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# Propagation of an electromagnetic lightwave through isolated or periodically arranged openings

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**Abstract:** The goal of this document is to provide a comprehensive description of the equations governing the propagation of an electromagnetic lightwave through an opening. The latter is first considered as isolated and then integrated within a periodically arranged array. The material in which the openings are drilled is supposed to be non-magnetic ( $\mu_{\text{material}} = \mu_{\text{air}}$ ). No other specific assumptions are made on the optical indices of the different regions involved in order to keep this study as general as possible.

**Keywords:** Maxwell's equation, leaky waveguide, leaky modes, periodic array

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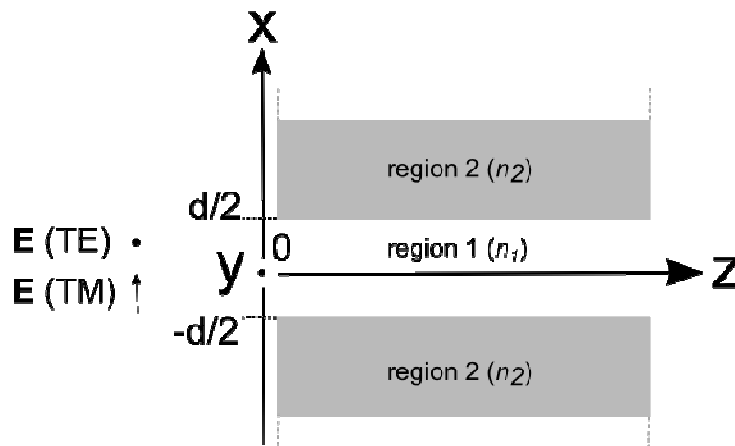
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## 1. Isolated Opening

The first case investigated is depicted on **Figure 1** and corresponds to an isolated opening (with a width and a complex optical index denoted “ $d$ ” and “ $n_1$ ”, respectively) drilled inside a material (with a complex optical index denoted “ $n_2$ ”). Both TE and TM polarizations are considered in the following. Please note that the subscript “1” (“2”) is attributed to the parameters of region 1 (region 2).



**Figure 1:** Isolated opening (region 1) drilled in a material with an infinite extent in the x, y and z directions (region 2). The lightwave is propagating along the z direction.

### 1.1 TE polarization

In TE polarization, only the  $E_y$  component exists. From Maxwell’s equations, one can obtain the equations below [1]:

**Equation 1**

$$\frac{\partial^2 E_y}{\partial x^2} + (k^2 - k_z^2) E_y = 0$$

, with:

**Equation 2**

$$k_x^2 = (k^2 - k_z^2)$$

**N.B:** The imaginary part of k is supposed to be negative

The general form of  $E_y$  is:

**Equation 3**

$$E_y = A e^{jx\sqrt{(k^2 - k_z^2)}} + B e^{-jx\sqrt{(k^2 - k_z^2)}}$$

### 1.1.1 General expression of the field

**For  $x < -d/2$**

**Equation 4**

$$E_y = A_2 e^{jxk_{x2}} + B_2 e^{-jxk_{x2}}$$

**For  $-d/2 < x < d/2$**

**Equation 5**

$$E_y = A_1 e^{jxk_{x1}} + B_1 e^{-jxk_{x1}}$$

**For  $x > d/2$**

**Equation 6**

$$E_y = A'_2 e^{jxk_{x2}} + B'_2 e^{-jxk_{x2}}$$

Because the system considered is not periodic (see **Figure 1**), the electrical field is decaying to zero when  $x$  tends to  $-\infty$  or  $+\infty$ . This implies that:

**For  $x < -d/2$**

**Equation 7**

$$E_y = A_2 e^{jxk_{x2}}$$

**For  $-d/2 < x < d/2$**

**Equation 8**

$$E_y = A_1 e^{jxk_{x1}} + B_1 e^{-jxk_{x1}}$$

**For  $x > d/2$**

**Equation 9**

$$E_y = B'_2 e^{-jxk_{x2}}$$

1.1.2 Continuity of the tangential component  $E_y$

**At  $x = -d/2$**

**Equation 10**

$$A_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{-j\frac{d}{2}k_{x1}} + B_1 e^{j\frac{d}{2}k_{x1}}$$

