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# Strain Gradient Plasticity Applied To Material Cutting

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**Abstract.** To better understand the complex phenomena involved in the cutting process is to better qualify the behaviour law used in the simulation of machining processes (analytical and finite element modeling). The aim of this paper is to present the choices made regarding the behaviour law in this context, indeed, commonly used behaviour laws such as Johnson-Cook can bring unsatisfactory results especially for high strain and large deformation processes. This study develops a large deformation strain-gradient theoretical framework with hypothesis linked with metal cutting processes. The emphasis of the theory is placed on the existence of high shear phenomena creating a texture in the primary shear band. To account for the texture, the plastic spin is supposed to be relevant in this theory. It is shown that the theory has the capability of interpreting the complex phenomena found in machining and more particularly in high speed machining.

## Introduction

Machining is a chip-forming process in which the chip is formed under severe and rapid deformation. The understanding of the phenomenon in play and the modeling of the machining process can be challenging as many aspects are still not well mastered. Cahuc in [1] showed that complex phenomena appear while the chip is formed. Those phenomena are the result of the plastic dissipated energy and the friction from the tool/workpiece interface. The phenomena were observed in that study through the complete action tensor measurement at the tool tip. This measurement was done by a six component dynamometer developed by Couétard [2]. One of the recent focus in the study of machining simulation is the definition of a behaviour law able to represent the complex deformation underlying the formation of the chip and allowing a precise numerical computation especially in regards to the width of the slip band.

**Moments in machining.** In most machining experiments, only the force components are measured, it is usually believed that those quantities are representative of the process. Many studies in the past 15 years have tried to show the importance of the other quantities found in the complete action tensor in the representation of the machining process. Toulouse and Cahuc in [3] and [1] have demonstrated the existence of a vector moment directly applied at the tool tip. It is important to note that this moment is not the sole result of a lever arm. Cahuc, in [5] then showed that the study of the complete tensor made possible a complete assessment of the power expenditure in the cutting process.

Another study which shows the importance of moments in the machining process is an experiment done by Cahuc, Darnis and Laheurte in [5] where a pin on disk friction test was done using a six component dynamometer. It was shown that moments also appear in the friction mechanism. The main deformation mechanism found in that experiment is shearing and it was concluded that the intense shearing was responsible for rotations strains appearing within the material. The development of a behaviour law for machining needs to have the capability of expressing the moment as seen in experimental studies. An option would be the couple stress theory developed by Cosserat [4] considered in [5]. The media was considered to be comprised of micro-

particles self rotating regardless of the displacement of the surrounding media. Furthermore, they were considered perfectly rigid and therefore this theory is not sufficiently representative of the phenomena studied in chip formation.

**Modeling considerations.** The modeling of machining has been a hot topic for some years, in fact, many manufacturers seek to optimize the processes of material forming in order to increase their profitability. In the case of machining, it is possible to model separately the process itself and the deformation mechanisms of the material that come into play during the formation of the chip. There are many phenomena to grab with the modeling of the deformation mechanisms in machining; the most challenging one being the shear band. A difficulty inherent to most models of the chip formation is the control of the width of the shear band and its localization. It poses a problem especially when remeshing because the width tends to diminish with the size of the elements. It then becomes necessary to impose a length scale to control the shear band. Most behaviour laws do not include such length scales.

**Cutting model.** The analysis of the tool-work piece-chip interface shows the existence of four zones as seen in Fig. 1. The focus of the study is in relation to the primary (resp. secondary shear) zone 1 (resp. 2) seen in Fig. 1. During chip formation, shear stresses are generated in the primary and secondary shearing zone. The micrograph (Fig. 2) shows a non-linear strain evolution. A texture in the material appears as the crystals orientation in the metal seem to have an orientation in link with the underlying strain evolution during the cutting process.

Therefore, strain modeling must take into account the non-linear evolution of strain. Thus, a suitable theory has to adequately represent the observed phenomena which is not the case with the classical theory. In mechanics of the continuous media, by classical theory, it is meant a linear small strain theory which is not taking into account plastic spin. Nevertheless, it is not sufficient in this case as substantial rotation phenomena (Fig. 2) cannot be expressed with the definitions of the displacement.

