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A simultaneous approach to the estimation of Risk Aversion and the Subjective Time Discount Rate*

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Abstract

In this paper we analyze a sample of 1,832 individuals who responded to six randomly generated lottery questions that differ with respect to chance, prize and the timing of the draw. Using a model that explicitly allows for consumption smoothing, we obtain an estimate of relative risk aversion of 82. Instead, assuming consumption to be immediate gives an estimate of 2, close to what is traditionally reported, while a model of full asset integration gives estimates higher by several orders of magnitude. Our results show that estimated risk aversion is sensitive to the assumptions made with respect to the consumption profile and that it is possible to determine the level of asset integration endogenously. The average subjective time discount rate, which includes a preference for the present, equals 6% per month. It is found that both parameters vary strongly over individuals and that the variation can be explained by income, age, gender, and entrepreneurship, consistent with the majority of previous evidence.

JEL classification: D12; D80; D90; E21
Keywords: Expected Utility; Asset Integration; Risk Aversion; Time Preference; Lotteries; Hypothetical Questions

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1. Introduction

In standard economic analysis, decisions that involve a risk or a time dimension are traditionally analyzed separately within the framework of expected- or discounted-utility respectively. Many choice situations, however, concern both dimensions. For example, if we observe an investor who is reluctant to invest in a project promising large but risky long run profits, should we infer from this that he is risk averse, or should we ascribe his reluctance to impatience, preferring consumption now to later? Similarly, are students who quit school early impatient, or are they more inclined to take the current, more certain wage offer because they are more risk averse?

The general problem that both risk and time preferences can affect choices simultaneously has been acknowledged for some time in the macro-economic literature, where both the discount rate and relative risk aversion play a role in the estimation of the Euler equation of aggregate consumption. The separation of risk and time preferences has received considerable attention in this field, both theoretically and empirically (Kreps and Porteus, 1978; Hall, 1988; Epstein and Zin, 1989; Weil, 1990). In the micro-econometric literature that is concerned with the elicitation of these preferences at the individual level, however, risk and time dimensions are almost always treated separately (see for instance Barsky et al., 1997; Harrison et al., 2005). In order to see how this can affect the results, consider the willingness to pay for a simple lottery. In this context risk aversion is often estimated without acknowledging that the potential gains from such a lottery, in case they are big, will not be spent immediately but will be spread out over time. Hence, the value of the lottery-prize, and consequently the lottery, will differ between patient- and impatient-individuals, making time preferences a confounding factor. Similarly, when estimating time preferences the uncertainty associated with delay, and also utility curvature, is often neglected, resulting in an estimation bias (Frederick et al., 2002; Andersen et al., 2008).

The aim of the present paper is to show how, under plausible assumptions on consumption, time preferences affect risky decision making. To illustrate this, we model the willingness to pay for a lottery in the discounted expected utility framework, acknowledging that a large prize will not be consumed immediately but spread optimally over time. Assuming that individuals are borrowing constrained, consumption will be spread over a finite period that is endogenously determined. This model forms an intermediate case between two extremes: (i) no smoothing, current consumption equals current income, and (ii) no capital constraints. In the first case only current consumption is affected, while in the second the prize is integrated into total wealth. In that case time preferences determine the shape of the optimal profile, but not the curvature of the utility of wealth, yielding time preferences irrelevant. We will see that these assumptions have a great influence on the estimated risk aversion because of their differing levels of asset integration. The novelty of our model is that the level of asset integration is endogenously determined and that risk and time preferences are estimated jointly.

To estimate the model, we use a large survey in which we ask for the willingness to pay for different lotteries that differ with respect to chance, prize, and timing of the draw. Using our model these data allow for the joint estimation of the coefficients of relative risk aversion and the time preference rate. The variation in these parameters can be explained by individual characteristics such as income, age, education, gender, intensity of religious participation, entrepreneurship, and other variables. In all estimated equations most effects are significant, plausible and consistent with the findings in most studies that relate risk and time preferences to demographics.

According to our knowledge there are only a few studies that simultaneously report estimates on risk and time preferences at the micro level. Even if these studies have data on

choices with a risk and/or a time dimension, the risk and time effects are analyzed separately (Barsky et al., 1997; Anderhub et al., 2001; Eckel et al., 2005; Harrison et al., 2005). The only exception is the study by Andersen et al. (2008), who simultaneously estimate risk and time preferences for a representative sample of the Danish population. Contrary to our model however, these authors do not model how the prize is consumed.

The structure of the present paper is as follows. Section 2 gives more background on the estimation of risk and time preferences in the literature. The model is outlined in section 3, followed by a description of the data in section 4. Section 5 presents an analysis of the survey results assuming homogeneity in preferences, an assumption that is dropped in section 6. Section 7 concludes, followed by appendices A and B that give derivations of the key mathematical equations in the paper.

2. Background

Decisions under uncertainty are, traditionally, described by the Von Neumann - Morgenstern expected utility (EU) model, which defines the utility to be maximized as the expectation of the utilities of the random alternatives. The expected utility model is not beyond discussion. We refer to Allais (1953), Kahneman and Tversky (1979), Tversky and Kahneman (1992) and Rabin and Thaler (2001) for critique and alternatives. An important ingredient in this framework is the specification of the utility function. The most popular one-parameter specification is the constant relative risk aversion (CRRA) function defined by $u(y) = y^{1-\gamma} / \left(1-\gamma\right)$, where the coefficient of relative risk aversion is constant and given by γ. Empirical estimates of this parameter vary greatly at the micro level, and they seem to be particularly sensitive to the magnitude of the stakes and whether outcomes are modeled in terms of final wealth or in terms of gains and losses (Rabin, 2000; Rabin and Thaler, 2002; Meyer and Meyer, 2005; Wakker, 2005). Most studies of risk aversion look at gambles, at decisions on the choice of risky assets in portfolios, or at the choice of insurance policies.[1] Another important source of individual risk aversion estimates are experiments.[2] Finally, measures of risk aversion obtained through hypothetical questions have also been used to explain choice under uncertainty.[3]

A very similar model, the discounted utility model (DU), describes the problem of decisions over time where utilities at different moments of time are exponentially weighted by a subjective time discount rate ρ. Also this model is not beyond discussion, as is clear from the extensive list of anomalies that have been reported in the literature (e.g. Loewenstein and Prelec, 1991; Frederick et al., 2002). The empirical estimates of the parameter ρ vary a great deal. They are mostly derived from consumption-smoothing models, experimental choice situations, or hypothetical questions.[4] In the first type of model, identification of time preferences relies on the assumption that agents have access to perfect capital markets and that they smooth their consumption according to their rate of time preferences. Data on an individual's consumption flow serve to assess this rate. Most studies of time preference, however, exploit the last two types of data. Here, time preferences are

[1] Two studies that look at gambles are Jullien and Salanié (2000) and Beetsma and Schotman (2001). Pållson (1996) looks at portfolio composition, and Halek and Eisenhauer (2001) examine insurance choices.

[2] Examples are Anderhub et al. (2001), Holt and Laury (2002) and Harrison et al. (2005).

[3] Examples of this approach can be found in Barsky et al. (1997), Donkers et al. (2001), Hartog et al. (2002), Guiso and Paiella (2006) and Dohmen et al. (2006).

[4] For example Trostel and Taylor (2001) estimate time preferences using a consumption smoothing model while Benzion et al. (1989), Coller and Williams (1999), Read (2001), Anderhub et al. (2001) and Harrison et al. (2002) use experiments and Barsky et al. (1997), Donkers and van Soest (1999), Lazaro et al. (2001) and Kapteyn and Teppa (2003) use hypothetical questions.

identified by looking at individual choices between *income* streams. Then restrictions have to be put on either the possibilities of intertemporal arbitrage or the subjects' optimizing behavior. The implicit assumption in most studies seems to be that individuals ignore the possibility of intertemporal arbitrage either because they are unaware of it or because they are unable to exploit it (Pender, 1996).[5] The fact that imputed interest rates do not converge to market interest rates justifies this assumption (Frederick et al., p. 381). Estimated discount rates per year vary from 10% to well over 100% (Frederick et al., pp. 377-381).

