HAL
open science

Principles of Uncertainty, by J.B. Kadane: A review

Christian Robert

To cite this version:

Christian Robert. Principles of Uncertainty, by J.B. Kadane: A review. 2012. hal-00659915

HAL Id: hal-00659915

https://hal.science/hal-00659915v1

Submitted on 14 Jan 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Christian P. Robert
Université Paris-Dauphine, CEREMADE, & IUF,

A review of

Principles of Uncertainty, by

Joseph B. Kadane. Chapman and Hall/CRC Press, 2011. ISBN 978-1-439-86161-5. xxvii+ 475.

> *"Bayes Theorem is a simple consequence of the axioms of probability, and is therefore accepted by all as valid. However, some who challenge the use of personal probability reject certain applications of Bayes Theorem." J. Kadane,* Principles of Uncertainty*, p.44*

Principles of Uncertainty by Joseph Kadane (from Carnegie Mellon University) is a profound and mesmerizing book on the foundations and principles of subjectivist or behavioristic Bayesian analysis. Kadane wrote *Principles of Uncertainty* over a period of several years and, more or less in his own words, it represents the legacy he wants to leave for the future. And this is a legacy Jay can certainly be proud of!

The book starts with a large section on Kadane's definition of a probability model, including rigorous mathematical derivations all the way to Lebesgue measure (or more exactly the McShane-Stieltjes measure). Terrific: I was somehow under the impression that measure theory had been given up in the US statistics curriculum. This section contains many gems as side derivations that pertain to mathematical analysis, in order to explain the subtleties of infinite countable versus uncountable sets, and the distinction between finitely additive and countably additive (probability) measures. Unsurprisingly, given the author's stance, the role of utility is strongly emphasized in a book that keeps stressing the personalistic entry to Bayesian statistics. *Principles of Uncertainty* also contains a formal development on the validity of Markov chain Monte Carlo methods that is superb and missing in most equivalent textbooks. Overall, the book is a pleasure to read as the author's reasoning unravels in a coherent and unitarian manner. I highly recommend *Principles of Uncertainty* for teaching as it can be used at so many different levels (except the most elementary ones).

> *"My desire to avoid the phrase 'it can be shown that' has led me to display more of the mathematical underpinnings of the subject than customary." J. Kadane,* Principles of Uncertainty*, p.xxv*

Indeed *Principles of Uncertainty* is (almost) self-contained from a mathematical point of view. Probability is defined from a betting perspective (no stabilization of frequencies à la von Mises!). Limits, series, uncountable sets, Riemann integrals (whose simultaneous use with and without integration domain confused me for a while), Stieltjes integrals, Fatou's lemma, Lebesgue's dominated convergence theorem, matrix algebra, Euler's formula, the Borel-Kolmogorov paradox, Taylor expansions—I dislike the use of HOT for "higher order terms" in math formulas!—, Laplace's approximation, the Weierstrass approximation, all those notions are covered in reasonable details within the 500 pages of the book. (However, I am uncertain about the discussion on the uniform distribution on the integers, in Section 3.2!) All standard distributions are covered and justified (incl. the Wishart distribution). Paradoxes like Simpson's, Monty Hall's, the Gambler's Ruin, Allais', the Prisoner dilemma, are processed in specific sections. As written above, the processing of the convergence of MCMC algorithms is quite nice and rather unique: the argument is based on a minorizing constraint (existence of a small set) and the use of the corresponding renewal process of Nummelin (1984), which, in my opinion (Robert, 1995), is a beautiful way of explaining most convergence properties of Markov chains. While the R code sprinkled along the book may appear superficial, I think on the opposite it relates to the same goal of Kadane to keep no step unjustified and hence to back graphs with the corresponding R code. The style is as personalistic as the message and very enjoyable, with little stories at the entry of some chapters to make a point. As I ravenously read the book within a few days, I cannot be certain I did not miss typos, but I saw very few. (A change of line within the first displayed formula of page 87 is rather surprising and, I think, unintentional. Some pages like p.215, p.230 or p. 324-326 also end up with several highly isolated formulas because of long displayed equations the page after. A Σ instead of a $\sum$ p. 302. Huge spaces after "=" signs on p. 322 and pp. 363-364. Nothing LaTeX cannot not fix.)