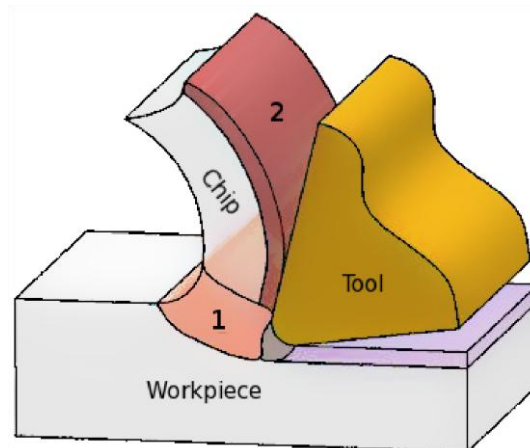


Fig. 1: Cutting model

A supposedly representative theory is the strain gradient theory. The displacement description gives the possibility to introduce the strain and the strain gradient of the media. The strain gradient enables the modelling of rotational strain phenomena as the rotation is a higher order term than linear displacements. This theory gives also the opportunity to introduce length scales within the material behaviour law [17].

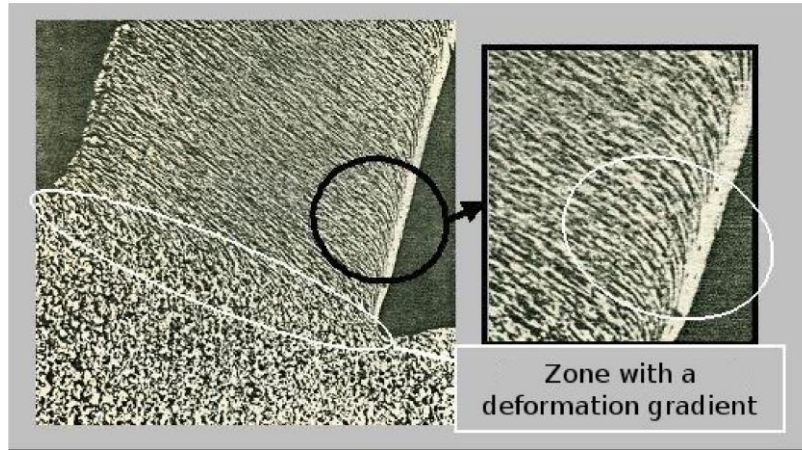


Fig. 2: Chip Micrograph

### Strain Gradient Plasticity

Strain gradient plasticity was developed because the behaviour of materials, and metals especially, can be difficult to explain when plastically deformed over different scales. A size-dependence tends to appear with such deformations and conventional plastic theories lack material length scales in their formulations.

This method was developed in the 1960s by Mindlin [6], Toupin [7] and Germain [8]. These works form the basis of this theory, but they only considered linear elasticity. Fleck and Hutchinson [9] developed the plasticity case. A strain gradient plasticity theory allows to take into account size dependence in a material behaviour law. Gudmundson [10] did an extensive review of different strain gradient theories.

Micro-rotations inside the studied material must first be taken into account. Finite strains must also be considered. The work is focused on developing a thermodynamically consistent finite deformation strain gradient theory accounting the temperature as a variable of the free energy.

The theoretical framework developed here is based on and is the continuation of the work done by different authors. Gurtin in [11] and [12] worked on a gradient theory which accounts for Burgers vector and more particularly for dissipations due to plastic spin in [11]. On the other hand, finite deformation developments of the strain gradient theory were done by different authors such as Polizzotto in [13] and Lele in [14]. Polizzotto presents in his work a constitutive model thermodynamically consistent for finite deformations. Lele develops a finite deformation viscoplastic strain gradient theory based on Gurtin's work on strain gradient plasticity and viscoplasticity. Finally, our work was influenced by articles done by Voyiadjis [15] and Abu Al-Rub [16] where they focus their work on developing a thermodynamically consistent strain-gradient formulation.

