

Scale invariant images in astronomy through the lens of multifractal modeling

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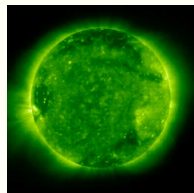
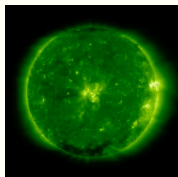
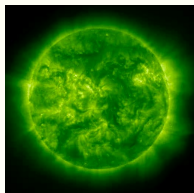
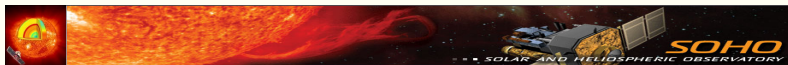
V. Delouille, J.-F. Hochedez

Royal Observatory of Belgium

Bruxelles - Belgium



The quiet Sun seen by E.I.T. in extreme UV



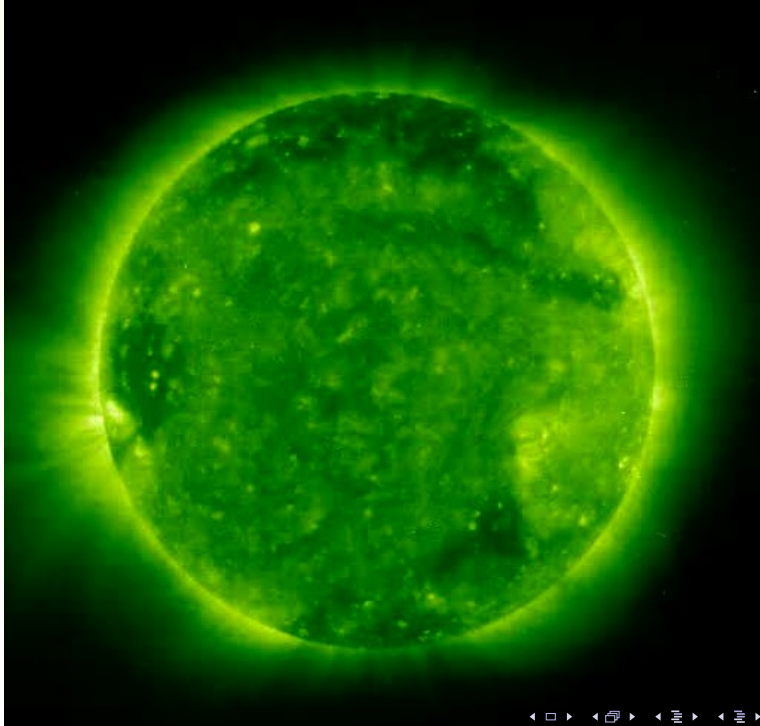
⇒ Active periods

- large number of active regions
- intense activity

⇒ Quiet periods

- very few active regions
- disordered turbulent background
- textured aspect

Royal Obs. of Belgium: V. Delouille, J.-F. Hochedez, S. Gissot,
LIMOS: E. Koenig, V. Barra, 2005-...



Position of the problem

⇒ **E.I.T. images** : 1 pixel = $(1800\text{km})^2$

⇒ **Future images** : 1 pixel = $(80\text{km})^2$

⇒ **Aims** : better understanding of the quiet Sun

Pb : resolution ↗, photons/pixel ↘, number of black pixels ↗

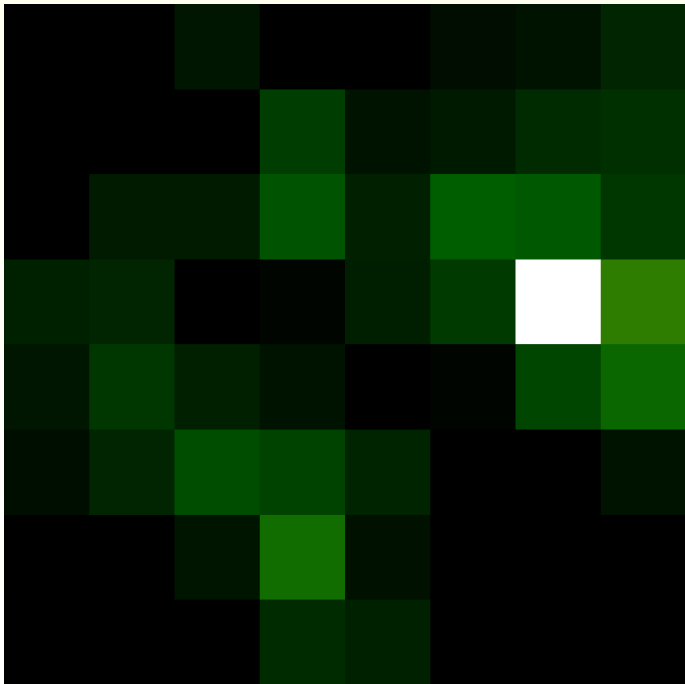
⇒ **Main thrust** : to model high resolution images