**At  $x = +d/2$**

**Equation 11**

$$B'_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{j\frac{d}{2}k_{x1}} + B_1 e^{-j\frac{d}{2}k_{x1}}$$

1.1.3 Continuity of the  $H_z$  component

$H_z$  is related to  $E_y$  by the Maxwell's equations through:

**Equation 12**

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

So the general form of  $H_z$  is:

**Equation 13**

$$H_z = \frac{k_{x2}}{\mu\omega} \left( B_2 e^{-jxk_{x2}} - A_2 e^{jxk_{x2}} \right)$$

**At  $x = -d/2$**

**Equation 14**

$$\frac{k_{x2}}{\mu_2} \left( -A_2 e^{-j\frac{d}{2}k_{x2}} \right) = \frac{k_{x1}}{\mu_1} \left( B_1 e^{j\frac{d}{2}k_{x1}} - A_1 e^{-j\frac{d}{2}k_{x1}} \right)$$

**At  $x = +d/2$**

**Equation 15**

$$\frac{k_{x2}}{\mu_2} B'_2 e^{-j\frac{d}{2}k_{x2}} = \frac{k_{x1}}{\mu_1} \left( B_1 e^{-j\frac{d}{2}k_{x1}} - A_1 e^{j\frac{d}{2}k_{x1}} \right)$$

1.1.4 Condition on the parity

It is then possible to combine the equations of continuity to derive the  $A_1/B_1$  ratio.

**At  $x=-d/2$**

$$A_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{-j\frac{d}{2}k_{x1}} + B_1 e^{j\frac{d}{2}k_{x1}}$$

$$\frac{k_{x2}}{\mu_2} \left( -A_2 e^{-j\frac{d}{2}k_{x2}} \right) = \frac{k_{x1}}{\mu_1} \left( B_1 e^{j\frac{d}{2}k_{x1}} - A_1 e^{-j\frac{d}{2}k_{x1}} \right)$$

, which gives rise to:

**Equation 16**

$$\frac{k_{x2}}{\mu_2} \left( A_1 e^{-j\frac{d}{2}k_{x1}} + B_1 e^{j\frac{d}{2}k_{x1}} \right) = \frac{k_{x1}}{\mu_1} \left( A_1 e^{-j\frac{d}{2}k_{x1}} - B_1 e^{j\frac{d}{2}k_{x1}} \right)$$

**Equation 17**

$$\frac{A_1}{B_1} = \frac{\left( \frac{k_{x1}}{\mu_1} + \frac{k_{x2}}{\mu_2} \right)}{\left( \frac{k_{x1}}{\mu_1} - \frac{k_{x2}}{\mu_2} \right)} e^{j d k_{x1}}$$

**At  $x=+d/2$**

$$B'_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{j\frac{d}{2}k_{x1}} + B_1 e^{-j\frac{d}{2}k_{x1}}$$

$$\frac{k_{x2}}{\mu_2} B'_2 e^{-j\frac{d}{2}k_{x2}} = \frac{k_{x1}}{\mu_1} \left( B_1 e^{-j\frac{d}{2}k_{x1}} - A_1 e^{j\frac{d}{2}k_{x1}} \right)$$

, which gives rise to:

**Equation 18**

$$\frac{B_1}{A_1} = \frac{\frac{k_{x2}}{\mu_2} + \frac{k_{x1}}{\mu_1}}{\frac{k_{x1}}{\mu_1} - \frac{k_{x2}}{\mu_2}} e^{j d k_{x1}}$$

From the equations above, one can notice that:

**Equation 19**

$$\frac{\frac{k_{x2}}{\mu_2} + \frac{k_{x1}}{\mu_1}}{\frac{k_{x1}}{\mu_1} - \frac{k_{x2}}{\mu_2}} e^{j d k_{x1}} = \frac{B_1}{A_1} = \frac{A_1}{B_1}$$

So:

**Equation 20**

$$B_1^2 = A_1^2. \text{ This leads to two cases:}$$

Even case:  $B_1 = A_1$

Odd case:  $B_1 = -A_1$

#### 1.1.5 Even case $B_1 = A_1$

**At  $x = -d/2$**

Using Euler's formula:

