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Valuing satellite systems to support fishing in a dynamic competitive model*

Fabrice Collard[†] Carole Haritchabalet[‡]

Abstract

This paper investigates the technology adoption decision of a new satellite system aimed at locating tuna shoals. We propose a dynamic imperfect competition model with vertical differentiation in which each firm acts as a Cournot oligopolist and takes the evolution of the natural resource into account. In this dynamic setting, the model cannot be solved analytically and we rely on a numerical approach. Results are derived for the northern bluefin tuna. We find that high quality firms value more the technology than low quality firms. A direct implication of this result is that the market value of the new technology can be maximized while serving only the highest quality firms. We then evaluate how individual quotas can be used to increase the value that firms attach to the technology.

JEL Class.: L13, Q22, Q55

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1 Introduction

Production in the fishery industry —more particularly in the tuna industry— is largely contingent on the migration of fish populations. Therefore, one of the prominent technological aspect of the fishing activity is the time spent to locate favorable fishing grounds. Any technology allowing for a sizeable acceleration of this localization process is usually viewed as a means to reduce operating costs. Several companies¹ have now launched new satellite systems that facilitate the detection of tuna shoals in fishing grounds and offer this new service to fishing firms. Such systems have enabled tuna purse seine and long line vessels worldwide to find good fishing locations faster, reducing fuel costs in the process. Adopting such a technology is thus a strategic decision for any firm acting in the fishing sector. This adoption process affects (i) market competition and (ii) the evolution of the stock of tuna, which in turn modifies the technological adoption decision. Our objective in this paper is to analyze the incentives of adopting these satellite systems in a setting that explicitly takes into account the dynamic aspects of technology adoption that the evolution of the natural resource implies.

The tuna industry is a vertically differentiated industry where the quality of output is determined by the type of boat used to fish tunas. More specifically, the two main types of boats used in this industry are long lines and purse seine, long line firms producing the best quality of tuna. The purse seine is a huge net that is cast into the sea, gathering fishes in its sweep. The fish is beaten with a stick so that the fish is stressed and damaged. The long line consists of cable to which smaller independent cables are attached at intervals of several meters. There is not much stress involved. Holding the stock of tuna constant —*i.e.* leaving aside the dynamic aspects— and given these characteristics, the technological adoption problem is akin to that analyzed in Bester and Petrakis (1994). Therefore, the model we consider heavily builds on their analysis. It however departs from their setting as we acknowledge the effects of technological adoption on the evolution of the natural resource. This makes the problem explicitly dynamic. More precisely, we focus on the relationship between the adoption of the new technology, the evolution of the stock and market competition.

We develop an imperfect competition model in which vertically differentiated firms compete *à la* Cournot. Following Bester and Petrakis (1994), firms are assumed to play a two-stage game. In the first stage, each fishing firm decides whether or not to adopt the satellite system technology. Adopting the technology yields a decrease in the marginal cost but triggers an adoption cost. In the second stage, market competition takes place. Production plans decided in the second stage of the game obviously affect the size of the fish population. Firms acknowledge this effect

¹For example Collecte Localisation Satellites (<http://www.cls.fr/>) and Orbimage (<http://www.orbimage.com/>) have both developed satellite system tracking tuna shoals over the world.

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7 and therefore take it into account when deciding their production plans. But, more importantly,
8 since the technology adoption decisions modifies firms' cost and thus competition, the first stage
9 game also affects the future size of the fish population.
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11 Our analysis is first concerned with the impact of cost reducing on firms' competition and focuses
12 on pure profit motives for the adoption of the technology leaving aside its implications for the
13 dynamics of fish population. In such a static framework, the model corresponds to the case
14 studied in Bester and Petrakis (1994) as far as incentives of technology adoptions are concerned.
15 In particular, marginal returns to the technology adoption are increasing and a market analysis
16 has to be undertaken to understand the value that each firm attaches to this cost reducing
17 technology. We then derive clear cut decisions regarding technology adoption for a given stock
18 of fish as a function of the technology price. Our results suggest that differences in firms' initial
19 costs or in firms' product quality are key determinants of incentives to adopt the technology.
20 This implies that each category of firms attaches more value to the new technology if the other
21 type of fishing firms does not possess it. We show that the demand for the new technology
22 is perfectly elastic piecewise and may be partially decreasing for some value of the technology
23 price.
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32 We then extend the analysis to a dynamic setting in order to take the impact of the new
33 technology on the level of the stock of fish into account. Adopting the cost reducing technology
34 affects market competition and the harvesting activity. This in turn modifies the evolution of
35 the renewable natural resource and therefore the future path of profits. In other words, adoption
36 technology affects the level of intertemporal profits, which obviously feeds back on technology
37 decisions. It is therefore critical to explicitly model dynamics in such a setting. The modeling
38 of the evolution of the renewable resource builds on the pioneering work of Clark (1971), Clark
39 and Munro (1975), and Levhari and Mirman (1980). The evolution of resource depends on its
40 natural, or biological, growth function and on the harvesting activity. The stock of fish evolves
41 over time and firms take the impact of their decisions on the future stock of fish into account.
42 In such a dynamic setting, the model cannot be solved analytically. We therefore rely on a
43 numerical approach. Results are derived for the northern bluefin tuna and the two categories of
44 firms that we consider (long line and purse seine firms). We derive the demand function for the
45 satellite systems from the market harvest equilibrium. In our vertically differentiated model,
46 we obtain that high quality firms (long line) produce less whereas low quality firms produce
47 more than the socially optimal production level. Therefore, high quality firms should be the
48 first category of firms encouraged in the adoption of the technology. We show that it may be
49 more profitable for the technology provider to serve only high quality firm than all the market.
50 Individual quotas can be used to increase the value that both firms attach to the technology
51 and to modify the technology provider decision to serve only high quality firms.
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Section 2 presents a static model of technology adoption in which two types of firms compete on the market but do not internalize the future effect of their decisions on the stock of fish. Section 3 extends the preceding analysis to a dynamic setting where the stock of tuna evolves over time according to biological constraints and fishing activities. Section 4 discusses the calibration of the model. Section 5 presents the numerical results for the dynamic version of the model. In particular it discusses cost reducing technology adoption both in equilibrium and at the social optimum. We then introduce quotas in the model to shed light on their role in the regulation of the market.

2 A static model

We consider an economy with two types of fishing firms that exploit a single constant stock of fish, x . The fishing industry consists of $n_1 \geq 1$ (resp. $n_2 \geq 1$) fishing firms of type 1 (resp. type 2). We consider a vertically differentiated industry where the quality of fish is specific to the type of firm and yields a specific market value. Type 1 firms (type 2 firms) are assumed to produce fish with quality u_1 (u_2), with $u_1 \geq u_2$. Hereafter, we will consider the quality levels as fixed. Given these assumptions, the system of inverse demand functions is given by²

$$p_1 = \bar{v}u_1 - u_1Q_1 - u_2Q_2 \quad (1)$$

$$p_2 = \bar{v}u_2 - u_2Q_1 - u_2Q_2 \quad (2)$$

where p_i is the price paid to firm i when the aggregate production of type i firms is Q_i and the aggregate production of type j firms is Q_j .

Let q_{ij} be the quantity harvested by firm j with type i . The aggregate harvest Q is thus given by

$$Q = \sum_{j=1}^{n_1} q_{1j} + \sum_{j=1}^{n_2} q_{2j} = Q_1 + Q_2.$$

The cost function of type i firm for harvesting a quantity q when the level of the stock of fish, x , is present is given by³

$$c_i q x^{\alpha_i}.$$

where $\alpha_i < 0$ is the catch stock elasticity. This latter effect aims at capturing the fact that it is easier and less expensive to catch fish when the population of fish is large. The parameter c_i in the cost function represents the harvest ability of each type of firm.

²See Motta (1993) for the detailed determination of the inverse demand functions.

³This cost function is derived from a Cobb Douglas production function with fishing effort and fish biomass as production inputs of the form $q = \theta_i e x^{-\alpha_i}$ where e is the effort of the firm. θ_i is productivity term that measures the catchability per unit of effort.

Given these specifications, the instantaneous profit function of a type i firm is written as

$$\pi_i(q, x) = p_i(Q)q - c_iqx^{\alpha_i}$$

We assume that before the market opens, each firm can invest, T , in a satellite system to support fishing by enhancing the ability of firm i to detect shoals. When implemented, the technology yields a cost reduction, which takes the form of a cut in the parameter c_i , which is then reduced by δ_i where $0 < \delta_i < c_i$. The investment decision of the firm is a discrete decision — to invest or not — denoted by μ_i where $\mu_i = \{0, \delta_i\}$. The value of the firm is then defined as the difference between its instantaneous variable profit and the cost of the technology, T , when implemented

$$V_i(\mu_i, x) = \pi_i(q, x) - T \mathbb{I}_{[\mu_i=\delta_i]} \quad (3)$$

where $\mathbb{I}_{[\mu=\delta]} = 1$ if $\mu = \delta$, 0 otherwise.

The behavior of type i firm is then determined by solving a two-stage game. In the first stage of the game, each firm decides whether or not to invest in the cost reducing technology. In the second stage, all firms compete in the level of harvest. We look for a symmetric Nash equilibrium in which each firm with the same type has the same technology adoption strategy and produces the same level of harvest. The game is solved by backward induction. We first solve the competition between firms as a function of the firms' decisions to invest in the cost reducing technology. We then analyze the decision to invest in the cost reducing technology as a function of the technology cost.