=

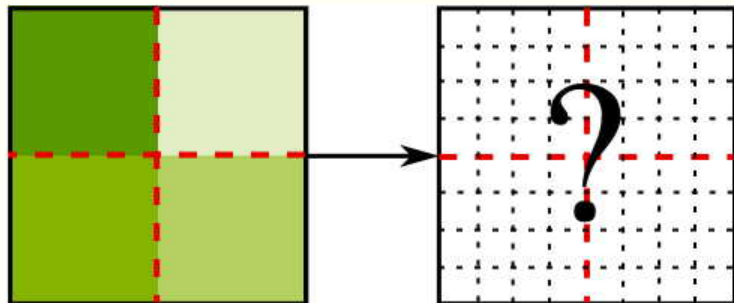
virtual super-resolution

⇒ **Applications** :

- validation & optimisation of compression algorithms
- SNR evaluation of better resolved images
- arguments, pros & cons for a new mission...



Position of the problem

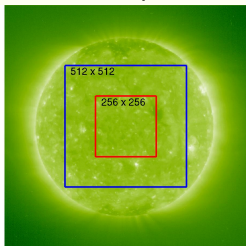


A plausible guess of what might be hidden under the available resolution ?

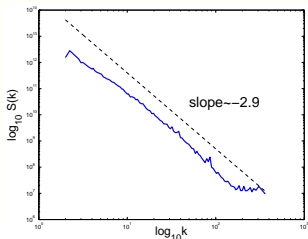
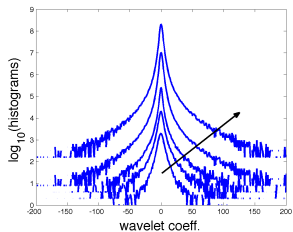
New subpixel details will be introduced, under some physical and statistical constraints...

Quiet Sun images are multifractal (higher order correlations)

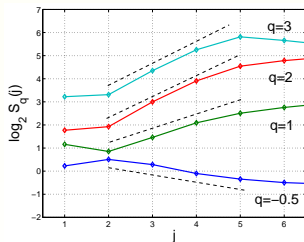
example



wav. coeff. pdf



$$1/||k||^{2.9}$$

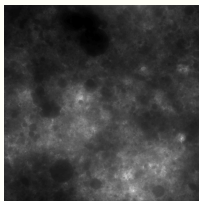


$$S_q(a) \propto a^{\zeta(q)}$$

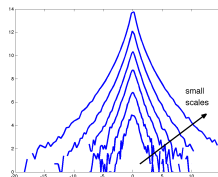
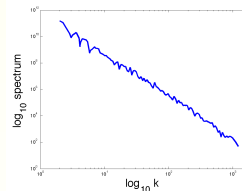
Scale invariance (by dilation)

Some coherence in the structure of disorder :

multifractal : scale invariant & non Gaussian



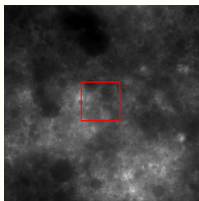
$$S_q(j) = \frac{1}{N_j} \sum_{k=1}^{N_j} |d(j, k)|^q \sim 2^{j\zeta(q)}$$



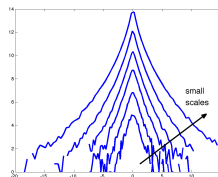
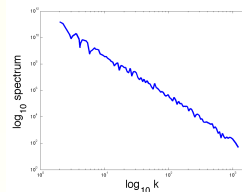
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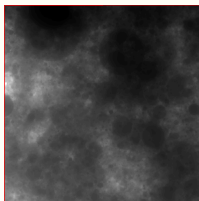
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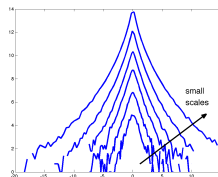
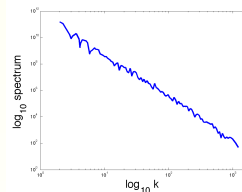
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$$S_q(j) = \frac{1}{N_j} \sum_{k=1}^{N_j} |d(j, k)|^q \sim 2^{j\zeta(q)}$$



Stochastic processes to model quiet Sun images

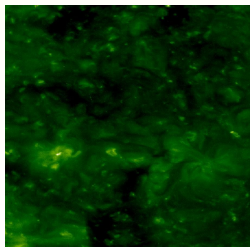
⇒ **Assumption** : multifractality persists at subpixel scales

⇒ **Multifractal exponents** :

