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► **To cite this version:**

Ouafae Bennis, Frédéric Kratz. Diagnostic de roulement à l'aide du filtrage Particulaire. Diagnostic de roulement à l'aide du filtrage Particulaire, May 2009, SKIKDA, Algérie. pp.171-177. hal-00657629

HAL Id: hal-00657629

<https://hal.science/hal-00657629>

Submitted on 10 Jan 2012

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Diagnosis of a Roller Bearing Using Deterministic Particle Filtering

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Abstract. In this article, the detection of a fault on the inner race of a roller bearing is presented as a problem of optimal estimation of a hidden fault, via measures delivered by a vibration sensor. First, we propose a linear model for the transmission of a vibratory signal to the sensor's diaphragm. The impact of shocks due to the default is represented by a stochastic drift term whose values are in a discrete set. To determine the state of the roller bearing, we estimate the value of this term using deterministic particle filtering.

Keywords: Deterministic Particle filtering, diagnosis.

1 Introduction

The diagnosis of the rotating machines became a stake major for industry, in particular aeronautical, petrochemical, pharmaceutical, specialty chemicals or iron and steel, because of the gigantic costs generated by a catastrophic failure and consequences: economic, safety and environmental and on the public image of the company impact.

In this study, we are interested with a detection of a failure on the inner race of a roller bearing on a turbo reactor. Many sensors are given at different bearings on the reactor except on the suspected one. Hence we have a partial observation of the state of the roller from the sensors.

The physical model used is developed in [1]. The state variables we consider are respectively the displacement and the speed of the vibration sensor's diaphragm. We observe the second variable across a propagation factor α , high level noise context. Under a state representation, this is close to an optimal filtering problem.

Our aim is to estimate the hypothesis of there being a fault and subsequence level deterioration.

2 State representation of the problem

Let $y(t)$ be the response of the sensor's diaphragm to an excitation $h(t)$. The vibratory signal next to it is [1]: $y(t) = e_d(t) * h(t)$, for $t > 0$, with $e_d(t) = \sin(\omega_r t) e^{-\frac{t}{T_c}}$, ω_r is the resonance pulsation, T_c is the time-constant of the sensor and $*$ the convolution product. This obeys to the differential equation:

$$\ddot{y}(t) + \frac{2}{T_c} \dot{y}(t) + \left(\omega_r^2 + \frac{1}{T_c^2} \right) y(t) = \omega_r h(t)$$

Let $X(t) = [y(t) \quad \dot{y}(t)]^T$, be the displacement and the speed vector, $z(t)$ the signal measured. For a healthy bearing, the membrane of the sensor receives the noises in the neighbourhood of the sensor, which one summarizes with a continuous Gaussian white noise $w(t)$ with intensity $Q(t)$. Then we have:

$$\begin{cases} \dot{X}(t) = AX(t) + Bw(t) \\ z(t) = HX(t) + v(t) \end{cases} \quad (2.1)$$

$$A = \begin{bmatrix} 0 & 1 \\ -\left(\omega_r^2 + \frac{1}{T_c^2} \right) & -\frac{2}{T_c} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \omega_r \end{bmatrix} \quad H = [\alpha \quad 0]$$

$v(t)$ represents the measurement noise. It is assumed that $v(t)$ can be described as white noise where R is the variance matrix. α is a factor of propagation in the sensor's diaphragm: $0 < \alpha < 1$.

In the case of a fault on the inner race, an impulse is generated when each element of the roller comes into contact with the fault. The sensor's diaphragm receives in addition to $w(t)$ a discontinuous component: $h_{shock}(t) = U \cdot \sum_n \delta(t - nT_d)$

T_d and U are respectively the period of the fault and the load applied at the contact point. T_d is assumed to be deterministic and known. We choose the sampling period T_e so that: $T_d = 72 \cdot T_e$. In this case $h_{shock}(t)$ is reduced to a maximum of a single shock in the period T_e . Consequently, the fault introduces a stochastic drift term in the second state equation each time k is a multiple of 72.