We first present the Cournot equilibrium in which each firm decides on its harvesting level so as to maximize its profit given the quantity harvested by its competitors. Therefore, the harvesting decision of firm j with type i maximizes profits subject to the demand it faces. The first order condition for each type is given by

$$\frac{\partial \pi_{1j}}{\partial q_{1j}} = \bar{v}u_1 - u_1q_{1j} - u_1 \sum_{j=1}^{n_1} q_{1j} - u_2 \sum_{j=1}^{n_2} q_{2j} - (c_1 - \mu_1)x^{\alpha_1} = 0 \quad (4)$$

$$\frac{\partial \pi_{2j}}{\partial q_{2j}} = \bar{v}u_2 - u_2q_{2j} - u_2 \sum_{j=1}^{n_2} q_{2j} - u_1 \sum_{j=1}^{n_1} q_{1j} - (c_2 - \mu_2)x^{\alpha_2} = 0 \quad (5)$$

In a symmetric equilibrium, we have $q_{1i} = q_{1j} = q_1^*$ and $q_{2i} = q_{2j} = q_2^*$ for all i, j . Then, we have $Q_1 = \sum_{j=1}^{n_1} q_{1j} = n_1q_1^*$ and $Q_2 = \sum_{j=1}^{n_2} q_{2j} = n_2q_2^*$. Since technology adoption is dictated by the firms' payoffs in the first stage of the game, we now characterize the profit attained by each firm in the symmetric equilibrium. It should be clear to the reader that firms' payoffs depend upon both firms' costs and thus on the first period decision to adopt or not the cost reduction

technology. Let us denote by $\pi_i(\mu_1, \mu_2)$ the profit attained by a type $i = 1, 2$ firm in a symmetric equilibrium given the first period decisions of each type of firm μ_1 and μ_2 . We have⁴

$$\pi_1^*(\mu_1, \mu_2) = \frac{u_1(\bar{v}((n_2 + 1)u_1 - n_2u_2) - (n_2 + 1)(c_1 - \mu_1)x^{\alpha_1} + n_2(c_2 - \mu_2)x^{\alpha_2})^2}{(u_1(n_1 + 1)(n_2 + 1) - u_2n_1n_2)^2} \quad (6)$$

$$\pi_2^*(\mu_1, \mu_2) = \frac{u_2(\bar{v}u_1 - (n_1 + 1)(u_1/u_2)(c_2 - \mu_2)x^{\alpha_2} + n_1(c_1 - \mu_1)x^{\alpha_1})^2}{(u_1(n_1 + 1)(n_2 + 1) - u_2n_1n_2)^2} \quad (7)$$

It can be readily checked, using the envelope theorem, that an increase in c_i (resp. c_j) decreases (resp. increases) type i firms' profits.

We now consider the first period stage and look for a symmetric Nash equilibrium in which each firm with the same type has the same technology adoption strategy.

Strategies and payoffs may be summarized by the following gain matrix where the strategy of a firm of type 1 is indicated in each row, while firms of type 2 refer to each column. A_i (resp. \bar{A}_i) indicates that type i firm adopts (resp. does not adopt) the new technology.

	A_2	\bar{A}_2
A_1	$(\pi_1^*(\delta_1, \delta_2) - T, \pi_2^*(\delta_1, \delta_2) - T)$	$(\pi_1^*(\delta_1, 0) - T, \pi_2^*(\delta_1, 0))$
\bar{A}_1	$(\pi_1^*(0, \delta_2), \pi_2^*(0, \delta_2) - T)$	$(\pi_1^*(0, 0), \pi_2^*(0, 0))$

The Nash equilibrium of the game then determines the choice of the two types of firm.

Let $\Delta_i(\mu_j) = \pi_i^*(\delta_1, \mu_j) - \pi_i^*(0, \mu_j)$. If for a given type of firm $\Delta_i(\delta_j)$ and $\Delta_i(0)$ are greater (resp. lower) than the technology cost T , then adopting (resp. not adopting) the new technology is a dominant strategy for the firm. Else, the choice of the firm is driven by the behavior of the other firm.

We have

$$\Delta_i(\mu_j) = \frac{u_i(n_j + 1)\delta_i(q_i^*(\delta_i, \mu_j) + q_i^*(0, \mu_j))}{(u_1(n_1 + 1)(n_2 + 1) - u_2n_1n_2)} \quad (8)$$

and

$$\Delta_i(\mu_j) - \Delta_i(0) = -\frac{u_1(n_1 + 1)(n_2 + 1)\delta_1\delta_2}{(u_1(n_1 + 1)(n_2 + 1) - u_2n_1n_2)^2} \quad (9)$$

As in Bester and Petrakis (1994), the marginal return on investment in the cost reducing technology is increasing:

- i) Δ_i is a decreasing function of c_i .
- ii) Δ_i an increasing function of c_j .

⁴In the sequel, we denote $\pi_i^*(\mu_1 = \delta_1, \mu_2 = \delta_2)$ by $\pi_i^*(\delta_1, \delta_2)$.

It is therefore more profitable for a firm to invest in the cost reducing technology when its production cost is low. Part *ii*) indicates that a firm has more incentives to adopt the new technology when the other firm has a high production cost — *i.e.* when the other firm is not equipped with the new technology ($\Delta_i(\delta_j) < \Delta_i(0)$). In other words, the technology is less profitable when the other firm also reduces its cost. These cost reducing strategies are strategic substitutes.

The equilibrium of the game then depends on the relation between $\Delta_i(\mu_j)$ and $\Delta_j(\mu_i)$ ($\mu_i = \{0, \delta_i\}$) and is specific to industry parameters. Assume that $\Delta_2(\delta_1) < \Delta_1(\delta_2)$. We have from (9) that $\Delta_1(\delta_2) - \Delta_1(0) = \Delta_2(\delta_1) - \Delta_2(0)$, so there exist two possible configurations: either $\Delta_2(\delta_1) < \Delta_2(0) < \Delta_1(\delta_2) < \Delta_1(0)$ or $\Delta_2(\delta_1) < \Delta_1(\delta_2) < \Delta_2(0) < \Delta_1(0)$.

The next proposition summarizes all possible cases and indicates, for each value of the technology price T , the equilibrium adoption decision of each firm.

Proposition 1 1. Assume that $\Delta_2(\delta_1) < \Delta_2(0) < \Delta_1(\delta_2) < \Delta_1(0)$. The possible equilibria of the game are given by

(a) (A_1, A_2) when $T \leq \Delta_2(\delta_1)$.

(b) $(A_1, \cancel{A_2})$ when $\Delta_2(\delta_1) < T < \Delta_1(0)$.

(c) $(\cancel{A_1}, \cancel{A_2})$ when $\Delta_1(0) < T$.

2. Assume that $\Delta_2(\delta_1) < \Delta_1(\delta_2) < \Delta_2(0) < \Delta_1(0)$. The possible equilibria of the game are given by

(a) (A_1, A_2) when $T \leq \Delta_2(\delta_1)$.

(b) $(A_1, \cancel{A_2})$ when $\Delta_1(\delta_2) < T < \Delta_2(0)$.

(c) When $\Delta_1(\delta_2) < T \leq \Delta_2(0)$, there are two equilibria in pure strategies $(\cancel{A_1}, A_2)$ and $(A_1, \cancel{A_2})$, and one equilibrium in mixed strategies. Type i firm probability to adopt the new technology is $\rho_i = \frac{\Delta_j(0) - T}{\Delta_j(0) - \Delta_j(\delta_i)}$.

(d) $(\cancel{A_1}, \cancel{A_2})$ when $\Delta_1(0) < T$.

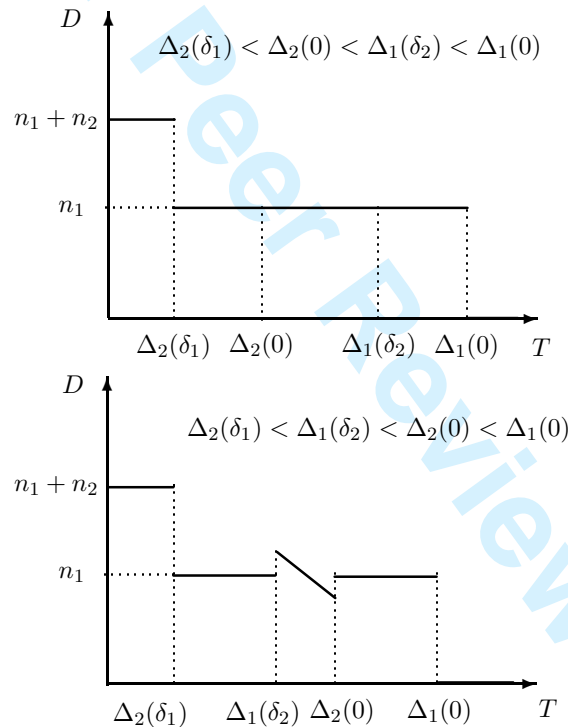
In case *a*, adopting the new technology is a dominant strategy for both firms and we observe a joint adoption. In this case, type i firms are willing to pay $\Delta_i(\delta_j)$ which represents the increases in firm i profit after the adoption given that firm j is also equipped. Conversely, in case 1.*c* or 2.*d*, not adopting the new technology is a dominant strategy for both types of firm.

In case 1.*b* and 2.*b*, adopting the new technology is a dominant strategy for type 1 firms and type 2 firms value positively the technology only if they invest alone. We thus obtain a unique adoption from type 1 firms.

In case 2.c, each type of firms value positively the technology only if they invest alone in the new technology. As a consequence, and because marginal returns in the technology increases with the competitor costs, there is a coordination problem between the two types of firms and the game admits two pure strategy equilibria. In the mixed equilibrium, the mean number of technology adoption is $n_2 \frac{\Delta_1(0)-T}{\Delta_1(0)-\Delta_1(\delta_2)} + n_1 \frac{\Delta_2(0)-T}{\Delta_2(0)-\Delta_2(\delta_1)}$. Observe that it is linearly decreasing with T , that it exceeds n_1 when $T = \Delta_1(\delta_2)$ and is strictly lower than n_1 when $T = \Delta_2(0)$.

Given these firms adoption decision choices, we obtain that the demand function for the new technology is perfectly elastic piecewise and may partially decrease over the interval $\Delta_1(\delta_2) < T \leq \Delta_2(0)$ when the game admits a coordination problem. This demand function is depicted in Figure 1.

Figure 1: Demand function for the technology



An attractive feature of this model is that it delivers clear cut decisions regarding technology adoption for a given stock of fish and thus allows to derive the demand function for the new technology. However, the model implies myopic decision from the firms as it does not take into account the impact of the new technology on the level of the stock. This is clearly a shortcoming of this approach and it calls for an explicitly dynamic model that let firms take the impact of the new technology on the reproduction rate of the stock of fish. The next section deals with this problem and proposes a dynamic model in which the stock of tuna evolves over time and depends on harvesting behavior.

3 A Dynamic Fishing Model

This section develops a model in the lines of the model of the previous section. We consider a two-stage game where, in the first stage, each firm decides whether or not to invest in the cost reducing technology and, in the second stage, all firms engage in a dynamic competition in the level of harvest. We however depart from the previous modeling in that we allow the stock of fish to evolve over time and let agents take the impact of their decisions on the future stock of fish into account. The model is then explicitly dynamic.