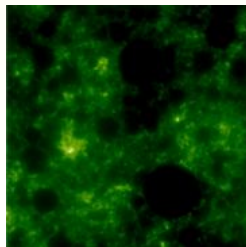
$$\zeta(q) = qH + \tau(q), \quad \tau(0) = \tau(1) = 0$$

fractional integration \mathcal{I}_H * **Compound Poisson Cascade**

⇒ **Multifractal model** : $I_1 = I_0 + \alpha \mathcal{I}_H \{ Q_{r_1}^{r_0} - 1 \}$

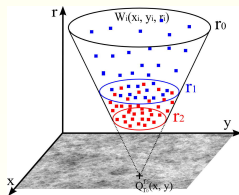


quiet Sun image



frac. integ. CPC

Virtual super-resolution based on CPC



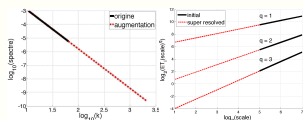
⇒ **Local augmentation of information** :

- multiscale decomposition :

$$\mathbf{Q}_{r_2}^{r_0} = \mathbf{Q}_{r_1}^{r_0} \cdot \mathbf{Q}_{r_2}^{r_1}$$

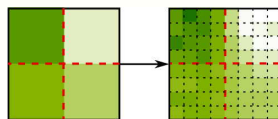
- global model :

$$\mathbf{I}_1 = \mathbf{I}_0 + \alpha \mathcal{I}_H \{ \mathbf{Q}_{r_1}^{r_0} - \mathbf{1} \}$$



⇒ **Statistical consistency**

- conservation of scale invariance
- extrapolation at finer scales

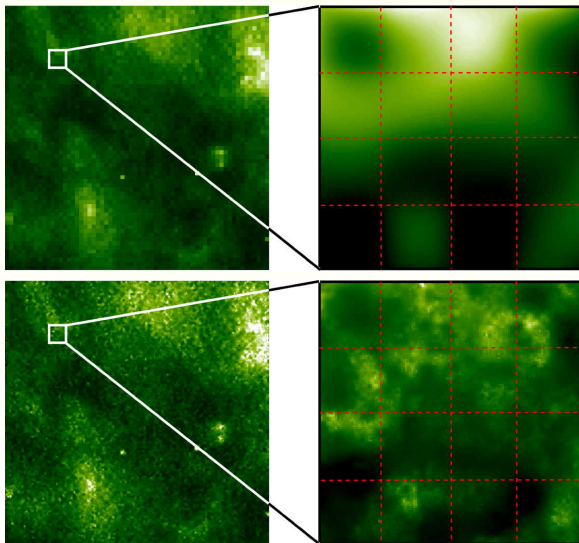


⇒ **Physical consistency**

- zooming out by grouping pixels gives the initial image back

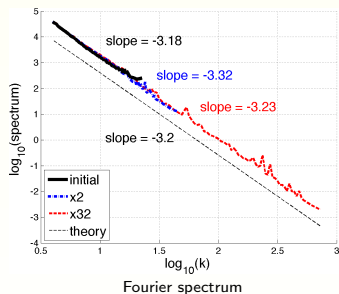
[J. Math. Imaging & Vision 2011]

Example : $x 32$, $1\text{px} = 1800\text{ km} \rightarrow 56\text{ km}$

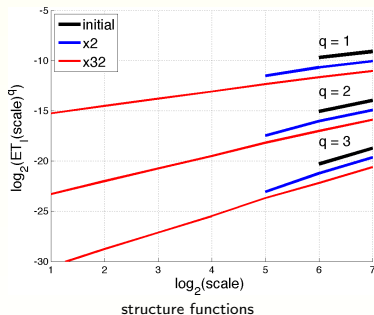


Extrapolation of scale invariance properties

⇒ Extrapolation of Fourier spectrum towards high frequencies

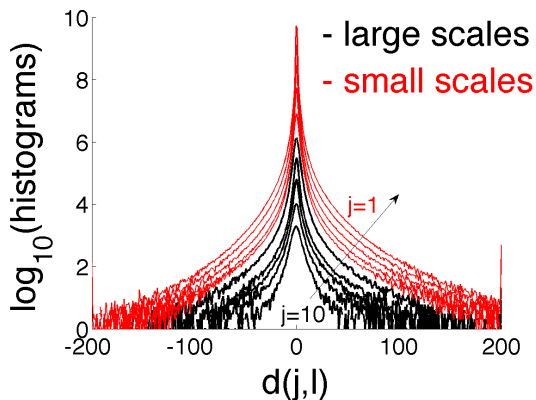


⇒ Extrapolation of multifractal scaling at small scales



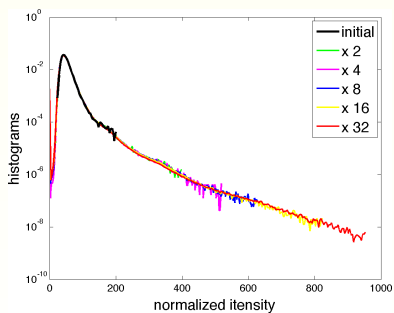
Extrapolation of scale invariance properties