### Kinematics

Let us consider a homogeneous solid body  $C$  of volume  $V$  and boundary surface  $SV$  occupying a region of space in a fixed reference configuration referred to a Cartesian orthogonal coordinate system. We denote  $X$  an arbitrarily chosen material point of  $C$ . We can then define the movement by its Lagrangian description given by:

$$x = f(X, t). \quad (1)$$

The deformation gradient is defined by:

$$F = \nabla f = \text{grad}_L f. \quad (2)$$

The velocity is then defined by:

$$\frac{df(X,t)}{dt} = v(X,t) \quad \text{with } X \text{ constant.}$$

The two velocity gradients are defined by:

$$\dot{F} = \text{grad}_L v \quad (4)$$

$$(5) \quad L = \text{grad}_E v$$

It can also be noted that:

$$L = \dot{F} F^{-1} \quad (6)$$

Our theory is based on the Kröner-Lee decomposition [18]. A representation can be seen in Fig. 3.

$$F = F^e F^p \quad (7)$$

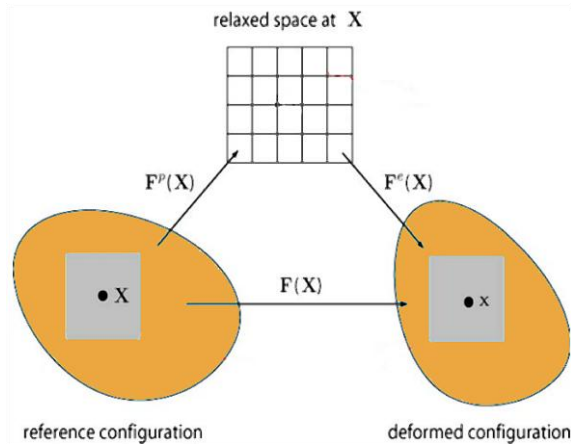


Fig. 3: Kröner-Lee decomposition

The Kröner-Lee decomposition is the separation of the deformation gradient into an elastic and plastic part.

$F^e$  represents the elastic mechanism within the material, i.e. stretching and rotation of the crystal structure. It will be referred as the plastic distortion.

$F^p$  represents a local plastic deformation of the material in  $X$  due to the distortion of the crystalline structure enforced by the formation of dislocations. It will be referred as the elastic distortion.

It is interesting to introduce a similar separation of the velocity gradient with the two following tensors:

$$L^e = \dot{F}^e F^{e-1} \quad (8)$$

$$L^p = \dot{F}^p F^{p-1} \quad (9)$$

Therefore, with these assumptions, the following relationship is found by using 6,7, 8 and 9.

$$L = L^e + F^e L^p F^{e-1} \quad (10)$$

It is possible to define the elastic and plastic stretching and spin tensors through a symmetric tensor  $D$  and an antisymmetric tensor  $W$ . In regards to the elastic distortion, it can be found that:

$$D^e = \frac{1}{2}(L^e + L^{eT}) \quad (11)$$

$$W^e = \frac{1}{2}(L^e - L^{eT}) \quad (12)$$

Using a similar treatment for the plastic distortion, we have:

$$D^p = \frac{1}{2}(L^p + L^{pT}) \quad (13)$$

$$W^p = \frac{1}{2}(L^p - L^{pT}) \quad (14)$$

As a reminder, we obtain the following relation:

$$L^e = D^e + W^e \quad (15)$$

$$(16) \quad L^p = D^p + W^p$$

**Hypotheses. The standard assumption that the plastic flow is incompressible is used. A classical direct result is that:**

$$\det F^p = 1 \quad (17)$$

$$\text{tr } L^p = 0 \quad (18)$$

Usually, it is widely assumed that the plastic flow is irrotational and plastic spin vanishes [11] and [10]. But in the currently developed theory, (and according to experimental measures) it was decided to fully account for the plastic distortion and suppose the plastic spin to be relevant.