As said before, the only study integrating both risk and time preferences at the individual level is Andersen et al. (2008).[6] Using responses to a list of (binary) choices between either lotteries or payoff profiles, these authors simultaneously identify risk and time preferences. Under the assumption of EU (over money gains) their risk aversion task pins down utility curvature, which is then used to obtain an estimate of the subjective time discount rate corrected for concavity of the utility function. The correction decreases the estimates, confirming the existence of an upward bias when utility curvature is neglected.

In their setup, risk aversion affects the estimated rate of time preference (through utility curvature), but not the other way round. Hence, the estimated level of risk aversion does not control for the fact that the consumption of the lottery gains will be spread over time. Since the stakes are relatively low in their study, this is unlikely to pose a big problem because consumption can be assumed to be approximately immediate in that case. When the stakes are large however, time preferences will also affect how outcomes are evaluated. In that setting also the consumption profile has to be modeled. This will be illustrated by the simple model in the next section.

3. The Model

In this section we consider the value of a lottery that is, the maximum amount an individual is willing to pay for a ticket for a specific lottery. First we will describe this problem using the standard expected utility model without a time dimension. Then we view the problem in an intertemporal framework. The standard model turns out to be a special case where individuals face no liquidity constraints, while the model that does not include wealth corresponds to the case where consumption is immediate and borrowing and saving are not possible. Then we present an intermediate case where saving is possible, but borrowing is not, and see that time preferences become an additional parameter in the problem.

Let us consider an individual with non-stochastic monthly income y, and let W denote the net present value of this income stream. Suppose that he gets an offer to participate in a lottery that will give prize Z with chance π. Moreover, let the price of a ticket be denoted by a and the individual's utility of wealth be denoted by $U(\bullet)$. Then, the expected utility of accepting the offer will be $(1-\pi)U(W-a)+\pi U(W-a+Z)$. The maximum amount an individual is prepared to pay for taking part in the lottery is the amount A, which solves the indifference equation

$$(1-\pi)U(W-A)+\pi U(W-A+Z)=U(W). \qquad (1)$$

We call A the *value of the lottery* or *the reservation price.* This is the traditional, timeless, (normative) framework that is used to model decisions under risk. In this model all money

[5] Coller and Williams (1999) and Harrison et al. (2002) form notable exceptions. In these studies of time preference the authors explicitly take censoring due to market interest rates into account.

[6] Technically, studies that estimate the reference wealth level (Heinemann, 2007; Harrison et al., 2007) can be thought to estimate a reduced form of a model that includes a time dimension. Since these studies do not explicitly model the time dimension, we do not consider them in the class of models simultaneously integrating risk and time preferences.

involved in the lottery is integrated into lifetime wealth, and the individuals' risk aversion is determined by the degree of concavity of utility with respect to wealth.[7]

Now we view the same problem within an intertemporal framework. We assume a discounted expected utility model in continuous time with a CRRA instantaneous utility function $u(\bullet)$ defined over present consumption $c\left(t\right)$, and (subjective) discount rate ρ. The utility of a consumption profile c is then given by $\int_{t=0}^{\infty} e^{-\rho t} u\left(c\left(t\right)\right) dt$.

If there are no liquidity constraints, the individual has preferences only over present discounted sums of money. This is true because, through intertemporal arbitrage, all income streams with the same discounted value give rise to the same consumption possibilities. Hence, in this setting time preferences do not matter and additional money is simply integrated with wealth. The utility of an amount of wealth W is given by the instantaneous utility function $u\left(W\right)$.[8] Hence, the indifference equation under the assumption of full consumption smoothing is

$$\left(1-\pi\right)u\left(W-A\right)+\pi u\left(W-A+Z\right)=u\left(W\right). \tag{2}$$

Thus, the *standard* expected utility model defined over wealth can be interpreted as describing decision under risk under full consumption smoothing.

If we now suppose the other extreme (i.e. that it is not possible to borrow or save) then current income equals current consumption. In that case only the present (month) is affected, with consumption equal to $y-A+Z$ if the prize is won, and $y-A$ otherwise. We will call this the *immediate* model. The indifference equation for this model is

$$\left(1-\pi\right)u\left(y-A\right)+\pi u\left(y-A+Z\right)=u\left(y\right). \tag{3}$$

In both cases the level of risk aversion is determined by the curvature of the instantaneous utility function. Let this function be defined over changes-in-wealth *x*, plus some reference level R, that is $u\left(R+x\right)$. The standard model then corresponds to the case where the reference point is wealth (i.e. $R=W$), whereas in the immediate model it is present consumption (i.e. $R=y$). This difference has serious repercussions for the inferred level of relative risk aversion. Various authors have noted that the estimate of relative risk aversion is very sensitive to the level of the outcome dimension. Wakker (2008), for example, states that the CRRA family is not invariant to the level of inputs, and Meyer and Meyer (2005) show more specifically that the inferred measure of relative risk aversion increases (almost) proportionally with the assumed origin of the input scale. A simple numerical example illustrates this.

Consider an individual earning *y=500* a month, with corresponding lifetime wealth *W=200.000*. Say this individual is prepared to pay $A=100$ for a lottery with $(\pi,Z)=(.5,1000)$. Then, if we assume the immediate model holds ($R=W$), we would infer that $\hat{\gamma}\approx 4$, while we get an estimate of $\hat{\gamma}\approx 1385$ if the standard model ($R=W$) holds! More generally, if we gradually increase *R*={500, 1000, 5000, 10000, 100.000,

[7] There is a debate in the literature on whether the expected utility model presupposes that outcomes are defined in terms of final wealth or not. Both Cox and Sadiraj (2006) and Rubinstein (2006) argue that it is not, and conclude that Rabin's (2000) critique of the model does not necessarily hold because of this. In a recent working paper Wakker (2005) argues that even though the fundamental axioms of EU do not say anything about the nature of outcomes, the model only has normative content if defined over final wealth. Hence, in this paper we assume the expected utility model to be defined over wealth.

[8] We refer to Schechter (2007) for the technical details on the intertemporal optimization problem.

200.000}, we find $\hat{\gamma}$ ={4, 7.5, 35, 70, 693, 1385} respectively.[9] Hence, the assumptions made about the consumption profile are of crucial importance. A practical example is provided by Schechter (2007), who reports estimates of relative risk aversion close to 2 if the reference level is daily consumption and estimates of over 2000 if outcomes are added to lifetime wealth.

The intuition behind this is that if the consumption of a given amount of small money is spread over the entire lifetime, the effect on each period's consumption will be negligible. To generate appreciable risk aversion, the per-period utility function must then be very concave, translating in a high estimate of relative risk aversion. Rabin (2000) shows that this has bizarre implications for high stake risk aversion. If consumption is confined to the present period, however, the relative impact of the same amount of money is much larger. In that case mild curvature can generate same small stake risk aversion, circumventing Rabin's extreme implication. Indeed, most studies that find appreciable small stake risk aversion do not integrate outcomes with wealth and, thereby, find moderate values of risk aversion, mostly below 10. Rabin and Thaler (2002) suggest that wealth is often neglected not only because it is hard to measure, but also because "it would make referees worry" if the extreme measures of relative risk aversion were reported that are implied by these studies had outcomes been added to wealth (see for example Holt and Laury 2002; Harrison et al. 2005; Dohmen et al. 2006).

An intermediate case: asset integration endogenously determined.

The assumptions underlying both previous models may be seen as extreme positions. If we take an intermediate position, as we do in this paper, where saving is assumed possible while borrowing is not, the consumption of the prize will be spread over time, albeit over a finite period. In that case the outcomes affect future periods, but they are not fully integrated into lifetime wealth as in the standard model. To see this, we consider the same model with a borrowing constraint. If it is not possible to borrow and individuals are assumed to be impatient (i.e. $\rho > 0$), then baseline consumption is equal to monthly income (i.e. $c(t) = y(t) = y$). Now we will make some specific assumptions about the timing of the income flows associated with the lottery. These assumptions are made for simplicity. A different specification will yield different quantitative results, but they do not affect the main qualitative message of the paper that if one introduces capital restrictions, both risk and time preferences operate simultaneously and the level of asset integration becomes endogenous.