- ⇒ Evolution of wavelet coefficients distributions :
less and less Gaussian



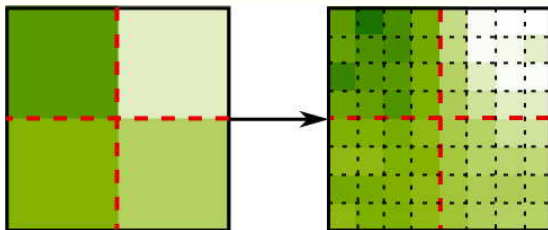
Extrapolation of the histogram of intensities

- ⇒ Conservation of the low resolution histogram,
- ⇒ Extrapolation towards extreme values



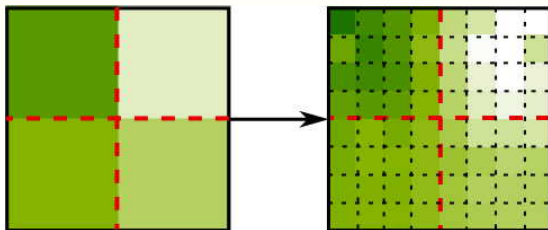
Stochastic extrapolation

- ⇒ **random details** inside original pixels
- ⇒ **infinite number of realizations** for some given initial image



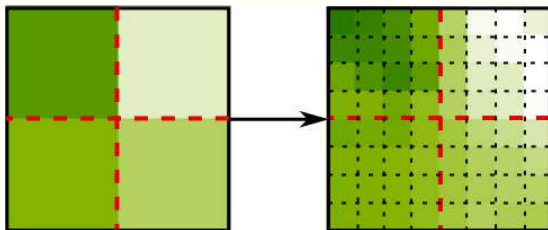
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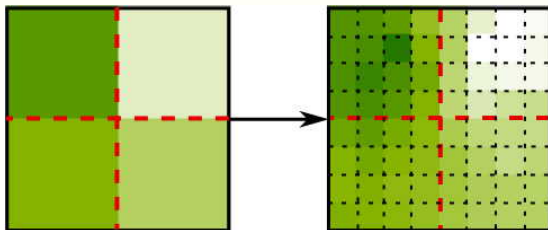
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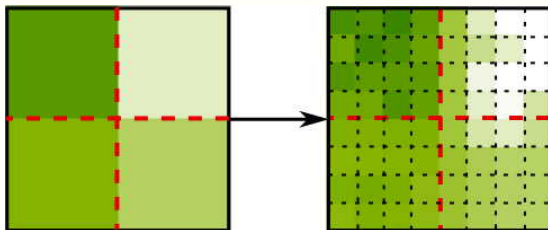
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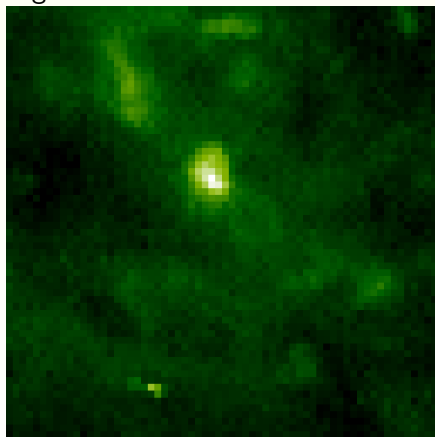
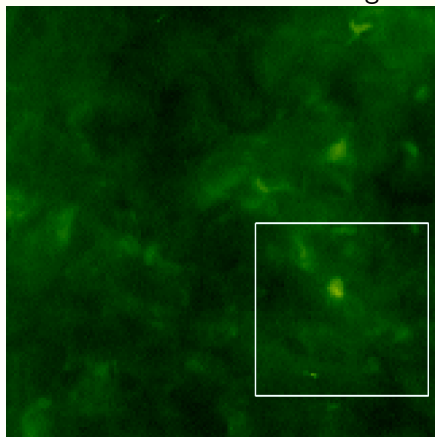
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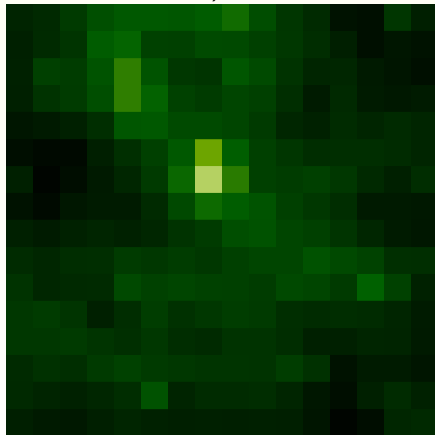
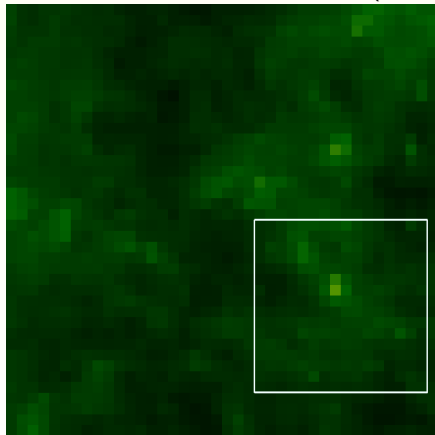
Test on actual images

