the dynamical system can be represented by the evolution operator *F*, which links one point to the next one, in a trajectory:

$$X(k+1) = F(X(k)). \tag{2}$$

If $Y(k)$ is a close neighbour of $X(k)$, and noting $\delta X_i = X(k+i) - Y(k+i)$, the first order Taylor expansion of *F* introduces the Jacobian matrix calculated at the point $X(k)$, i.e. $J[X(k)]$:

$$\begin{aligned} \delta X_1 &= J[X(k)]\delta X_0 + o\|\delta X_0\|, \\ \delta X_2 &= J[X(k+1)]J[X(k)]\delta X_0 + o\|\delta X_0\|. \end{aligned} \tag{3}$$

To evaluate the Jacobian matrix with experimental data, the first steps of the method used to estimate the Lyapunov exponents are employed [10]. The algorithm presented hereafter is illustrated in Figure 1.

A fiducial point $X(k)$ is chosen in the reconstructed state space, and its r nearest neighbors, noted $Y_{nn}^{i}(k)$, $i=1\ldots r$, are selected to form a neighborhood. The difference vectors are constructed as follows

$$\delta X_{nn}(k)=\left\{X(k)-Y_{nn}^{i}(k)\middle|i=1,\ldots,r\right\}. \tag{4}$$

The least-square method is used to evaluate the Jacobian matrix as the best linear mapping between the neighborhoods k and $k+1$ (Eq.4 and Eq.5).

$$\begin{aligned}\delta X_{nn}(k+1)&=J_1\left[\delta X_{nn}(k)\right]\\ \delta X_{nn}(k+2)&=J_2\left[\delta X_{nn}(k+1)\right].\end{aligned} \tag{5}$$

A previous study [11] has shown that the sensitivity to damage is improved if the mapping does not include the initial neighborhood. Hence, the components of J_2 (Eq.5) are used to form a feature vector which will be referred to as the Jacobian Feature Vector (JFV).

$$JFV=J_2(:). \tag{6}$$

This process is repeated for 100 fiducial points across the state-space. The number of neighbors, r, is set to twice the number of parameters to be estimated in the Jacobian matrix.

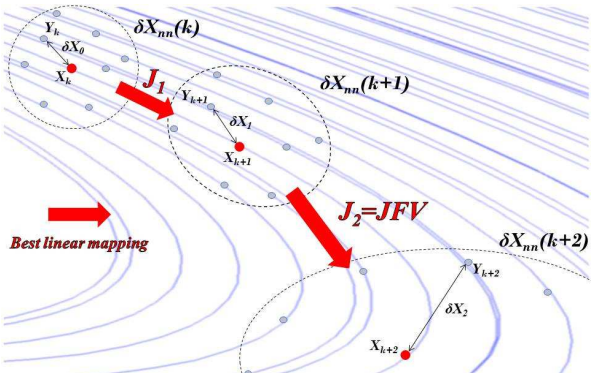


Figure 1. Illustration of the JFV calculation algorithm

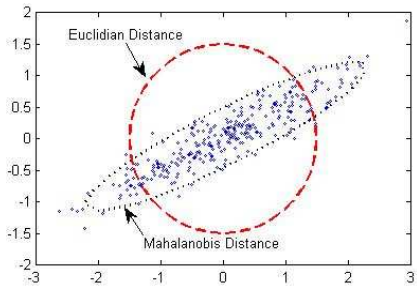


Figure 2. Mahalanobis vs Euclidian distance

Novelty Detection

In order to compare a point from a set of points, the calculation of the Mahalanobis distance (MD) is common. Unlike Euclidian distance, it takes into account the correlations between coordinates (Figure 2). Its use is widely spread in outlier analysis or novelty detection [15]. It converts a multivariate DSF to a scalar value which is easier to analyse with statistical modelling.

