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On Cardinal Numbers and Self Adjusting \mathcal{M} -sets. A Mathematical Letter.

J.I. Pillay¹

Introduction

This is a technical letter where we outline certain considerations involving cardinal numbers. Notations and terminology follow similarly from our previous article[1]. The mechanistic approach to the logical development of arithmetic in our previous article had proven so useful, that we wish here to extend its application to cardinality. Our Approach involves a mechanistic approach similar to that of our previous article with no reference to formal set theory[2].

Outline

We will again form our analysis on \mathcal{B}_i as these can be paired off with the naturals N . A self adjusting \mathcal{M} -set is best seen via the following example:

If it were required, via the existing mechanisms surrounding arithmetic operations, that a representation to be formed expressing $\frac{1}{11}$, is met with recursion, as can be seen from the multiplication $1 \approx (0.1)(11) = 1.1$. As such, the only means of removing the 0.1 in 1.1 via the multiplicative mechanism is by adjusting the term to which 11 is multiplied. i.e (0.011)(11). To remove the 0.01 we multiply (0.01011)..(0.0101011) and so on.

This necessary recursion or 'adjusting' of the (0, 1) symbols is informally defined to be a self adjusting mechanism.

There are two ways in which it seems possible to form an irrational number.

A) Via operations on N .

B) Via Construction.

The famous diagonalisation argument of Cantor can be seen as a form of constructing irrationals.

To see the mechanisms at play in the formation of rationals we introduce the following notation for a number followed by a decimal value :

$$\mathfrak{S} := n : \overline{P\{p\}}$$

P here denotes the decimal portion of the number and $\{p\}$, a value indicating the current position of the decimal if n, P were combined. From this one can define a rational number as :

$$\sum_{p \in pos(P)}^M n : \overline{P\{p\}} \quad (1)$$

¹With fond memories of a true friend and advisor without whom many endeavors would not have been possible. For Dr.W.E.Meyer, Many thanks.

Where it is necessarily required that M be finite. This tells us that in order for \mathfrak{S} to be rational, it is required that $\forall p_i, p_{i+1} \in pos(P)$ there is a finite maximum ℓ such that $p_{i+1} - p_i \leq \ell$. For if this were not the case then no finite summation of the form (1) would be capable of forming a natural number. This brings us to the conclusion that The only other mutually exclusive case of mechanisms forming P is the case where no finite ℓ exists, specifically, no maximum measure between elements $p_i, p_{i+1} \in pos(P)$ exists. This set will form the set of irrational numbers, as a **finite** summation of the form (1) will not be capable of reducing this mechanistic set to an integer or rational.

We wish now to consider all possible partitions one can form on the set $\mathfrak{S}_{\mathcal{R}}$. By doing this one can perhaps study better the cardinality associated with the partition sets.

We now have a clear definition in terms of ℓ -measure, the procedure for forming irrational sets. Using this, one can form partitions in many different ways and it is the manner in which this can be done that is of interest. This is because the nature of such partitions (as we shall see) gives indication to the types of mappings possible with the naturals.

Before we proceed on with forming partitions, consider the following. as was demonstrated by Cantor, algebraic fields are in one to one correspondence with the naturals. Given our new insights, a question is, what property of the reals might enable these to map to the naturals? Given the nature of irrational numbers as described above, it is easy to see that only multiplications between the irrationals can form rationals, as the multiplication itself constitutes an infinite summation.

furthermore, it is trivial to see that, forming a partition via any steady change in $prog\mathcal{I}$ will result in a one to one mapping with the naturals, as such, the partition formed for this not to be the case must have the exact properties as those of the set $\mathfrak{S}_{\mathcal{R}}$

The above considerations brings us to this one single method of forming a partition that is not of the previous types.

Any infinite set of the form :

$$\{p_o \in pos((I))\} | prog(p_o) | \forall \ell_i \in \ell, \exists \ell_k \in \ell | \ell_k > \ell_i \quad (2)$$

forms an irrational number. A partition P of such a set which is also of the same type, would be all elements belonging to a set of the type (2) such that $\forall \ell'_i \in P, \ell_i > \ell'_i$. However due to the progression of ℓ' being divergent, both sets are actually the same.

Another way of forming a similar partition P_g is by leaving gaps in the set P . i.e. a Set of the form with ℓ forming between alternate pairs of p_j .

Note however that one can still form the one-to-one mapping $\ell(P_g) \mapsto \ell_{\mathfrak{S}}$, thus expressing the same cardinality as that of \mathfrak{S} .

It is for these reasons that it is conceivable that cardinalities between

$$\aleph_0 < C < 2^{\aleph_0}$$

are unlikely.

References

[1] J.I.Pillay A New Discourse on Incompleteness and Proof Systems (2012). HAL

[2] Cohen, Paul J. (December 15, 1963). "The Independence of the Continuum Hypothesis". *Proceedings of the National Academy of Sciences of the United States of America* 50 (6): 1143-1148. doi:10.1073/pnas.50.6.1143. JSTOR 71858. PMC 221287. PMID 16578557.

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