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Jean-Baptiste Fournier

► To cite this version:

Jean-Baptiste Fournier. Comment on "Are stress-free membranes really "tensionless"?" by Schmid F.. EPL - Europhysics Letters, 2012, 97, pp.18001. 10.1209/0295-5075/97/18001 . hal-00658501

HAL Id: hal-00658501

<https://hal.science/hal-00658501>

Submitted on 10 Jan 2012

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Comment

Comment on “Are stress-free membranes really ‘tensionless’?” by Schmid F.

JEAN-BAPTISTE FOURNIER

Université Paris Diderot, Sorbonne Paris Cité, Laboratoire Matière et Systèmes Complexes (MSC), UMR 7057 CNRS, F-75205 Paris, France

PACS 87.16.dj – dynamics and fluctuations

PACS 68.03.Kn – Dynamics (capillary waves)

PACS 68.35.Md – Surface thermodynamics, surface energies

In recent years, it has been argued that the tension parameter driving the fluctuations of fluid membranes should differ from the imposed lateral stress, the “frame tension” [1, 2]. In particular, stress-free membranes were predicted to have a residual fluctuation tension. Some experimental evidence of this property has been reported in [3]. In a recent paper [4], Schmid argued that the reasoning published in [2] (where the frame tension is calculated by averaging the stress tensor and by differentiating the free energy), is inherently inconsistent—in the sense that a linearized theory, the Monge model, was used to predict a non-linear effect. In the present comment, we show that the criticism argued in [4] does not hold.

The correct way to proceed in the calculations is to be consistent at first-loop order, i.e., at first order in the small parameter $k_B T / \kappa$, where $k_B T$ is the temperature in energy units and κ the membrane bending rigidity. In [4], however, all energies were given in units of $k_B T$, and thus all tracks of the small parameter in which the expansion should be made were lost.

Restoring the $k_B T$ factors, eq. (3) of [4] reads

$$G(N, \sigma_{\text{int}}, A_p) = \sigma_{\text{int}} A_p + k_B T \frac{N-1}{2} \left[\ln \left(\frac{2\sigma_{\text{int}} \lambda^2}{k_B T} \right) - 2 + \left(1 + \frac{\sigma_{\text{int}} A_p}{4\pi\kappa(N-1)} \right) \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) \right]. \quad (1)$$

Here, G is the total free-energy of the membrane described at the Gaussian level in the Monge gauge, N the total number of fluctuating degrees of freedom (proportional to the total number of lipids), σ_{int} the “internal” tension appearing in the Gaussian Hamiltonian $H = \frac{1}{2} \int d^2 r_{\perp} [\sigma_{\text{int}} (\nabla h)^2 + \kappa (\nabla^2 h)^2]$ functional of the membrane height function $h(\mathbf{r}_{\perp})$, A_p the fixed projected area above which the membrane stands, and λ a small distance cutoff in the direction orthogonal to the membrane.

Differentiating the free-energy with respect to the projected area, one obtains the frame tension $\sigma_f = \partial G / \partial A_p$, yielding

$$\frac{\sigma_f}{\sigma_{\text{int}}} = 1 + \frac{k_B T}{8\pi\kappa} \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) - \frac{k_B T}{8\pi\kappa} \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} + \mathcal{O}(\epsilon^2), \quad (2)$$

which corresponds to eq. (4) of [4]. Note that this result is only valid at first-order in the small parameter $\epsilon = k_B T / \kappa$ because H is only the Gaussian approximation of the complete Helfrich Hamiltonian.

The excess area $(A - A_p)/A_p$ can be obtained from the relation $A = \partial G / \partial \sigma_{\text{int}}$, yielding

$$\frac{A - A_p}{A_p} = \frac{k_B T}{8\pi\kappa} \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) + \mathcal{O}(\epsilon^2), \quad (3)$$

which corresponds to eq. (5) of [4]. Thus $(A - A_p)/A_p$ is also a first-order quantity.

The argumentation in [4] begins by defining

$$y = \ln \left(1 + \frac{4\pi\kappa(N-1)}{\sigma_{\text{int}} A_p} \right) \simeq \frac{8\pi\kappa}{k_B T} \frac{A - A_p}{A_p} = \mathcal{O}(1). \quad (4)$$

Schmid, however, fails to notice that this is a zeroth-order quantity (in ϵ): omitting $k_B T$ in the denominator, Schmid considers y as a first-order quantity, like $(A - A_p)/A_p$. However, $(A - A_p)/A_p = \mathcal{O}(\epsilon)$ is small, but $8\pi\kappa/(k_B T) = \mathcal{O}(\epsilon^{-1})$ is large, and so the product is $\mathcal{O}(\epsilon^0 = 1)$.

Then, from eqs. (2) and (4), Schmid writes

$$\frac{\sigma_f}{\sigma_{\text{int}}} = 1 - \frac{k_B T}{8\pi\kappa} (e^y - y - 1) \approx 1 - \frac{4\pi\kappa}{k_B T} \left(\frac{A - A_p}{A_p} \right)^2 + \dots \quad (5)$$

and says: “Hence the frame tension is found to differ from the internal tension to second order in $(A - A_p)/A_p$. On the other hand, the Monge model approximates planar interfaces only up to the first order in $(\nabla h)^2$ or $(A - A_p)/A_p$. Thus the results (6) and (4)¹ are not rigorous”.

However, since $y = \mathcal{O}(1)$, using the power series expansion $e^y - y - 1 = \frac{1}{2}y^2 + \mathcal{O}(y^3)$ in eq. (5) is *not* justified. Even the result is in contradiction with the conclusion drawn: since $4\pi\kappa/(k_B T) = \mathcal{O}(\epsilon^{-1})$ and $[(A - A_p)/A_p]^2 = \mathcal{O}(\epsilon^2)$, the correction $(\sigma_f - \sigma_{\text{int}})/\sigma_{\text{int}}$ appearing in eq. (5) would be a first-order correction.

The correct answer, at first-loop order (at first-order in $k_B T / \kappa$), is thus the one given in eq. (2). In conclusion, the reasoning in [4] arguing that the results of Fournier and Barbeta are erroneous does not hold.

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¹These equations correspond here to eqs. (5) and (2)