If the reference baseline is characterized by a mean vector $\overline{JFV}$ and a covariance matrix C_{JVF}, the square MD of the ith JFV vector is:

$$D_i^2=\left(JFV_i-\overline{JFV}\right)^T\left(C_{JFV}\right)^{-1}\left(JFV_i-\overline{JFV}\right). \tag{7}$$

Statistical Modelling

The use of control charts implies the setting of control limit. Using the normal hypothesis to set the limit can lead to wrong value since extreme events reside in the tails which are poorly modelled when the nature of the distribution in unknown. To overcome this problem, it is possible to consider the variable Y, which represents the exceedances of D over a threshold u. Regardless the distribution of D, if u is in the tail

of D, the distribution of Y will follow a Generalized Pareto Distribution (GPD) [16]. This is an equivalent of the central limit theorem for extreme values. A special case of GPD is the exponential distribution which is easy to fit. Thus, for $d \geq u$:

$$F(d) = P(D \leq d) \approx 1 - (1 - F(u)) \exp\left(\frac{u - x}{\sigma}\right) \tag{8}$$

with σ, the scale parameter.

The calibration of the model starts by the determination of the threshold, u. It is done by choosing a number of exceedances k, among a sample of size m taken from the undamaged database $\{d_1, \ldots, d_m\}$. These values are sorted in descending order $\{d_{1:m} \geq d_{2:m} \geq \ldots \geq d_{m:m}\}$. Then, u is associated with the k order statistics $d_{k:m}$ and $F(u)$ is estimated by the empirical distribution of D.

$$u = d_{k:m} \quad k < m \quad and \quad F(u) = 1 - \frac{k}{m+1}. \tag{9}$$

The parameter σ is estimated using maximum loglikehood method [16].

$$\hat{\sigma} = \frac{1}{k} \sum_{i=1}^{k} d_{i:m} \ . \tag{10}$$

Finally, since only data from the undamaged state are available, the classification of a new sample d as damaged or undamaged is done with a hypothesis test. To improve the robustness of control charts, it is common to consider a run of p measurements instead of each measurement individually [18]. Thereby, the null hypothesis (undamaged state) is rejected if p consecutives values of d exceed the control limit:

$$H_0 : d_i \leq CL, \quad H_1 : d_i > CL \quad i = 1 \ldots p. \tag{11}$$

The control limit is fixed with respect to a probability α of type I error. So, with the verified assumption that d_i are independent, α is the probability that p successive values exceed the control limit when the state is undamaged.

$$\alpha = P(D > CL)^p = \left[1 - F(CL)\right]^p \Rightarrow F(CL) = 1 - \alpha^{\frac{1}{p}}. \tag{12}$$

CL is calculated using Eq.12 and inversing Eq.8.

Database repartition

When a learning process is involved in pattern recognition, data are usually split in three parts to control the quality of the training.

- The reference baseline is composed of half of the undamaged database. It is used as reference for Mahalanobis distance calculation.
- The classification base is composed of a quarter of the undamaged database. It is used to estimate the parameters of the statistical model and fixes the control limit.
- The test base is composed of a quarter of the undamaged database and all damaged base. It is used to test the damage detection method.

CASE STUDIES

Case study 1

The first case study (CS1) is a 4 dof mass/spring/damper numerical model, excited by a white noise. The system is solved by a fourth order Runge-Kutta method sampled at 30Hz. Damage is introduced by reducing the stiffness in traction of the spring k_1 to

simulate opening and closing of a crack (Figure 3). The database is composed of 120 times series of 8192 values at the undamaged state, and 90 damaged one with a reduction of stiffness by 10% every 10 measurements. This case is a first test for the damage detection algorithm, with very clean data and no external variations.

Case study 2

To test the robustness of the approach, it has to be tested on more realistic data. In civil engineering structures, environmental variations cause large fluctuations of damage indexes making damage detection very difficult.

This second case study is a 200x12x10cm fibre reinforced concrete beam (Figure 4). To simulate environmental variations, 4 infra-red lights are used to heat the beam on one face, up to 50°C. Damage is introduced by four points flexion cycles with an increasing load. While temperature fluctuates, vibration measurements are recorded through 4 uniaxial accelerometers placed on the top. Accelerometers will be referred as #1 to #4 from the right to the left of the beam. The excitation is an 800Hz band-limited noise and is produced by an electrodynamic shaker. For each measurement sequence, 5 times series are recorded. Each time series counts 8192 points at 2048Hz. The undamaged and the damaged databases are composed of 210 times series each (42 sequences).