3.1 The Evolution of the Stock of Fish

We now consider that the stock of fish, x , is a dynamic variable. The resource depends on its natural, or biological, growth function and on the harvesting activity. More precisely, denoting by x_t the stock of fish at date t , the stock in period $t + 1$ is the difference between the biological evolution of the stock and the harvest

$$x_{t+1} = F(x_t) - Q_t$$

Q_t denotes the aggregate production of the fishery industry in the economy. It therefore accounts for the harvesting activity of firms and exerts a negative effect on the stock of fish. $F(x_t)$ corresponds to the natural evolution of the stock of fish, and hence accounts for the biological component. This latter component has been extensively studied in the biological literature (See e.g. Clark and Conrad (1987)). In the sequel, $F(\cdot)$ is borrowed from this literature and takes the form

$$F(x_t) = rx_t \left(1 - \frac{x_t}{k}\right) + x_t$$

where r is the intrinsic growth rate of the resource stock and k denotes the environmental carrying capacity. Note that the knowledge of the current period stock, x_t and the production level suffice to fully characterize the next period stock of tuna. x_t is then a predetermined variable. Hence, the dynamics of the stock of fish are described by the following law of motion

$$x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - Q_t \quad (10)$$

3.2 The Nash–Cournot Equilibrium

We first solve the dynamic competition between firms to compute the present value of each firm profit. We then perform the same exercise as in the static problem: we consider the choice of each firm to adopt the cost reducing technology through a matrix that indicates the value of firms' profits according to the strategy of each type of firm.

In a dynamic setting the firm takes into account the impact of its action on its current profit but also on the discounted flow of profits it will get in the future. Therefore, the objective function of a type i firm is given by

$$\Pi(\mu_i) = \sum_{t=1}^{\infty} \beta^{t-1} (p_i(Q_t)q_{it} - (c_i - \mu_i)q_{it}x_t^{\alpha_i}) \quad (11)$$

where $\beta \in (0, 1)$ is the discount factor of the firm. The optimal production plan of the firm — the sequence $\{q_{it}\}_{t=1}^{\infty}$ of harvest levels — maximizes (11) subject to the demand functions it faces, the biological constraint, positivity constraints on harvests and the stock of fish, and taken the harvest of type j firms as given. Therefore, the program of the firm writes

$$\max_{q_{ijt}} \Pi(\mu_i) = \sum_{t=1}^{\infty} \beta^{t-1} (p_i(Q_t)q_{ijt} - (c_i - \mu_i)q_{ijt}x_t^{\alpha_i}) \quad (12)$$

$$s.t. \quad p_1(Q_t) = \bar{v}u_1 - u_1 \sum_{j=1}^{n_1} q_{1jt} - u_2 \sum_{j=1}^{n_2} q_{2jt} \quad (13)$$

$$p_2(Q_t) = \bar{v}u_2 - u_2 \sum_{j=1}^{n_1} q_{1jt} - u_2 \sum_{j=1}^{n_2} q_{2jt} \quad (14)$$

$$q_{ijt} \geq 0 \quad (15)$$

$$q_{ijt} \leq k_i \quad (16)$$

$$x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - \left(\sum_{j=1}^{n_1} q_{1jt} + \sum_{j=1}^{n_2} q_{2jt} \right) \quad (17)$$

$$x_{t+1} \geq 0 \quad (18)$$

We thus consider that fishing firms are rational and take the appropriate notice of all variables and relationships affecting their profit functions including the resource growth constraint and each other's fishing activity.⁵ Note that the behavior of competitors is constrained by the fact that they have to keep next period stock of fish positive. The price at time t , $p_i(Q_t)$, is determined by total harvest Q_t . Though each firm will manipulate the price schedule via the quantities it send to the market, each firm takes the price vector as given. These prices may be thought of as determined in future markets under conditions of perfect foresight and are given by the inverse demand function as determined in the static model. Also note that, through equation (16), we include the possibility that the industry is regulated by means of individual catch quotas. As a consequence, firms may be constrained in their harvest level.

The set of all first order conditions for this problem is reported in Appendix B. Although we will not discuss all of them in details, some — evaluated in a symmetric equilibrium — call for

⁵In the fisheries economic literature, there is some ambiguity about the firms' perception of the biomass constraint. Some studies retain the assumption that firms compete in the level of harvest and ignore the biomass growth constraint. Firms thus behave myopically. See for instance Okugushi (2003), Sandal and Steinshamn (2004) or Szidarovsky, Ilieva and Okuguchi (2002).

some comments

$$\lambda_{1t} = vu_1 - u_1(1 + n_1)q_{1t} - n_2u_2q_{2t} - (c_1 - \mu_1)x_t^{\alpha_1} \quad (19)$$

$$\lambda_{2t} = vu_2 - u_2n_1q_{1t} - (1 + n_2)u_2q_{2t} - (c_2 - \mu_2)x_t^{\alpha_2} \quad (20)$$

$$\lambda_{1t} = \beta \left[\lambda_{1t+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \alpha_1q_{1t+1}(c_i - \mu_i)x_{t+1}^{\alpha_1-1} \right] \quad (21)$$

$$\lambda_{2t} = \beta \left[\lambda_{2t+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \alpha_2q_{2t+1}(c_i - \mu_i)x_{t+1}^{\alpha_2-1} \right] \quad (22)$$

for all i and t for which $q_{it} > 0$, $x_{t+1} > 0$ and ignoring quotas. Parameter λ_{it} is type i firms marginal valuation of an additional unit of biomass. The first two equations therefore indicate that the firm will fish tuna up to the point where the marginal net profit gain from extracting one additional tuna from the stock equates its marginal valuation. The marginal net profit gain is the sum of three components: (i) a direct effect stemming from the fact that the firm sells an additional unit, (ii) an indirect competition effect pertaining to the manipulation of the price scheme by the oligopolist and (iii) a negative marginal cost effect. The two last relations actually rule the dynamics of the marginal valuation of the stock of fish and reflect the forward looking behavior of firms. This actually determines the dynamic fishing decision. The firm can fish an additional tuna in period t , in which case it can obtain λ_{it} extra units of net profits. But it may also wait an additional period and let tuna reproduce, in which case it will be able to fish $(1 + r(1 - \frac{2}{k}x_{t+1}))$ extra tunas in the next period which will both raise its net profits by $\lambda_{i,t+1}$. Beside this direct effect, by letting the fish stock renew, postponing fishing triggers a reduction in the cost that makes it more beneficial to wait an extra period. The non-arbitrage principle implies that the intertemporal allocation of the fishing activity is such that the current and the next period marginal valuation of the stock of fish equate.

Just alike in the static model, the quantity harvested by each firm is a function of the current value of the stock and the technology decision of each type of firms. Strategies and payoffs of each type of firms may be then summarized, as in the previous section, by a matrix indicating firms' values in the four possible cases : (A_i, A_j) , (A_i, \bar{A}_i) , (\bar{A}_i, A_i) and (\bar{A}_i, \bar{A}_j) .

3.3 The social optimum

For comparative purposes it is interesting to investigate the dynamic properties of the social optimum. We thus define the instantaneous consumer surplus by

$$S(Q) = \int_{v_{\emptyset 2t}}^{v_{12t}} (vu_2 - p_{2t})dv + \int_{v_{12t}}^{\bar{v}_t} (vu_1 - p_{1t})dv$$

with $v_{12t} - v_{\emptyset 2t} = n_2q_{2t}$ and $\bar{v}_t - v_{12t} = n_1q_{1t}$.

Social welfare at date t is then given by

$$W(Q_t) = S(Q_t) - C(Q_t) \quad (23)$$

where $C(Q_t)$ is the total production cost.

The problem of the social planner is (i) to find the sequence of harvest that maximizes the present value of social surplus subject to the biological constraints and (ii) to decide whether firms should be equipped with the new technology or not given the production cost of the technology.⁶ More formally, denoting $\Gamma(n)$ the total cost to equip a number of n firms with the technology,⁷ the problem of the social planner is

$$\begin{aligned} \max_{q_{1t}, q_{2t}, \mu_1, \mu_2} \quad & \sum_{t=1}^{\infty} \beta^{t-1} \left(S(n_1 q_{1t} + n_2 q_{2t}) - n_1 (c_1 - \mu_1) q_1 x^{\alpha_1} + n_2 (c_2 - \mu_2) q_2 x^{\alpha_2} \right. \\ & \left. - \Gamma(n_1 \mathbb{I}_{[\mu_1 = \delta_1]} + n_2 \mathbb{I}_{[\mu_2 = \delta_2]}) \right) \\ \text{s.t.} \quad & x_{t+1} = r x_t \left(1 - \frac{x_t}{k}\right) + x_t - Q_t \\ & x_{t+1} \geq 0 \end{aligned}$$

The problem can be decomposed in two steps and solved by backward induction. First, we determine socially optimal harvest levels as a function of productions costs. Second, given the value of social welfare associated to each costs levels, we determine the optimal equipment of each type of firms. First order conditions that determine optimal harvest levels are reported in Appendix C. Although we will not discuss all of them in details, some — evaluated in a symmetric equilibrium — are worth discussing:

$$\begin{aligned} \lambda_t &= v u_1 - u_1 n_1 q_{1t} - n_2 u_2 q_{2t} - n_1 (c_1 - \mu_1) x_t^{\alpha_1} \\ \lambda_t &= v u_2 - u_2 n_1 q_{1t} - n_2 u_2 q_{2t} - n_2 (c_2 - \mu_2) x_t^{\alpha_2} \\ \lambda_t &= \beta \left[\lambda_{t+1} \left(1 + r \left(1 - \frac{2}{k} x_{t+1}\right)\right) - \sum_{i=1}^2 \left(\alpha_i q_{it+1} n_i (c_i - \mu_i) x_{t+1}^{\alpha_i - 1}\right) \right] \end{aligned}$$

for all i and t for which $q_{it} > 0$ and $x_{t+1} > 0$. Parameter λ_t represents the social current shadow value of an additional unit of stock along the optimal path. Observe that the first order conditions pertaining to the harvesting policy departs from the corresponding decisions in an equilibrium in at least two aspects. First, no matter the type of the firm we consider, firms have the same marginal valuation of an extra unit of fish, which dynamics is given by the last equation. Second, firms are not allowed to manipulate the price schedule, therefore eliminating the distortion stemming from imperfect competition. This therefore fundamentally alters the harvesting policy of firms. Finally, the central planner chooses an intertemporal allocation of production that minimizes the distortions stemming from coordination failures on the stock of fish. This is reflected by (i) the fact that the marginal valuation of the stock is equated across firms ($\lambda_{1t} = \lambda_{2t} = \lambda_t$) and (ii) the fact that the central planner acknowledges the total impact

⁶Note that this cost differs from the price paid by firms to obtain the technology.

⁷ Γ is assumed to be increasing with n

of postponing fishing on the total production cost (last part of the right hand side of the Euler equation). This can be simply illustrated looking at the steady state of the model. Indeed, for a given stock of fish \bar{x} and two production levels $(q_1, q_2) = (\bar{q}_1, \bar{q}_2)$, it can be easily checked that the social shadow value of an additional unit of biomass, λ , is a linear combination of the private shadow values, λ_1 and λ_2

$$\lambda = n_1\lambda_1 + n_2\lambda_2 \quad (24)$$

Since $n_i \geq 1$, $i = 1, 2$, *Ceteris Paribus*, the steady state social marginal valuation of the stock of fish is greater than the private marginal valuation of the stock.