**Equation 21**

$$\frac{A_2}{A_1} = -2 \frac{k_{x1}}{\mu_1} \frac{\mu_2}{k_{x2}} e^{j \frac{d}{2} k_{x2}} j \sin\left(\frac{d}{2} k_{x1}\right)$$

**At  $x = +d/2$**

**Equation 22**

$$\frac{B'_2}{A_1} = -2 \frac{k_{x1}}{\mu_1} \frac{\mu_2}{k_{x2}} e^{j \frac{d}{2} k_{x2}} j \sin\left(\frac{d}{2} k_{x1}\right)$$

#### 1.1.6 Odd case $B_1 = -A_1$

**At  $x = -d/2$**

**Equation 23**

$$\frac{A_2}{A_1} = 2 \frac{k_{x1}}{\mu_1} \frac{\mu_2}{k_{x2}} \cos\left(\frac{d}{2} k_{x1}\right) e^{j \frac{d}{2} k_{x2}}$$

**At  $x = +d/2$**

**Equation 24**

$$\frac{B'_2}{A_1} = -2 \frac{k_{x1}}{\mu_1} \frac{\mu_2}{k_{x2}} \cos\left(\frac{d}{2} k_{x1}\right) e^{j \frac{d}{2} k_{x2}}$$

#### 1.1.7 Transcendental equations

##### 1.1.7.a Transcendental equations: Even case

**At  $x = -d/2$**

Starting from:

$$\frac{A_2}{A_1} = -2 \frac{k_{x1}}{\mu_1} \frac{\mu_2}{k_{x2}} e^{j \frac{d}{2} k_{x2}} j \sin\left(\frac{d}{2} k_{x1}\right)$$

And:

$A_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{-j\frac{d}{2}k_{x1}} + A_1 e^{j\frac{d}{2}k_{x1}}$ , which can be rewritten as:

$$\frac{A_2}{A_1} = 2 \cos\left(\frac{d}{2}k_{x1}\right) e^{j\frac{d}{2}k_{x2}}$$

We now find:

**Equation 25**

$$k_{x2} = -j \times k_{x1} \frac{\mu_2}{\mu_1} \tan\left(\frac{d}{2}k_{x1}\right)$$

#### 1.1.7.b Transcendental equations: Odd case

**At  $x=-d/2$**

Starting from:

$$\frac{A_2}{A_1} = 2 \frac{k_{x1}}{\mu_1} \frac{\mu_2}{k_{x2}} \cos\left(\frac{d}{2}k_{x1}\right) e^{j\frac{d}{2}k_{x2}}$$

And:

$A_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{-j\frac{d}{2}k_{x1}} - A_1 e^{j\frac{d}{2}k_{x1}}$ , which can be rewritten as:

$$\frac{A_2}{A_1} = -2j \sin\left(\frac{d}{2}k_{x1}\right) e^{j\frac{d}{2}k_{x2}}$$

We now find:

**Equation 26**

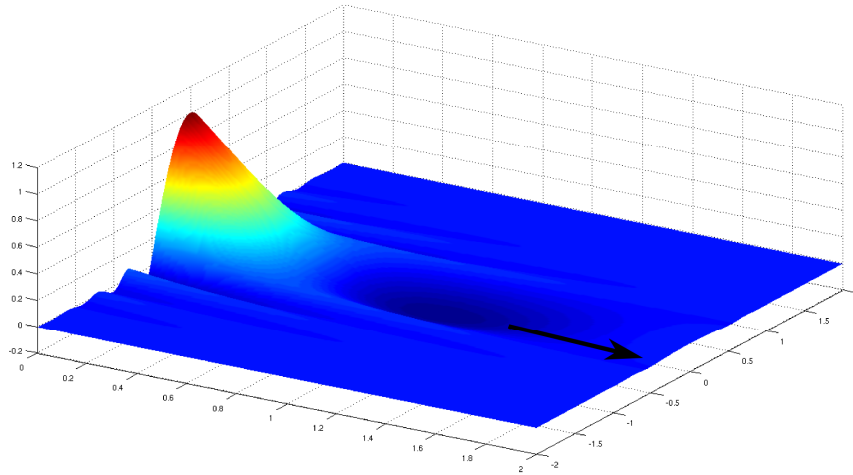
$$k_{x2} = j k_{x1} \frac{\mu_2}{\mu_1} \cot\left(\frac{d}{2}k_{x1}\right)$$