We can then define two tensors, plastic flow directions, and two scalars, plastic flow rates.

$$N^p = \frac{D^p}{|D^p|}$$

$$v^p = |D^p|$$

Because of our assumption to consider the plastic spin to be relevant, we have to define a plastic flow direction and a plastic flow rate in regards to .

$$M^p = \frac{W^p}{|W^p|}$$

$$\eta^p = |W^p|$$

Using equations 6, 8 and 10, we obtain

$$\dot{F} F^{-1} = \dot{F}^e F^{e-1} + F^e L^p F^{e-1} \quad (23)$$

and using 2, 19, 20, 21 and 22, we have

$$(\nabla f) F^{-1} = \dot{F}^e F^{e-1} + v^p F^e N^p F^{e-1} + \eta^p F^e M^p F^{e-1} \quad (24)$$

**Burgers' Tensor.** It was then decided to fully account for the Burgers vector in the herein developed strain-gradient theory. The use of Burgers vector in a strain gradient theory allows to account for internal dislocations in crystals; its tensor field, the Burgers tensor, is a measure of geometrically necessary dislocations and is precisely defined in continuum mechanics [11].

It is important to note that when a metal is plastically deformed, dislocations are generated, displaced and accumulated. It is possible to define plastic deformation in regards to geometrically necessary dislocations or statistically stored dislocations, or a combination of both. Statistically stored dislocations refer to dislocations trapping each other in a random way while the material is deformed [20]. Geometrically necessary deformations are dislocations which are necessary to accommodate the geometry induced by the plastic deformations; they are related to the gradients of plastic strains [21].

We use the definition of the Burgers tensor field given by Cermelli and Gurtin in [19].

$$G = F^p \text{rot}(F^p)$$

For our constitutive theory, we need to introduce a scalar constitutive variable related to the accumulation of geometrically necessary dislocations defined in terms of the Burgers tensor. We develop our work in a similar way than Lele in [14]. We will define the plastically convected rate  $\square G$  of the Burgers tensor. First, we have

$$\dot{G} = \dot{F}^p \text{rot}(F^p) + F^p \overline{\dot{(F^p)}} \quad (26)$$

We can define

$$G^* = F^p \overline{\dot{(F^p)}} \quad (27)$$

therefore

$$\dot{G} = \dot{F}^p F^{p-1} F^p \text{rot} F^p + G^* \quad (28)$$

and

$$\dot{G} = L^p G + G^* \quad (29)$$

Using indices, we have

$$G_{ij}^* = F_{im}^p \epsilon_{mrs} F_{js,r}^p \quad (30)$$

$$(31) \quad G_{ij}^* = F_{im}^p \epsilon_{mrs} F_{qs,r}^p L_{jq}^p + F_{im}^p \epsilon_{mrs} F_{qs}^p L_{jq,r}^p$$

$$G_{ij}^* = G_{ij} + F_{im}^p \epsilon_{mrs} F_{qs,r}^p L_{jq,r}^p$$

We define the plastically convected rate of G as

$$\overset{\square}{G}_{ij} = \epsilon_{mrs} F_{im}^p F_{qs,r}^p L_{jq,r}^p \quad \text{Thus} \quad (33)$$

$$\overset{\square}{G}_{ij} = \dot{G} - L^p G - G L^p \quad (34)$$

Using indices to simplify the comprehension

$$\overset{\square}{G}_{ij} = \epsilon_{iaq} L_{jq,b}^p F_{ba}^{p-1} \quad (35)$$

We need to separate the contribution of the plastic spin from the rest

$$\overset{\square}{G}_{ij} = \epsilon_{iaq} L_{jq,b}^p F_{ba}^{p-1} \quad (36)$$

Using the definitions of the plastic flow rates, we obtain

$$\overset{\square}{G}_{ij} = \epsilon_{iaq} v^p N_{jq,b}^p F_{ba}^{p-1} + \epsilon_{iaq} v_{,b}^p N_{jq}^p F_{ba}^{p-1} + \epsilon_{iaq} \eta^p M_{jq,b}^p F_{ba}^{p-1} + \epsilon_{iaq} \eta_{,b}^p M_{jq}^p F_{ba}^{p-1} \quad (37)$$

