



**HAL**  
open science

# Bistatic Polarimetric SAR Decomposition in Terms of Roll-Invariant Parameters

Lionel Bombrun

► **To cite this version:**

Lionel Bombrun. Bistatic Polarimetric SAR Decomposition in Terms of Roll-Invariant Parameters. 5th International Workshop on Science and Applications of SAR Polarimetry and Polarimetric Interferometry (POLinSAR 2011), 2011, Frascati, Italy. pp.1-5. <hal-00661711>

**HAL Id: hal-00661711**

**<https://hal.science/hal-00661711v1>**

Submitted on 23 Jan 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

# BISTATIC POLARIMETRIC SAR DECOMPOSITION IN TERMS OF ROLL-INVARIANT PARAMETERS

Lionel Bombrun

Université de Bordeaux, UBI, IPB, ENSEIRB-Matmeca, Laboratoire IMS, UMR 5218  
Groupe Signal et Image, Talence, France  
lionel.bombrun@ims-bordeaux.fr

## ABSTRACT

This paper introduces a new bistatic Polarimetric SAR decomposition in terms of roll-invariant parameters. The proposed decomposition is an extension of the Target Scattering Vector Model to the bistatic case, where the cross-polarization terms of the scattering matrix are not necessary equal.

Key words: Bistatic Polarimetry, Polarimetric Synthetic Aperture Radar, Roll-invariant decomposition, Target Scattering Vector Model.

## 1. INTRODUCTION

The polarimetric information has been widely used to interpret the Synthetic Aperture Radar (SAR) scene. Hence, many coherent and incoherent target decompositions have been proposed in the literature to extract polarimetric parameters with a physical meaning (1) (2). Nevertheless, for most of them, the reciprocity assumption is assumed. For a bistatic Polarimetric SAR (Pol-SAR) sensor, the cross-polarization terms of the scattering matrix are generally not equal ( $S_{HV} \neq S_{VH}$ ).

This paper presents a generalization of the Target Scattering Vector Model (TSVM) to the bistatic case: the bistatic TSVM (3). Five roll-invariant parameters (independent of the orientation angles) are necessary for an unambiguous description of the target scattering mechanism:  $\mu$ ,  $\alpha_s$ ,  $\Phi_{\alpha_s}$ ,  $\tau_1$  and  $\tau_2$ .

This paper is organized as follows. In Section 2, the extension of Target Scattering Vector Model to the bistatic case is first introduced. Then, a presentation of the bistatic incoherent target decomposition in terms of roll-invariant parameters and a study of the influence of the processing window size on the estimation of the bistatic TSVM parameters are achieved in Section 3. Next, some results are shown in Section 4. Finally, some conclusions of this work are discussed in Section 5.

## 2. THE TARGET SCATTERING VECTOR MODEL

### 2.1. The Kennaugh-Huynen con-diagonalization

Coherent targets are fully described by their scattering matrix  $\mathbf{S}$ . In the context bistatic polarimetry,  $\mathbf{S}$  is a complex  $2 \times 2$  matrix,  $\mathbf{S} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$  where the cross-polarization elements  $S_{HV}$  and  $S_{VH}$  are not equal in general.

Kennaugh and Huynen have proposed to apply the characteristic decomposition on the scattering matrix to retrieve physical parameters (4) (5). The Kennaugh-Huynen decomposition is parametrized by means of 8 independent parameters:  $\theta_R$ ,  $\tau_R$ ,  $\theta_E$ ,  $\tau_E$ ,  $\nu$ ,  $\mu$ ,  $\kappa$  and  $\gamma$  by (4) (6) (7):

$$\mathbf{S} = e^{-j\theta_R\sigma_3} e^{-j\tau_R\sigma_2} e^{j\nu\sigma_1} \mathbf{S}_0 e^{j\nu\sigma_1} e^{-j\tau_E\sigma_2} e^{j\theta_E\sigma_3} \quad (1)$$

where:

$$\mathbf{S}_0 = \mu e^{j\kappa} \begin{bmatrix} 1 & 0 \\ 0 & \tan^2 \gamma \end{bmatrix} \text{ and } e^{j\alpha\sigma_k} = \sigma_0 \cos \alpha + j\sigma_k \sin \alpha. \quad (2)$$

