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Some considerations on acoustic resonator phase noise modeling and recent short-term stability experimental results

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Abstract— This paper presents a critical review of different acoustic resonator models exposed in the past, in the light of recent experimental results. Among the few models attempting to explain the $1/f$ behavior of the quartz crystal resonator, the $1/f$ quantum noise theory of P. Handel and the thermal boson theory of M. Planat, are recalled. The latest published experimental results are analyzed taking into account these models. In the case of Handel's model, the noise depends on the resonator volume under the electrodes and the unloaded quality factor of the resonator. Handel's model is supposed to give an order of magnitude of the lowest possible noise level, but now some experimental results are under its prediction. An approach using the vibrating volume of the resonator is proposed to renew the lowest noise level estimation and found to be compatible with the latest experimental results.

I. INTRODUCTION

High-precision quartz oscillators are mainly used in telecommunications, metrology and space applications such as local oscillators in satellites. The short-term stability of such oscillators is essential.

In 1994, J. Norton [1] has measured the best result of short-term stability of a quartz crystal resonator. It was a 5 MHz SC cut mounted in a BVA structure. The result in term of Allan standard deviation was $3.74 \cdot 10^{-14}$ for τ equals to 10 s. In 1999, at 10 MHz, R.J. Besson [2] measured two BVA quartz crystal oscillators with $5 \cdot 10^{-14}$ Allan standard deviation. Then, for ten years, there had been no significant improvement of quartz oscillators in the short term stability domain and no real solution had been found to improve the phase noise of BAW resonators. J. Norton's result represented the best acoustic resonator noise performance that could be found in the literature, pretty close to the lowest theoretical estimates. However, recently, better experimental results [3-4] have shown that quartz crystal oscillators may still have not said their last word in the short-term stability domain.

The oscillator limitation is theoretically imposed by the resonator. However, few models attempt to explain the $1/f$ behavior of the quartz crystal resonator. In this paper, we will briefly recall the $1/f$ quantum noise theory of P. Handel and the thermal boson theory of M. Planat in order to compare the latest published experimental results to the lower limitations computed with these two models. In the case of Handel's model, the noise depends on the resonator volume under the electrodes and the unloaded quality factor (Q) of the resonator. It gives an order of magnitude of the noise, but now some experimental results are below its prediction. In this paper, we will show that a lower theoretical estimate can nonetheless be extracted from Handel's model provided one uses the vibrating volume of the resonator instead of the volume under the electrodes.

II. MODELS

The following part presents a summary of the two models trying to elucidate the physical origin of the $1/f$ flicker noise of a quartz crystal resonator with a focus on the origin of the $1/f$ formula. We note, that both models can be seen as the two opposite sides of the fluctuation-dissipation theorem [5], since Planat studies the energy fluctuations around statistical equilibrium of a population of massless degenerate thermal bosons, whereas Handel proposes an origin for the dissipation occurring when the oscillator comes back to its mode after a small perturbation.

A. Planat's Model

In 2003, M. Planat proposed a statistical thermodynamics based model to explain the $1/f$ noise in quartz crystal resonators [6]. In this paper, Planat considered the case of a population of massless bosons with a degenerate quantum harmonic oscillator energy spectrum which predict a hyperbolic fluctuation spectrum at low frequencies. According to the equipartition law of statistical mechanics, the partition function is introduced using a degeneracy parameter of the

energy level nhf with n the number of bosons, h the Planck constant and f the frequency. The free energy F may be approximated in the low frequency part of the spectrum by:

$$\frac{F}{kT} \approx -\frac{\pi^2}{6x} - \frac{1}{2} \ln\left(\frac{x}{2\pi}\right) + \frac{x}{24} \quad (1)$$

with $x = hf/kT$

Subsequently, the exposed model keeps only the first term of (1).

According to the Einstein's conventional approach, the energy fluctuations of a system in equilibrium within a larger system of temperature T are:

$$\varepsilon^2 = \langle (E - \langle E \rangle)^2 \rangle = kT^2 \frac{\partial \langle E \rangle}{\partial T} \quad (2)$$

One can reformulate this relation for the fluctuations of the energy $dE = u(f,T)df$ in the bandwidth df :

$$d\varepsilon^2 = kT^2 \frac{\partial u}{\partial T} df = S_u(f) df \quad (3)$$

with u the energy spectral density and $S_u(f)$ the power spectral density of the fluctuations of u .