Original image



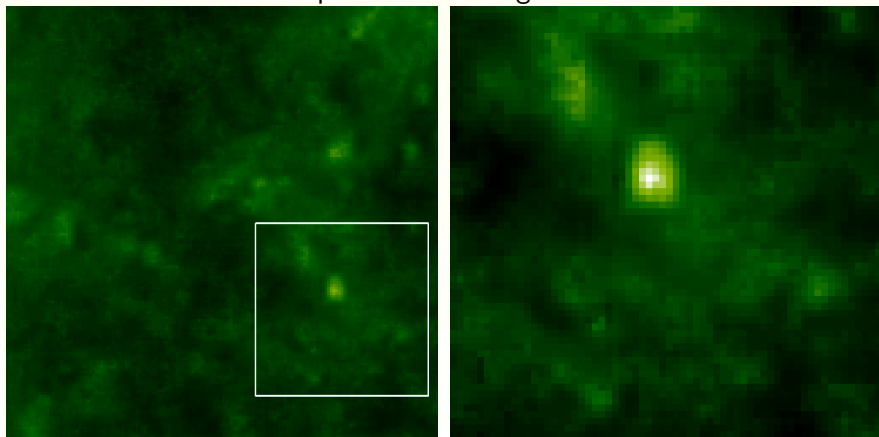
Test on actual images

Degraded image (resolution divided by 4)



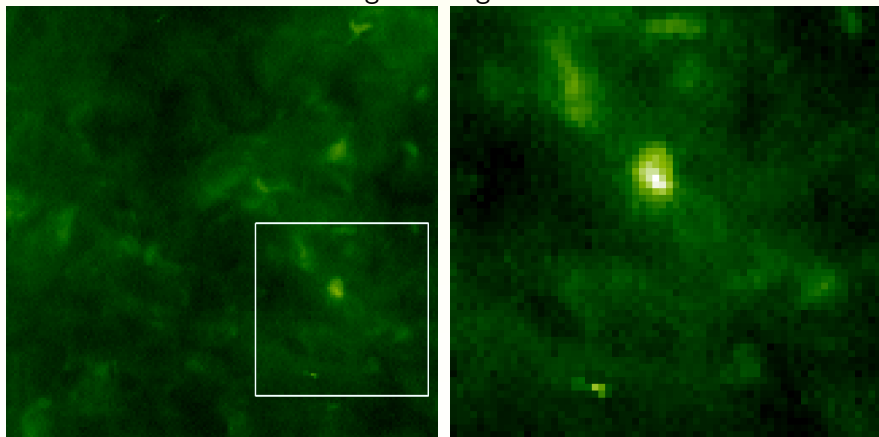
Test on actual images

Super-resolved image

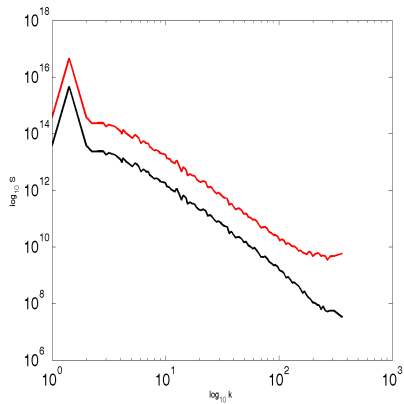
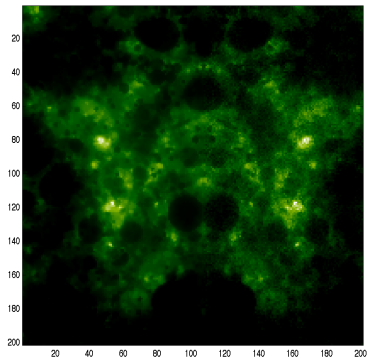