To simplify we define a new notation

$$\overset{\square}{G}_{ij} = \epsilon_{iaq} D_{jq,b}^p F_{ba}^{p-1} + \epsilon_{iaq} W_{jq,b}^p F_{ba}^{p-1} \quad (38)$$

Therefore, we have

$$\overset{\square}{G}_{ij} = [(F^{p-T} \nabla v^p) \times] N^p + [(F^{p-T} \nabla v^p) \times] \nabla N^p - [(F^{p-T} \nabla v^p) \times] M^p - [(F^{p-T} \nabla v^p) \times] \nabla M^p \quad (39)$$

We finally define the scalar constitutive variable related to the accumulation of geometrically necessary dislocations

$$\Phi^p(X, t) = \int_0^t \overset{\square}{G}(X, \zeta) d\zeta \quad (40)$$

$$(41) \quad \dot{F} F^{-1} = \dot{F}^e F^{e-1} + F^e L^p F^{e-1}$$

and using the previous relations:

$$(\nabla \dot{f}) F^{-1} = \dot{F}^e F^{e-1} + v^p F^e N^p F^{e-1} + \eta^p F^e M^p F^{e-1} \quad (42)$$

**Frame indifference.** When developing the framework of a behaviour law, it is necessary to check for the objectivity (or frame indifference) of the tensors introduced. The changes in the frame or the observer are smooth time-dependent rigid transformations in the Euclidean space. The theory developed here requires to be invariant under a transformation of the form:

$$f^*(X, t) = Q(t) f(X, t) + q(t) \quad (43)$$

Q is a proper-orthogonal rotation tensor and q(t) a vector The deformation gradient is transformed:

$$F^* = QF$$



(44)

We obtain the following results,

$$F^* = (QF)^{-1} = F^{-1} Q^T \quad (45)$$

$$(46) \quad \dot{F}^* = Q\dot{F} + \dot{Q}F$$

Therefore,

$$\dot{F}^* = Q\dot{F} + \dot{Q}F \quad (47)$$

$$L^* = (Q\dot{F} + \dot{Q}F)F^{-1}Q^T$$

$$L^* = Q\dot{F}F^{-1}Q^T + \dot{Q}FF^{-1}Q^T \quad (49)$$

$$L^* = QLQ^T + \dot{Q}Q^T$$

Concerning the Kröner-Lee decomposition, we have:

$$(F^e F^p)^* = QF^e F^p \quad (51)$$

As we can see in Fig. 3, the Kröner-Lee decomposition implies that because the observer is only aware of the deformed configuration that is invariant by a frame change. Therefore, we have:

$$F^{e*} = QF^e \quad (52)$$

$$(53) \quad F^{p*} = F^p$$

As a result

$$L^{p*} = L^p \quad (54)$$

and

$$D^{p*} = L^{p*} + L^{p*\top} = D^p \quad (55)$$

$$W^{p*} = L^{p*} - L^{p*\top} = W^p$$

In a similar manner, just like  $F^p$ ,  $D^p$  and  $W^p$ ,  $G$  and  $\overset{\square}{G}$  are also both frame indifferent.

**Virtual Powers.** The principle of virtual power is used to determine the correct energy balances. It is then assumed that the power expended by each "rate-like" kinematical descriptor can be expressible in terms of an associated mechanical action. It is important to note that the "rate-like" descriptors are constrained by equation 24. Let  $\Omega$  denote an arbitrary part of the body with  $\mathbf{n}$  the outward unit normal on the boundary  $\partial\Omega$  of  $\Omega$ . The following mechanical action system is associated:

Elastic stress	$S^e \Leftrightarrow \dot{F}^e$
Traction stress	$\pi \Leftrightarrow v^p$
Traction stress gradient	$\zeta \Leftrightarrow \nabla v^p$

Rotation stress	$\tau \Leftrightarrow \eta^p$
Rotation stress gradient	$\mu \Leftrightarrow \nabla \eta^p$