Thus, energy fluctuations in the bandwidth df are given by:

$$\left[\frac{S_u(f)}{u^2} \right]_{LF} = \frac{9hc_{ph}^3}{4\pi^3 V} \frac{1}{kTf} = \frac{A_{ph}}{Vf} \quad (4)$$

With $3/c_{ph}^3 = 1/c_C^3 + 1/c_B^3 + 1/c_A^3$, c_A , c_B and c_C the velocities of the longitudinal mode A and the shear modes B and C, respectively.

Considering [7-8], the fractional frequency fluctuations in a quartz crystal resonator are:

$$S_y(f) = \frac{S_\omega(f)}{\omega^2} = \frac{1}{Q^4} \frac{A_{ph}}{Vf} = \frac{h_{-1}}{f} \quad (5)$$

(5) has been used to give an order of magnitude of the power spectral density of the fractional frequency fluctuations of a 5 MHz 5th overtone (OT 5), AT-cut quartz crystal resonator.

B. Handel's Model

P. H. Handel proposes in [9] a summary of his $1/f$ quantum noise theory. According to him, the general origin of the intrinsic $1/f$ noise is due to a quantum self-interference process caused by very small spontaneous bremsstrahlung energy losses. The rate Γ of phonon interactions which remove phonons from the main quartz resonator mode is modulated by the quantum $1/f$ effect. The time-derivative of the polarization vector P of the quartz crystal is affected. According to bremsstrahlung, when the phonon is removed, a constant energy is radiated away per unit frequency interval at any frequency, its expression is:

$$E = \frac{1}{(4\pi\varepsilon_0)} \frac{(\Delta\dot{P})^2}{3c^3} \quad (6)$$

With $\Delta\dot{P}$, the variation of current density of the quartz crystal.

Then the number of photon emitted as bremsstrahlung of frequency f , per Hertz is given by dividing (6) by the energy of photon hf :

$$Z = \frac{2\alpha(\Delta\dot{P})^2}{3\pi f e^2 c^2} \quad (7)$$

With $\alpha = (1/4\pi\varepsilon_0)(2\pi e^2/hc) = 1/137$, the Sommerfeld's fine structure constant.

Thus the one side spectrum is inversely proportional to f :

$$S_{\delta\Gamma/\Gamma}(f) = \frac{4\alpha(\Delta\dot{P})^2}{3\pi f e^2 c^2} \quad (8)$$

If the main resonant mode is described as a forced harmonic oscillator with losses then the quantum $1/f$ fluctuations are present in the loss coefficient Γ of the following equation:

$$\frac{d^2x}{dt^2} + 2\Gamma \frac{dx}{dt} + \omega_0^2 x = F(t) \quad (9)$$

The quality factor of the resonator is defined as:

$$Q = \omega_r / 2\Gamma \quad (10)$$

The resonant frequency is given by:

$$\omega_r^2 = \omega_0^2 - 2\Gamma^2 \quad (11)$$

Thus

$$S_y(f) = \frac{1}{4Q^4} S_{\delta\Gamma/\Gamma}(f) \quad (12)$$

Where $y = \Delta f/f_0$ is the relative frequency fluctuation.

The vibrational energy of the crystal can be written as:

$$W = n\hbar\langle\omega\rangle = 2\left(\frac{Nm}{2}\right)\left(\frac{dx}{dt}\right)^2 = \left(\frac{m}{Ne^2}\right)\chi^2(\dot{P})^2 \quad (13)$$

With m the reduced mass of the elementary oscillating dipoles, e their charge, N their number in the crystal, χ a polarization constant, $\langle\omega\rangle$ the average circular frequency of a thermal phonon interacting with phonons in the main resonator mode and n the average number of phonons in that typical thermal phonon mode.