Let the lottery ticket be bought at price A, the cost of which is assumed to be spread evenly during the period $[0,\alpha]$ before the draw of the lottery, which occurs later, at time α. Hence, in the period before the draw of the lottery, consumption is reduced to $(y - \frac{A}{\alpha})$. If the prize, an amount Z, has been won, it becomes available at time α and may be gradually spent over the period in the future $[\alpha,\infty)$. Let $P(t)$ denote the fraction of Z that is spent up to time $t \in (\alpha,\infty)$ and let $p(t)$ denote its derivative. Trivially, we may define $p(t) \equiv 0$ for $0 \leq t < \alpha$. We have the constraints

$$\int_0^\infty p(t)\,dt = 1 \text{ and } p(t) \geq 0 \text{ for all } t > 0. \qquad (4)$$

[9] For large wealth levels it can be shown that $\hat{\gamma} \approx 1 + \frac{-\log(1-\pi)}{A} W$, which is nearly proportional in W.

Finally, the lottery buyer may or may not take into account that his prize may bear interest at a rate *r* when deposited in a savings account. Given these assumptions, the value A of the lottery is found by equating the utility value without buying a ticket to the expected value when buying a ticket. We have

$$\begin{aligned}\int_0^{\alpha} e^{-\rho t}u(y-A/\alpha)\,dt+\pi\int_{\alpha}^{\infty} e^{-\rho t}u\left(y+e^{r(t-\alpha)}p(t)Z\right)dt\\ +(1-\pi)\int_{\alpha}^{\infty} e^{-\rho t}u(y)\,dt=\int_0^{\infty} e^{-\rho t}u(y)\,dt.\end{aligned} \tag{5}$$

It follows that the value of A also depends on the spending pattern $p(t)$ according to which the prize is spent over time. If we assume that the consumption pattern for the windfall gain can be chosen at will, there will be an optimal spending pattern $\hat{p}(t)$. Then, the value *A* of the lottery is found from the equation

$$\begin{aligned}&\tfrac{1}{\rho}(1-e^{-\rho\alpha})\left(u\left(y\right)-u\left(y-A/\alpha\right)\right)\\ &\quad=\pi e^{-\rho\alpha}\max_{p(\bullet)}\int_0^{\infty} e^{-\rho\tau}\left\{u\left(y+e^{r\tau}p\left(\tau+\alpha\right)Z\right)-u(y)\right\}d\tau\\ &\quad=\pi e^{-\rho\alpha}\int_0^{\infty} e^{-\rho\tau}\left\{u\left(y+e^{r\tau}\hat{p}\left(\tau+\alpha\right)Z\right)-u(y)\right\}d\tau,\end{aligned} \tag{6}$$

where $\tau=t-\alpha$ denotes time, with the draw of the lottery taken as the origin. This equation basically says that the utility loss associated with the payment of the lottery ticket should equal the expected future gains from it. We can derive the optimum pattern $\hat{p}(t)$ of how the prize Z should be spent over future periods from the Euler condition for this problem. This is given by

$$e^{-\rho\tau}u'(y+e^{r\tau}\hat{p}(\tau+\alpha)Z)e^{r\tau}Z=C\,,\ \forall\tau\geq 0\,, \tag{7}$$

where C stands for a constant. This equation and the constraints in (4) imply the optimal spending pattern

$$\hat{p}(\tau+\alpha)=\begin{cases}ce^{-B\tau}-\psi e^{-r\tau} & \text{if}\quad 0\leq\tau\leq T_{\max}\\ 0 & \text{if}\quad \tau>T_{\max}\end{cases}, \tag{8}$$

with $B\equiv\frac{\rho-r}{\gamma}+r$, $\psi\equiv\frac{y}{Z}>0$, and an unknown constant, c. The spending path is decreasing and intersects the horizontal axis in finite time $T_{\max}$. This point is endogenously determined and depends on B, which measures the psychological tradeoff between diminishing instantaneous utility and delay, the 'relative prize' $\psi^{-1}\equiv\frac{Z}{y}>0$, and the (monthly) interest rate r. Hence, the winnings only affect a finite period $\left[0,T_{\max}\right]$. This is a key difference with both previous models that can be viewed as having $T_{\max}=\infty$ and $T_{\max}=1$, respectively. In appendix A we present more details about the determination of the unknown constants c and $T_{\max}$ that pin down the optimal path.

In order to get some idea of how $T_{\max}$ varies with the three parameters, we present Table 1. We see that $T_{\max}$ increases with the relative prize. For instance, let us consider an individual with a monthly income of € 2000 with γ equal to 2, a time discount ρ of 2% (per month),

and an interest rate $r = 4\%$ per year, that is 0.32% per month. Consequently, his B is calculated to be 0.012. The spending period of a prize of one hundred times his monthly income, that is € 200,000, will be 134 months, that is, approximately 11 years. Notice that for most configurations the spending period will be fairly short. The prize is then considered as a windfall profit, to be consumed almost immediately.

Table 1: $T_{\max}$ for different preferences and different relative prizes

$r = 0\%$		ψ				
		0.01	0.1	1	10	100
B	100	0.09	0.07	0.05	0.03	0.01
	10	0.69	0.47	0.26	0.11	0.04
	1	4.66	2.61	1.15	0.42	0.14
	0.1	26.11	11.46	4.16	1.38	0.44
	0.01	114.6	41.62	11.82	4.44	1.41

$r = 0.32\%$		ψ				
		0.01	0.1	1	10	100
B	100	0.09	0.07	0.05	0.03	0.01
	10	0.69	0.47	0.26	0.11	0.04
	1	4.68	2.62	1.15	0.42	0.14
	0.1	26.90	11.75	4.25	1.41	0.45
	0.01	153.7	52.74	17.07	5.44	1.72

From calculations with a variety of realistic interest rates, we found that the effect of *r* on $T_{\max}$ is negligible for small prizes and reasonable ratios ρ / γ. Intuitively, a higher interest rate affects consumption smoothing behavior only when the (relative) prize is very large or when an individual is very patient or risk averse (which makes consumption smoothing more attractive).

Now that we know the optimal spending path, it is possible to evaluate the first part of the integral in (6) and subsequently determine the value of the lottery A. More precisely, we have

$$A = A(\gamma, \rho; y, \pi, Z, \alpha, r). \tag{9}$$

An explicit analytical expression cannot be given (see appendix B for more details). Nevertheless, the question arises whether it would be possible to derive information on γ and ρ from equation (9). More precisely, let us assume an individual n characterized by a specific $(\gamma_n, \rho_n; y_n, r_n)$ combination, where r_n is known. If we offer this individual two different lotteries with different payoff dates, say (π_i, Z_i, α_i) $(i = 1,2)$, and ask for the reservation prices (A_{1n}, A_{2n}) of both lotteries, it will be possible to derive the values (γ_n, ρ_n) from his answers A_{in} by solving the system

$$A_{in} = A(\gamma_n, \rho_n; y_n, \pi_{in}, Z_{in}, \alpha_{in}, r_n),\ i = 1,2. \tag{10}$$

Indeed (10) is a system of two equations in (γ_n, ρ_n). It stands to reason that the system is highly non-linear. Nevertheless, the two unknown parameters are identifiable.

