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► **To cite this version:**

Doudou Badiane, Alain Gasser, Eric Blond, Luc Bellière. Vibrating beams and application to viscometers. Pan American Congress of Applied Mechanics 2012, Jan 2012, Port of Spain, Trinidad and Tobago. pp.6. hal-00658851

**HAL Id: hal-00658851**

**<https://hal.science/hal-00658851>**

Submitted on 12 Jan 2012

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## VIBRATING BEAM IN VISCOUS FLUID FOR VISCOSITY SENSING: APPLICATION TO AN INDUSTRIAL VIBRATING VISCOMETER

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**Abstract.** This paper presents a finite element model of an apparatus based on the phenomenon of resonance, for measuring the viscosity of newtonian fluids. The study is based on a cantilever beam in a viscous fluid excited by an electromagnetic force. The Bernoulli-Euler equation is used to model the beam and linearized Navier-Stokes equations for the fluid. The Maxwell equations are used for the magnetic-structure interaction. The action of the fluid on the beam is modeled by hydrodynamic resistance coefficients. A semi numerical model of a vibrating beam is obtained and results are used to model the vibrating viscometer.

**Keywords:** Vibration, Fluid-structure interaction, Viscosity, Viscometer, Magneto-structure interaction,...

### 1. INTRODUCTION

Viscometers that are used in the oil, food or cosmetic industries to control on-line processes can be modeled by cantilever or doubly clamped beams commonly used in atomic force microscopy "Sader (1998)" as mass-sensitive devices "Villarroya et al. (2005)", and biosensors "Vancura et al. (2005)", in both liquid and gaseous environments. Several types of viscometers are available "Streeter (1955); Blinder (1955); Baley (1958); Pao (1961); Wazer et al. (1963)", among which: rotational viscometers, capillary viscometers, etc. This study focuses on the vibrating viscometer in a flexural mode, in which excitation is achieved by the Laplace force. Generally a beam vibrating in viscous flowing fluid is subjected to two forces: the force generated by natural unsteadiness flow (turbulent forces or dysphasic) that does not depend on the beam's vibration movements and the elastic interaction forces that depend on the vibration movements. Multiphase flow and flow with solid particles are not investigated in this study. Viscometer design is a multi-physics problem. The aim of this work is to construct a numerical model of the viscometer based on cantilever structures, including all the physics (i.e. magnetism, vibration, fluid mechanics) to assist in enhancing Sofraser's vibrating viscometers "Fig. 1" that has been mainly developed through empirical work.

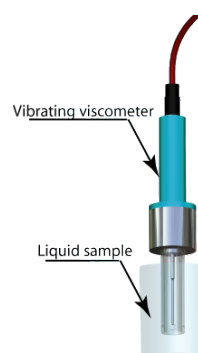


Figure 1. Vibrating viscometer.

Thereby the gain of this work is the modeling and the semi numerical computation of a fluid structure interaction problem, where commercial finite element programs show huge difficulties in an adequate resolution of both fluid and beam dynamics due to computational limitations. Beyond the semi numerical model that is developed, the final aim is to miniaturize the existing viscometer.

The following study is a multi-physics problem, comprising a structural part (the sensor), and a fluid part, which changes the vibration characteristics of the sensor, so that we are dealing with a problem of fluid-structure interaction "Fig. 2".

The vibration of the mechanical oscillator is due to an electric current through a coil which is under a static magnetic field. When the mechanical oscillator is immersed in a viscous fluid, the magnitude of the vibration decreases because of the energy dissipation. In these conditions, the quality factor  $Q$  of the oscillator can be significantly reduced by viscous

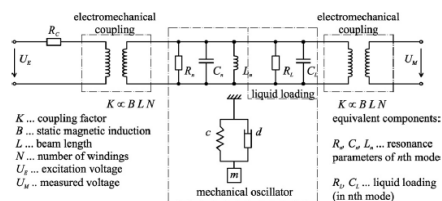


Figure 2. Electrical equivalent circuit for a vibrating viscometer "Reichel et al. (2008)".

dissipation "Kojro et al. (1996)". In mechanical oscillator, many physical mechanisms can be responsible for the energy dissipation "Vignola et al. (2006)". The  $Q$  is typically dominated by radiation of elastic energy into the attachments "Judge et al. (2007)", or by one of three dissipation mechanisms associated with the surrounding fluid medium "Vignola et al. (2006)": acoustic, squeeze-film and viscous losses. The energy dissipated per cycle is then the sum of the energy dissipated by each of these mechanisms. Acoustic radiation can be the dominant loss for plate-like structures vibrating out-of-plane "Willams (1983)". Squeeze-film loss can be significant when a narrow gap is formed between vibrating and stationary elements and the vibratory motion squeezes the fluid in the gap "Hansen et al. (2000)". For slender structures viscous drag is typically the dominant loss mechanism. The excitation of the sensor involves a magneto-structure interaction. Each physical phenomenon is described by its equations and modeled by the finite element method.