$\sigma_i$  are the spin Pauli matrices defined by:

$$\begin{aligned} \sigma_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \\ \sigma_2 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}. \end{aligned} \quad (3)$$

$\theta_R$  and  $\theta_E$  are the tilt angles.  $\tau_R$  and  $\tau_E$  are the helicity. The subscript  $R$  and  $E$  stand respectively for reception and emission.  $\mu$  is the maximum amplitude return.  $\gamma$  and  $\nu$  are respectively referred as the characteristic and skip angles.  $\kappa$  is the absolute phase of the target, this term is generally ignored except for interferometric applications. Moreover, it can be shown that:

$$\begin{aligned} e^{j\nu\sigma_1} \mathbf{S}_0 e^{j\nu\sigma_1} &= \begin{bmatrix} \mu e^{2j(\nu+\kappa/2)} & 0 \\ 0 & \mu \tan^2 \gamma e^{-2j(\nu-\kappa/2)} \end{bmatrix} \\ &= \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \end{aligned} \quad (4)$$

where  $\lambda_1$  and  $\lambda_2$  are the two complex con-eigenvalues of  $S$ .

## 2.2. The bistatic TSVM model

The TSVM consists in the projection in the Pauli basis of the scattering matrix con-diagonalized by the Takagi method. It yields that  $\mathbf{k}_P = 1/\sqrt{2} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, S_{HV} + S_{VH}, j(S_{HV} - S_{VH})]^T$ . After some mathematical manipulations, it yields:

$$\mathbf{k}_P = e^{j\Phi_s} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) & 0 \\ 0 & \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_2) & 0 & 0 & -\sin(\theta_2) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -j \sin(\theta_2) & 0 & 0 & -j \cos(\theta_2) \end{bmatrix} \mathbf{k}_P^{\text{roll-inv}}, \quad (5)$$

where the expression of the roll-invariant target vector  $\mathbf{k}_P^{\text{roll-inv}}$  is given by (3):

$$\mathbf{k}_P^{\text{roll-inv}} = \mu \begin{bmatrix} \cos \alpha_s \cos(\tau_1) \\ \sin \alpha_s e^{j\Phi_{\alpha_s}} \cos(\tau_2) \\ -j \cos \alpha_s \sin(\tau_1) \\ -j \sin \alpha_s e^{j\Phi_{\alpha_s}} \sin(\tau_2) \end{bmatrix}, \quad (6)$$

where  $\tau_1 = \tau_R + \tau_E$  and  $\tau_2 = \tau_R - \tau_E$ .  $\alpha_s$  and  $\Phi_{\alpha_s}$  are the scattering type magnitude and phase parameters, given by:

$$\tan(\alpha_s) e^{j\Phi_{\alpha_s}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}. \quad (7)$$

It has been pointed out that Huynen's parameters are not unique and need to be reevaluated. If the tilts angles ( $\theta_1 = \theta_R + \theta_E, \theta_2 = \theta_R - \theta_E$ ) are solution of (6), then  $(\theta_1 \pm \pi, \theta_2 \pm \pi)$ ,  $(\theta_1 \pm \pi, \theta_2)$  and  $(\theta_1, \theta_2 \pm \pi)$  are also solutions of (6). It yields the following three relations (3):

$$\begin{aligned} \mathbf{k}_P &= \mathbf{k}_P(\Phi_s \pm \pi, \theta_1 \pm \pi, \theta_2 \pm \pi, \tau_1, \tau_2, \mu, \alpha_s, \Phi_{\alpha_s}) \\ &= \mathbf{k}_P(\Phi_s, \theta_1 \pm \pi, \theta_2, -\tau_1, -\tau_2, \mu, \alpha_s, \Phi_{\alpha_s} \pm \pi) \\ &= \mathbf{k}_P(\Phi_s \pm \pi, \theta_1, \theta_2 \pm \pi, -\tau_1, -\tau_2, \mu, \alpha_s, \Phi_{\alpha_s} \pm \pi). \end{aligned} \quad (8)$$