#### 1.1.8 Example

As an example, one can assume that region 1 is air ( $n_1=1$ ) and that region 2 is an absorbing medium with an optical index  $n_2=5+i$ , which roughly corresponds to the indices of amorphous silicon in the blue region of the visible spectrum. The wavelength is set to 1 and the width of the hole is set to 0.5 (in lambda units). Using **Equation 25**, it is then possible to compute the x and z components of the wave vector in regions 1 and 2, which then gives:

- $k_{x(\text{region } 1)} = 6.039 - 0.729i$
- $k_{x(\text{region } 2)} = 31.335 + 6.156i$
- $k_{z(\text{region } 1)} = k_{z(\text{region } 2)} = 2.552 + 1.726i$

The normalized field profile of the lightwave propagating through this subwavelength opening is reported in **Figure 2**.



**Figure 2:** Normalized field profile of a lightwave propagating in the direction indicated by the arrow (First order depicted). The field amplitude is maximum in the middle of region 1 (air) and is decaying both laterally, towards the absorbing medium, and in the propagation direction.

## 1.2 TM polarization

In TM polarization, only the  $H_y$  component exists. From Maxwell's equations, one can obtain the equations below:

**Equation 27**

$$\frac{\partial^2 H_y}{\partial x^2} + (k^2 - k_z^2) H_y = 0$$

, with:

**Equation 28**

$$k_x^2 = (k^2 - k_z^2)$$

The general form of  $H_y$  is:

**Equation 29**

$$H_y = A e^{jx\sqrt{(k^2 - k_z^2)}} + B e^{-jx\sqrt{(k^2 - k_z^2)}}$$

### 1.2.1 General expression of the field

**For  $x < -d/2$**

**Equation 30**

$$H_y = A_2 e^{jxk_{x2}} + B_2 e^{-jxk_{x2}}$$

**For  $-d/2 < x < d/2$**

**Equation 31**

$$H_y = A_1 e^{jxk_{x1}} + B_1 e^{-jxk_{x1}}$$

**For  $x > d/2$**

**Equation 32**

$$H_y = A'_2 e^{jxk_{x2}} + B'_2 e^{-jxk_{x2}}$$

Because the system considered is not periodic (see **Figure 1**), the electrical field is decaying to zero when  $x$  tends to  $-\infty$  or  $+\infty$ . This implies that:

**For  $x < -d/2$**

**Equation 33**

$$H_y = A_2 e^{jxk_{x2}}$$

**For  $-d/2 < x < d/2$**

**Equation 34**

$$H_y = A_1 e^{jxk_{x1}} + B_1 e^{-jxk_{x1}}$$

**For  $x > d/2$**

**Equation 35**

$$H_y = B'_2 e^{-jxk_{x2}}$$

### 1.2.2 Continuity of the tangential component $H_y$

**At  $x = -d/2$**

**Equation 36**

$$A_2 e^{-j\frac{d}{2}k_{x2}} + B_2 e^{j\frac{d}{2}k_{x2}} = A_1 e^{-j\frac{d}{2}k_{x1}} + B_1 e^{j\frac{d}{2}k_{x1}}$$

**At  $x = +d/2$**

**Equation 37**

$$A'_2 e^{j\frac{d}{2}k_{x2}} + B'_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{j\frac{d}{2}k_{x1}} + B_1 e^{-j\frac{d}{2}k_{x1}}$$

### 1.2.3 Continuity of the $E_z$ component

$E_z$  is related to  $H_y$  by the Maxwell's equations through:

**Equation 38**

$$\frac{\partial H_y}{\partial x} = +j\omega\epsilon E_z$$

So the general form of  $E_z$  is:

**Equation 39**

$$E_z = \frac{k_{x2}}{\epsilon\omega} \left( A_2 e^{jxk_{x2}} - B_2 e^{-jxk_{x2}} \right)$$

**At  $x=-d/2$**

**Equation 40**

$$\frac{k_{x2}}{\epsilon_2} A_2 e^{-j\frac{d}{2}k_{x2}} - \frac{k_{x2}}{\epsilon_2} B_2 e^{j\frac{d}{2}k_{x2}} = \frac{k_{x1}}{\epsilon_1} A_1 e^{-j\frac{d}{2}k_{x1}} - \frac{k_{x1}}{\epsilon_1} B_1 e^{j\frac{d}{2}k_{x1}}$$