It is possible to define the power expenditure within  $\Omega$ :

$$W_{\text{int}}(\Omega) = \int_{\Omega} (S^e : \dot{F}^e + \pi : v^p + \zeta : \nabla v^p + \tau : \eta^p + \mu : \nabla \eta^p) dV \quad (57)$$

The power expended on  $\Omega$  by material external to  $\Omega$  can be defined using:

- $\zeta$  Force density
- $T$  Macroscopic force
- $\chi$  Microscopic force
- $M$  Microscopic Moment

with:

$$W_{\text{ext}}(\Omega) = \int_{\Omega} \zeta \cdot \dot{f} dV + \int_{\partial\Omega} T \cdot \dot{f} dS + \int_{\partial\Omega} \chi \cdot v^p dS + \int_{\partial\Omega} M \cdot \eta^p dS \quad (58)$$

It is assumed that, at some arbitrarily chosen but fixed time, the fields  $f$ ,  $F^e$ ,  $N^p$  and  $M^p$  are known, and the fields  $\dot{f}$ ,  $\dot{F}^e$ ,  $v^p$  and  $\eta^p$  are considered as virtual velocities to be specified independently in a consistent manner with (24). This virtual field is denoted as:

$$V = (\dot{f}, \dot{F}^e, v^p, \eta^p) \quad (59)$$

The compatibility equation becomes

$$(\nabla \dot{f}) F^{-1} = \dot{F}^e F^{e-1} + v^p F^e N^p F^{e-1} + \eta^p F^e M^p F^{e-1} \quad (60)$$

The external and internal power expenditure of virtual power is obtained:

$$W_{\text{int}}(\Omega) = \int_{\Omega} (S^e : \overline{\dot{F}^e} + \pi : v^p + \zeta : \nabla v^p + \tau : \eta^p + \mu : \nabla \eta^p) dV \quad (61)$$

$$(62) \quad W_{\text{ext}}(\Omega) = \int_{\Omega} \zeta \cdot \dot{f} dV + \int_{\partial\Omega} T \cdot \dot{f} dS + \int_{\partial\Omega} \chi \cdot v^p dS + \int_{\partial\Omega} M \cdot \eta^p dS$$

The principal of virtual power is the requirement that the internal and external powers be balanced for any given  $\Omega$ :

$$W_{\text{int}}(\Omega, V) = W_{\text{ext}}(\Omega, V) \quad \text{for any } V \quad (63)$$

It is possible to choose any  $V$  consistent with equation (60). It will allow us to deduce the consequences of the principle of virtual powers. The following virtual velocity field is considered with  $v^p = 0$  and  $\eta^p = 0$ .

$$(\nabla \dot{f}) F^{-1} = \dot{F}^e F^{e-1} \quad (64)$$

and therefore,

$$(\nabla \dot{f}) F^{p-1} = \dot{F}^e \quad (65)$$

$$\int_{\Omega} \zeta \cdot \dot{f} dV + \int_{\partial\Omega} T \cdot \dot{f} = \int_{\Omega} S^e : \dot{F}^e dV$$

(66)

$$\int_{\Omega} \zeta \cdot \dot{f} dV + \int_{\partial\Omega} T \cdot \bar{f} = \int_{\Omega} S^e : (\nabla f) F^{p-1} dV \quad (67)$$

$$\int_{\Omega} \zeta \cdot \dot{f} dV + \int_{\partial\Omega} T \cdot \dot{f} = \int_{\Omega} S^e F^{p-T} : \nabla f dV \quad (68)$$

We note

$$S = S^e F^{p-T} \quad (69)$$

Therefore, it is possible to write

$$\int_{\partial\Omega} T \cdot \dot{f} ds = \int_{\Omega} (S : \dot{f} - \zeta \cdot \dot{f}) dV \quad (70)$$

$$\left( \int_{\partial\Omega} T \cdot \dot{f} ds = \int_{\Omega} (-\text{Div } S \cdot \dot{f} - \zeta \cdot \dot{f}) dV + \int_{\partial\Omega} S \mathbf{n} \cdot \dot{f} ds \right)$$