After this step, the tilt angles  $\theta_1$  and  $\theta_2$  belong to the interval  $[-\pi/2, \pi/2]$  and the decomposition is unique. In the context of bistatic polarimetry, five parameters (namely  $\mu, \tau_1, \tau_2, \alpha_s$  and  $\Phi_{\alpha_s}$ ) are necessary for an unambiguous description of a coherent target. The scattering type magnitude  $\alpha_s$  and phase  $\Phi_{\alpha_s}$  contain information on the scattering type mechanism. The target helicity  $\tau_1$  is a measure of the target symmetry, *i.e.*  $\tau_1 = 0$  for a symmetrical target. The target helicity  $\tau_2$  contains information on the asymmetrical part of the scattering matrix, *i.e.*  $\tau_2 = 0$  corresponds to  $S_{HV} = S_{VH}$ .

## 3. BISTATIC INCOHERENT TARGET DECOMPOSITION IN TERMS OF ROLL-INVARIANT PARAMETERS

### 3.1. Principle

The roll-invariant bistatic incoherent target decomposition is based on the characteristic decomposition of the coherency matrix. For a bistatic polarimetric radar, the coherency matrix  $[T]$  can be represented as the incoherent sum of four coherency matrix  $[T]_i$  of rank 1. The four coherency matrices  $[T]_i$  are weighted by their real positive eigenvalue  $\mu_i$  by:

$$[T] = \sum_{i=1}^4 \mu_i [T]_i. \quad (9)$$

Each coherency matrix  $[T]_i$  of rank 1 is associated to one single scatterer.  $[T]_i$  can be rewritten as a function of the target scattering eigenvector  $\mathbf{k}_i$  as:

$$[T]_i = \mathbf{k}_i \mathbf{k}_i^H. \quad (10)$$

The principle of the roll-invariant bistatic incoherent target decomposition can be decomposed into three steps as follows:

1. Estimation of the coherency matrix  $[T]$  on a sliding window, denoted  $[\hat{T}]$ .
2. Application of the characteristic decomposition on  $[\hat{T}]$ , and extraction of the four eigenvectors  $\mathbf{k}_i$  with their respective eigenvalues  $\mu_i$ .
3. Application of the bistatic TSVM on each eigenvector  $\mathbf{k}_i$  and extraction of the five roll-invariant parameters ( $\mu_i, \tau_{1i}, \tau_{2i}, \alpha_{s_i}$  and  $\Phi_{\alpha_{s_i}}$ ) to fully characterized the scatterers.

To apply the incoherent target decomposition, the covariance matrix  $[T]$  should be estimated. For homogeneous regions, the target vector  $\mathbf{k}$  is generally modeled by a zero-mean multivariate circular complex Gaussian vector. In this case, the maximum likelihood estimator of the polarimetric covariance matrix  $[T]$  is the sample covariance matrix  $[\hat{T}]_{SCM}$  given by:

$$[\hat{T}]_{SCM} = \frac{1}{N} \sum_{i=1}^N \mathbf{k}_i \mathbf{k}_i^H. \quad (11)$$

### 3.2. Influence of the processing window size

As PolSAR images are corrupted by the speckle noise, incoherent target decomposition parameters can be biased. A study of the influence of the processing window size on

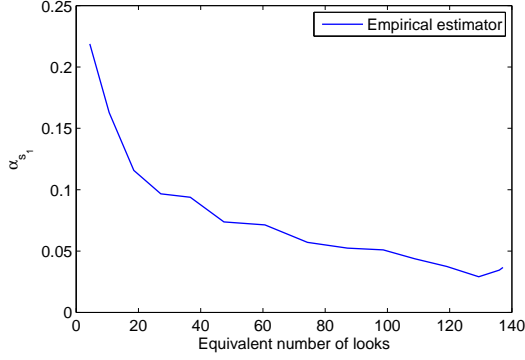


Figure 1. Influence of the number of looks  $L$  on the estimation of the dominant scattering type magnitude parameter  $\alpha_s$ .

the estimation of the bistatic TSVM parameters is necessary to find the minimum window size required for nearly unbiased parameter estimation.