4. The data

The data source used for the empirical analysis is the NIPO Post Initial Schooling Survey that was administered by TNS NIPO, a Dutch (commercial) market research company, in December 2005. It is the third in a series of surveys jointly commissioned by the project group SCHOLAR of the Faculty of Economics and the Max Goote Centre, both of the University of Amsterdam, focusing on issues of educational attainment of employed individuals. Unlike the previous surveys, where individuals where contacted by phone, the 2005 sample was obtained using an internet-questionnaire. Apart from a cost reduction, internet-based surveys make randomization of questions relatively easy, a feature that was

used to obtain more variation in the data. TNS NIPO has a large database of about 200,000 people who have indicated that they are willing to take part in TNS questionnaires. Because a large amount of background characteristics of these respondents is readily available, it is possible to focus on specific groups in the population (i.e. to draw a random sample conditional on these characteristics). As the NIPO Post Initial Schooling Survey focuses mainly on education of working individuals, a random sample of just over three thousand ($N = 3026$) *employed* individuals between the ages of 16 and 65 was drawn.

Although the sample is a good reflection of the Dutch working population, it is, by construction, not representative for the population as a whole. This is not a major problem since there is sufficient heterogeneity in the sample to make it suitable for econometric analysis of the relationship between most variables. Whether the sample is selective with respect to our variables of interest, risk and time preferences, is hard to say since we have no out-of-sample statistics on these variables to compare with. The response to NIPO questionnaires is generally very high, however, because respondents have already indicated a willingness to cooperate. Hence, we assume that our results will not be dramatically affected by self–selection of participation in the survey. Whether the results also hold for *unemployed* is an open question that can only be resolved by using additional data. Using weights we can make the sample more representative for the total population with respect to the dimensions of age, income, and education. Finally, regarding the accuracy of the data, we should keep in mind that most respondents will not have spent too much time on answering the questions, given the overall size of the questionnaire. Consequently, for some questions there may be a considerable random error in the answers.

The questionnaire consisted of 70 questions focusing mainly on education and educational attainment while being employed. The question module on which we are concentrating for this paper is that of the lottery-questions, posed at the end of the survey.[10] This module runs as follows.

> **Question 70 – *i***
>
> Suppose that a lottery ticket is offered to you for a lottery in which N_i people participate (so you have a chance of 1 in N_i that you will win). The prize is a money amount equal to € Z_i. The draw of the lottery will be in α_i months, but you will have to buy the ticket now in order to participate. What is the maximum amount you are willing to pay for the ticket? €…..

Each respondent was confronted with six ($i = 1,...,6$) of these questions, where the parameter-triplets (N_i, Z_i, α_i) were randomly and independently drawn from discrete distributions with $N_i \in \{100,10,5,4,3,2\}$, $Z_i \in \{1000,3000,5000,10000,50000,1000000\}$ and $\alpha_i \in \{1,3,12\}$. No individual was given the same question twice. That is, per individual the parameter-triplet (N_i, Z_i, α_i) was drawn without replacement. The six lotteries differ with respect to the chances of winning, p_i, which are an element of $\{\frac{1}{100}, \frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}\}$, with respect to the size of the prize Z, and also with respect to the time delay between the payment of the lottery ticket and the draw of the prize, α. This randomization gives a lot of variation, both between and within subjects, and allows for the estimation of our parameters of interest, risk aversion and time preference. All 3026 subjects in the sample answered the six lottery questions that were posed to them, but 626 of them did not show any variation in their answers, which is likely due to a lack of interest or to misunderstanding of the lottery

[10] € 1 was equivalent to about $1.26 at the moment (2006) of surveying.

question.[11] We dropped these individuals from the sample, together with those who did not state their income. A total of $N = 1832$ individuals and 10992 usable answers remain, spread randomly over the $6 \cdot 6 \cdot 3 = 108$ different questions. This gives rise to an average of about 102 answers for each lottery. Summary statistics of the answers are given in table 2.

[11] This amount of non-response is quite common in large scale-hypothetical questions on risky choices, where subjects have to perform a matching task that is cognitively quite demanding. For instance, Guiso and Paiella (2006) and Dohmen et al. (2006) were forced to drop 57% and 61% of their observations, respectively.

Table 2: Summary statistics of the lottery questions

		α																	
		1 month						3 months						12 months					
		Z						Z						Z					
p		€ 1000	€ 3000	€ 5000	€ 10000	€ 50000	€ 1000000	€ 1000	€ 3000	€ 5000	€ 10000	€ 50000	€ 1000000	€ 1000	€ 3000	€ 5000	€ 10000	€ 50000	€ 1000000
1/100	mean	9.9	12	11	19	47	38	6.2	8.6	15	24	28	181	7.6	9.5	13	22	29	103
	sd	20.7	12	11	22	150	49	6.1	9.6	20	31	54	1002	14	14	31.11	31	73	487
	median	5	10	10	10	10	15	5	5	10	10	10	20	5	5	5	10	10	15
	n	102	118	98	106	91	93	110	95	102	98	113	104	111	87	97	86	100	114
1/10	mean	22	24	37	72.	198	1083	16	35	40	73	78	140	20	24	39.46	47	179	1177
	sd	53	46	68	172	770	9611	17	70	82	163	174	497	28	44	115.64	76	705	9614
	median	10	10	21	25	20	37.5	10	10	10	20	25	50	10	10	10	15	25	40
	n	98	107	94	91	67	108	90	104	90	87	108	111	100	86	100	97	98	109
1/5	mean	31	43	46	87	158	91	38	66	55	83	169	336	28	29	33.14	72	152	278
	sd	43	68	132	160	328	5464	157	143	113	185	521	117	42.28	39	41.61	164	326	1100
	median	15	17.5	15	25	25	50	10	20	19	25	40	50	10	10	20	25	50	50
	n	103	100	95	84	98	84	94	96	94	93	103	94	104	95	99	93	95	106
1/4	mean	29	43	92	83	335	292	27	41	83	105	345	485	19.63	45	45.38	90	279	453
	sd	48	65	195	227	159	1113	38	80	272	325	1278	2115	26.56	82	116.48	178	1040	1664
	median	10	20	25	25	30	50	10	15	20	25	25	100	10	15	14	25	50	50
	n	109	97	99	103	99	84	102	88	111	103	93	95	109	101	87	94	100	114
1/3	mean	37	59	78	100	213	2149	29	55	78	67	325	524	39.48	54	102.06	104	376	1190
	sd	58	86	181	242	1009	19503	33	124	181	97	1358	1664	115.79	103	312.40	225	1366	9762
	median	10	25	25	25	30	100	15	15	20	25	50	100	10	20	20	25	30	50
	n	95	110	91	94	103	105	99	97	93	92	106	103	98	103	112	99	113	105
1/2	mean	67	119	201	330	457	1125	61	117	143	270	534	2316	42.06	6	135.88	213	737	313
	sd	107	232	375	765	2092	5410	118	263	311	797	1534	12746	77.28	167	252.77	603	2690	754
	median	25	30	50	50	60	100	25	25	50	50	62.5	100	15	20	30	50	75	100
	n	99	100	94	120	97	91	100	101	81	108	100	123	109	104	90	93	93	100

The table shows the great diversity in the proposed lotteries, with expected values ranging from just € 10 to € 500,000. Some of these fall within the range of Dutch popular lotteries (LOTTO, State Lottery) that are likely to be the frame of reference for most of our respondents. These lotteries typically have a very large prize and a very low probability of winning, with an expected value well below € 100.[12] Most of the lotteries proposed in the table however, have a higher probability of success and consequently also a higher mathematical expectation.

The statistics of the answers given to the various questions reveal three things: (1) both the mean and the median answers demonstrate very high risk aversion, with the level of risk aversion (defined as the fraction of the lottery expectation that individuals are willing to pay for it) increasing with the size of the prize, (2) the mean and average answers show an increasing pattern in both chances and monetary outcomes, which means that the average person behaves rationally in the sense that he complies with first order stochastic dominance; the results for discounting appear more mixed, and (3) there is considerable variation in the answers, which is a first indication of heterogeneity in preferences.