**2. METHODS AND MATERIALS**

In this theoretical analysis, we first consider a circular cantilever beam vibrating in air, and then the same beam immersed in a viscous fluid. It is assumed that both the beam and the fluid satisfy the following criteria "Sader (1998)":

- The cross section of the beam is uniform over its entire length;
- The diameter  $D$  of the beam is far smaller than its length;
- The beam is an isotropic linearly elastic solid and internal frictional effects are negligible;
- The amplitude of vibration  $X(x, \omega)$  of the beam is smaller than its length  $L$  and diameter;
- The fluid is newtonian, incompressible and in case the beam is vibrating we assume that far from it the velocity of the fluid is zero.

**2.1 The euler-bernoulli equation**

The motion of a beam clamped on one side and free on the other "Fig. 3", can be described in time harmonic terms by

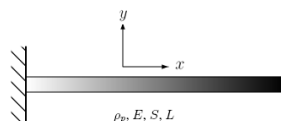


Figure 3. Clamped-free beam vibrating in the air.

the following equation "Landau and Lifshitz (1970)", with respect to the Euler-Bernoulli hypothesis

$$\rho_p S \frac{\partial^2 w}{\partial t^2} = -EJ \frac{\partial^4 w}{\partial x^4} + f \tag{1}$$

Where  $\rho_p$  is the mass density of the beam,  $S$  its cross section,  $E$  the Young modulus,  $J$  the quadratic momentum,  $w(x, t)$  the deflection in  $y$ -direction,  $f$  the external forces on the beam.

Considering a frequency domain study, the equation "Eq. (1)" in terms of the modal amplitude becomes:

$$\frac{\partial^4 X}{\partial x^4} - \omega^2 \frac{\rho_p S}{EJ} X = \frac{F}{EJ} \tag{2}$$

Where  $X(x, \omega)$  is the amplitude of the harmonic-beam deflection in  $y$ -direction,  $k$  the wave number,  $F_{ext}$  the external forces that include the magnetic driving force,  $\omega$  the angular vibration frequency of the beam. When the source term  $F_{ext}$  is zero, solving equation "Eq. (2)" gives the vibration modes of the clamped beam in vacuum. The wave number is:

$$k^4 = \omega^2 \frac{\rho_p S}{EJ} \tag{3}$$

### 2.2 The Navier-Stokes equations

The fluid can be described by the Navier-Stokes equations "Lovesey (1986)":

$$\begin{cases} \rho \frac{\partial V}{\partial t} + \rho(V \cdot \nabla)V = -\nabla p + \mu \Delta V & (4) \\ \nabla \cdot V = 0 & (5) \end{cases}$$

Where  $\rho$  is the density of the fluid,  $V$  the velocity field,  $p$  the pressure,  $\mu$  the kinematic viscosity of the fluid. It is assumed that the fluid is initially stabilized and the amplitudes of vibrations are small, it follows that the non linear convective effects are null:  $(V \cdot \nabla)V = 0$ .

### 2.3 The fluid-structure interaction

When a structure is vibrating in a viscous fluid, the fluid exerts on it a load called hydrodynamic force "Fig. 4". In

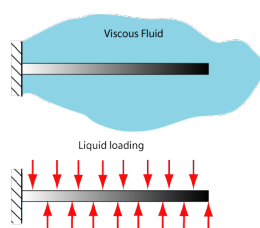


Figure 4. The hydrodynamic force acting on the vibrating beam.

case of small amplitude of vibration, one found "Weiss et al. (2008); Tuck (1969)" that the hydrodynamic force is a function of the amplitude of vibration  $F_{hydro} = \Lambda(\omega)X(x, \omega)$ . "Tuck (1969)" shows that:

$$\Lambda(\omega) = (k_m - jk_d)\pi\rho R^2\omega^2 \tag{6}$$

Where  $k_m$  reflects the added mass coefficient and  $k_d$  the damping coefficient,  $R$  is the radius of the beam. In these conditions the Reynolds number is:

$$Re = \frac{\rho\omega D^2}{4\mu} \tag{7}$$

where  $\rho$  is the density of the fluid,  $D$  the diameter of the beam and  $\mu$  the kinematic viscosity. Stokes "Rosenhead (1963)" provided a solution to Tuck's formulation :

$$k_m - jk_d = 1 + \frac{4iK_1(-i\sqrt{iRe})}{\sqrt{iRe}K_0(-i\sqrt{iRe})} \tag{8}$$

where  $K_0, K_1$  are modified bessel functions of second kind,  $\Gamma(\omega) = k_m - jk_d$  is a hydrodynamic coefficient.