**At  $x=+d/2$**

**Equation 41**

$$\frac{k_{x2}}{\epsilon_2} A'_2 e^{j\frac{d}{2}k_{x2}} - \frac{k_{x2}}{\epsilon_2} B'_2 e^{-j\frac{d}{2}k_{x2}} = \frac{k_{x1}}{\epsilon_1} A_1 e^{j\frac{d}{2}k_{x1}} - \frac{k_{x1}}{\epsilon_1} B_1 e^{-j\frac{d}{2}k_{x1}}$$

#### 1.1.4 Condition on the parity

It is then possible to combine the equations of continuity to derive the  $A_1/B_1$  ratio.

**At  $x=-d/2$**

$$A_2 e^{-j\frac{d}{2}k_{x2}} + B_2 e^{j\frac{d}{2}k_{x2}} = A_1 e^{-j\frac{d}{2}k_{x1}} + B_1 e^{j\frac{d}{2}k_{x1}}$$

$$\frac{k_{x2}}{\epsilon_2} A_2 e^{-j\frac{d}{2}k_{x2}} - \frac{k_{x2}}{\epsilon_2} B_2 e^{j\frac{d}{2}k_{x2}} = \frac{k_{x1}}{\epsilon_1} A_1 e^{-j\frac{d}{2}k_{x1}} - \frac{k_{x1}}{\epsilon_1} B_1 e^{j\frac{d}{2}k_{x1}}$$

Assuming  $B_2=0$ , this gives rise to:

**Equation 42**

$$\frac{A_1}{B_1} = \frac{\left( \frac{k_{x1}}{\epsilon_1} + \frac{k_{x2}}{\epsilon_2} \right)}{\left( \frac{k_{x1}}{\epsilon_1} - \frac{k_{x2}}{\epsilon_2} \right)} e^{j d k_{x1}}$$

**At  $x=+d/2$**

$$A_2' e^{j\frac{d}{2}k_{x2}} + B_2' e^{-j\frac{d}{2}k_{x2}} = A_1 e^{j\frac{d}{2}k_{x1}} + B_1 e^{-j\frac{d}{2}k_{x1}}$$

$$\frac{k_{x2}}{\epsilon_2} A_2' e^{j\frac{d}{2}k_{x2}} - \frac{k_{x2}}{\epsilon_2} B_2' e^{-j\frac{d}{2}k_{x2}} = \frac{k_{x1}}{\epsilon_1} A_1 e^{j\frac{d}{2}k_{x1}} - \frac{k_{x1}}{\epsilon_1} B_1 e^{-j\frac{d}{2}k_{x1}}$$

Assuming  $A_2=0$ , this gives rise to:

**Equation 43**

$$\frac{\left(\frac{k_{x2}}{\epsilon_2} + \frac{k_{x1}}{\epsilon_1}\right)}{\left(\frac{k_{x1}}{\epsilon_1} - \frac{k_{x2}}{\epsilon_2}\right)} e^{j d k_{x1}} = \frac{B_1}{A_1}$$

From the equations above, one can notice that:

**Equation 44**

$$\frac{\left(\frac{k_{x2}}{\epsilon_2} + \frac{k_{x1}}{\epsilon_1}\right)}{\left(\frac{k_{x1}}{\epsilon_1} - \frac{k_{x2}}{\epsilon_2}\right)} e^{j d k_{x1}} = \frac{B_1}{A_1} = \frac{A_1}{B_1}$$

So:

**Equation 45**

$B_1^2 = A_1^2$ . This leads to 2 cases:

Even case:  $B_1 = A_1$

Odd case:  $B_1 = -A_1$

1.2.5 Even case  $B_1=A_1$

**At  $x=-d/2$**

Using Euler's formula:

**Equation 46**

$$\frac{k_{x2}}{\epsilon_2} \frac{A_2}{A_1} e^{-j\frac{d}{2}k_{x2}} = -2j \frac{k_{x1}}{\epsilon_1} \sin\left(\frac{d}{2}k_{x1}\right)$$