The first finding, that of high risk aversion, is not uncommon with hypothetical questions on the willingness to pay for simple lotteries. For instance, using similar hypothetical questionnaires Guiso and Paiella (2006) and Hartog et al. (2002) find average willingness to pay of 36% and 20% respectively. In another study Dohmen et al. (2006) find that 60% of subjects are not willing to invest anything in a hypothetical asset yielding a 200% or 50% return with equal probability, when given an initial endowment of € 100,000. This again points to high risk aversion, even when there are no potential losses. At first glance the high levels of risk aversion found in these studies may appear unrealistic; one might think that if the subjects were presented with the same choice in reality, they might display less risk aversion. Both Guiso and Paiella and Dohmen et al. show, however, that the obtained risk aversion measure has significant explanatory power in predicting risky behaviors, which suggests that simple lottery questions do provide reliable information about risk attitudes. Moreover, Dohmen et al. validate the answers of the simple lottery question by relating them to the choices made in a risk aversion experiment using real incentives and find that both responses correlate well. Also, there is evidence that real incentives do not affect mean results in simple choice tasks, but simply make responses more noisy (Camerer and Hogarth, 1999). Finally, if individuals have an inclination not to reveal their true value, this so called hypothetical bias is generally found to be positive (List and Gallet, 2001), that is, in the direction of overestimation. In our case, this would imply *more* risk aversion when real incentives are used, an effect that has also been found in choice experiments with risky prospects (e.g. Holt and Laury, 2002, 2005). Given that subjects' responses to the survey questions are already quite conservative, we suspect that such an effect is unlikely in our case.

Although we are unaware of studies that show a hypothetical bias in the direction of more risk aversion, some observations suggest this could be the case here. In particular, the finding that the median person does not want to spend more than a hundred euros on a lottery yielding a million euros in a month with 50% probability suggests that people may simply think in terms of € 10 to € 100 to spend on a lottery. It is unlikely that this is driven by liquidity constraints, but it could be explained by the supposed familiarity of the respondents with popular lotteries. Without any learning opportunity, the choice heuristic adopted may be that of buying a "normal" lottery ticket. The discovered preference hypothesis says that,

[12] The Dutch National State Lottery, for instance, with a monthly clientele of 3.5 million tickets, sold at a price of €13.50 in an adult population of about 13 million, offers a chance of success of about 1 in 10 million, with a prize between 1 and 10 million. An interesting comparison of the Dutch popular lotteries has been published in the January 2003-issue of the Dutch consumer monthly *De Consumentengids.*

when given an opportunity to learn, individuals will discover their true preferences and act upon them (Plott, 1996). This would probably yield less risk aversion in our case. Hence, we conjecture that what we measure is an estimate of single shot risk attitudes without any learning opportunities. Interestingly however, higher educated individuals, who can be expected to act upon their true preferences with fewer learning opportunities, show the same median values.

The second notable feature of table 2 is that the median answers are mostly (weakly) increasing in both chance and money outcomes. This principle is violated only four times for outcomes and six times for probabilities, and it is an indication that people comply with dominance. The results with respect to discounting appear more mixed, with some later dated prizes valued less and others higher. At the individual level, consistency can be tested by comparing the willingness to pay for pairs of lotteries where one dominates the other. If we have two lotteries $L_1 = (p_1, Z_1, \alpha_1)$ and $L_2 = (p_2, Z_2, \alpha_2)$ with $p_1 \leq p_2$, $Z_1 \leq Z_2$ and $\alpha_1 \geq \alpha_2$, then we call L_1 (weakly) dominated by L_2. Because the lotteries were drawn randomly, the number of within-individual lottery pairs where one lottery dominates the other differs between respondents. For the whole sample (N=3026), thus including individuals who did not report their income or showed no variation in their answers, there are, on average, 6.22 possible comparisons per individual. Of these, 94,5% comply with the dominance prediction.[13] This rate of consistency is very high, suggesting that the subjects took the questions seriously and thought them through, which strengthens the case that what we find are unbiased answers in a context without learning. It also suggests that the mixed pattern with respect to the time delay observed in the median data is due to individual heterogeneity.

The final observation that can be made from the table is that there is considerable variability in the answers. This can have two causes: (1) between-subject variation, caused by heterogeneity in preferences and (2) within-subject random error (Hey, 2005). Both sources of variation have received considerable attention in the literature, but for different reasons. The first strand of literature is aimed at explaining and predicting risk attitudes (Barsky et al., 1997; Harrison et al., 2005), while the second has focused on the implications of different error specifications on statistical inference and model comparison (Carbone and Hey, 2000; Loomes, 2005).[14] The present study falls within the first class of articles. In section 5 we will analyze whether there is structural variation in the answers that can be explained by background characteristics.

5. The estimation procedure and first results

The answers A_i that are given by the respondent to the lottery questions $\{(\pi_i, Z_i, \alpha_i)\}_{i=1}^{6}$ may thus be seen as the respondent's solutions to the above equation (6). With six lottery questions we have, in principle, six solutions A_{in} for each respondent n. Given that we have more than two observations per individual, we are not in the situation of equation system (10) where we have only two equations that exactly identify the parameters (γ_n, ρ_n). It is obvious that there will not be an exact solution to the system in this case, so we add an i.i.d. normally distributed error term $\varepsilon_{in} \sim N(0, \sigma_n)$ to the model. This gives the non-linear model

$$\ln A_{in} = \ln A\left(\gamma_n, \rho_n; y_n, \pi_i, Z_{in}, \alpha_{in}, r_n\right) + \varepsilon_{in}, \ \forall i \in \{1,...,6\}, \ n = 1,...,N. \qquad (11)$$

[13] The proposed consistency test gives a rough indication of individual rationality. See Choi et al. (2006) for a more elaborate test for consistency at the individual level.

[14] Only more recently have authors used within individual randomness to explain observed violations of EU (Schmidt and Hey, 2004; Blavatskyy, 2007).

We may consider (11) as consisting of N systems of six non-linear equations in the unknowns (γ_n, ρ_n), where the A_{in} stand for the observed responses to the lottery questions and the unit of time is one month. The parameters (γ_n, ρ_n) could be estimated for all $n = 1,...,N$ separately. These estimates will not be very precise, however, because the number of observations per individual is at most six. Also, estimators of non-linear models are, in general, not unbiased for small samples. Assuming homogeneity in preferences (i.e. $(\gamma_n, \rho_n) = (\gamma, \rho)$) decreases the number of parameters dramatically, which makes the use of asymptotic theory more appropriate. We specify the log-likelihood of the model by

$$\ell(\gamma, \rho) = \sum_{n=1}^{N} \sum_{i=1}^{6} -\tfrac{1}{2}\ln 2\pi - \ln\sigma - \tfrac{1}{2}\left(\frac{\ln a_{in}}{\sigma}\right)^2, \qquad (12)$$

with $\ln a_{in} \equiv \ln A_{in} - \ln A(\gamma, \rho; y_n, \pi_i, Z_{in}, \alpha_{in}, r)$. Because we cannot observe the interest rate on savings, we estimate the model for different interest rates, ranging from 0 to 4% per year. Estimation of the parameters (γ, ρ) by means of maximum likelihood is straightforward.[15] The results are presented in table 3, together with estimates of relative risk aversion for the immediate- and the standard-model.[16]

Table 3: Estimated preference parameters

	Immediate case (R=y)	Intermediate case					Standard model (R=W)
r	4%	0 %	1 %	2 %	3 %	4 %	
γ	2.10***	81.71***	81.73***	81.77***	81.84***	81.95***	338.4***
	(0.002)	(4.219)	(4.223)	(4.234)	(4.254)	(4.283)	(0.050)
ρ		0.0603***	0.0603***	0.0602***	0.0602***	0.0601***	
		(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	
σ	2.640***	1.767***	1.767***	1.767***	1.767***	1.767***	2.807***
	(0.028)	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)	(0.028)
N	1832	1832	1832	1832	1832	1832	1832
ℓ	-19544.1	-16262.7	-16262.7	-16262.6	-16262.6	-16262.6	-20045.9

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The models are presented from left to right in increasing order of intertemporal flexibility. The first column reports the most constraining model, where present income equals present consumption, followed by the intermediate model where saving is allowed, given various assumed interest rates. The last column reports estimates of the standard model, where the lottery is integrated into lifetime wealth and consumption is spread over the entire lifetime. The table shows that the results for the intermediate case are insensitive to the choice of the interest rate. This is due to the fact that the estimated discount rate is higher by an order of magnitude. For convenience we take r = 4%, which is close to the rate of interest on government bonds in the Netherlands.