U depends of the deterioration level of the bearing roller; its value grows slowly in time. Hence, we consider it stochastic with values in a discrete set $\mathfrak{S} = \{u_1, u_2, u_3, \dots, u_N\}$. The filtering model is still linear Gaussian verifying:

$$\begin{cases} \dot{X}(t) = AX(t) + Bw(t) + B.U \cdot \sum \delta(t - nT_d) \\ z(t) = HX(t) + v(t) \end{cases} \quad (2.2)$$

Discrete equations. The measurements z are collected at discrete times, it's more natural to deal with the discrete time system:

$$\begin{cases} X_{k+1} = FX_k + Gw_k + Gu_k^i, \\ z_k = HX_k + v_k \end{cases}, \quad k > 0 \quad (2.3)$$

$$F = (I + AT_e), \quad G = \begin{bmatrix} 0 \\ T_e \omega_r \end{bmatrix}, \quad u_k^i = U \delta_{k,72i}$$

where δ represents the Kronecker symbol (i.e. $\delta_{a,b} = 1$ if $a = b$ $\delta_{a,b} = 0$ otherwise).

3 Roller bearing diagnosis

The deterioration of a roller bearing is a slow phenomenon, except in case of deficient conditions of use. We suppose that the value U is constant on a limited time interval. So a diagnosis of roller bearing comes down to estimating the possibility of shock occurrence and its amplitude U . The evolution of the amplitude U value is defined by the transition matrix P_{tr} . An element $p(i,j)$ of the matrix P_{tr} defines the probability of the change from U_i to U_j . The diagonal elements are more important than the others. The probability of a change is lower than the probability of remaining in the same state and the probability of going backward to a faultless case is close to 0. Hence P_{tr} is a triangular matrix. The diagnosis of the state of the bearing is equivalent to estimate the parameter U , the measurements z_k on the trajectory from 0 to k .

4 Particle filtering principle:

General model:

Consider $x(t)$ a Markov process, partially observed by $y(t)$,

$$\begin{cases} x_t = f(x_{t-1}, w_t) \\ y_t = h(x_t, v_t) \end{cases}$$

This system can be linear, non linear, Gaussian, Dirac... To apply particle filtering we must be able to:

1°) simulate the noise: w_t

2°) Calculate explicitly: $p(y_t | x_t)$

At the step (t-1), we represent the conditional probability by a Dirac measures sum:

$$p(x_{t-1} | Y_0^{t-1}) \cong \frac{1}{N} \sum_{i=1}^N \delta_{x_{t-1}^i}(x_{t-1})$$

At t, we calculate recursively this probability on two steps:

Prediction: Monte-Carlo dynamic Simulation:

$$\tilde{x}_t^i = f(x_{t-1}^i, w_t^i) \quad \text{where} \quad \{w_t^i\}_{i=1}^N$$

are sorted independently with respect to:

$$p(x_t | Y_0^{t-1}) \cong \frac{1}{N} \sum_{i=1}^N \delta_{\tilde{x}_t^i}(x_t)$$

This step represents the signal x(t) time evolution,

Correction: By Bayes Formulae:

$$p(x_t | Y_0^t) \cong \frac{p(y_t | x_t) \frac{1}{N} \sum_{i=1}^N \delta_{\tilde{x}_t^i}(x_t)}{\int p(y_t | x_t) \frac{1}{N} \sum_{i=1}^N \delta_{\tilde{x}_t^i}(x_t) dx_t} = \frac{\frac{1}{N} \sum_{i=1}^N p(y_t | \tilde{x}_t^i) \delta_{\tilde{x}_t^i}(x_t)}{\frac{1}{N} \sum_{i=1}^N p(y_t | \tilde{x}_t^i)}$$

Hence,

$$p(x_t | Y_0^t) \cong \sum_{i=1}^N p_i^t \delta_{\tilde{x}_t^i}(x_t)$$

This step represents the observation signal y(t) time evolution,

Redistribution:

Sort the N new particles

$$\{p_t^1, \dots, p_t^N\}$$

with respect to the multinomial density:

$$p(x_t | Y_0^t) \cong \frac{1}{N} \sum_{i=1}^N \delta_{x_t^i}(x_t)$$

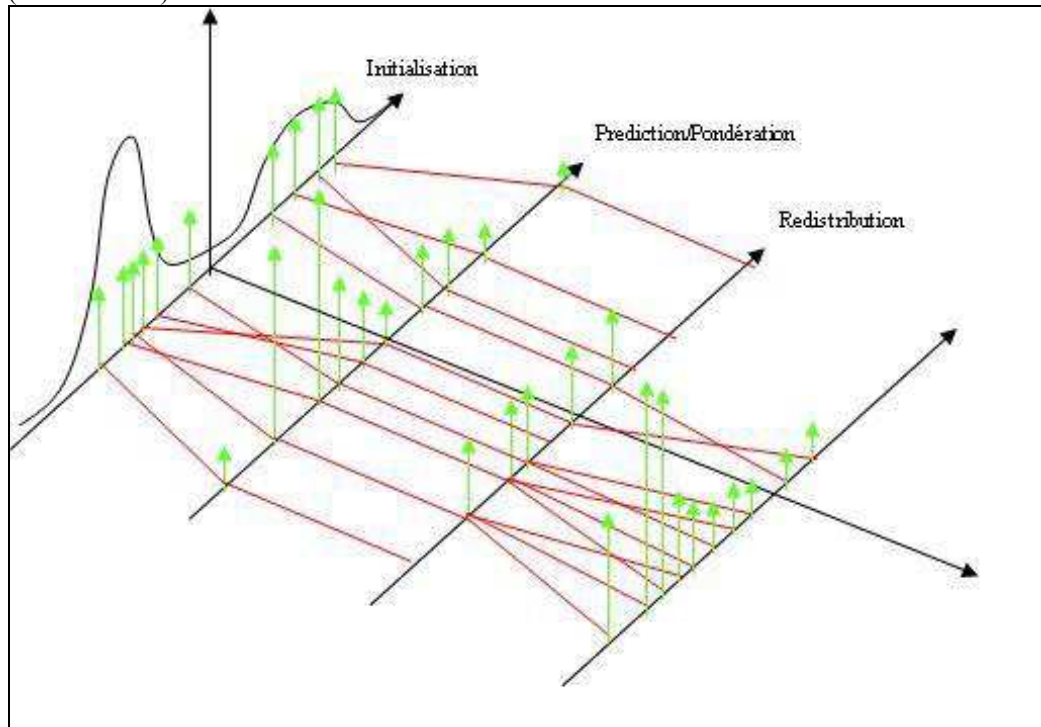
That we summarize in the schema (4.1)

This technique generates some numerical problems:

- N must be high number,

- The distribution may come with a reduced particle number support...

(...Aervoir ...)



5 Deterministic particle filtering

Principle:

The state space is represented via an N inter-dependent evolving meshing (N is constant) with respect to the observations. We privilege the most a posteriori probable areas in the state space. From the N initial conditions 'well chosen' (particles), we systematically explore the tree structure generated by the system dynamic, with a minimal M discrete points of the state noise. This gives $N \cdot M$ possible particles.

To hold the cardinal N constant, we select the N particles which have the greatest likelihood, with respect to the observations; which reduces calculation time and memory space.

General algorithm:

- 1°) Particle support initialization by N initials particles,
- 2°) *Deterministic branching*: with respect to the dynamic model and discret noise
- 3°) Local prediction of the nominal state trajectory:
- 4°) Local correction with respect to the nominal state trajectory:
- 5°) Weighting: likelihood calculus for every particle
- 6°) Deterministic selection of the N particles which have the greatest likelihood
- 7°) The best estimation is:

$$\hat{X}_t = \arg \max_{X_t} V(X_t | Y_t) = \arg \max_{X_t} \log(p(X_t | Y_t))$$

6 Deterministic Particle filtering applied to diagnosis

Let fix U at the value \hat{U}_k , least estimate of U, in (2.3). The problem is hence linear and Gaussian.