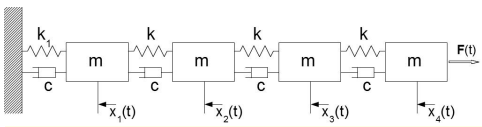


Figure 3. CS1 : Numerical model

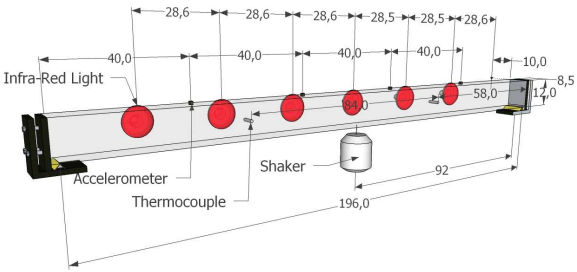


Figure 4. CS2 : Concrete beam dimensions [cm]

RESULTS AND DISCUSSION

The reconstruction procedure suggests that the optimal delays are 6 and 3 for respectively CS1 and CS2, and reconstruction dimension of 7 for both. This leads to JFV formed by 49 components.

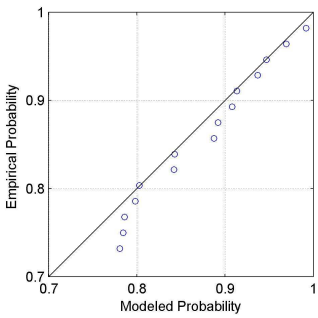


Figure 5. P-P plot for accelerometer 2

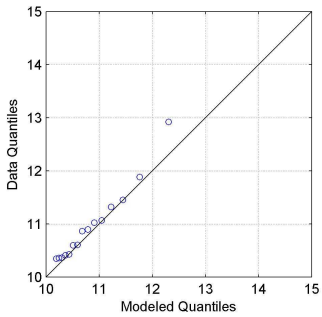


Figure 6. Q-Q plot for accelerometer 2

To determine the control limit, the probability of type I error is set to 0.01 and between 15 exceedances are used to calibrate the statistical models. The quality of the model can be assessed by the quantile and probability plot (Figure 5 and Figure 6). As most of the points lay on the central line for accelerometer #2 (CS2), the fit is acceptable. Similar results are obtained for other models.

The Figure 7 presents the Mahalanobis distance as function of the percentage of stiffness decrease. The filled markers indicates that the null hypothesis is rejected (three consecutive points exceed the control limit). Only one false alarm is detected. From 30% of stiffness reduction, all measurements are correctly classified (except one point). Furthermore, the MD presents a monotonic trend as damage increase; this could lead to a quantification of the degree of degradation. This simple case, with very clean data, proves that the proposed DSF offers some promising abilities.

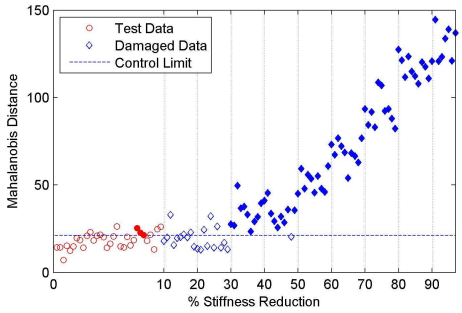
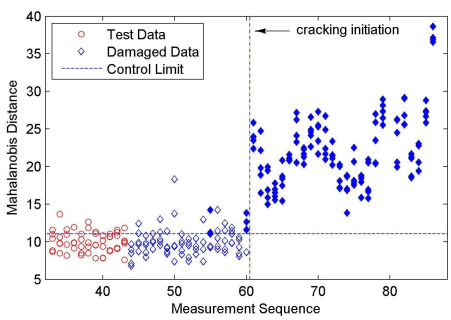