> *"A hierarchical model divides the parameters into groups that permit the imposition of assumptions of conditional independence." J. Kadane,* Principles of Uncertainty*, p.337*

The hierarchical chapter of *Principles of Uncertainty* is also well designed, with connections to the James-Stein phenomenon (my personal sesame to the Bayesian world!). And an inclusion of the famous New Jersey turnpike lawsuit (Kadane and Terrin, 1997). In the model choice section (pp.343-344), Kadane comes the closest to defining a Bayesian test, even though he does not call it this way. He will only return to tests in the final chapter (see below). The MCMC chapter that comes right after, while being highly

enjoyable on the theoretical side, is missing an illustration of MCMC implementation and convergence (or lack thereof).

> *"A claim of possession of the objective truth has been a familiar rhetorical move of elites, social, religious, scientific, or economic. Such a claim is useful to intimidate those who might doubt, challenge, or debate the 'objective' conclusions reached. History is replete with the unfortunate consequences, nay disasters, that have ensued. To assert the possession of an objective method of analyzing data is to make a claim of extraordinary power in our society. Of course it is annoyingly arrogant, but, much worse, it has no basis in the theory it purports to implement." J. Kadane,* Principles of Uncertainty*, p.446*

The above quote is the concluding sentence of the one but last chapter. It reflects on the opinion of the author in such a belligerent way that I fear this chapter does not belong to a general audience book: the chapter "Exploration of Old Ideas" in *Principles of Uncertainty* is too antagonistic to be understandable by neophytes, sounding rather like a backyard fight between small gangs with esoteric names and goals. I of course object to the few lines dismissing 'objective' Bayes and Jeffreys' (1939) heritage as ungrounded, but also to the too hasty processing of Fisher's and Neyman-Pearson's "flavors of testing". (The unfamiliarity of Kadane with frequentist testing shows in the choice of the bound 2.36 for the one sided-test at level 0.05 p.439, since it should be either 1.68 at level 0.05 or 2.36 at level 0.01?.) While being unhappy with this chapter, I obviously consider it as a very minor scratch on an otherwise superb and uniquely coherent book. (Esp. when considering the very final sentence "It is precisely to explain the reasons why I find certain methodologies appropriate, and others less so, that I undertook to write this book" (p.448).) A must-read for sure!

In addition, *Principles of Uncertainty* is freely available in pdf on the author's website `http://uncertainty.stat.cmu.edu`, which is a terrific feature. It should ensure a wider dissemination of Kadane's perspective, especially among the graduate students.

As a postscript, the reader may wonder at my lack of comparison with existing books like Berger (1985), Bernardo and Smith (1994) or even Robert (2001). As the author of the latter, it is rather delicate for me to propose an 'objective' ranking wrt *Principles of Uncertainty*. However, those reference textbooks are all of the 'objective' Bayes flavor so fiercely criticized by Kadane. They any and all enjoy a coherent presentation of the 'objective' approach and may thus serve as a solid counterpoint to Kadane's own 'Art of Fugue' in a graduate course.

REFERENCES

Berger, J.O. (1985), *Statistical Decision Theory and Bayesian Analysis*, New York: Springer-Verlag.

Bernardo, J.M. and Smith, A.F.M. (1994), *Bayesian Theory,* New York: J. Wiley.

Jeffreys, H. (1939), *Theory of Probability*, Cambridge: The Clarendon Press.

Kadane, J.B. and Terrin, N. (1997), "Missing data in the forensic context," *Journal of the Royal Statistical Society Series A*, 160, 351–357.

Kalman, M. (2007), *The Principles of Uncertainty*, New York: Penguin Group.

Nummelin, E. (1984), *General Irreducible Markov Chains and Non-Negative Operators*, Cambridge: Cambridge University Press.

Robert, C.P. (1995), "Convergence control techniques for MCMC algorithms," *Statistical Science*, 10(3), 231–253.

Robert, C.P. (2001), *The Bayesian Choice*, New York: Springer-Verlag.