Technology adoption decisions are then taken by evaluating and comparing the value of the social optimum in the four possible cases we described in the previous sections.

It should be noted that the model is fundamentally non-linear and forward looking. It cannot be solved analytically. We therefore rely on a numerical evaluation of the model which requires to assign values to the structural parameters of the model.⁸

4 Parametrization

The model is parameterized for the Northern Atlantic bluefin tuna fisheries using data collected by Barry and Kirkwood (1997) from the ICCAT for the year 1995. The Northern Atlantic bluefin tuna fishery is rather complex, as it is harvested by a variety of fishing gears. The fishing gears are the long line, purse seine, trap, and baitboat. The two types of fishing firms we consider are long line (type 1 firms) and purse seine (type 2 firms) firms as they are representative of the tuna fishing industry. Indeed, the aggregate production of these two types of firms exceeds 75% of the total catch of the 1995 year. In the following LL refers to long line firms and PS refers to purse seine firms. The values of the structural parameters are reported in Table 1.

The discount factor β is set such that firms discount the future at a 4% annual rate, implying β equals 0.96. This is in accordance with other applied studies, using similar investment horizons, such as the US department of Commerce, National Marine and Fisheries Service (1995). The taste parameter \bar{v} and the quality parameters u_1 and u_2 are not directly observable. We therefore assume that the economy was at its steady state in 1995 and set these parameters such that the model generates the observed catch of the 1995 year.

The number of each type of firms targeting bluefin tuna is relatively small, and we assume that $n_1 = n_2 = 10$. We checked the robustness of our results against alternative numbers of firms and

⁸We use a projection method that is detailed in Appendix C.

Table 1: Calibration

Parameter	Mnemonic	Value	Units
<i>Preferences:</i>			
Discount factor	β	0.96	
Taste parameter	\bar{v}	100000	
High quality parameter	u_1	0.029	
Low quality parameter	u_2	0.0179	
<i>Long line firms:</i>			
price	p_1	17000	USD/ton
number of firms	n_1	10	
marginal cost	c_1	440	USD/ton
cost reducing	δ_1	44	
catch stock elasticity	α_1	-0.2	
<i>Purse Seine firms:</i>			
price	p_2	9000	USD/ton
number of firms	n_2	10	
marginal cost	c_2	233	USD/ton
cost reducing	δ_2	23	
catch stock elasticity	α_2	-0.2	
<i>Resource stock:</i>			
intrinsic growth rate	r	0.25	
environmental carrying capacity	k	1200000	tons

did not find any major change in the quantitative implications of the model to a change in either n_1 or n_2 . The fishing technology, as specified in the model, requires some parameters to be set. Just like preferences, most of the parameters are not all directly observable. We therefore follow the approach we used to set preference parameters and set the costs parameters, c_i , so as to match observed costs and prices in the tuna market for the year 1995. Data pertaining to prices and costs behavior are borrowed from the applied literature that has used a similar economic model (see Pintassilgo and Costa Duarte (2002) or Bjørndal, and Brasão (2004)). In the bluefin tuna fishery, LL and PS use advanced method of detection so that catches do not depend much upon the stock. Therefore, a low catch stock elasticity, α_i , is often used in the literature. We follow this line and assume $\alpha_1 = \alpha_2 = -0.2$. Using this information together with the observed price levels and costs, c_1 and c_2 are set to the values reported in Table 1. Using satellite systems to support fishing allows firms to increase their catch per unit effort around 10 %. This implies that marginal production costs can decrease, δ_i/c_i by 10 %. This led to values for δ_i , $i = 1, 2$, of, respectively, 44 and 23.

The model of population dynamics borrows from Bjørndal et al. (2004) who propose a model of the evolution of the resource. According to their calculation, the total biomass should increase until 2040 and stabilize thereafter at a steady state level of about 1200000 MT. This led us to set $k=1200000$. Then, the intrinsic growth rate, r , is set so as to match the average growth rate over the period they consider.

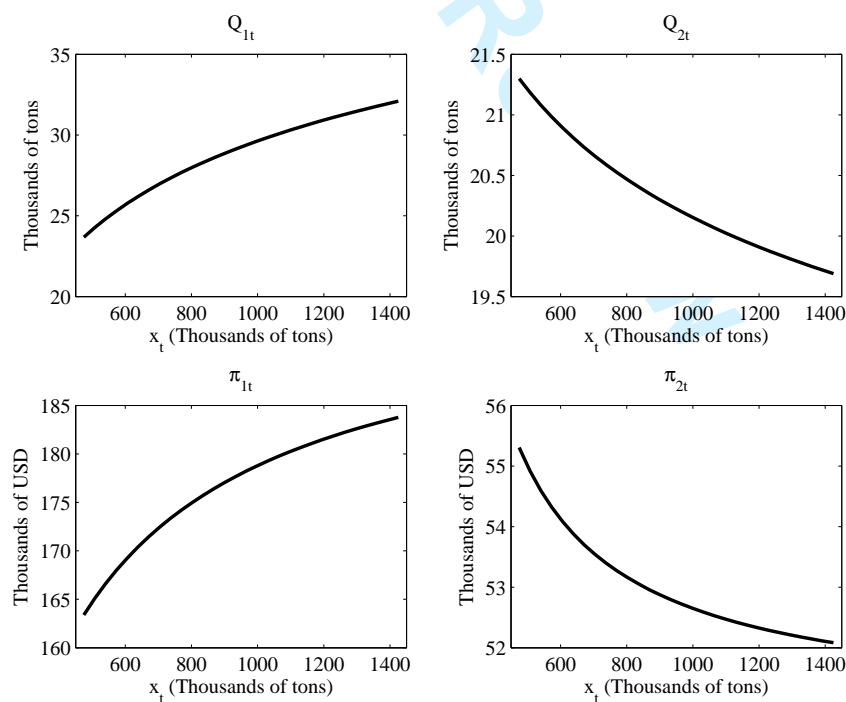
Using this set of values, the model is then solved using a modified minimum weighted residuals method as advocated by Judd (1992) and Christiano and Fisher (2000).⁹

5 The results

5.1 The unregulated economy

We first consider a benchmark situation in which none of the firms has access to the new technology and cannot therefore implement a cut in its costs. Figure 2 presents the harvesting decision rule of each type of firm and the associated present value.¹⁰ More precisely, it reports, for each level of the stock of fish, x_t , the equilibrium level of harvesting for each type of firm, $n_i q_{it}$.

Figure 2: Market equilibrium



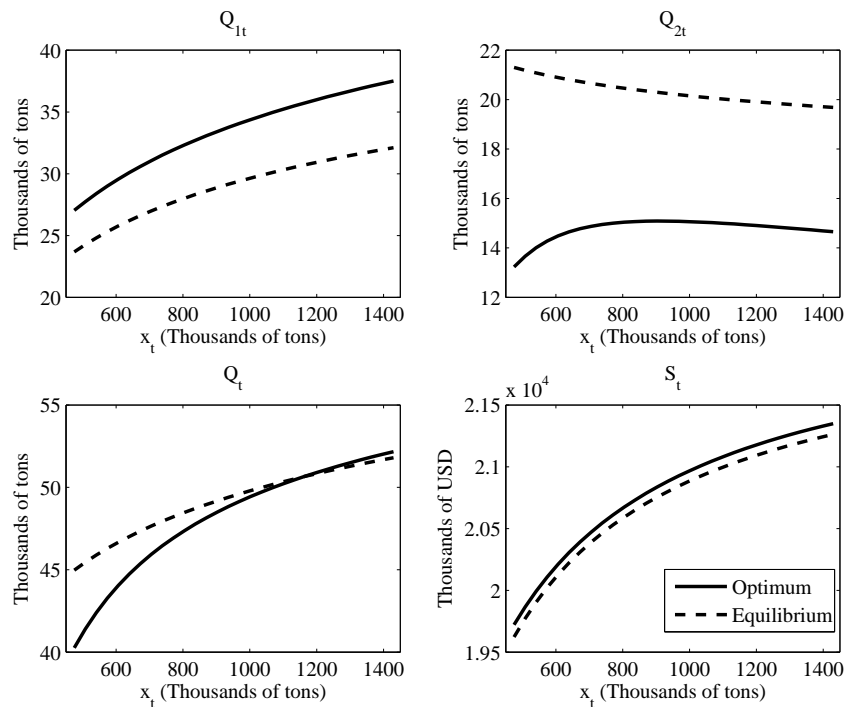
⁹The reader is left to refer to Appendix D for a detailed description of the method.

¹⁰The evolution of equilibrium market prices that supplements this benchmark can be found in Appendix A.

Two main observations can be drawn from Figure 2. First, LL firms perform better than PS firms both in terms of production and in terms of present value. Second, LL firms benefit from an increase in the resource stock while PS firms experience both production and profit losses as the biomass increases. These two observations are actually accounted for by productivity differences. Indeed, as the resource increases, production costs decrease for both type of firm ($\alpha_i < 0$) and therefore affect the degree of competition in the economy. Since LL firms are less productive than PS firms ($c_1 \geq c_2$), LL firms loose part of their oligopolistic power and become more competitive. They therefore have to produce more in order to maintain their profits. On the contrary, *Ceteris Paribus*, PS firms can charge a high price while lowering their harvesting level.

Figure 3 reports the comparison of optimal and equilibrium level of harvesting as well as the implied social surplus. Such a comparison is of great interest as it is well known that as soon

Figure 3: Optimal versus Equilibrium harvest



as several agents simultaneously exploit a productive common-property resource, the resulting externality generally leads to over-exploitation of the resource relative to the social optimum. This is so-called *tragedy of the commons* is important as it calls for a regulation of the equilibrium to bring the economy back to efficiency. In this competition model with quality differentiation, LL firms are found to under-produce whereas PS firms over-produce. The global effect is however ambiguous as over-production obtains for relatively low levels of the biomass and turns into under-production as the stock of fish is high. This latter under-exploitation of the resource

has already been underlined and discussed by Dutta and Sundaram (1993). In this model, this phenomenon arises because firms internalize the effect of their own production level on the price schedule. Indeed, when the resource is relatively abundant, firms limit their harvest so as to avoid the price cut — which would limit the rent they can extract from consumers — that would follow an increase in their production.