Finally, for any part of  $c$  and any virtual velocity field with the chosen hypotheses:

$$\int_{\partial\Omega} (T - S \mathbf{n}) \cdot \dot{f} ds + \int_{\Omega} (\text{Div } S + \zeta) \cdot \dot{f} dV \quad (72)$$

The following boundary conditions are obtained:

$$T = S \mathbf{n} \quad (73)$$

$$\text{Div } S + \zeta = 0 \quad (74)$$

The assumption that  $\dot{f} = 0$  and  $v^p = 0$  is made:

$$\dot{F}^e = -\eta^p F^e M^p \quad (75)$$

Therefore,

$$S^e : \dot{F}^e = -\eta^p S^e : F^e M^p \quad (76)$$

$$S^e : \dot{F}^e = -\eta^p ((F^{eT} S^e) : M^p)$$

$$S^e : \dot{F}^e = -\eta^p (K^e : M^p) \quad \text{with } K^e = F^{eT} S^e \quad (78)$$

$$(79) \quad \dot{F}^e = -\eta^p F^e M^p$$

It implies

$$\int_{\partial\Omega} \chi \cdot v^p dS = \int_{\Omega} ((\bar{\pi} - k^m) v^p + \bar{\zeta} (\nabla v^p)) dV \quad (80)$$

Therefore

$$\int_{\partial\Omega} (\chi - \bar{\zeta} \mathbf{n}) v^p dS + \int_{\Omega} (k^m - \bar{\pi} + \text{Div } \bar{\zeta}) v^p dV = 0 \quad (81)$$

Finally,

$$\chi = \bar{\zeta} \mathbf{n}$$

(82)

$$k^m = \pi - \text{Div } \zeta \quad (83)$$

In a similar way, with the assumption that  $\dot{f} = 0$  and  $\eta^p = 0$ , the following boundary equations are obtained:

$$M = \mu \mathbf{n} \quad (84)$$

$$k^n = \pi - \text{Div } \mu \quad \text{with} \quad k^n = K^e : N^p \quad (85)$$

### Thermodynamics and Flow rule

With the final objective of determining a flow law for materials used in machining, thermodynamics provide all the necessary information. It is possible to use the Clausius-Duhem inequality which derives from the first and second principles of thermodynamics. The following equation in local form is therefore obtained:

$$S^e : \dot{F}^e + \pi : \mathbf{v}^p + \zeta : \nabla \mathbf{v}^p + \tau : \eta^p + \mu : \nabla \eta^p - \rho(\dot{\Psi} + s \dot{\Theta}) - \frac{q \nabla \Theta}{\Theta} \geq 0$$

$\Psi$  represents the free energy of the system, measured per unit volume in the structural space. The assumption is made that the free energy can be separated into two parts: elastic free energy  $\Psi^e$  and plastic free energy  $\Psi^p$ .

$$\Psi = \Psi^e + \Psi^p \quad (87)$$

Both free energies are chosen to depend on temperature and variables in relation to the internal phenomena in play. For the plastic free energy, a dependence on the accumulation of geometrically necessary dislocations will be taken. It is possible to then choose a form for the free energy and therefore fully express the different equations to obtain the flow rule.

### Conclusion

During cutting processes, complex phenomena arise and thus the behaviour law of the material must be suitable to model the phenomena. In regards to both the existence of moments at the tip of the tool and for meshing considerations, the strain gradient theory is the most suitable to describe the complete behaviour of the material. In this paper is presented the complete architecture of the development of a theoretical framework of a flow rule applicable in the simulation of machining processes. It was decided to fully account for the plastic spin and incorporate this assumption in the expression of a scalar variable representing the accumulation of geometrically necessary dislocations in the material. Following that study, a finite element implementation will be done in order to refine and test the behaviour law and to confront it to experiments representative of the cutting process. The objective is to use the results from the simulation and experimentation to quantify the different material parameters. As a long term objective, precise predictive simulation of machining processes usable by industrial partners is aimed at.

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