Figure 1 shows the evolution of the scattering type magnitude  $\alpha_s$  parameter associated to the dominant scattering mechanism (of eigenvalue  $\lambda_1$ ) as a function of the equivalent number of looks  $L$ . For this purpose, an homogeneous region containing fields located on the south west of Figure 2 has been extracted and bistatic TSVM parameters are computed on square window. As observed in Figure 1,  $\hat{\alpha}_{s_1}$  is biased for a relatively small window size ( $7 \times 7$ ) which corresponds approximately to equivalent number of looks  $L$  of 18. For sufficiently large windows size ( $15 \times 15$ , *i.e.*  $L \approx 60$ ), the bias vanishes and  $\hat{\alpha}_{s_1}$  converges to  $\alpha_{s_1}$  for the sample covariance matrix estimator. A processing window containing at least 60 equivalent number of looks is therefore necessary to derive nearly unbiased incoherent decomposition parameters as discussed in (8; 9).

### 3.3. Degree of coherence for the assessment of target scattering phase

As observed in (6), the target scattering type phase  $\Phi_{\alpha_s}$  is the trihedral-dihedral channel phase difference. This roll-invariant parameter can be exploited only under coherence conditions. By following the same procedure as described in (10), the degree of coherence of  $\Phi_{\alpha_s}$  (denoted  $p_{\Phi_{\alpha_s}}$ ) is therefore introduced. Its expression is given by:

$$p_{\Phi_{\alpha_s}} = \frac{\sqrt{\left(\langle |a|^2 - |b|^2 \rangle\right)^2 + 4|\langle a \cdot b^* \rangle|^2}}{\langle |a|^2 + |b|^2 \rangle}, \quad (12)$$

where  $a = \cos \alpha_s \cos \tau_1$  and  $b = \sin \alpha_s e^{j\Phi_{\alpha_s}} \cos \tau_2$  for a bistatic polarimetric radar. Therefore, a partially coherent scatterer is represented as a point inside the Poincaré sphere at a distance  $p_{\Phi_{\alpha_s}}$  from the sphere center.

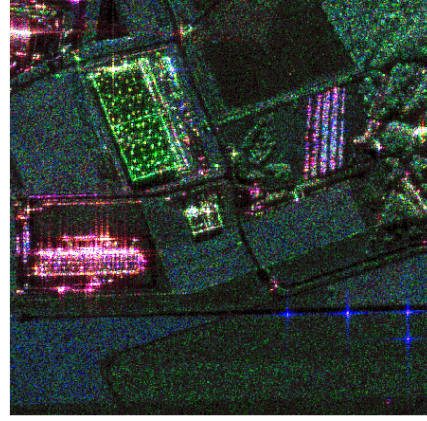


Figure 2. RAMSES X-band PolSAR data over Brétigny, France ( $501 \times 501$  pixels). Colored composition of the target vector in the Pauli basis  $[k]_2$ - $[k]_3$ - $[k]_1$ .

## 4. RESULTS

To illustrate the roll-invariant bistatic target scattering vector model, a PolSAR RAMSES X-band data-set acquired over Brétigny, France is analyzed. This data-set is composed by three buildings surrounded by agricultural fields. A colored composition of the target vector in the Pauli basis is given in Figure 2. This PolSAR data-set has been acquired in a monostatic mode. As the four polarimetric SAR channel are recorded by the sensor, the incoherent bistatic TSVM parameters can be computed. Nevertheless, the usefulness of the proposed method will not be fully emphasized as  $S_{HV}$  and  $S_{VH}$  are quite similar in the monostatic case. To evaluate the influence of the processing window size on the roll-invariant parameters, three different window size ( $7 \times 7$ ,  $15 \times 15$  and  $31 \times 31$ ) are analyzed. Figure 3 shows the dominant target scattering type magnitude  $\alpha_s$  issued from the first eigenvector of the estimated coherency matrix. Moreover, for relatively small window size ( $7 \times 7$ , *i.e.*  $L \approx 18$ ), the estimated parameters are biased. An unbiased estimation of target scattering parameters is achieved for a sufficiently large window ( $31 \times 31$  pixels, *i.e.*  $L \approx 137$ ) as shown in Figure 1. This yields to a blur effect. Nevertheless, for such window size, the region might involve scatterers having different polarimetric responses and might lead to a misinterpretation of the bistatic TSVM parameters. In the following, a window size of  $15 \times 15$  pixels will be considered as a good trade-off.