Thus with:

$$(\Delta\dot{P})^2 = \frac{N\hbar\langle\omega\rangle}{n} \frac{e^2}{4\chi^2} \quad (14)$$

We can write:

$$S_y(f) = \frac{\beta}{f} \left(\frac{V}{Q^4} \right) \quad (15)$$

With: $\beta = n_0 \frac{\alpha \hbar \langle \omega \rangle}{12n\pi mc^2 \chi^2} = 1/cm^3$, n_0 the number of dipoles per volume unit and V in cm^3 .

III. ACOUSTIC VOLUME ESTIMATION

The acoustic volume estimation is carried out using Tiersten's definitions as *e. g.* in [10].

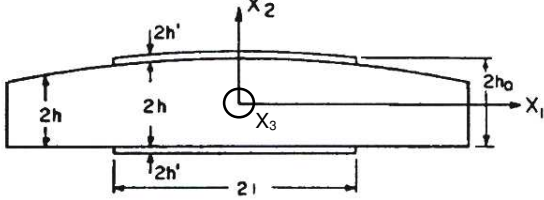


Figure 1. Geometric definition of a plano-convex resonator

Due to the plano-convex shape, the thickness of the resonator $2h$ is given by the following expression (Fig. 1):

$$2h \cong 2h_0 \left[1 - \frac{(x_1^2 + x_2^2)}{4Rh_0} \right] \quad (17)$$

The amplitude of the mechanical displacement in the resonator is:

$$u_{nmp} = A \sin \frac{n\pi x_2}{2h} H_m(\sqrt{\alpha_n} x_1) H_p(\sqrt{\beta_n} x_3) \cdot e^{-\alpha_n(x_1^2/2)} e^{-\beta_n(x_3^2/2)} e^{j\omega_{nmp}t} \quad (18)$$

with:

$$\alpha_n^2 = \frac{n^2 \pi^2 \hat{c}}{8Rh_0^3 M_n} \quad \text{and} \quad \beta_n^2 = \frac{n^2 \pi^2 \hat{c}}{8Rh_0^3 P_n} \quad (19)$$

Where $n = 1, 3, 5, \dots$ the overtone number, $m, p = 0, 2, 4, \dots$, label the different mode shapes in the plane of the resonator, H_m and H_p are Hermit polynomials, R is the curvature radius, $2h_0$ is the thickness of the resonator center, M_n and P_n are the dispersion constants and \hat{c} is the effective elastic constant associated to the propagation of the (C) mode.

For the 3rd overtone (OT 3), we have:

$$u_{300} = A \sin \frac{3\pi x_2}{2h} e^{-\alpha_3(x_1^2/2)} e^{-\beta_3(x_3^2/2)} e^{j\omega_{300}t} \quad (20)$$

$$\alpha_3^2 = \frac{9\pi^2 \hat{c}}{8Rh_0^3 M_3} \quad \text{and} \quad \beta_3^2 = \frac{9\pi^2 \hat{c}}{8Rh_0^3 P_3} \quad (21)$$

In the case of SC-cut quartz resonator [11], the cut is defined by a doubly rotation ($\varphi = 22^\circ 45'$, $\theta = 34^\circ$) and in N/m^2 :

$$\hat{c} \cong 34.6 \cdot 10^9, M_3 \cong 57 \cdot 10^9, P_3 \cong 67 \cdot 10^9$$

The energy of the acoustic wave is given by:

$$W_1 \cong \frac{1}{2} \hat{c} \left(\frac{3\pi}{2h} \right)^2 \iiint_V \left[\cos \left(\frac{3\pi}{2h} x_2 \right) e^{-\alpha_3 \frac{x_1^2}{2}} e^{-\beta_3 \frac{x_3^2}{2}} \right]^2 dV \quad (22)$$

To define the acoustic volume, we can consider the vibration amplitude which remains constant over the surface S_{eq} , then we compute a cylindrical volume that has the same energy than the acoustic wave.

$$W_2 \cong \frac{1}{2} \hat{c} \left(\frac{3\pi}{2h} \right)^2 \int \left[\cos \left(\frac{n\pi}{2h} x_2 \right) \right]^2 dx_2 \times S_{eq} \quad (23)$$

Thus:

$$W_1 = W_2 \Rightarrow S_{eq} = \iint_S \left[e^{-\alpha_3 \frac{x_1^2}{2}} e^{-\beta_3 \frac{x_3^2}{2}} \right]^2 dx_1 dx_3 \quad (24)$$