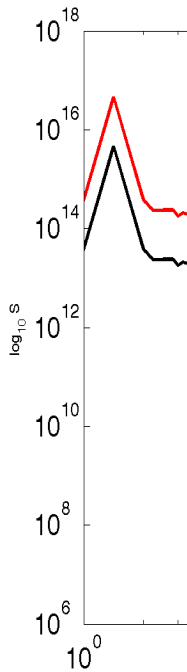
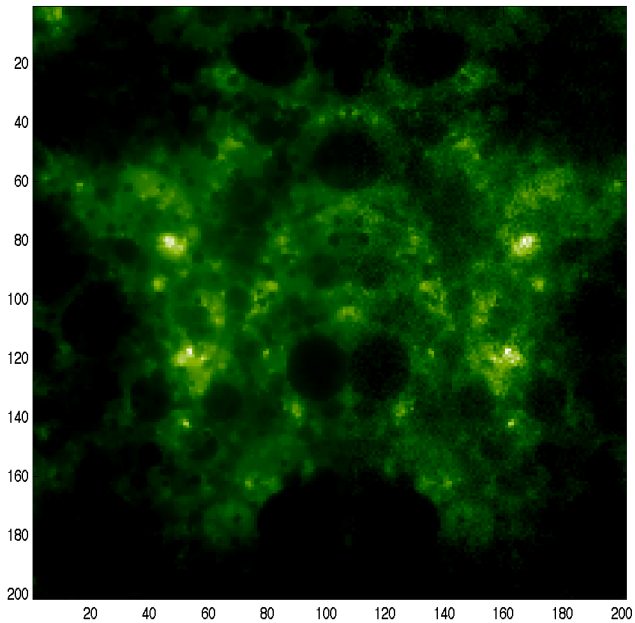
Test on actual images

Original image

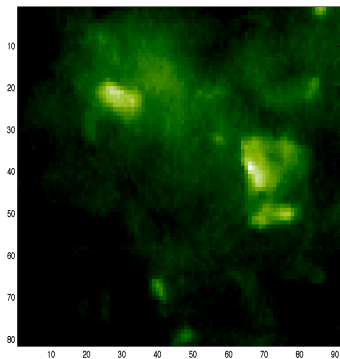
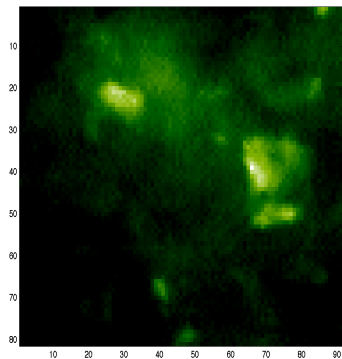