Table 2 reports steady state harvests contingent on the technology adoption strategy of firms. In the table, A_{LL} (resp. \bar{A}_{LL}) indicates that type i firm adopts (resp. does not adopt) the new technology. The technology adoption clearly increases aggregate harvest but leaves the

Table 2: Steady state values

	LL harvest	PS harvest	Aggregate harvest	Resource Stock
$(\bar{A}_{LL}, \bar{A}_{PS})$	29250	20220	49470	949979
(A_{LL}, \bar{A}_{PS})	35780	14280	50060	945927
(\bar{A}_{LL}, A_{PS})	26020	25740	51760	933976
(A_{LL}, A_{PS})	32550	19790	52340	929719

Note: All quantities are expressed in tons.

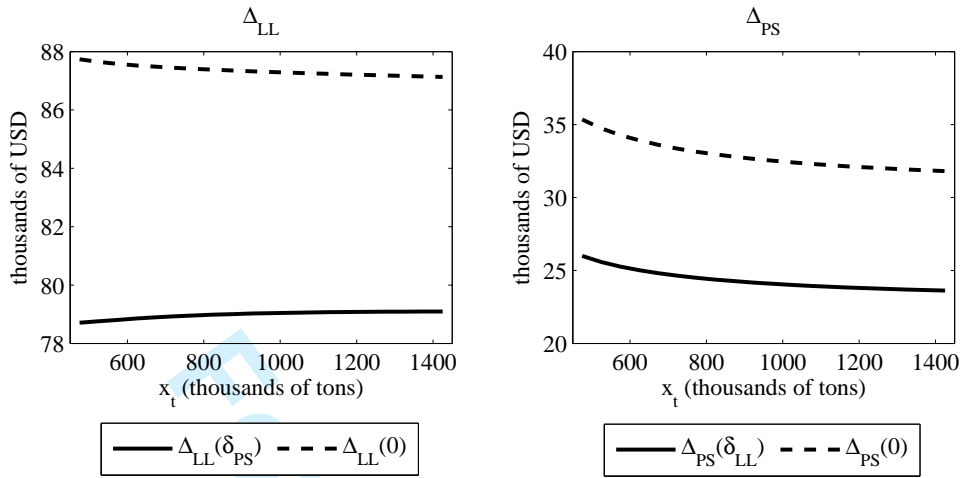
asymptotical behavior of the steady state fish stock essentially unaffected.¹¹ At the firm level, an increase in production depends on market competition. A single adoption makes the firm more competitive and increases its production level. When all firms adopt the new technology, LL firms increase their production but PS firms produce less than without the technology. Such an increase in aggregate harvest ought to lower social surplus. But, the adoption of the new technology has important consequences in terms of efficiency. Indeed, this induces a reallocation of harvests from less efficient toward more efficient firms. The over and under production results described above are therefore softened. This suggests a trade-off in the technology adoption decision, which we now investigate.

5.1.1 Technology adoption in an equilibrium

Figure 4 illustrates the effects of technology adoption on the payoffs of the firms in an equilibrium situation. The figure reports the gains in terms of net present value of adopting the new technology for each type of firm — *i.e.* the firms willingness to pay for the new technology for each possible value of the stock of fish. For instance, $\Delta_{LL}(\delta_{PS})$ (resp. $\Delta_{LL}(0)$) plots the gains in terms of net present value for the long line firm (LL) of adopting the new technology when the Purse Seine has (resp. not) already adopted. As expected from the static analysis, the marginal returns of the cost reducing technology are larger when the other firm has not yet

¹¹Szidarovsky et al. (2002) perform a static comparative analysis of the steady state to a change in the harvest quantity. They find that an increase in the aggregate production either does not affect the asymptotic behavior of the steady state stock of fish or makes it more unstable.

Figure 4: cost reducing technology and increase in firms' profits



adopted the new technology. It is worth noting that the gains are decreasing functions of the stock of fish as the larger the stock the lower the need for a new technology since fish abundance lowers the cost of harvesting.

Strategies and payoffs (in thousands of \$) in the steady state can be represented by the following gain matrix,

	A_{PS}	\bar{A}_{PS}
A_{LL}	$(220.9 - T, 50.4 - T)$	$(265.8 - T, 26.3)$
\bar{A}_{LL}	$(141.9, 85.1 - T)$	$(178.5, 52.7)$

where the strategy of a LL firm is indicated in each row, while firms of type PS refer to each column. A_{LL} (resp. \bar{A}_{LL}) indicates that type i firm adopts (resp. does not adopt) the new technology. T represents the adoption cost of the new technology. The increase in firms' profits due to the technology adoption are then given by

$$\Delta_{LL}(0) = 87.3$$

$$\Delta_{LL}(\delta_{PS}) = 79.0$$

and

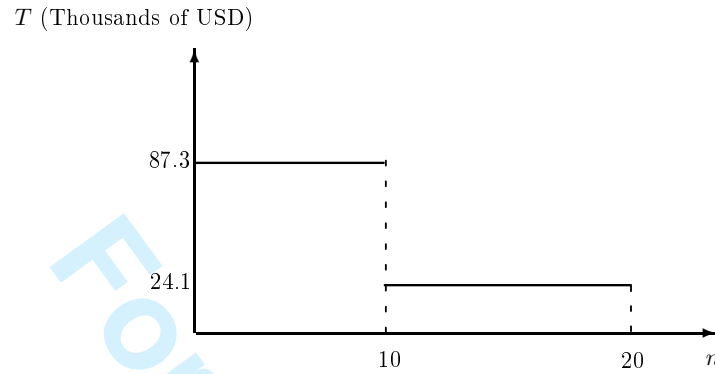
$$\Delta_{PS}(0) = 32.4$$

$$\Delta_{PS}(\delta_{LL}) = 24.1$$

LL firms are willing to pay 87.3 \$M to be equipped with the new system when PS firms do not possess it. Their willingness sharply drops to 79\$M when the other type of firm is already equipped. Likewise, PS firms are willing to pay 32.4\$M when LL firms are not equipped with the new technology while their valuation of the technology gain reduces to 24.1\$M when LL

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7 firms are also equipped. This analysis allows to derive the following inverse demand function
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9 for the new technology.

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11 Figure 5: The inverse demand function for the new technology



26 Given the discontinuity in the inverse demand function, it may be more profitable to serve only
27 high quality firms. Indeed, serving all the market yields a maximal revenue of 482\$M whereas
28 serving PS firms only allows for 873 \$M.
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30

31 32 5.1.2 Socially Optimal Technology Adoption

33
34 As in the equilibrium case, the technology adoption decision in the social optimum depends on
35 the social value of the cost reducing technology. The next gain matrix reports the social surplus
36 attained at the optimal harvest in the four possible cases
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	A_{PS}	\tilde{A}_{PS}
A_{LL}	$24002.4 - \Gamma(n_{LL} + n_{PS})$	$23650.4 - \Gamma(n_{LL})$
\tilde{A}_{LL}	$21641.3 - \Gamma(n_{PS})$	20945.7

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45 As can be seen from the matrix, the joint adoption of the technology yields an increase in the
46 social surplus of $[24002.4 - \Gamma(n_{LL} + n_{PS})] - 20945.7 = 3056.7 - \Gamma(n_{LL} + n_{PS})$ relative to the
47 use of the old technology. This therefore suggests that the joint adoption of the technology is
48 preferred to a *status quo* as long as the total production cost of the technology does not exceed
49 3056.7\$M.
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53 Since there are as many LL as PS firms, the adoption of the new technology by the sole LL firms
54 is always preferred to the adoption by the sole PS firms ($23650.4 - \Gamma(n_{LL}) \geq 21641.3 - \Gamma(n_{PS})$).
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57 The choice between a joint and a single adoption is driven by the level and the form of the
58 technology production cost. Technology adoption by LL firms only yields an increase in the
59 total surplus of $2704.7 - \Gamma(n_{LL})$. The joint adoption increases the surplus by $352 - \Gamma(n_{LL} +$
60 $n_{PS}) + \Gamma(n_{LL})$. Therefore, the joint adoption of the technology is preferred to a situation where

LL firms only adopt it as long as the increase in the surplus (352) exceeds the increase in the cost $\Gamma(n_{LL} + n_{PS}) + \Gamma(n_{LL})$. Assuming that the market value of the technology exceeds its production cost, a joint adoption is preferable.

This analysis rests on a first best evaluation of the surplus. Social surplus is evaluated at the optimal level of harvest. It is however informative to evaluate the social value of the technology when harvesting levels are set to their equilibrium values. The gain matrix then takes the form

	A_{PS}	\bar{A}_{PS}
A_{LL}	$23875 - \Gamma(n_{LL} + n_{PS})$	$23450 - \Gamma(n_{LL})$
\bar{A}_{LL}	$21611 - \Gamma(n_{PS})$	20884

A technology adoption by LL firms only yields an gain in the total surplus of $2566 - \Gamma(n_{LL})$ which is smaller than in the first best analysis. But the joint adoption increases this surplus to a larger extent as it amounts to $425 - \Gamma(n_{LL} + n_{PS}) + \Gamma(n_{LL})$. Again, assuming that the market value of the technology exceeds its production cost, joint adoption is preferable.

5.2 Quotas and the market value of the technology

Fishing is a highly regulated economic activity. In particular, international agreements impose some quotas on harvesting. It is then important to assess the impact of these quotas on the market valuation of the cost reducing technology. We therefore replicate the preceding analysis introducing quotas on one or both type of firms. More precisely, we consider three cases. In case 1, PS firms only are restricted by the quota. In case 2, LL firms only are restricted by the quota. In case 3, both types of firm are restricted by the quotas. Figure 6 depicts the market equilibrium in case 2. As can be seen from the figure, imposing quotas on LL firms not only affect their production level but also the production level of PS firms. More precisely, as soon as LL firms hit the quotas, PS firms are able to capture a larger share of the market such that (i) they increase their production level in order to satisfy the demand side of the market and (ii) are able to increase their profit. This contrasts with the situation where no quotas are implemented as in this case PS firms profits are decreasing. Indeed, as soon as the quota is hit by the LL firm, only PS firms can indeed manipulate the price schedule. Therefore, only PS firms can fully benefit from any increase in the biomass. Their profit increase. Not to say that LL firms do not benefit from an increase in the stock of fish. Indeed, even if LL firms cannot increase their production, they benefit from the fact that the market price, as fixed by PS firms increase. Their profits do increase too.

Technology adoption is also affected by the existence of quotas. This is investigated in Figure 7 that represents the inverse demand function of the technology for the different cases.