Figure 4 shows the five roll-invariant bistatic target scattering target vector parameters ( $\mu$ ,  $\alpha_s$ ,  $\Phi_{\alpha_s}$ ,  $\tau_1$  and  $\tau_2$ ) issued from the first eigenvector of the sample covariance matrix estimator  $[\hat{T}]_{SCM}$  computed on a  $15 \times 15$  window. The degree of coherence  $p_{\Phi_{\alpha_s}}$  is also computed and shown in Figure 4(f). The maximum amplitude return  $\mu$  characterizes the power of the dominant scattering mechanism. This parameter is equal to the largest eigenvalue  $\lambda_1$  of the covariance matrix  $[\hat{T}]_{SCM}$ . The  $\alpha_s$  quantity

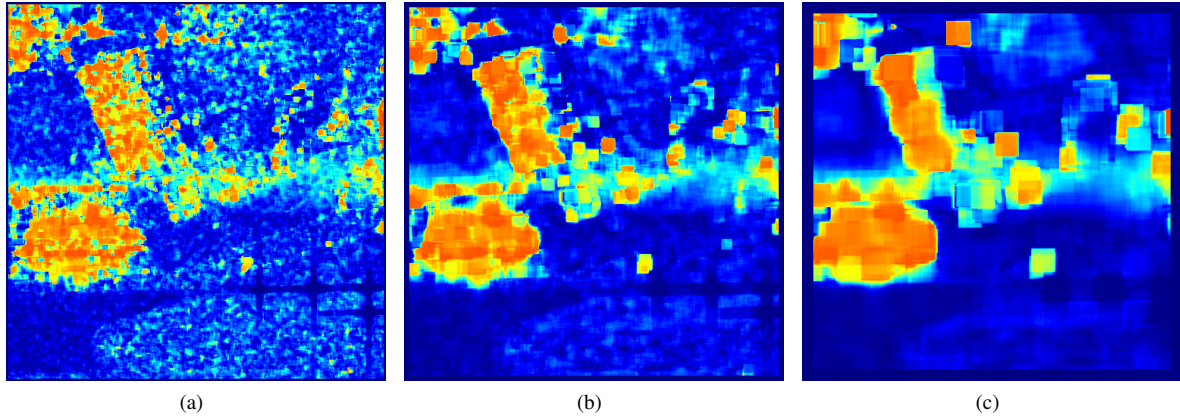


Figure 3. Dominant target scattering type magnitude  $\alpha_{s1}$ , issued from the Sample Covariance Matrix estimator  $[\hat{T}]_{SCM}$  on a (a)  $7 \times 7$  window ( $L \approx 18$ ), (b)  $15 \times 15$  window ( $L \approx 60$ ) and (c)  $31 \times 31$  window ( $L \approx 137$ ).

describes the type of scattering mechanism (1). It varies from 0 for surface scattering (see fields on the bottom left of Figure 4(b)) to  $\pi/2$  for double bounce scattering as observed on the three buildings.  $\Phi_{\alpha_s}$  is the target scattering type phase. This term can be exploited only under coherence conditions. For this purpose, the degree of coherence  $p_{\Phi_{\alpha_s}}$  is computed. For high values of  $p_{\Phi_{\alpha_s}}$  (near 1), the target scattering type phase can be used as an additional parameter to interpret PolSAR data. The target helicity  $\tau_1 = \tau_R + \tau_E$  is a measure of the target symmetry. It allows to separate symmetric ( $\tau_1 = 0$ ) from asymmetric ( $\tau_1 \neq 0$ ) scatterers that have the same scattering type magnitude and phase. For natural media, the target helicity  $\tau_1$  is relatively low ( $\tau_1 \approx 0$ ) as observed on fields (see Figure 4(c)). The target helicity  $\tau_2$  characterizes the asymmetrical part of the scattering matrix. This term indicates if the reciprocity assumption holds (for  $\tau_2 = 0$ ) or not (for  $\tau_2 \neq 0$ ). As the studied PolSAR data-set has been acquired under monostatic condition, the cross-polarization terms  $S_{HV}$  and  $S_{VH}$  are expected to be similar, which will lead to  $\tau_2 \approx 0$ . This is confirmed by Figure 4(d) where the target helicity  $\tau_2$  is very low.