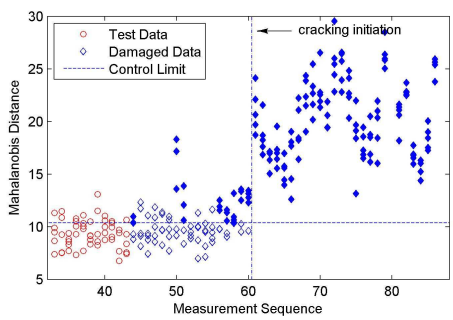
Figure 7. CS1 : Mahalanobis distance as function of the percentage of stiffness reduction

The task is more complicated in more realistic conditions. Figure 8 shows the control charts for each accelerometer. There are almost no differences between sensors, it is not possible to locate the damage. This means that it affects the global behavior of the beam significantly.

All the undamaged points in the test base are correctly classified since no false alarms are detected. On the other hand, between measurement sequences 44 and 60 only few points are classified as damaged (1 for accelerometer #1 to 7 for accelerometer #2). This indicates that the damage induced by the first cycles of loading does not affect significantly the dynamic of the beam. It is likely that only micro-cracking is developing during the early loadings. However, the few true detections alert that a slight change occurs in the beam. Beyond the 60th sequence, all points are detected as damaged and the change is strong between 60 and 61. It is related to the apparition of the first macro-crack which modifies considerably the behavior of the beam.

Once the macro-cracking appears, the MD presents variations which are not related to the severity of damage but more likely to the temperature fluctuations. Since it is not present before the sequence 60, the cracking somehow amplifies the sensitivity to environmental variations. In spite of this increased variability, the damage detection algorithm keeps correctly classifying the measurements.





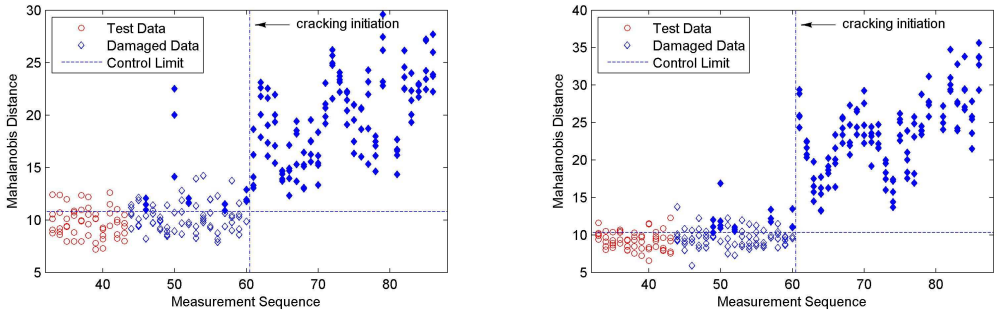


Figure 8. From top right, to bottom left : Accelerometer #1 to #4 ; Mahalanobis distance for each sequence of measurement

CONCLUSION

A damage detection strategy is proposed. It is formed by a new damage sensitive feature based on state-space embedding of time series. More precisely, the damage sensitive feature is formed by the evaluation of the Jacobian matrix of the evolution of trajectories in the state space. Then, the Mahalanobis distance compares a new vector to a reference database, converting the multidimensional vector information into a unique scalar value. The classification is finally carried out through control charts, in which the control limit is set using extreme value statistics modeling.

The global approach is tested successfully on a simple 4 dof mass/spring numerical model. It is able to detect damage from 30% of stiffness reduction, with no false alarms. When tested on a concrete beam subjected to loading cycles, as well as temperature variations, only few true detections are observed during the micro-cracking. As soon as macro-cracking occurs, all points are classified as damaged in spite of a greater dispersion due to temperature fluctuations.

To enhance the method, further investigations on the statistical modeling are needed. Indeed, it appears that in spite of the small type I error probability selected, some false alarms can occur depending on the repartition of measurements in the three bases (train, classification and test). It is important to work on the selection of these bases and on their minimum size required to achieve good statistical modeling.

The robustness of the algorithm has to be confirmed on different case studies and compared to other damage sensitive features like time series models (AR) or modal analysis.

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