Consequently, the acoustic volume is defined by:

$$V_{ac} = S_{eq} 2h_0 \quad (25)$$

IV. APPLICATION OF THE MODELS TO EXPERIMENTAL MEASUREMENTS

As seen in [4], the best measured resonators are the 5 MHz BVA SC-cut, OT 3, resonator from Oscilloquartz (Neuchatel, Switzerland). In [12], a succinct description of the resonator is given which allows us to compute an order of magnitude of our approach. The diameter of the resonant part of the resonator is around 20 mm, the electrodes diameter D is about 11 mm. The thickness of the resonator is chosen to vibrate on the OT 3 at 5 MHz. The acoustic velocity for the thickness shear C-mode of a SC-cut in quartz crystal is about 3610 m/s. Thus, the thickness at the center of the resonator $2h_0$ is given by $3\lambda/2$ formula and is approximately equal to 1.15 mm.

The energy trapping is quasi-independent of the electrodes dimension, in contoured resonators. It influences the motional resistance and the static capacitor. The displacement can be computed from the center of the resonator to the external edge of the resonator. The radius of curvature is one of the most important parameter. Usually, sufficient energy trapping is obtained when the lateral boundary has no effect on the vibration. Thus, we will consider an attenuation of the mechanical displacement of about 10^{-7} between the center and the boundary of the resonator. This trapping corresponds to a curvature radius R of about 290 mm (fig. 2).

The Q factor of this kind of resonator can be found from the dissipative matrix η_{ij} of the quartz crystal [13]. These constants must be expressed in the SC-cut orientation. Then, the values are considered as the imaginary part of the effective elastic coefficients. Resolution of the Christoffel equations [11] gives the complex eigenvalue $\hat{c} + i\hat{\eta}$. Then, an order of magnitude of the Q factor can be obtained using the following equation:

$$Q = \frac{\hat{c}}{\hat{\eta}2\pi f} \quad (26)$$

In our case, $\hat{\eta} \cong 3.95 \cdot 10^{-4} \text{Ns/m}^2$. A SC-cut quartz crystal resonator, at 5 MHz, OT 3, gives a maximum of quality factor of about $2.8 \cdot 10^6$. This value corresponds with the one given in [3].

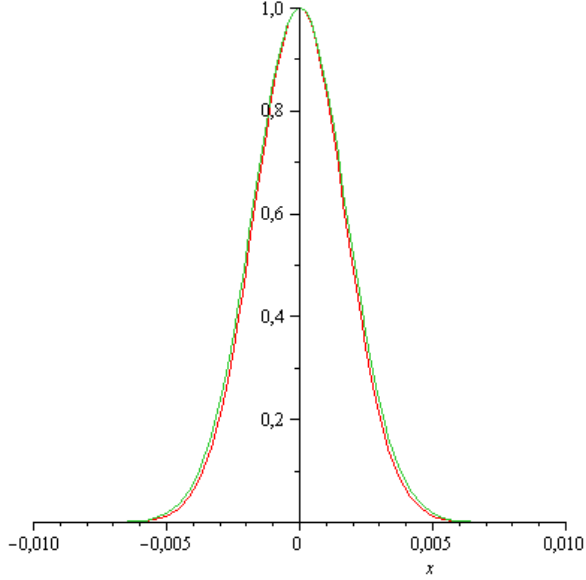


Figure 2. Energy trapping of a plano-convex resonator, SC-cut 5 MHz, OT3, $R = 290 \text{ mm}$. (red curve: along x_3 axis, green curve: along x_1 axis).

Consequently, we can compute the motional resistor R_{mot} from the motional capacitance C_{mot} using:

$$R_{mot} = \frac{l}{QC_{mot}2\pi f} = \frac{L_{mot}2\pi f}{Q} \quad (27)$$

In the case of OT 3, C_{mot} is given by [10]:

$$C_{mot} = \frac{64\hat{e}_{26}^2 \left(\int_0^{\frac{D}{2}} e^{-\alpha_3 \frac{x_1^2}{2}} dx_1 \int_0^{\frac{D}{2}} e^{-\beta_3 \frac{x_3^2}{2}} dx_3 \right)^2 \sqrt{\alpha_3} \sqrt{\beta_3}}{9\pi^3 h_0 \hat{c}} \quad (28)$$

With D the electrode diameter, $\hat{e}_{26} \cong -0.0576 \text{ C/m}^2$ the effective piezoelectric constant.