## 5. CONCLUSION

This paper has presented a roll-invariant target decomposition in the context of bistatic PolSAR imagery: the bistatic target scattering vector model. Based on the Kennaugh-Huynen characteristic decomposition, five parameters, namely  $\alpha_s$ ,  $\Phi_{\alpha_s}$ ,  $\tau_1$ ,  $\tau_2$  and  $\mu$ , are necessary for an unambiguous description of a coherent target. This new PolSAR model is an extension of the TSVM, introduced by Touzi in 2007 (1), to the general case of bistatic polarimetry where the cross-polarization elements  $S_{HV}$  and  $S_{VH}$  are not equal. A new parameter has been introduced: the target helicity  $\tau_2$ , which contains information on the asymmetrical part of the scattering matrix.

## ACKNOWLEDGMENT

The author wish to thank the French Research Agency (ANR) for supporting this work through the EFIDIR project (ANR-2007-MDCO-004-03, <http://www.efidir.fr>).

## REFERENCES

- [1] R. Touzi, "Target Scattering Decomposition in Terms of Roll-Invariant Target Parameters," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 45, no. 1, pp. 73–84, January 2007.
- [2] C. Titin-Schnaider, "Physical Meaning of Bistatic Polarimetric Parameters," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 5, pp. 2349 – 2356, May 2010.
- [3] L. Bombrun, "Extension of the Target Scattering Vector Model to the Bistatic Case," in *Geoscience and Remote Sensing, IGARSS'10, Hawaii, USA*, 2010, pp. 4047–4050.
- [4] J.R. Huynen, *Phenomenological Theory of Radar Targets*, Academic Press, 1978.
- [5] K. Kennaugh, "Effects of Type of Polarization on Echo Characteristics," *Ohio State Univ., Research Foundation Columbus Antenna Lab, Tech. Rep. 389-4, 381-9*, 1951.
- [6] A.-L. Germond, *Théorie de la Polarimétrie Radar en Bistatique*, Ph.D. thesis, Université de Nantes, Nantes, France, 1999.
- [7] Z.H. Czyz, "Fundamentals of Bistatic Radar Polarimetry Using the Poincare Sphere Transformations," Technical report, Telecommunications Research Institute, <http://airex.tksc.jaxa.jp/pl/dr/20010100106/en>, 2001.