### 2.4 The finite-element Model

The FSI problem "Fig. 5" is time consuming and sometimes problems related to the convergence of the solution are encountered because of the resolution of Navier-Stokes equations. The effects of the fluid on the structure are modeled

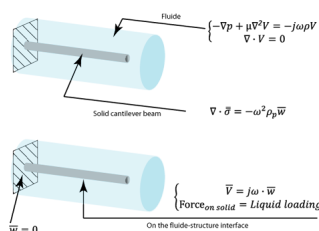


Figure 5. The finite-element model of the vibrating beam under viscous fluid.

as a force "Fig. 6" which is computed in a parametric frequency study. Since the fluid phase and the solid phase are solved separately, the common boundary condition is the force considered in section "(2.3)", it follows then solving the Navier-Stokes equation is not important. The model becomes semi numerical with the solid domain in which the action due to the fluid phase is implemented as impedance term.

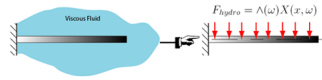


Figure 6. Vibrating beam under viscous fluid.

**2.5 The magneto-structural model**

Generally, the force exerted on a particle  $\vec{q}$  moving with velocity  $\vec{v}$  in an electrostatic field  $\vec{E}$  and magnetic field  $\vec{B}$  has the form:

$$\vec{F}^E = q(\vec{E} + \vec{v} \wedge \vec{B}) \tag{9}$$

The integral of these forces that is called Laplace force Lacheisserie (2000a,b) and it can be written as a tensor; the Maxwell tensor:

$$\sigma^M = \frac{1}{\epsilon_0} \vec{E} \otimes \vec{E} + \frac{1}{\mu_0} \vec{B} \otimes \vec{B} - \frac{1}{2} \left( \frac{E^2}{\epsilon_0} + \frac{B^2}{\mu_0} \right) I \tag{10}$$

where  $\epsilon_0$  is vacuum permeability and  $\mu_0$  the magnetic permeability of vacuum. This Maxwell tensor is computed and applied to the moving part of the viscometer as boundary condition concerning the magnetic excitation system.

**3. RESULTS AND DISCUSSIONS**

The FSI model of the clamped beam "Fig. 5" is computed in frequency domain "Fig. 7".

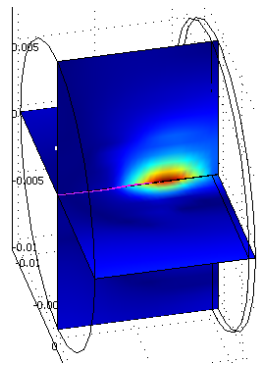


Figure 7. Vibrating beam in viscous fluid: interaction between the fluid and the structure.

The convergence study considers only the mesh density in the frequency domain. The radius of the surrounding fluid domain is a determining factor. Fluid vorticity also plays a significant role in the immediate vicinity of the vibrating structure; it is possible to simplify Stokes equations in the region of the fluid domain sufficiently far from the cantilever. For  $R_e \in [10^{-1} \dots 10^4]$  the semi numerical model holds "Fig. 8" "Fig. 9". When  $R_e \rightarrow \infty$  the hydrodynamic coefficient follows the asymptotic behavior of the inviscid potential flow solution. In the inviscid flow, damping is zero and the added mass  $k_m \pi \rho R^2$  equals that of the liquid contained in the circular cylinder which circumscribes the beam.

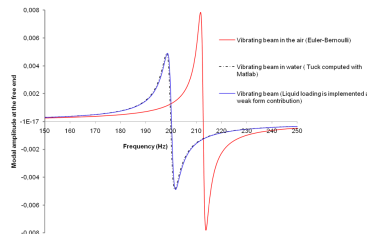


Figure 8. Modal amplitude at the free end of the beam: the fluid domain is replaced by an impedance term.

Results obtained from the clamped beam are used to overcome the vibration problem of the viscometer. Liquid loading is implemented on the vibrating viscometer "Fig. 10".

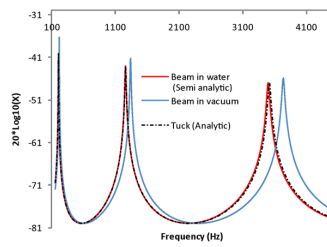


Figure 9. First free resonance frequencies of the beam.



Figure 10. First eigenmode of the viscometer in the viscous fluid: the fluid domain is replaced by an impedance term.

Regarding the vibration amplitude at the free-end of the sensor, its frequency response in both vacuum and fluid is represented in "Fig. 11".

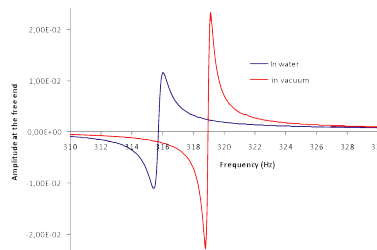


Figure 11. Frequency response of the viscometer in both vacuum and water.