A firm that hits the quota can benefit from the cost reducing technology since each units are

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Figure 6: Market equilibrium — $k_1 = 2500$

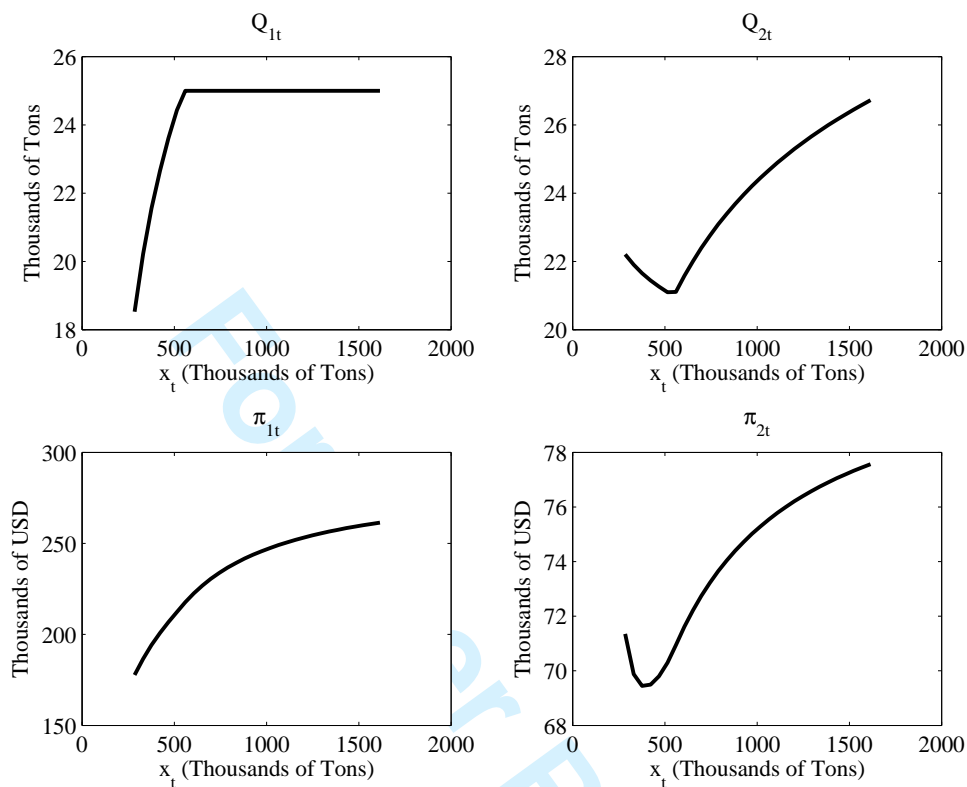
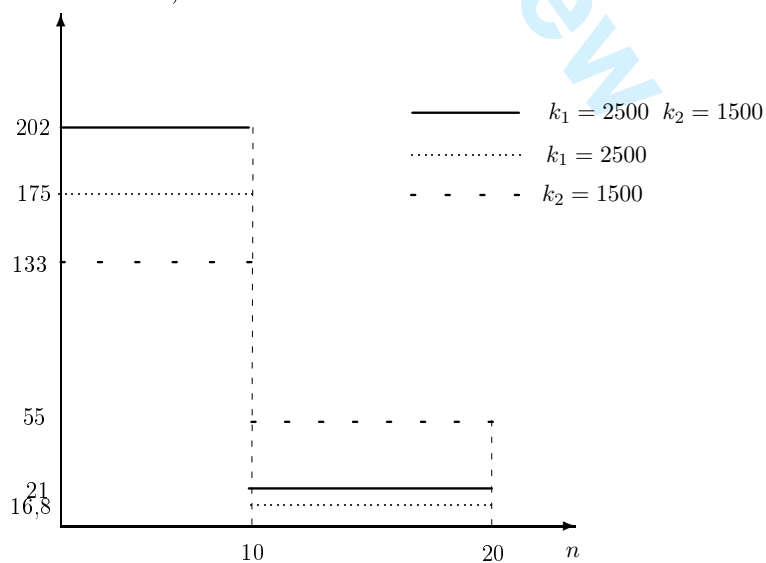


Figure 7: Quotas and inverse demand functions for the new technology
 T (Thousands of USD)



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7 produced at a lower cost without any negative impact on the price. The willingness to pay for
8 the new technology then increases. The opposite result obtains when the adoption of the new
9 technology initially allows for a significative increase in the production. In this case, quotas
10 prevent firms from increasing their production level, which reduces the value that they attach
11 to the cost reducing technology. Quotas can thus be used to finance firms in their technology
12 adoption when there are not too strict so that the first effect dominates the second. Observe
13 also that quotas can decrease the difference between firms' willingness to pay. The maximal
14 revenue from serving all the market may then exceed the revenue obtained from PS firms.
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19 20 **6 Conclusion**

21
22 This paper provides a framework to evaluate satellite systems to support tuna fishing. Valuing
23 this technology requires to evaluate the market equilibrium because marginal returns in the
24 technology adoption are not constant.
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29 We obtain that high quality firms value more the technology than low quality firms. The socially
30 optimal analysis yields similar results : the technology is more valuable for high quality firms
31 than for low quality firms. If the technology cost is not too important, both types of firms should
32 be equipped. Else, only high quality firms should adopt it. Finally, we show that individual
33 quotas can be used to increase the value that both firms attach to the technology.
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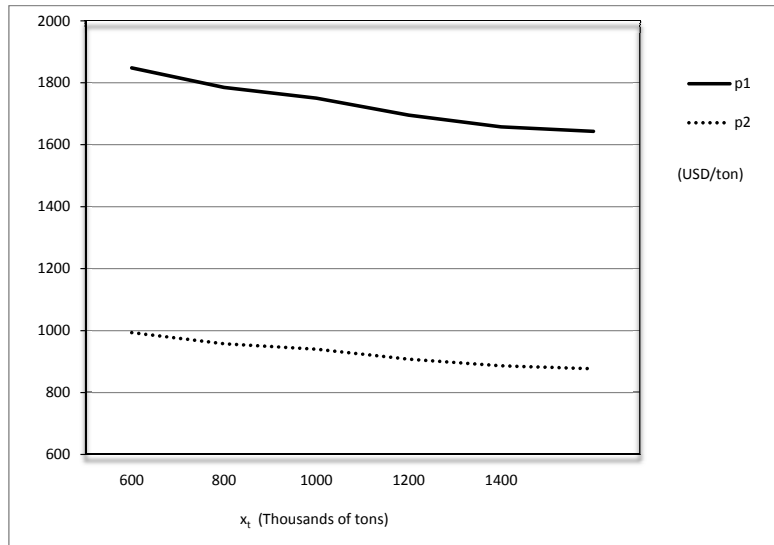
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For Peer Review

— APPENDIX —

A Equilibrium market prices evolution



B Equilibrium conditions

Recall that the profit maximization problem of firm i is given by

$$\max_{q_{it}} \Pi(\mu_i) = \sum_{t=1}^{\infty} \beta^{t-1} (p_i(Q_t)q_{it} - (c_i - \mu_i)q_{it}x_t^{\alpha_i}) \quad (\text{B.1})$$

$$\text{subject to} \quad (\text{B.2})$$

$$p_1(Q_t) = \bar{v}u_1 - u_1Q_{1t} - u_2Q_{2t} \quad (\text{B.3})$$

$$p_2(Q_t) = \bar{v}u_2 - u_2Q_{1t} - u_2Q_{2t} \quad (\text{B.4})$$

$$0 \leq q_{it} \quad (\text{B.5})$$

$$q_{it} \leq k_i \quad (\text{B.6})$$

$$x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - Q_t \quad (\text{B.7})$$

$$0 \leq x_{t+1} \quad (\text{B.8})$$

The set of optimality conditions is given by

$$\lambda_{1t} - \nu_{1t} + \zeta_{1t} = vu_1 - u_1(1 + n_1)q_{1t} - n_2u_2q_{2t} - (c_1 - \mu_1)x_t^{\alpha_1} \quad (\text{B.9})$$

$$\lambda_{2t} - \nu_{2t} + \zeta_{2t} = vu_2 - u_2n_1q_{1t} - (1 + n_2)u_2q_{2t} - (c_2 - \mu_2)x_t^{\alpha_2} \quad (\text{B.10})$$

$$\lambda_{it} - \varphi_t = \beta \left[\lambda_{it+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \alpha_i q_{it+1} (c_i - \mu_i) x_{t+1}^{\alpha_i - 1} \right] \quad (\text{B.11})$$

$$\varphi_t x_{t+1} = 0 \quad (\text{B.12})$$

$$\nu_{it} q_{it} = 0 \quad (\text{B.13})$$

$$\zeta_{it} (q_{it} - k_i) = 0 \quad (\text{B.14})$$

$$\varphi_t \geq 0 \quad (\text{B.15})$$

$$\nu_{it} \geq 0 \quad (\text{B.16})$$

$$\zeta_{it} \geq 0 \quad (\text{B.17})$$

for $i = \{1, 2\}$.

C Socially optimal conditions

Recall that the socially optimal quantities are derived from.

$$\max_{q_{1t}, q_{2t}} \sum_{t=1}^{\infty} \beta^{t-1} (S(n_1q_{1t} + n_2q_{2t}) - n_1(c_1 - \mu_1)q_{1t}x^{\alpha_1} + n_2(c_2 - \mu_2)q_{2t}x^{\alpha_2}) \quad (\text{C.1})$$

subject to

$$x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - Q_t \quad (\text{C.2})$$

$$0 \leq x_{t+1} \quad (\text{C.3})$$

The set of optimality conditions is given by

$$\lambda_t = vu_1 - u_1n_1q_{1t} - n_2u_2q_{2t} - n_1(c_1 - \mu_1)x_t^{\alpha_1} \quad (\text{C.4})$$

$$\lambda_t = vu_2 - u_2n_1q_{1t} - n_2u_2q_{2t} - n_2(c_2 - \mu_2)x_t^{\alpha_2} \quad (\text{C.5})$$

$$\lambda_t - \varphi_t = \beta \left[\lambda_{t+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \sum_{i=1}^2 [\alpha_i q_{it+1} n_i (c_i - \mu_i) x_{t+1}^{\alpha_i - 1}] \right] \quad (\text{C.6})$$

$$\varphi_t x_{t+1} = 0 \quad (\text{C.7})$$

$$\varphi_t \geq 0 \quad (\text{C.8})$$

D Approximation Method

Recall that the set of optimality conditions for the problem is given by

$$\lambda_{1t} - \nu_{1t} + \zeta_{1t} = vu_1 - u_1(1 + n_1)q_{1t} - n_2u_2q_{2t} - (c_1 - \mu_1)x_t^{\alpha_1} \quad (\text{D.1})$$

$$\lambda_{2t} - \nu_{2t} + \zeta_{2t} = vu_2 - u_2n_1q_{1t} - (1 + n_2)u_2q_{2t} - c_2x_t^{\alpha_2} \quad (\text{D.2})$$