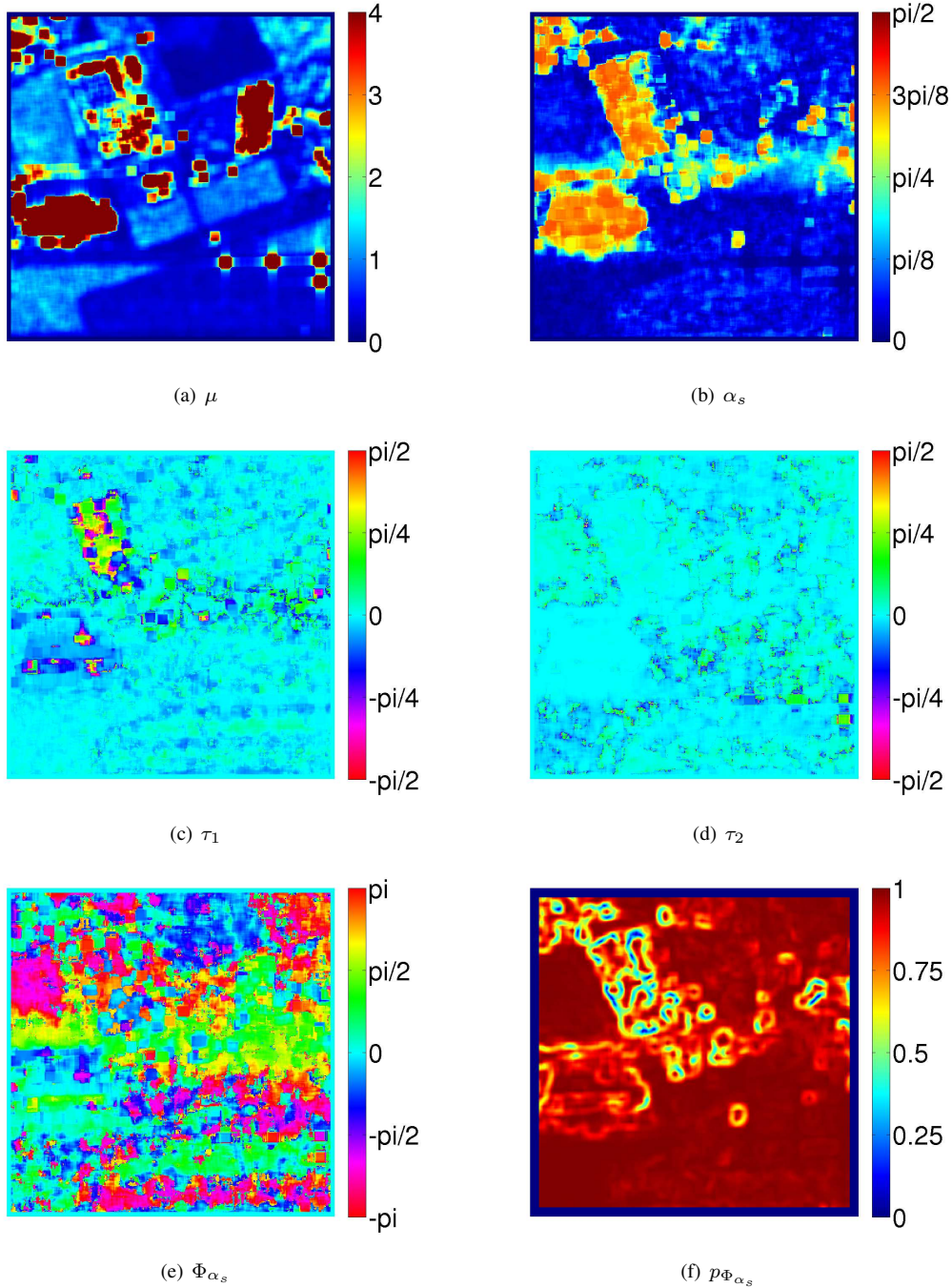


Figure 4. Dominant bistatic target scattering target vector parameters computed on a  $15 \times 15$  window with the sample covariance matrix estimator  $[\hat{T}]_{SCM}$ : (a) maximum amplitude return  $\mu$ , (b) target scattering type magnitude  $\alpha_s$ , (c) target helicity  $\tau_1$ , (d) target helicity  $\tau_2$ , (e) target scattering type phase  $\Phi_{\alpha_s}$  and (f) degree of coherence  $p_{\Phi_{\alpha_s}}$ .

- [8] C. López-Martínez, E. Pottier, and S.R. Cloude, "Statistical Assessment of Eigenvector-Based Target Decomposition Theorems in Radar Polarimetry," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 43, no. 9, pp. 2058–2074, 2005.
- [9] R. Touzi, "Speckle Effect on Polarimetric Target Scattering Decomposition of SAR Imagery," *Canadian Journal of Remote Sensing*, vol. 33, pp. 543–

551, February 2007.

- [10] R. Touzi, A. Deschamps, and G. Rother, "Phase of Target Scattering for Wetland Characterization Using Polarimetric C-band SAR," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 47, no. 9, pp. 3241–83261, September 2009.