The computing value of C_{mot} is around 280 fF. We obtain R_{mot} equals to 42 Ω , L_{mot} is about 3.6 H. These results are closed to the measured value in [3].

The static capacitor C_0 can be also computed with the following formula [9]:

$$C_0 = \frac{\pi D^2}{4} \left(1 + \frac{(D/2)^2}{8Rh_0} \right) \frac{\epsilon_{22} + \frac{\hat{e}_{26}^2}{\hat{c}}}{2h_0} \quad (29)$$

With $\epsilon_{22} = 39.78 \cdot 10^{-12} \text{ F/m}$ the permittivity constant in SC-cut.

We have C_0 around 3.4 pF.

Short-term stability is usually given in the time domain with an Allan variance. The Allan variance is computed from the Fourier frequency and from the power spectral density of the frequency fluctuations [14]:

$$\sigma_y^2(\tau) = \int_0^{\infty} S_y(f) \frac{2 \sin^4 \pi f \tau}{(\pi f \tau)^2} df \quad (30)$$

In the case of the flicker frequency noise (which is the resonator noise), relation between the Allan standard deviation and the power spectral density (PSD) of relative frequency fluctuations $S_y(f)$, is given by [15]:

$$\sigma_{y_floor} = \sqrt{2 \ln(2) S_y(1\text{Hz})} \quad (31)$$

Table 1 presents the comparison between measurements and the results proposed by both models. For Planat's model, only the velocity of the C-mode of the acoustic wave is used to the computation. The result is really higher than the experiment. Using the acoustic volume in this model is not relevant because of the invert proportionality.

TABLE I. COMPARISON OF THE SHORT-TERM STABILITY IN TERM OF ALLAN STANDARD DEVIATION σ_{y_floor} OF A SC-CUT, 5 MHz, OT 3 RESONATOR

Frequency (MHz)	5
Quality factor Q (10^6)	2.8
R_{mot} (Ω)	42
C_{mot} (fF)	280
L_{mot} (H)	3.6
C_0 (pF)	3.4
Volume under electrodes V_{elec} (cm^3)	$1.093 \cdot 10^{-1}$
Acoustic volume V_{ac} (cm^3)	$1.076 \cdot 10^{-2}$
Volume ratio	10
$\sigma_{y_floor_planat}$ (vol. under electrodes)	$5.2 \cdot 10^{-12}$
$\sigma_{y_floor_Handel}$ (vol. under electrodes)	$5.4 \cdot 10^{-14}$
$\sigma_{y_floor_Handel}$ (Acoustic vol.)	$1.67 \cdot 10^{-14}$
$\sigma_{y_floor_exp}$ [1] (oscillator measurement)	$3.75 \cdot 10^{-14}$
$\sigma_{y_floor_exp}$ [4] (oscillator measurement)	$2.5 \cdot 10^{-14}$

The flicker noise seems to be the limit of the short-term stability of a quartz crystal resonator. An order of magnitude of this limit is given by the previous calculus around $1.6 \cdot 10^{-14}$ with the Handel's model using the definition of the acoustic volume.

If it was possible to eliminate this noise, the noise limit should be given by the thermal noise of the resonator. In term of Allan standard deviation, its expression is usually [16]:

$$\sigma_{y_therm}(\tau) = \frac{1}{Q_L} \sqrt{\frac{kT}{2P\tau}} \quad (32)$$

With a loaded Q-factor Q_L around $1.6 \cdot 10^6$ for the previous resonator and a excitation power of $60 \mu\text{W}$, we get around $T = 80^\circ\text{C}$:

$$\sigma_{y_therm}(\tau) \approx 4 \cdot 10^{-15} \cdot \tau^{-1/2} \quad (33)$$

At 1s, without $1/f$ noise, the short-term stability limit should be $4 \cdot 10^{-15}$.