For the immediate model we find a moderate degree of risk aversion equal to 2, an estimate that seems to be most prevalent in the macroeconomic literature (Bliss and Panigirtzoblou, 2004). Within this strand of literature, however, outcomes are taken in terms of wealth, as in

[15] Robust standard errors were calculated correcting for within-subject correlations in the answers due to possible unobserved heterogeneity by using Stata's clustering option.

[16] Lifetime wealth is calculated as the present value of the income stream, $W = y / r$. Likewise, the lottery prize is taken at present value (i.e. $\exp(-r\alpha)Z$).

the standard model. The risk aversion that we find in that case is higher by two orders of magnitude (i.e. 338). This result is similar to that of Schechter (2007) and it corresponds to Rabin and Thaler (2002) who note that the small-stake risk aversion that is found in the experimental literature (for example Holt and Laury, 2002; Harrison et al. 2005) would translate into extreme values of relative risk aversion if integrated with wealth. The estimated risk aversion for the intermediate model, where the level of asset integration is endogenously determined, falls between the two extremes. The estimate of $\hat{\gamma} = 82$ is high compared to the usual experimental estimates, but much lower than the estimates suggested by the 'standard model'. With the small prizes that are usually involved in the experimental literature, it may be argued that consumption is immediate, justifying the neglect of lifetime wealth. With larger stakes however, such as the ones applied here, consumption will not be immediate but spread over time. This increases the estimated level of relative risk aversion towards (but still short of) that of the standard model. This shows that we can obtain lower, more reasonable estimates of risk aversion than those implied by the standard model while retaining the plausible assumption that consumption is not immediate. The estimates of the intermediate model are sensitive to the assumptions made with respect to the baseline income profile and the precise form of liquidity constraints. The qualitative conclusion, however, that a model with borrowing constraints yields estimates between the two extreme cases, holds in general.

The estimated rate of time preference of $\hat{\rho} = 6\%$ per month is high, but falls within the range of values that have been found in the empirical literature. As our questions include a tradeoff between the immediate present and the future, the estimated discount rate includes preferences for immediate gratification. These preferences have been found to be strong compared to those of delayed rewards, yielding discount rates of a hundred or even a thousand percent (Frederick et al., 2002). This phenomenon has led researchers to formulate different models of time preferences that allow the discount rate to vary with time (Laibson, 1997; Read, 2001). We employ the simple exponential discounting model because we want to show how risk and time preferences interact in the standard model under different assumptions. Hence, we need to qualify our rate of time preference as including preferences for the present that have been found to be strong.

The large variation in the answers that is observed in table 2 is captured by the estimated standard deviation of the error process, $\hat{\sigma} = 1.76$. Indeed there appears to be much variation which is not explained by the model. Fortunately, the estimated standard errors of the parameters, that are robust to individual heterogeneity, are quite small due to the large sample size and variation in (π, Z, α). Some of the unexplained variation may be due to heterogeneity in preferences. Hence, in the next section we will parameterize risk and time preferences by a linear combination of individual characteristics.

6. Explanations by assuming heterogeneity

In this section we attempt to explain the individual parameters (γ_n, ρ_n) by means of some independent variables (for other examples of studies that relate risk and/or time preferences to individual characteristics see Binswanger 1980; Pålsson, 1996; Barsky et al., 1997; Coller and Williams, 1999; Donkers and van Soest, 1999; Halek and Eisenhauer, 2001; Donkers et al., 2001; Harrison et al., 2002; Hartog et al., 2002; Kapteyn and Teppa, 2002; 2003; Harrisson et al., 2005; Tu, 2005; Dohmen et al., 2006).[17] To allow for heterogeneity in

[17] There are also studies that reverse the relation and view risk and time preferences as explanatory variables for different kind of behaviors. For instance Wärneryd (1996) and Guiso and Paiella (2006) try to explain portfolio holdings by a measure of risk aversion, while Borghans and Golsteyn (2006) try to explain obesity by people's level of impatience. Kapteyn and Teppa (2002), Barsky et al. (1997), Donkers and van Soest (1999) and Dohmen et al. (2006) apply both approaches to risk attitudes, disentangling risk aversion by background

preferences, we parameterize the preference parameters by $\left(\gamma_n, \rho_n\right) = \left(\boldsymbol{\beta}'_\gamma \boldsymbol{x}_{\gamma,n}, \boldsymbol{\beta}'_\rho \boldsymbol{x}_{\rho,n}\right)$. The log-likelihood of the model then becomes

$$\ell\left(\boldsymbol{\beta}'_\gamma, \boldsymbol{\beta}'_\rho\right) = \sum_{n=1}^{N}\sum_{i=1}^{6} -\tfrac{1}{2}\ln 2\pi - \ln\sigma - \tfrac{1}{2}\left(\frac{\ln a_{in}}{\sigma}\right)^2, \tag{13}$$

with $\ln a_{in} \equiv \ln A_{in} - \ln A\left(\boldsymbol{\beta}'_\gamma \boldsymbol{x}_{\gamma,n}, \boldsymbol{\beta}'_\rho \boldsymbol{x}_{\rho,n}; y_n, \pi_i, Z_{in}, \alpha_{in}, 4\%\right)$. Estimation of the parameters $\left(\boldsymbol{\beta}'_\gamma, \boldsymbol{\beta}'_\rho\right)$ by maximum likelihood is, again, straightforward.

Demographic variables that can be thought to be exogenous in this model are the respondent's *gender* (*Male,* a dummy equal to 1) and *age* (*Age*). Males are expected to be less risk-averse than females, which is one of the most consistent findings in the literature on heterogeneity in risk attitudes (see Charness and Gneezy, 2007, for a recent investigation into this issue). The results with respect to time preferences vary, but studies that report a significant effect find women to be more patient than men when making decisions between sooner smaller and later larger rewards (Coller and Williams, 1999; Donkers and van Soest, 1999; Read and Read, 2004; Tu, 2005).

The effect of *age* (*Age*) on risk attitudes has not been the main point of focus of any study, but this variable has been included in most of the above-mentioned analyses. Risk aversion is generally found to be either increasing or U-shaped in age (Pålsson, 1996; Donkers and van Soest, 1999; Halek and Eisenhauer, 2001; Hartog et al., 2002). It is unclear whether this is a cohort effect or a pure age effect since none of these studies exploits panel data, which would enable the separation of the two effects. With respect to the time discount rate, we observe that with aging the remaining lifetime shortens. This would suggest a shrinking time horizon and hence stronger time discounting.[18] This effect is indeed found in most studies (Trostel and Taylor, 2001; Kapteyn and Teppa, 2003; Read and Read, 2004; Harrison et al., 2002), although some studies also find young individuals to be more impatient, attributed to a lack of self-control.

Other socio-economic and behavioral variables are potentially endogenous such that their coefficients should be interpreted as a measure of association useful for detecting individual heterogeneity and for prediction, but not for causal inference. For instance *education* (*Edu*), measured as the number of years spent on regular education, can reduce attitudes towards risks because individuals with more schooling are better able to judge the risks they are facing. On the other hand, schooling attainment can be seen as a risky investment, which will cause risk- neutral individuals to select themselves into higher education, assuming that wage dispersion is higher (Hartog et al., 2004). Both effects imply a negative relation between risk-aversion and the level of schooling, but with a different direction of causality. For both reasons we expect that more educated people are less risk-averse, an effect that is found in most studies (Donkers et al., 2001; Hartog et al., 2002; Kapteyn and Teppa, 2002; Dohmen et al., 2006), but certainly not in all (Halek and Eisenhauer, 2001, and Harrison et al., 2005, find the opposite). Similarly, education is a long-term investment and such a long-term investment is triggered by a long time horizon. Hence, we expect that more education goes hand-in-hand with a lower discount rate (Harrison et al., 2002; Kapteyn and Teppa, 2003).