**At  $x=+d/2$**

**Equation 47**

$$\frac{k_{x2}}{\epsilon_2} \frac{B'_2}{A_1} e^{-j\frac{d}{2}k_{x2}} = -2j \frac{k_{x1}}{\epsilon_1} \sin\left(\frac{d}{2}k_{x1}\right)$$

1.2.6 Odd case  $B_1=-A_1$

**At  $x=-d/2$**

**Equation 48**

$$\frac{k_{x2}}{\epsilon_2} \frac{A_2}{A_1} e^{-j\frac{d}{2}k_{x2}} = 2 \frac{k_{x1}}{\epsilon_1} \cos\left(\frac{d}{2}k_{x1}\right)$$

**At  $x=+d/2$**

**Equation 49**

$$\frac{k_{x2}}{\epsilon_2} \frac{B'_2}{A_1} e^{-j\frac{d}{2}k_{x2}} = -2 \frac{k_{x1}}{\epsilon_1} \cos\left(\frac{d}{2}k_{x1}\right)$$

1.2.7 Transcendental equations

1.2.7.a Transcendental equations: Even case

**At  $x=-d/2$**

Starting from:

$$\frac{A_2}{A_1} = -2j \frac{k_{x1}\epsilon_2}{\epsilon_1 k_{x2}} \sin\left(\frac{d}{2}k_{x1}\right) e^{j\frac{d}{2}k_{x2}}$$

And:

$$A_2 e^{-j\frac{d}{2}k_{x2}} = A_1 e^{-j\frac{d}{2}k_{x1}} + B_1 e^{j\frac{d}{2}k_{x1}}, \text{ which can be rewritten as:}$$

$$\frac{A_2}{A_1} = 2 \cos\left(\frac{d}{2}k_{x1}\right) e^{j\frac{d}{2}k_{x2}}$$

We now find:

**Equation 50**

$$\frac{k_{x2}}{\epsilon_2} = -j \frac{k_{x1}}{\epsilon_1} \tan\left(\frac{d}{2}k_{x1}\right)$$

### 1.2.7.b Transcendental equations: Odd case

**At  $x=-d/2$**

Starting from:

$$\frac{A_2}{A_1} = 2 \frac{k_{x1} \epsilon_2}{\epsilon_1 k_{x2}} \cos\left(\frac{d}{2} k_{x1}\right) e^{j \frac{d}{2} k_{x2}}$$

And:

$$A_2 e^{-j \frac{d}{2} k_{x2}} = A_1 e^{-j \frac{d}{2} k_{x1}} + B_1 e^{j \frac{d}{2} k_{x1}}, \text{ which can be rewritten as:}$$

$$\frac{A_2}{A_1} = -2j \sin\left(\frac{d}{2} k_{x1}\right) e^{j \frac{d}{2} k_{x2}}$$

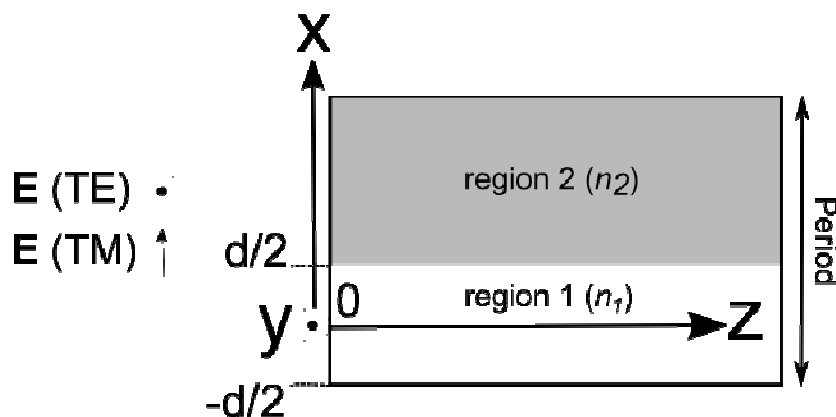
We now find:

**Equation 51**

$$\frac{k_{x2}}{\epsilon_2} = j \frac{k_{x1}}{\epsilon_1} \cot\left(\frac{d}{2} k_{x1}\right)$$

## 2. Periodically Arranged Openings

In this section, we study the propagation of an electromagnetic lightwave through an opening of width “d” which is placed within a periodic array of openings with a periodicity denoted “P” (See **Figure 3**). As previously mentioned, the subscript “1” (“2”) is attributed to the parameters of region 1 (region 2) and two polarization states (TE and TM) will be successively considered. Once again, the goal is to describe the different steps leading to the transcendental equations, as can be found in [2]. In the following, we introduced the continuity of the field at the interfaces by assuming a null dephasing.