V. CONCLUSION

Regarding the experimental results, the $1/f$ story of the quartz crystal is not over. The considered models should be the fundamental limit of the flicker noise of a resonator. Values of Handel's model are under the experimental measurements if the volume is considered as the acoustic volume. The experimental results seem to be now very close to the fundamental limit. The model validity could be proved by varying the acoustic volume while keeping the Q factor of the resonator constant.

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REFERENCES

- [1] J. R. Norton, "Performance of ultrastable quartz oscillators using BVA resonators," Proc. European Frequency and Time Forum, Weihenstephan, Germany, March 1994, pp. 457-465.
- [2] R. J. Besson, M. Mourey, S. Galliou, F. Marionnet, F. Gonzalez, P. Guillemot, R. Tjoelker, W. Diener, A. Kirk, "10 MHz Hyperstable quartz oscillators," Proc. Joint Meeting IEEE Ann. Freq. Cont. Symp. and European Frequency and Time Forum, Besançon, France, april 2009, pp. 326-330.
- [3] J. Chauvin, P. Weber, J.P. Aubry, F. Lefebvre, F. Sthal, S. Galliou, E. Rubiola and X. Vacheret, "A new generation of very high stability BVA oscillators," Proc. Joint Meeting IEEE Ann. Freq. Cont. Symp. and European Frequency and Time Forum, Genova, Switzerland, June 2007, pp. 1261-1268.
- [4] P. Salzenstein, A. Kuna, L. Sojdr and J. Chauvin, "Significant step in ultra high stability quartz crystal oscillators," Electron. Lett., vol. 46, no. 21, pp. 1433-1434, (2010).
- [5] R. Kubo, "The fluctuation-dissipation theorem," Rep. Prog. Phys., vol. 29, pp. 255-284, (1966).
- [6] M. Planat, "Thermal $1/f$ noise from the theory of partitions: application to a quartz resonator," Physica A, vol. 318, pp. 371-386, (2003).
- [7] J. J. Gagnepain, J. Uebersfeld, G. Goujon, P. Handel, "Relation between $1/f$ noise and Q-factor in quartz resonators," Proc. IEEE Ann. Freq. Cont. Symp., Ft. Monmouth, New Jersey, May 1981, pp. 476-483.
- [8] P. H. Handel, "Nature of $1/f$ frequency fluctuations in quartz resonators," Solid State Electron., vol. 22, pp. 875-876, (1979).
- [9] P. H. Handel, "Quantum $1/f$ effect in resonant biochemical piezoelectric and MEMS sensors," IEEE Trans. on Ultrason., Ferroelec. and Freq. Cont., vol. 52, no. 9, Sept., pp. 1461-1467, (2005).
- [10] D. S. Stevens, H. F. Tiersten, "An analysis of doubly rotated quartz resonators utilizing essentially thickness modes with transverse variation," J. Acous. Soc. Am., vol. 79, n° 6, pp. 1811-1826, (1986).
- [11] R. Bourquin, B. Dulmet, J. J. Boy, "SC-cut resonator operating in anharmonic modes with B-mode reduction," Proc. European Frequency and Time Forum, Brighton, United Kingdom, March 1996, pp. 239-243.
- [12] D. Schnewly, Y. Schwab, "Quartz clocks," 3rd Int. Telec. Sync. Forum, London, UK, Oct. 2005.
- [13] J. Lamb, J. Richter, "Anisotropic acoustic attenuation with new measurements for quartz at room temperatures," Proc., Roy. Soc. Series A, vol. 293, pp. 479-492, (1966).
- [14] L.S. Cutler, C. L. Searle, "Some aspects of the theory and measurement of frequency fluctuations in frequency standards," Proc. IEEE, vol. 54, no. 2, Feb., pp. 136-154, (1966).
- [15] F. Sthal, M. Mourey, F. Marionnet, W. F. Walls, "Phase noise measurements of 10 MHz BVA quartz crystal resonators," IEEE Trans. on Ultrason., Ferroelec. and Freq. Cont., vol. 47, no. 2, March, pp. 369-373, (2000).
- [16] F. L. Walls, J. Vig, Phase Noise "Fundamental limits on the frequency stabilities of crystal oscillators," IEEE Trans. on Ultrason., Ferroelec. and Freq. Cont., vol. 42, no. 4, July, pp. 576-588, (1995).