It was already hypothesized by Arrow (1965) and van Praag (1971) that *absolute* risk premia are decreasing with *wealth* (the hypothesis of decreasing absolute risk aversion (DARA)) because the same monetary risk becomes relatively less important when wealth

characteristics and using the risk aversion measure as an explanatory variable in portfolio holdings, home ownership or risky behaviors.

[18] A more theoretical reasoning is given by Becker and Mulligan (1997) and Trostel and Taylor (2001).

increases. There is no *a priori* reason why *relative* risk premiums, that are closely linked to relative risk aversion, should be increasing (IRRA) or decreasing (DRRA) with wealth, and the empirical evidence seems to support neither hypothesis (Gollier, 2001; Halek and Eisenhauer, 2001). Hence we are agnostic about the sign of the coefficient of monthly income *y* (measured in euros), which we take as proxy for wealth. The results with respect to time preferences also vary, but most studies that report a significant effect find impatience to decrease with income (Pender, 1996; Kapteyn and Teppa, 2003; Read and Read, 2004). Liquidity constraints may provide a reason for this, individuals with a larger income tend to be wealthier and can 'afford' to wait.

Since our sample consists of employed individuals only, we cannot test whether workers and non-workers have different risk and time preferences. The respondents in the sample do differ in the type of employment they have. Some interesting work-related variables are whether someone is a *government employee* or an *entrepreneur*. The former group is typically thought to have a higher risk aversion (Hartog et al., 2002), whereas the latter group is thought to be more inclined to take risks (Cramer et al., 2002). We hypothesize that entrepreneurs are more forward- looking, since these individuals typically undertake large investments that entail (expected) returns in the future.

Religion is also a way to cope with the risks of life. Accepting religious rules gives a feeling of safety and security. Hence, we expect religion to be positively associated with risk aversion. Moreover, religious individuals are hypothesized to be more forward-looking (Becker and Mulligan, 1997). We included a measure of religiousness, which varies over five categories, where 1 stands for non-religious and 5 for very religious.

Other demographic variables that are often related to risk and time preferences are whether the respondent is married, has children, lives in a small or large community, or belongs to an ethnic minority. No robust differences in preference parameters appear to have been found in this domain. Behavioural variables of an economic nature that are likely to be related to attitudes towards time delay and risk are whether someone buys insurance, plays the lotto, has savings, or possesses risky assets. Papers that investigate this mostly find the expected relations, albeit often not significant (Wärneryd, 1996; Barsky et al., 1997; Guiso and Paiella, 2006; Dohmen et al., 2006).

There has been some research on the relation between time preferences and unhealthy behaviours such as smoking, drinking, overeating and using drugs (see for instance Fuchs, 1991; Bretteville-Jensen, 1999; Komlos et al., 2003; Picone et al., 2004; Borghans and Golsteyn, 2006). These studies view unhealthy behaviors, and consequently health, as a decision outcome, dependent on either risk or time preferences. Fuchs for instance, argues that impatient individuals have a shorter time horizon and, hence, do not think about the future consequences of unhealthy behaviors. We included several behavioral variables such as smoking, drinking and being overweight (measured by the Body Mass Index (*BMI*)). The estimates are presented in table 4.

Table 4: Maximum-likelihood estimates

	Summary stats[a]		coef.	s.e.	coef.	s.e.	coef.	s.e.
$\ln \gamma$	mean	s.d.						
Male	0.58	0.49	-0.677**	(0.270)	-0.715**	(0.284)	-0.705**	(0.326)
ln(*Age*)	39.14	10.54	-0.921	(0.571)	-0.844	(0.581)	-0.637	(0.845)
ln(*Edu*+1)	12.75	2.62	-0.822***	(0.316)	-0.816**	(0.324)	-0.885**	(0.404)
ln(*y*)	2127	1189	0.952***	(0.130)	0.988***	(0.132)	0.991***	(0.130)
Religion[b]	2.49	1.74	0.208	(0.205)	0.227	(0.221)	0.258	(0.197)
Entrepreneur	0.03	0.18	1.783***	(0.613)	1.790***	(0.619)	1.820***	(0.675)
Ln(*BMI*)	25.53	3.91	2.852**	(1.282)	2.792**	(1.305)	2.732**	(1.066)

Constant			-5.910	(5.188)	-6.180	(4.893)	-6.179	(4.836)
Demo. Controls			No		Yes		Yes	
Beh. Controls			No		No		Yes	
$\ln \rho$								
Male	0.58	0.49	0.599*	(0.309)	0.645*	(0.331)	0.661*	(0.395)
ln(*Age*)	39.14	10.54	1.34**	(0.641)	1.245*	(0.648)	1.049	(0.979)
ln(*Edu*+1)	12.75	2.62	0.592	(0.463)	0.595	(0.481)	0.722	(0.606)
ln(*y*)	2127	1189	-.258*	(0.139)	-0.297**	(0.140)	-0.277**	(0.138)
Religion[b]	2.49	1.74	-.220	(0.250)	-0.251	(0.275)	-0.281	(0.251)
Entrepreneur	0.03	0.18	-3.445*	(1.790)	-3.434*	(1.814)	-3.529	(2.175)
Ln(*BMI*)	25.53	3.91	-2.420	(1.669)	-2.337	(1.708)	-2.436*	(1.420)
Constant			0.014	(6.393)	0.195	(6.089)	0.566	(6.058)
Demo. Variables			No		Yes		Yes	
Beh. Variables			No		No		Yes	
σ			1.692***	(0.022)	1.691***	(0.022)	1.687***	(0.022)
ℓ			-15463		-15450		-15437	
N			1767		1767		1767	

a: Summary statistics of untransformed variables.
b: This ordinal variable has been mapped on the real axis using a monotonic transformation described by van Praag et al. (2003).
Note: Calculated standard errors robust to unobserved heterogeneity. Significance levels: *: 10%; **: 5%; ***: 1%.

We see that most coefficients have the expected sign, but not all are statistically significant. Most of the demographic and behavioral variables are not significant and are, therefore, not reported in the table. The coefficients, reported in the second and third column of point estimates, are subject to control for these covariates.

One of the most robust effects found in the empirical literature, the difference in risk aversion between males and females, is also found in our dataset; that is, males are much less risk-averse than females. The gender effect on time discounting is also strong in magnitude, woman being more patient, but this effect is only marginally significant. Growing older is associated with a lower degree of risk aversion, contrary to what is usually found, but this effect is not significant. A higher age is also associated with a higher level of impatience, consistent with previous evidence. Using dummies for age classes did not reveal a non-monotonic relation, hence there does not appear to be a U-shaped pattern in our data.

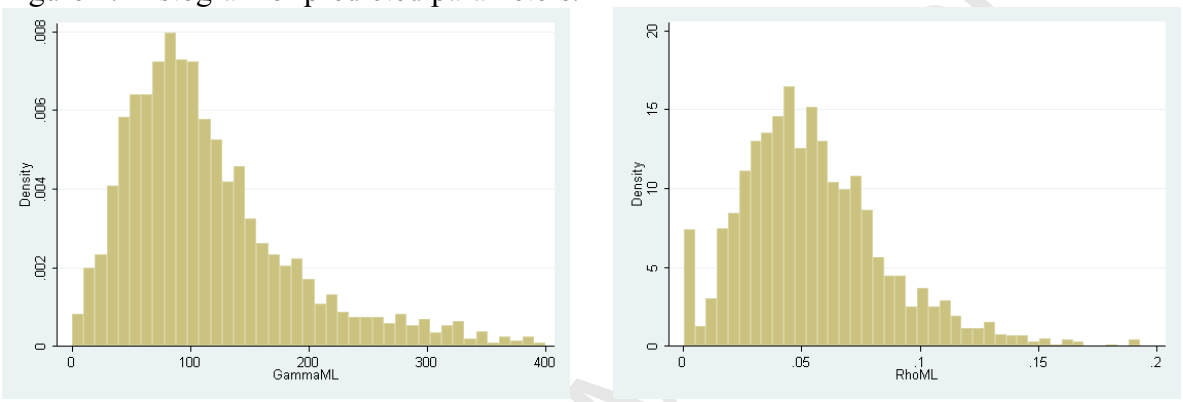
A higher level of schooling of the respondent is associated with a lower risk aversion and a lower patience level. The latter effect is not as predicted, but insignificant. The estimated coefficient of relative risk aversion increases with income, which means that as wealth increases, gambles proportional to wealth become less attractive. Impatience is associated with a lower income level, consistent with what is mostly found, but this result is not significant.