**Figure 3:** Opening (region 1) drilled in a material (region 2) with an infinite extent in the z direction, depicted over one period. The lightwave is propagating along the z direction.

## 2.1 TE polarization

### 2.1.1 General expression of the field

**For  $-d/2 < x < +d/2$**

**Equation 52**

$$E_y = A_1 \cos(xk_{x1})$$

**For  $x > +d/2$**

**Equation 53**

$$E_y = A_2 \cos\left(\left(x - \frac{P}{2}\right)k_{x2}\right)$$

### 2.1.2 Continuity of the tangential component $E_y$

**At  $x = +d/2$**

**Equation 54**

$$A_1 \cos\left(\frac{d}{2}k_{x1}\right) = A_2 \cos\left(\left(\frac{d}{2} - \frac{P}{2}\right)k_{x2}\right)$$

### 2.1.3 Continuity of the $H_z$ component

$H_z$  is related to  $E_y$  by the Maxwell's equations through:

**Equation 55**

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

This implies that:

**For  $-d/2 < x < +d/2$**

**Equation 56**

$$H_z = -j \frac{k_1}{\omega\mu_1} A_1 \sin(xk_{x1})$$

**For  $x > +d/2$**

**Equation 57**

$$H_z = -\frac{A_2}{\omega\mu_2} k_{x2} \sin\left(\left(x - \frac{P}{2}\right)k_{x2}\right)$$

Due to the continuity of  $H_z$ :

**At  $x = +d/2$**

**Equation 58**

$$k_{x1} A_1 \sin\left(\frac{d}{2} k_{x1}\right) = A_2 k_{x2} \sin\left(\left(\frac{d}{2} - \frac{P}{2}\right) k_{x2}\right)$$

**2.1.4 Transcendental equations**

By dividing term by term the following formula:

$$k_{x1} A_1 \sin\left(\frac{d}{2} k_{x1}\right) = A_2 k_{x2} \sin\left(\left(\frac{d}{2} - \frac{P}{2}\right) k_{x2}\right)$$

$$A_1 \cos\left(\frac{d}{2} k_{x1}\right) = A_2 \cos\left(\left(\frac{d}{2} - \frac{P}{2}\right) k_{x2}\right)$$

One can find:

**Equation 59**

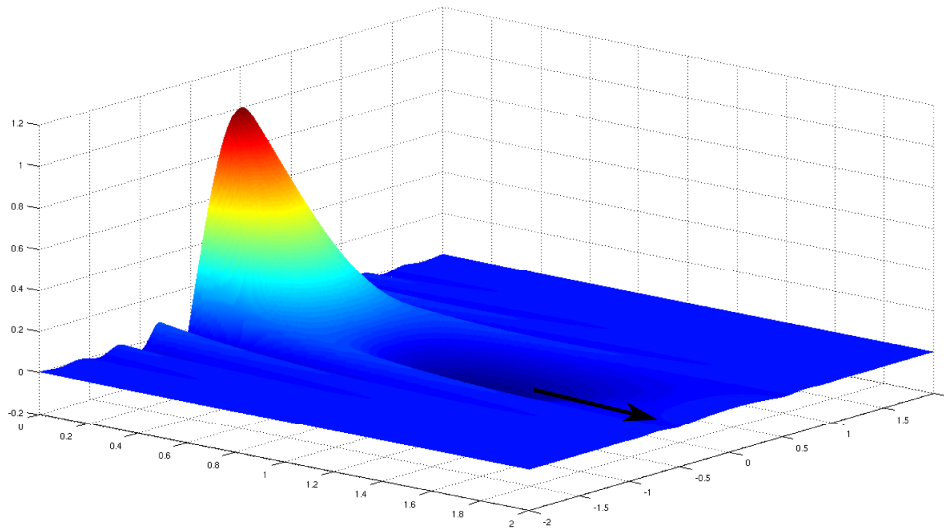
$$k_{x1} \tan\left(\frac{d}{2} k_{x1}\right) = -k_{x2} \tan\left(\left(\frac{d}{2} - \frac{P}{2}\right) k_{x2}\right)$$