$$\lambda_{1t} - \varphi_t = \beta \left[\lambda_{1t+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \alpha_1q_{1t+1}(c_1 - \mu_1)x_{t+1}^{\alpha_1-1} \right] \quad (\text{D.3})$$

$$\lambda_{2t} - \varphi_t = \beta \left[\lambda_{2t+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \alpha_2q_{2t+1}(c_2 - \mu_2)x_{t+1}^{\alpha_2-1} \right] \quad (\text{D.4})$$

$$\varphi_t x_{t+1} = 0 \quad (\text{D.5})$$

$$\nu_{1t}q_{1t} = 0 \quad (\text{D.6})$$

$$\nu_{2t}q_{2t} = 0 \quad (\text{D.7})$$

$$\zeta_{1t}(q_{1t} - k_1) = 0 \quad (\text{D.8})$$

$$\zeta_{2t}(q_{2t} - k_2) = 0 \quad (\text{D.9})$$

$$\varphi_t \geq 0 \quad (\text{D.10})$$

$$\nu_{1t} \geq 0 \quad (\text{D.11})$$

$$\nu_{2t} \geq 0 \quad (\text{D.12})$$

$$\zeta_{1t} \geq 0 \quad (\text{D.13})$$

$$\zeta_{2t} \geq 0 \quad (\text{D.14})$$

The algorithm is very close to the procedure suggested in Marcet and Lorenzoni (1999) and Christiano and Fisher (2000). The basic idea of this approach is to approximate the expectation function — the right hand side of (D.3)–(D.4) — of each player — rather than attempting to recover directly the decision rules — by a smooth function (a polynomial function). This amounts to replace (D.3)–(D.4) by

$$\lambda_{1t} - \varphi_t = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} - \varphi_t = \Phi_2(x_t, \Theta_2)$$

where $\Phi_1(x_t, \Theta_1)$ and $\Phi_2(x_t, \Theta_2)$ approximate the quantities

$$\beta \left[\lambda_{1t+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \alpha_1q_{1t+1}(c_1 - \mu_1)x_{t+1}^{\alpha_1-1} \right]$$

and

$$\beta \left[\lambda_{2t+1} \left(1 + r \left(1 - \frac{2}{k}x_{t+1} \right) \right) - \alpha_2q_{2t+1}(c_2 - \mu_2)x_{t+1}^{\alpha_2-1} \right]$$

respectively.

The two approximating functions are given by linear combinations of Chebychev polynomials

$$\Phi_\ell(x_t; \Theta_\ell) \equiv \sum_{j=0}^n \theta_{\ell,j} T_j(\Psi(x_t)) \quad \text{for } \ell = 1, 2$$

where $T_j(\cdot)$ is the Chebychev polynomial of order j . This class of polynomial possesses a discrete orthogonality property which has proven extremely efficient and useful in approximation theory. Since Chebychev polynomials are defined over $[-1; 1]$, while x_t is not, x_t is transformed by means of function

$$\Psi(x) = 2 \frac{x - \underline{x}}{\bar{x} - \underline{x}} - 1$$

which maps $[\underline{x}; \bar{x}]$ into $[-1; 1]$.

The parameters of the approximating functions, Θ_1 and Θ_2 , are then revealed by imposing identifying restrictions dictated by economic theory. More precisely, they are determined by imposing an orthogonality condition on the residuals of the Euler equations.

Note that in the sequel a ' will denote a variables evaluated in the next period ($t+1$).

The algorithm then works as follows.

1. Choose an order of approximation n , compute the $m > n$ roots of the Chebychev polynomial of order m_x as

$$z_i = \cos\left(\frac{(2i-1)\pi}{2(n+1)}\right) \text{ for } i = 1, \dots, m$$

2. Compute the matrix $\mathcal{T}(z)$

$$\mathcal{T}(z) = \begin{pmatrix} T_0(z_1) & \cdots & T_0(z_m) \\ \vdots & \ddots & \vdots \\ T_n(z_1) & \cdots & T_n(z_m) \end{pmatrix}$$

3. Compute x_i as

$$x_i = \log(\underline{x}) + (z_i + 1) \frac{\log(\bar{x}) - \log(\underline{x})}{2} \text{ for } i = 1, \dots, m$$

to map $[-1; 1]$ into $[\underline{x}; \bar{x}]$.

4. Compute the approximating expectation function of each player, ℓ , at each node x_i , $i = 1, \dots, m$

$$\Phi_\ell(x_i; \Theta_\ell) \equiv \sum_{j=0}^n \theta_{\ell,j} T_j(z_i) \text{ for } \ell = 1, 2$$

5. Using this expectation function, obtain quantities fished by each player in an unconstrained equilibrium for each value of the stock, x_i . In such a situation, we have $0 < q_{1i} < k_1$, $0 < q_{2i} < k_2$, and $x'_i > 0$. This implies that $\nu_{1i} = \nu_{2i} = \zeta_{1i} = \zeta_{2i} = \varphi_i = 0$. A direct consequence of this is that λ_{1i} and λ_{2i} are simply given by

$$\lambda_{1i} = \Phi_1(x_i, \Theta_1)$$

$$\lambda_{2i} = \Phi_2(x_i, \Theta_2)$$

and $q_{1i} \equiv Q_1(x_i, \Theta_1, \Theta_2)$ and $q_{2i} \equiv Q_2(x_i, \Theta_1, \Theta_2)$ are obtained by solving the simple linear system

$$u_1(1 + n_1)q_{1i} + n_2 u_2 q_{2i} = v u_1 - \lambda_{1i} - (c_1 - \mu_1)x_i^{\alpha_1} \quad (\text{D.15})$$

$$u_2 n_1 q_{1i} - (1 + n_2)u_2 q_{2i} = v u_2 - \lambda_{2i} - (c_2 - \mu_2)x_i^{\alpha_2} \quad (\text{D.16})$$

Then the unconstrained future value of tuna stock, $x'_i \equiv (x_i, \Theta_1, \Theta_2)$ is given by

$$x'_i = r x_i \left(1 - \frac{x_i}{k}\right) + x_i - n_1 q_{1i} - n_2 q_{2i}$$

6. Check whether any constraint, on q_{1i} , q_{2i} or x'_i is binding for each x_i , $i = 1, \dots, n$

- If the constraint is not binding then keep the computed values for q_{1i} , q_{2i} , x'_i , and set $\nu_{1i} = \nu_{2i} = \zeta_{1i} = \zeta_{2i} = \varphi_i = 0$
- If any constraint is binding, then adapt the equilibrium values accordingly.¹²

¹²This is described in appendix E.

At this step we have equilibrium values for each quantity and price in the current period. We also have next period tuna stock. We are then in a position to compute the next period values.

7. For each value $x'_i = X(x_i, \Theta_1, \Theta_2)$ apply the transformation

$$\Psi(x) = 2\frac{x - \underline{x}}{\bar{x} - \underline{x}} - 1$$

which maps $[\underline{x}; \bar{x}]$ into $[-1; 1]$. Then build the matrix

$$\mathcal{T}'(x) = \begin{pmatrix} T_0(\Psi(x_1)) & \cdots & T_0(\Psi(x_m)) \\ \vdots & \ddots & \vdots \\ T_n(\Psi(x_1)) & \cdots & T_n(\Psi(x_m)) \end{pmatrix}$$

8. Compute next period approximating expectation function of each player, ℓ , at each node x'_i , $i = 1, \dots, m$

$$\Phi_\ell(x'_i; \theta) \equiv \sum_{j=0}^n \theta_{\ell,j} T_j(\Psi(x'_i)) \text{ for } \ell = 1, 2$$

and obtain next period equilibrium values as obtained in steps 5 and 6.

9. Evaluate the residuals $R_1(x_i, \Theta_1, \Theta_2)$ and $R_2(x_i, \Theta_1, \Theta_2)$ obtained as

$$R_1(x_i, \Theta_1, \Theta_2) = \beta \left[\lambda'_{1i} \left(1 + r \left(1 - \frac{2}{k} x'_i \right) \right) - \alpha_1 q'_{1i} (c_1 - \mu_1) x_i^{\alpha_1 - 1} \right] - \Phi_1(x_i; \Theta_1)$$

$$R_2(x_i, \Theta_1, \Theta_2) = \beta \left[\lambda'_{2i} \left(1 + r \left(1 - \frac{2}{k} x'_i \right) \right) - \alpha_2 q'_{2i} (c_2 - \mu_2) x_i^{\alpha_2 - 1} \right] - \Phi_2(x_i; \Theta_2)$$

for each $i = 1, \dots, m$.

10. Compute the inner products

$$\mathcal{I}(z)R_1(x, \Theta_1, \Theta_2) = 0$$

$$\mathcal{I}(z)R_2(x, \Theta_1, \Theta_2) = 0$$

11. if all inner products are close enough to zero then stop, else update Θ_1 and Θ_2 and go back to 4.

Note that, as in the standard PEA algorithm, we treat ν_{it} , ζ_{it} and φ_t as technical variables which are just used to compute the residuals. We therefore do not need to compute explicitly its interpolating function.

From a practical point of view, we use 30th order Chebychev polynomials and 1000 nodes. These values are actually very high but allow us to achieve high accuracy of the approximation over the support $[(1 - \Delta)\bar{x}; (1 + \Delta)\bar{x}]$ where \bar{x} is the steady state of the model and $\Delta = 0.5$.

E Constrained Equilibrium

This section presents the possible equilibria that may occur, given the constraints imposed on the problem.