The viscometer is excited to its first resonance frequency which is about  $316\text{ Hz}$  "Fig. 11" for a viscosity value that is below  $1cP$ , the Reynolds number obtained in these conditions is less than 1000, so there is no need to take account the turbulence phenomenon, the only forces taken for the study remains the elastic interaction forces discussed previously.

#### 4. CONCLUSION

The operation of the viscometer involves many physical phenomena that are coupled. Any variation of one phenomenon therefore necessarily changes the others; the overall problem cannot be handled by treating each phenomenon separately. In this study, the behavior of a beam vibrating in a viscous fluid was first addressed. An analytical model based on that of Tuck approach was then introduced. A finite element model was defined in order to test the relevance of the analytical model. However, depending on the mesh and the diameter of the fluid sub domain, the results may be different. In an attempt to reduce the computational cost, the actions of the viscous fluid on the beam have been implemented in the finite-element model. A finite-element model of the sensor based on this previous work has been developed and will be validated by experiments.

#### 5. ACKNOWLEDGEMENTS

This work is funded by the General Council of Loiret (France) and SOFRASER society.

#### 6. REFERENCES

Baley, F. J., 1958. An Introduction to Fluid Dynamics. G. Allen and Unwin, New York, London.  
 Blinder, R. C., 1955. Fluid mechanics. Prentice-Hall, New York.

- Hansen, S. T., Turo, A., Degertekin, F. L., Khuri-Yakub, B. T., 2000. Characterization of capacitive micromachined ultrasonic transducers in air using optical measurements. *Ultrasonics Symposium 1*, 947–950.
- Judge, J. A., Photiadis, D. M., Vignola, J. F., Houston, B. H., Jarzynski, J., 2007. Attachment loss of micromechanical and nanomechanical resonators in the limits of thick and thin support structures. *Journal of Applied Physics* 101.
- Kojro, Z., v. d. Burg, E., Zinke, J., Hillmann, K., Grill, W., 1996. Viscosity shear waves and mass drag effect in liquids. *Zeitschrift für Physik B Condensed Matter* 101, 433–439.
- Lacheisserie, E. D. T. D., 2000a. *Magnétisme*. Vol. 1. Grenoble Sciences.
- Lacheisserie, E. D. T. D., 2000b. *Magnétisme*. Vol. 2. Grenoble Sciences.
- Landau, L. D., Lifshitz, E. M., 1970. *Theory of Elasticity*. Pergamon, Oxford.
- Lovesey, S. W., 1986. *Condensed matter Physics: Dynamics Correlations*. The Benjamin/Cummings Publishing Company Menlo Park, California, Amsterdam.
- Pao, R. H. F., 1961. *Fluid Mechanics*. Wiley, New York.
- Reichel, E. K., Riesch, C., Weiss, B., Jakoby, B., 2008. A vibrating membrane rheometer utilizing electromagnetic excitation. *Sensors and Actuators A: Physical* 145-146, 349 – 353.
- Rosenhead, L., 1963. *Laminar boundary layers*. Clarendon, Oxford.
- Sader, J. E., 1998. Frequency response of cantilever beams immersed in viscous fluids with applications to the atomic force microscope. *Journal of Applied Physics* 84 (1), 64–76.
- Streeter, V., 1955. *Fluid Mechanics*. McGraw-Hill, New York.
- Tuck, E. O., 1969. Calculation of unsteady flows due to small motions of cylinders in a viscous fluid. *Journal of Engineering Mathematics* 3 (1), 29–44.
- Vancura, C., Li, Y., Josse, F., 2005. Fully integrated cos resonance cantilever sensor for biochemical detection in liquid environments. *Transducers05, Digest of Tech 1*, 1142–1145.
- Vignola, J. F., Judge, J. A., Jarzynski, J., Zalalutdinov, M., 2006. Effect of viscous loss on mechanical resonators designed for mass detection. *Applied Physics Letters* 88.
- Villarroya, M., Verd, J., Teva, J., 2005. Cantilever based mems for multiple mass sensing. *Res. Microelectron Electron.* 1, 197–200.
- Wazer, V., Lyons, J. W., Kim, K. Y., 1963. *Viscosity and Flow Measurement*. Interscience.
- Weiss, B., Reichel, E. K., Jakoby, B., 2008. Modeling of a clamped-clamped beam vibrating in a fluid for viscosity and density sensing regarding compressibility. *Sensors and Actuators A: Physical* 143 (2), 293 – 301.
- Willams, E. G., 1983. A series expansion of the acoustic power radiated from planar sources. *J Acoust Soc Am* 73, 1520–1524.