**2.1.5 Example**

So as to illustrate the equation above, we can study the same configuration as in section 1.1.8 (region 1 is air ( $n_1=1$ ), region 2 is a material with an optical index  $n_2=5+i$  and the width of the hole is set to 0.5 (in lambda units)) but in this example, a periodic condition along the x axis is introduced as shown on **Figure 3**. By setting  $P=2$  (in lambda units), the x and z components of the wave vector can be computed again, which gives:

- $k_{x(\text{region } 1)} = 6.040 - 0.730i$
- $k_{x(\text{region } 2)} = 31.335 + 6.158i$
- $k_{z(\text{region } 1)} = k_{z(\text{region } 2)} = 2.552 + 1.729i$

The normalized field profile of the lightwave propagating through this periodic array of subwavelength opening is reported in **Figure 4**.



**Figure 4:** Normalized field profile of an electromagnetic lightwave propagating in the direction indicated by the arrow (represented over one period for the first order). The field amplitude is maximum in the middle of region 1 (air) and is decaying both laterally, towards the absorbing medium, and in the propagation direction.

## 2.2 TM polarization

### 2.2.1 General expression of the field

**For  $-d/2 < x < +d/2$**

**Equation 60**

$$H_y = A_1 \cos(xk_{x1})$$

**For  $x > +d/2$**

**Equation 61**

$$H_y = A_2 \cos\left(\left(x - \frac{P}{2}\right)k_{x2}\right)$$

### 2.2.2 Continuity of the tangential component $H_y$

**At  $x = +d/2$**

**Equation 62**

$$A_1 \cos\left(\frac{d}{2}k_{x1}\right) = A_2 \cos\left(\left(\frac{d}{2} - \frac{P}{2}\right)k_{x2}\right)$$

### 2.2.3 Continuity of the $E_z$ component

$E_z$  is related to  $H_y$  by the Maxwell's equations through:

**Equation 63**

$$\frac{\partial H_y}{\partial x} = +j\omega\epsilon E_z$$

This implies that:

**For  $-d/2 < x < +d/2$**

**Equation 64**

$$E_z = j \frac{k_{x1}}{\omega\epsilon_1} A_1 \sin(xk_{x1})$$

**For  $x > +d/2$**

**Equation 65**

$$E_z = j \frac{k_{x2}}{\omega\epsilon_2} A_2 \sin\left(\left(x - \frac{P}{2}\right)k_{x2}\right)$$

Due to the continuity of  $H_z$ :

**At  $x = +d/2$**

**Equation 66**

$$\frac{k_{x1}}{\epsilon_1} A_1 \sin\left(\frac{d}{2}k_{x1}\right) = \frac{k_{x2}}{\epsilon_2} A_2 \sin\left(\left(\frac{d}{2} - \frac{P}{2}\right)k_{x2}\right)$$

#### 2.2.4 Transcendental equations

By dividing term by term the following formula:

$$\frac{k_{x1}}{\epsilon_1} A_1 \sin\left(\frac{d}{2}k_{x1}\right) = \frac{k_{x2}}{\epsilon_2} A_2 \sin\left(\left(\frac{d}{2} - \frac{P}{2}\right)k_{x2}\right)$$

$$A_1 \cos\left(\frac{d}{2}k_{x1}\right) = A_2 \cos\left(\left(\frac{d}{2} - \frac{P}{2}\right)k_{x2}\right)$$

One can find:

**Equation 67**

$$\frac{k_{x1}}{\epsilon_1} \tan\left(\frac{d}{2}k_{x1}\right) = -\frac{k_{x2}}{\epsilon_2} \tan\left(\left(\frac{d}{2} - \frac{P}{2}\right)k_{x2}\right)$$

### **3. References**

- [1] Tremblay P. Guide d'onde diélectrique à 3 couches. 38–49, 2007.
- [2] Vadim Karagodsky, Forrest G. Sedgwick, and Connie J. Chang-Hasnain. Theoretical analysis of subwavelength high contrast grating reflectors. *Opt. Express*, 18(16):16973–16988, Aug 2010.

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