Poisson noise in EIT images

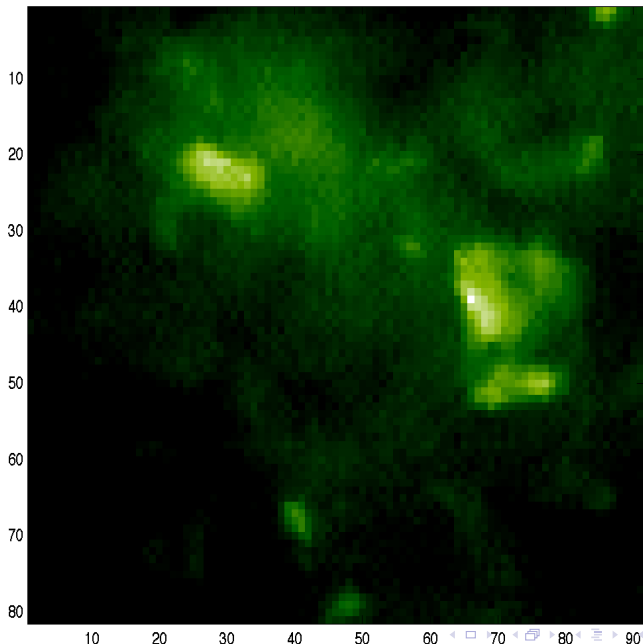




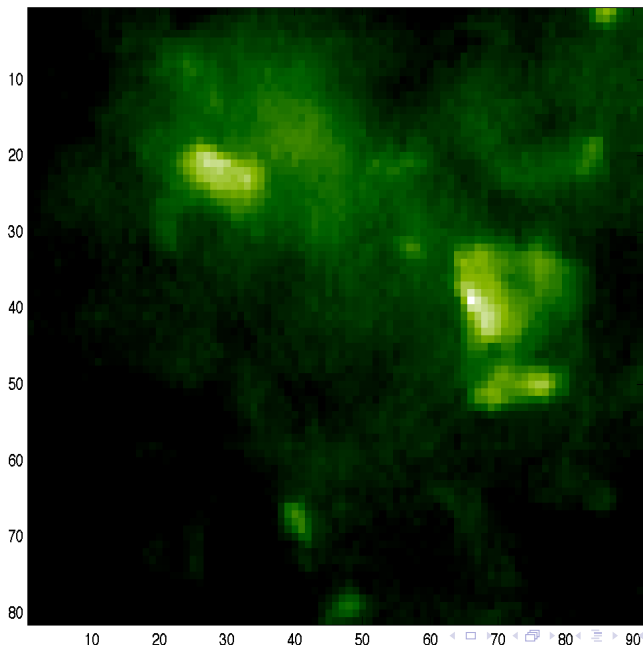
Scale invariant Wiener filtering of EIT images



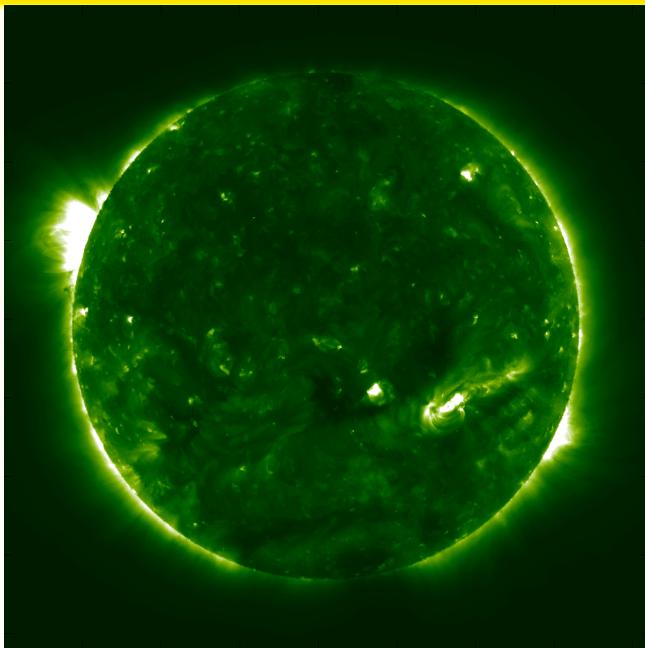
Scale invariant Wiener filtering of FLT images



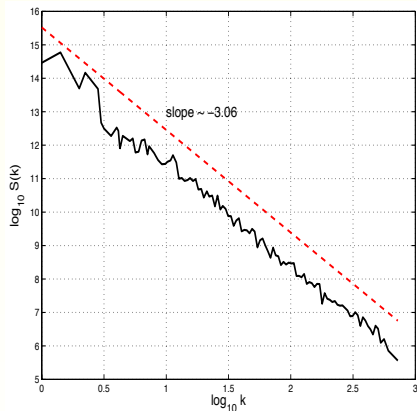
Scale invariant Wiener filtering of FLT images



Recent SDO observations : 4096x4096



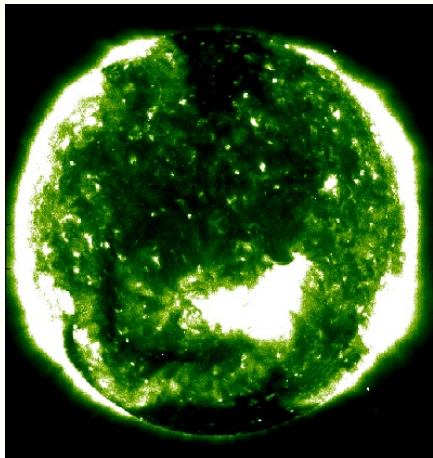
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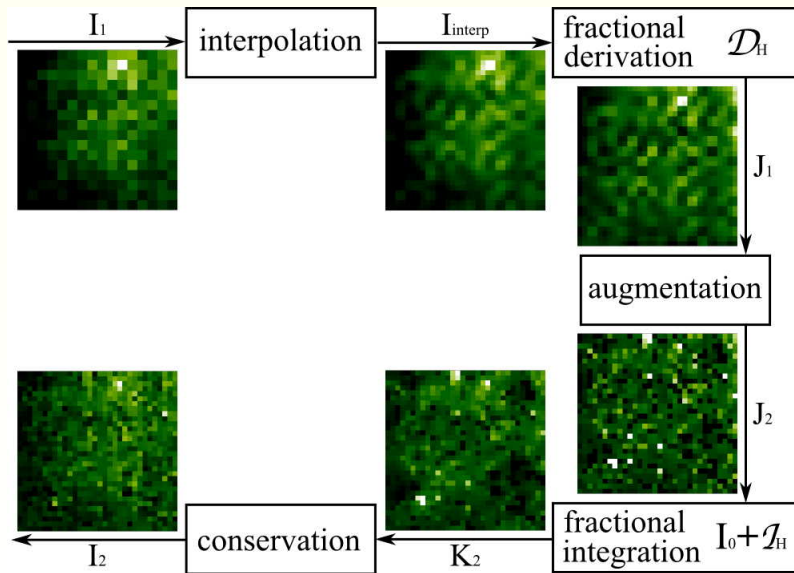
Conclusion