Case 1: $0 < q_{1t} < k_1$, $0 < q_{2t} < k_2$, and $x_{t+1} > 0$. This implies that $\nu_{1t} = \nu_{2t} = \zeta_{1t} = \zeta_{2t} = \varphi_t = 0$ and

$$\lambda_{1t} = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} = \Phi_2(x_t, \Theta_2)$$

Then $q_{1i} \equiv Q_1(x_i, \Theta_1, \Theta_2)$ and $q_{2i} \equiv Q_2(x_i, \Theta_1, \Theta_2)$ are obtained by solving the simple linear system

$$u_1(1 + n_1)q_{1i} + n_2u_2q_{2i} = vu_1 - \lambda_{1i} - (c_1 - \mu_1)x_i^{\alpha_1} \quad (\text{E.1})$$

$$u_2n_1q_{1i} - (1 + n_2)u_2q_{2i} = vu_2 - \lambda_{2i} - (c_2 - \mu_2)x_i^{\alpha_2} \quad (\text{E.2})$$

Case 2: Given the unconstrained equilibrium: $0 < q_{1t} < k_1$, $0 < q_{2t} < k_2$, and $x_{t+1} < 0$. This implies that $\nu_{1t} = \nu_{2t} = \zeta_{1t} = \zeta_{2t} = 0$ and $\varphi_t > 0$. Then we set

$$x_{t+1} = 0$$

The other variables are a solution of the following system

$$\begin{cases} \lambda_{1t} - \nu_{1t} + \zeta_{1t} = vu_1 - u_1(1 + n_1)q_{1t} - n_2u_2q_{2t} - (c_1 - \mu_1)x_t^{\alpha_1} \\ n_1q_{1t} + n_2q_{2t} = rx_t\left(1 - \frac{x_t}{k}\right) + x_t \\ \lambda_{2t} - \nu_{2t} + \zeta_{2t} = vu_2 - u_2n_1q_{1t} - (1 + n_2)u_2q_{2t} - (c_2 - \mu_2)x_t^{\alpha_2} \\ \lambda_{1t} - \varphi_t = \Phi_1(x_t, \Theta_1) \\ \lambda_{2t} - \varphi_t = \Phi_2(x_t, \Theta_2) \end{cases}$$

Case 3: Given the unconstrained equilibrium: $q_{1t} < 0$, $0 < q_{2t} < k_2$, and $x_{t+1} > 0$. This implies that $\nu_{1t} > 0$, $\nu_{2t} = \zeta_{1t} = \zeta_{2t} = \varphi_t = 0$. Then we set

$$q_{1t} = 0 \text{ and } q_{2t} = \frac{vu_2 - (c_2 - \mu_2)x_t^{\alpha_2} - \lambda_{2t}}{u_2(1 + n_2)}$$

with

$$\lambda_{1t} = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} = \Phi_2(x_t, \Theta_2)$$

Case 4: Given the unconstrained equilibrium: $q_{1t} < 0$, $0 < q_{2t} < k_2$, and $x_{t+1} < 0$. This implies that $\nu_{1t} > 0$, $\nu_{2t} = \zeta_{1t} = \zeta_{2t} = 0$ and $\varphi_t > 0$. Then we set

$$q_{1t} = 0 \text{ and } q_{2t} = \frac{1}{n_2} \left(rx_t \left(1 - \frac{x_t}{k} \right) + x_t \right)$$

and

$$\lambda_{2t} = vu_2 - (1 + n_2)u_2q_{2t} - (c_2 - \mu_2)x_t^{\alpha_2}$$

$$\varphi_t = \lambda_{2t} - \Phi_2(x_t, \Theta_2)$$

$$\lambda_{1t} = \varphi_t + \Phi_1(x_t, \Theta_1)$$

Case 5: Given the unconstrained equilibrium: $q_{1t} < 0$, $q_{2t} < 0$, and $x_{t+1} > 0$. This implies that $\nu_{1t} > 0$, $\nu_{2t} > 0$, $\zeta_{1t} = \zeta_{2t} = \varphi_t = 0$. Then we set

$$q_{1t} = q_{2t} = 0 \text{ and } x_{t+1} = rx_t \left(1 - \frac{x_t}{k} \right) + x_t$$

and

$$\lambda_{1t} = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} = \Phi_2(x_t, \Theta_2)$$

Case 6: Given the unconstrained equilibrium: $q_{1t} < 0$, $q_{2t} < 0$, and $x_{t+1} < 0$. This is clearly not an equilibrium

Case 7: Given the unconstrained equilibrium: $q_{1t} < 0$, $q_{2t} > k_2$, and $x_{t+1} > 0$. This implies that $\nu_{1t} > 0$, $\zeta_{2t} > 0$, $\zeta_{1t} = \nu_{2t} = \varphi_t = 0$. Then we set

$$q_{1t} = 0, \quad q_{2t} = k_2 \quad \text{and} \quad x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - n_2 k_2$$

and

$$\lambda_{1t} = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} = \Phi_2(x_t, \Theta_2)$$

Case 8: Given the unconstrained equilibrium: $q_{1t} < 0$, $q_{2t} > k_2$, and $x_{t+1} < 0$. This case is equivalent to case 4.

Case 9: Given the unconstrained equilibrium: $q_{1t} > k_1$, $0 < q_{2t} < k_2$, and $x_{t+1} > 0$. This implies that $\zeta_{1t} > 0$, $\nu_{1t} = \nu_{2t} = \zeta_{2t} = \varphi_t = 0$. Then we set

$$q_{1t} = k_1, \quad q_{2t} = \frac{vu_2 - n_1 u_2 k_1 - (c_2 - \mu_2)x_t^{\alpha_2} - \lambda_{2t}}{u_2(1 + n_2)} \quad \text{and} \quad x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - n_1 k_1 - n_2 q_{2t}$$

and

$$\lambda_{1t} = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} = \Phi_2(x_t, \Theta_2)$$

Case 10: Given the unconstrained equilibrium: $q_{1t} > k_1$, $0 < q_{2t} < k_2$, and $x_{t+1} < 0$. This implies that $\zeta_{1t} > 0$, $\nu_{1t} = \nu_{2t} = \zeta_{2t} = 0$ and $\varphi_t > 0$. Then we set

$$q_{1t} = k_1 \quad \text{and} \quad x_{t+1} = 0$$

We have

$$q_{2t} = \frac{1}{k_2} \left(rx_t \left(1 - \frac{x_t}{k}\right) + x_t - n_1 k_1 \right) \lambda_{2t} = vu_2 - n_1 u_2 k_1 - (1 + n_2) u_2 q_{2t} - (c_2 - \mu_2) x_t^{\alpha_2}$$

$$\varphi_t = \lambda_{2t} - \Phi_2(x_t, \Theta_2)$$

$$\lambda_{1t} = \varphi_t + \Phi_1(x_t, \Theta_1)$$

Case 11: Given the unconstrained equilibrium: $q_{1t} > k_1$, $q_{2t} > k_2$, and $x_{t+1} > 0$. This implies that $\zeta_{1t} > 0$, $\zeta_{2t} > 0$, $\nu_{1t} = \nu_{2t} = \varphi_t = 0$. Then we set

$$q_{1t} = k_1, \quad q_{2t} = k_2 \quad \text{and} \quad x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - n_1 k_1 - n_2 k_2$$

We have

$$\lambda_{1t} = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} = \Phi_2(x_t, \Theta_2)$$

Case 12: Given the unconstrained equilibrium: $q_{1t} > k_1$, $q_{2t} > k_2$, and $x_{t+1} < 0$. This case is identical to case 2.

Case 13: Given the unconstrained equilibrium: $0 < q_{1t} < k_1$, $q_{2t} < 0$, and $x_{t+1} > 0$. This implies that $\nu_{2t} > 0$, and $\nu_{1t} = \zeta_{2t} = \zeta_{1t} = \varphi_t = 0$. Then we set

$$q_{1t} = \frac{vu_1 - (c_1 - \mu_1)x_t^{\alpha_1} - \lambda_{1t}}{u_1(1 + n_1)}, \quad q_{2t} = 0 \quad \text{and} \quad x_{t+1} = rx_t \left(1 - \frac{x_t}{k}\right) + x_t - n_1 q_{1t}$$

We have

$$\lambda_{1t} = \Phi_1(x_t, \Theta_1)$$

$$\lambda_{2t} = \Phi_2(x_t, \Theta_2)$$

Case 14: Given the unconstrained equilibrium: $0 < q_{1t} < k_1$, $q_{2t} < 0$, and $x_{t+1} < 0$. This implies that $\nu_{2t} > 0$, $\nu_{1t} = \zeta_{2t} = \zeta_{1t} = 0$ and $\varphi_t > 0$. Then we set

$$q_{1t} = \frac{1}{n_1} \left(rx_t \left(1 - \frac{x_t}{k} \right) + x_t \right), \quad q_{2t} = 0 \text{ and } x_{t+1} = 0$$

We have

$$\begin{aligned} \lambda_{1t} &= vu_1 - (1 + n_1)u_1q_{1t} - (c_1 - \mu_1)x_t^{\alpha_1} \\ \lambda_{2t} &= \Phi_2(x_t, \Theta_2) \\ \varphi_t &= \lambda_{1t} + \Phi_1(x_t, \Theta_1) \end{aligned}$$

Case 15: Given the unconstrained equilibrium: $q_{1t} > k_1$, $q_{2t} < 0$, and $x_{t+1} > 0$. This implies that $\zeta_{1t} > 0$, $\nu_{2t} > 0$, $\nu_{1t} = \zeta_{2t} = \varphi_t = 0$. Then we set

$$q_{1t} = k_1, \quad q_{2t} = 0 \text{ and } x_{t+1} = rx_t \left(1 - \frac{x_t}{k} \right) + x_t - n_1k_1$$

We have

$$\begin{aligned} \lambda_{1t} &= \Phi_1(x_t, \Theta_1) \\ \lambda_{2t} &= \Phi_2(x_t, \Theta_2) \end{aligned}$$

Case 16: Given the unconstrained equilibrium: $q_{1t} > k_1$, $q_{2t} < 0$, and $x_{t+1} < 0$. This case is identical to case 14.

Case 17: Given the unconstrained equilibrium: $0 < q_{1t} < k_1$, $q_{2t} > k_2$, and $x_{t+1} > 0$. This implies that $\zeta_{2t} > 0$, $\nu_{1t} = \zeta_{1t} = \nu_{2t} = \varphi_t = 0$. Then we set

$$q_{1t} = \frac{vu_1 - n_2u_2k_2 - (c_1 - \mu_1)x_t^{\alpha_1} - \lambda_{1t}}{u_1(1 + n_1)}, \quad q_{2t} = k_2 \text{ and } x_{t+1} = rx_t \left(1 - \frac{x_t}{k} \right) + x_t - n_1q_{1t} - n_2k_2$$

We have

$$\begin{aligned} \lambda_{1t} &= \Phi_1(x_t, \Theta_1) \\ \lambda_{2t} &= \Phi_2(x_t, \Theta_2) \end{aligned}$$

Case 18: Given the unconstrained equilibrium: $0 < q_{1t} < k_1$, $q_{2t} > k_2$, and $x_{t+1} < 0$. This implies that $\zeta_{2t} > 0$, $\nu_{1t} = \zeta_{1t} = \nu_{2t} = 0$ and $\varphi_t > 0$. Then we set

$$q_{1t} = \frac{1}{n_1} \left(rx_t \left(1 - \frac{x_t}{k} \right) + x_t - n_2k_2 \right), \quad q_{2t} = k_2 \text{ and } x_{t+1} = 0$$

We have

$$\begin{aligned} \lambda_{1t} &= vu_1 - u_1(1 + n_1)q_{1t} - n_2u_2k_2 - (c_1 - \mu_1)x_t^{\alpha_1} \\ \lambda_{2t} &= \Phi_2(x_t, \Theta_2) \\ \varphi_t &= \lambda_{1t} + \Phi_1(x_t, \Theta_1) \end{aligned}$$