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Comment

Comment on “Are stress-free membranes really ‘tensionless’?” by Schmid F.

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In recent years, it has been argued that the tension parameter driving the fluctuations of fluid membranes should differ from the imposed lateral stress, the “frame tension” [1, 2]. In particular, stress-free membranes were predicted to have a residual fluctuation tension. Some experimental evidence of this property has been reported in [3]. In a recent paper [4], Schmid argued that the reasoning published in [2] (where the frame tension is calculated by averaging the stress tensor and by differentiating the free energy), is inherently inconsistent—in the sense that a linearized theory, the Monge model, was used to predict a non-linear effect. In the present comment, we show that the criticism argued in [4] does not hold.

The correct way to proceed in the calculations is to be consistent at first-loop order, i.e., at first order in the small parameter $k_B T / \kappa$, where $k_B T$ is the temperature in energy units and κ the membrane bending rigidity. In [4], however, all energies were given in units of $k_B T$, and thus all tracks of the small parameter in which the expansion should be made were lost.

Restoring the $k_B T$ factors, eq. (3) of [4] reads

$$G(N, \sigma_{\text{int}}, A_p) = \sigma_{\text{int}} A_p + k_B T \frac{N-1}{2} \left[\ln \left(\frac{2\sigma_{\text{int}} \lambda^2}{k_B T} \right) - 2 + \left(1 + \frac{\sigma_{\text{int}} A_p}{4\pi\kappa(N-1)} \right) \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) \right]. \quad (1)$$

Here, G is the total free-energy of the membrane described at the Gaussian level in the Monge gauge, N the total number of fluctuating degrees of freedom (proportional to the total number of lipids), σ_{int} the “internal” tension appearing in the Gaussian Hamiltonian $H = \frac{1}{2} \int d^2 r_{\perp} [\sigma_{\text{int}} (\nabla h)^2 + \kappa (\nabla^2 h)^2]$ functional of the membrane height function $h(\mathbf{r}_{\perp})$, A_p the fixed projected area above which the membrane stands, and λ a small distance cutoff in the direction orthogonal to the membrane.

Differentiating the free-energy with respect to the projected area, one obtains the frame tension $\sigma_f = \partial G / \partial A_p$, yielding

$$\frac{\sigma_f}{\sigma_{\text{int}}} = 1 + \frac{k_B T}{8\pi\kappa} \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) - \frac{k_B T}{8\pi\kappa} \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} + \mathcal{O}(\epsilon^2), \quad (2)$$

which corresponds to eq. (4) of [4]. Note that this result is only valid at first-order in the small parameter $\epsilon = k_B T/\kappa$ because H is only the Gaussian approximation of the complete Helfrich Hamiltonian.

The excess area $(A - A_p)/A_p$ can be obtained from the relation $A = \partial G/\partial \sigma_{\text{int}}$, yielding

$$\frac{A - A_p}{A_p} = \frac{k_B T}{8\pi\kappa} \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) + \mathcal{O}(\epsilon^2), \quad (3)$$

which corresponds to eq. (5) of [4]. Thus $(A - A_p)/A_p$ is also a first-order quantity.

The argumentation in [4] begins by defining

$$y = \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) \simeq \frac{8\pi\kappa}{k_B T} \frac{A - A_p}{A_p} = \mathcal{O}(1). \quad (4)$$

Schmid, however, fails to notice that this is a zeroth-order quantity (in ϵ): omitting $k_B T$ in the denominator, Schmid considers y as a first-order quantity, like $(A - A_p)/A_p$. However, $(A - A_p)/A_p = \mathcal{O}(\epsilon)$ is small, but $8\pi\kappa/(k_B T) = \mathcal{O}(\epsilon^{-1})$ is large, and so the product is $\mathcal{O}(\epsilon^0 = 1)$.

Then, from eqs. (2) and (4), Schmid writes

$$\frac{\sigma_f}{\sigma_{\text{int}}} = 1 - \frac{k_B T}{8\pi\kappa} (e^y - y - 1) \approx 1 - \frac{4\pi\kappa}{k_B T} \left(\frac{A - A_p}{A_p} \right)^2 + \dots \quad (5)$$

and says: “Hence the frame tension is found to differ from the internal tension to second order in $(A - A_p)/A_p$. On the other hand, the Monge model approximates planar interfaces only up to the first order in $(\nabla h)^2$ or $(A - A_p)/A_p$. Thus the results (6) and (4)¹ are not rigorous”.

However, since $y = \mathcal{O}(1)$, using the power series expansion $e^y - y - 1 = \frac{1}{2}y^2 + \mathcal{O}(y^3)$ in eq. (5) is *not* justified. Even the result is in contradiction with the conclusion drawn: since $4\pi\kappa/(k_B T) = \mathcal{O}(\epsilon^{-1})$ and $[(A - A_p)/A_p]^2 = \mathcal{O}(\epsilon^2)$, the correction $(\sigma_f - \sigma_{\text{int}})/\sigma_{\text{int}}$ appearing in eq. (5) would be a first-order correction.

The correct answer, at first-loop order (at first-order in $k_B T/\kappa$), is thus the one given in eq. (2). In conclusion, the reasoning in [4] arguing that the results of Fournier and Barbeta are erroneous does not hold.

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¹These equations correspond here to eqs. (5) and (2)