- ⇒ multifractals : usually for description and analysis,
 - ⇒ **example of a constructive model**
 - **Quiet Sun images are multifractal**
(confirmed by more recent and finer observations)
 - **FI-CPC for virtual super-resolution**
(statistically, physically & visually reasonable)
- ⇒ **prediction of histograms** / level of exposure of future high resolution images
- ⇒ **test and optimization of**
 - **compression algorithms** [S. Gissot]
(based on JPEG2000)
 - **scale invariance** based Wiener filter **denoising**
- ⇒ **on the sphere** : statistical characterization of the performances of elevation estimates (STEREO)

Application to some quiet Sun image

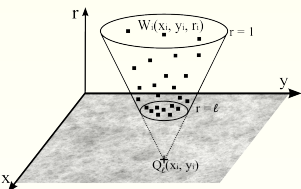


Virtual super-resolution $\times 2$



Compound Poisson Cascades (CPC)

With cylindrical kernel $\mathbf{1}_{\mathcal{D}(x_i, r_i)}$:

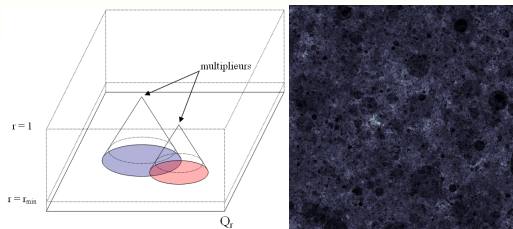


$$Q_\ell(x) = \frac{\prod_i W_i \mathbf{1}_{\mathcal{D}(x_i, r_i)}}{\mathbf{E} \left[\prod_i W_i \mathbf{1}_{\mathcal{D}(x_i, r_i)} \right]}, \quad \mathbf{E} W^q \leftrightarrow \tau(q)$$

Taking the logarithm :

$$\log Q_\ell(x) = \sum_i \log W_i \cdot \mathbf{1}_{\mathcal{D}(x_i, r_i)} + K$$

\Rightarrow superposition of 3D transparent cylinders



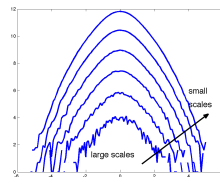
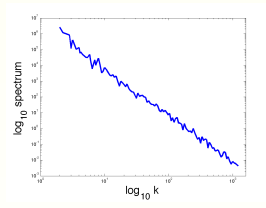
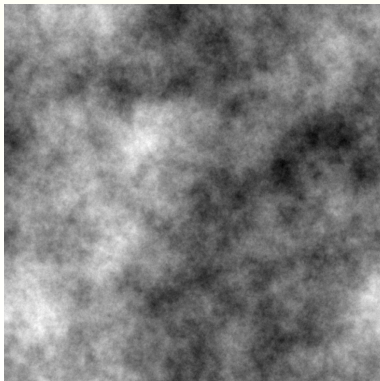
Scale invariance (by dilation)

Some coherence in the structure of disorder :

scale invariance

self-similar & (often) Gaussian :

$$\forall a \in \mathbb{R}^+, \quad \{I(ax), \mathbf{x} \in \mathbb{R}^2\} \stackrel{d}{=} a^H \{I(\mathbf{x}), \mathbf{x} \in \mathbb{R}^2\}$$



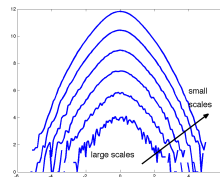
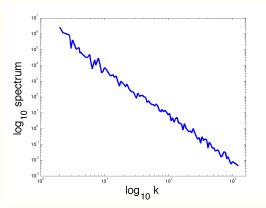
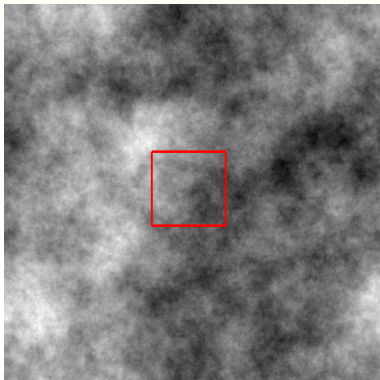
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