The intensity of being religious has no significant effect on either parameter, but the signs are as expected. Entrepreneurs display more utility curvature than employees, which is surprising since they are generally thought to be more risk tolerant. Background risk may be an explanation for this result; it could be that entrepreneurs are more risk averse because they generally face more uncertainty in their income than employees. Halek and Eisenhauer (2001) report the same effect. Entrepreneurs are found to be more patient, which is what we expected. We did not find an effect for other types of employment. Finally, obesity (being over weight) reduces the willingness to take risks, which is also what Dohmen et al. (2006) find, and increases the subject's time horizon. This last result is insignificant and contrasts with the hypotheses of Komlos et al. (2003), who argue that impatience could lead to over-

eating and consequently being obese, but the empirical robustness of this effect has yet to be established (Borghans and Golsteyn, 2006).

Clearly our findings correspond to most of the hypotheses and fit in well with the existing literature, except for the negative effect of age and the positive effect of entrepreneurship on risk aversion. It must be noted, however, that apart from the gender effect on risk aversion, there do not appear to be many other robust empirical findings in the literature that explain risk and/or time preferences. That is, there is variation across studies in the sign and significance of the estimated effect of most variables. Figure 1 plots the predicted preference parameters. Both distributions have a log-normal shape. The peak at $\hat{\rho}$ near zero is caused by the group of entrepreneurs that have a significantly higher time horizon.

Figure 1: Histogram of predicted parameters.



(a) Histogram of $\hat{\gamma}$. (b) Histogram of $\hat{\rho}$.

7. Summary and conclusions

This paper starts from the basic premise that many economic decision problems have both a risk and a time dimension. This was illustrated in the context of the valuation of simple lotteries. Traditionally, behavior in this context is modeled by looking at the risk dimension of decisions, neglecting the fact that the evaluation of the prize not only depends on the absolute amount of the prize, but also on the way in which the prize is spent over time. To illustrate how, in this context, the standard model can be extended to accommodate the additional time dimension, we formulated a simple discounted expected utility model where we take account of the opportunity to spread consumption optimally over time, while making the plausible assumption that individuals are borrowing constrained. In that case the consumption of the prize is spread over a finite period that is endogenously determined and depends on time preferences. This model forms an intermediate case between the expected utility model defined over wealth (the standard model) and defined over income (the immediate model). These models have dominated the literature on the measurement of risk aversion for large and small stakes respectively.

The empirical tractability of the model was shown by simultaneously estimating the coefficient of relative risk aversion and the subjective time discount rate, using a sample of 1,832 subjects who were asked to state their willingness to pay for six different, randomly assigned lotteries. Most of the answers were consistent, with 94.5% of all possible within-subject comparisons complying with dominance and discounting. This suggests that, even though we did not provide monetary incentives, the subjects took the questions seriously.

The average coefficient of relative risk aversion γ was estimated to be 82. While this estimate is high compared to what is usually reported, it falls between the estimates of the usual models. If consumption is assumed to be immediate, the inferred relative risk aversion

is 2, while we find an estimate of 338 if full asset integration is assumed. This shows that we get lower, more reasonable estimates of risk aversion than those implied by the standard model if we assume that individuals are borrowing constrained, while retaining the plausible assumption that consumption is not immediate. The subjective time discount rate was estimated at 6% per month on average, which is high but falls within the range of values that has been found. The quantitative values of these estimates depend on the assumptions made about baseline consumption, the timing of the lottery, and the exact form of the liquidity constraints, but the quantitative conclusion hold in general.

Both γ and ρ appeared to vary strongly over individuals. This variation could be explained by income, age, gender and entrepreneurship, consistent with the majority of previous evidence. It suggests that the parameters of the model indeed capture preferences towards risk *and* time.

Our analysis shows that the estimates of relative risk aversion are sensitive to the assumptions made about consumption and that it is possible to accommodate for the effects of both risk and time dimensions in subject's decisions when considering simple lotteries. This finding generalizes to many other settings, where we may think of risky assets, portfolios, and so on. Obviously, this also holds inversely. If we try to estimate subjective time discount rates from the evaluation of risky assets over time, we cannot do this without simultaneously taking the attitude towards risk into account (see Andersen et al., 2008). We hope that this analysis will stimulate researchers of risk attitudes and time preferences to consider both the risk and time dimensions simultaneously when analyzing subject's decisions.

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Appendix A: Determination of $(c, T_{\max})$

Dividing the Euler equation (eq. (7)) by that when evaluated at $\tau = 0$ and rearranging yields

$$u'(y + e^{r\tau}\hat{p}(\tau+\alpha)Z) = \tilde{c}e^{(\rho-r)\tau}, \forall\tau > 0, \tag{14}$$

with $\tilde{c} = C/Z$. Using the CRRA-specification and solving for the optimal profile we get

$$\hat{p}(\tau+\alpha) = ce^{-B\tau} - \psi e^{-r\tau}, \forall\tau > 0, \tag{15}$$

with $B \equiv \frac{\rho-r}{\gamma} + r$, $\psi \equiv \frac{y}{Z} > 0$ and the constant $c = \tilde{c}^{-\frac{1}{\gamma}}/Z > 0$ to be determined. We note that $B > r$ if $\rho > r$, irrespective of the value of $r > 0$. Hence, eventually $\hat{p}(\tau+\alpha)$

will become negative, which violates the non-negativity condition. The moment $T_{\max}$ at which this occurs is found by solving the equation

$$\hat{p}\left(T_{\max}+\alpha\right)=0 \Leftrightarrow c=\psi e^{\left(B-r\right)T_{\max}}, \tag{16}$$

for $T_{\max}$. We see that $T_{\max}$ depends on the unknown c. The additional constraint in (4), which states that the spending fractions should sum to one, can be used to identify the model. Substituting c from (16) into (8) we may rewrite this as

$$\psi e^{(B-r)T_{\max}}\int_0^{T_{\max}} e^{-B\tau}d\tau-\psi\int_0^{T_{\max}} e^{-r\tau}d\tau=1, \tag{17}$$

from which $T_{\max}$ and subsequently c may be solved numerically[19]. An analytical solution cannot be given because this equation contains both exponential end linear terms. For completeness, we notice that if $\rho<r$, then $\hat{p}(\tau+\alpha)$ would start being zero and become positive and increasing after $T_{\max}$. If that were the case, the integral in (17) would not converge. Therefore, we assume $\rho>r$.

Appendix B: Determination of A

With the optimal profile $\hat{p}(\tau+\alpha)$ fully specified, the maximum of the integral in (6) can be evaluated. To this end we can use equation (14), the fact that $c=\left(c'Z\right)^{-\gamma}$, and the relation $u=\frac{1}{1-\gamma}(u')^{\frac{\gamma-1}{\gamma}}$, which holds for a CRRA utility-function u and its derivative u'. This yields

$$\int_0^{T_{\max}} e^{-\rho\tau}u(y+e^{r\tau}\hat{p}\left(\tau+\alpha\right)Z)dt=\frac{\left(cZ\right)^{1-\gamma}}{1-\gamma}\int_0^{T_{\max}} e^{-B\tau}d\tau=\frac{\left(cZ\right)^{1-\gamma}}{-B\left(1-\gamma\right)}\left(e^{-BT_{\max}}-1\right). \tag{18}$$

The willingness to pay A can be solved from (6) by substitution of this expression.

[19] Equation (17) has two solutions in $T_{\max}$, but only one root is positive as required.