6.1 The linear Gaussian component estimation

The optimal estimation of X is performed by the Kalman filter equations:

Prediction step:

$$\left\{ \begin{array}{l} \hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1} + G\hat{U}_k \\ P_{k|k-1} = FP_{k-1|k-1}F^T + GQ_kG^T \end{array} \right. \quad (6.1)$$

a priori estimate

Correction step:

$$\left\{ \begin{array}{l} \hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k \tilde{z}_k \\ K_k = P_{k|k-1}H^T [HP_{k|k-1}H^T + R_k]^{-1} \\ P_{k|k} = [I - K_kH]P_{k|k-1} \\ \tilde{z}_k = z_k - \hat{X}_{k|k-1} \end{array} \right. \quad (6.2)$$

a posteriori estimate

6.2 Default detection/estimation

To perform the diagnosis problem, we have to estimate the parameter U. This will be accomplished by the particle filtering techniques.

Every step k, we realise N deterministic draws in the set $\mathfrak{S} = \{u_1, u_2, u_3, \dots, u_N\}$. The N realisations (i.e. Particles) will be used in the equation (2.3) to generate N different trajectories. For every particle U_i , we calculate the likelihood of the trajectory: $V_k = V(X_k, U_k | Z_{[0:k]}) = \ln(p(X_k, U_k | Z_{[0:k]}))$. The optimal estimate of U is: $\hat{U}_k = \arg\left(\max_{U_i}(V_k)\right)$, with respect to (6.1) and (6.2).

7 Estimation/detection algorithm

At every step k, let explore all the U values in the set $\mathfrak{S} = \{u_1, u_2, u_3, \dots, u_N\}$. The aim is to determinate the value maximising the likelihood V_k .

7.1 Initialisation

With N initial particles which are filled according to some probability measure, the initial likelihoods are: $V_0^i = \ln(p(X_0))$ for $i = 1$ to N.

7.2 Prediction/correction of the Gaussian component X

For every particle U_i , N Kalman filters are engaged to perform the estimate $\hat{X}_{k|k-1}^i$ corresponding to every U value, from the formulas (6.1), (6.2).

5.3 Evolution of the Gaussian component X, new particles

At the step k+1, for every initial condition $\hat{X}_{k|k}^i$, we pull M particles in the set $\mathfrak{S} = \{u_1, u_2, u_3, \dots, u_N\}$, which gives M*N particles and M*N trajectories from (2.3).

74 Likelihood ratio

Equations (4.1), (4.2) give a priori estimates of X and P. The innovation process leads to the calculus of V_k . The system in (2.3) is a Markov process with independent noises, hence V_k can be derived recursively by the formulas, for $i=1$ to M*N:

$$V_k^i = V_{k-1}^i + \ln(p(z_k | x_k, U_k = u_k^i)) = V_{k-1}^i + \ln(p(u_k^i)) + v_k^i$$

$$v_k^i = -\frac{1}{2} \left[\ln \left(HP_{k|k-1} H^T + R \right) + \left(z_k - H \hat{X}_{k|k-1} \right)^T \left(HP_{k|k-1} H^T + R \right)^{-1} \left(z_k - H \hat{X}_{k|k-1} \right) \right]$$

7.5 Estimation/ detection of U

The value of U, defining the deterioration level of the roller bearing is:

$$U_k^{\hat{\phi}} = \arg \left(\max_{U_i} (V_k^i) \right), \text{ with respect to (4.1) and (4.2)}$$

7.6 Redistribution

For the next iteration, we must sort N from M*N particles, whose have the best likelihood ratio. The algorithm will branch to **5.2**.

8 Simulations and results

The system equations have been simulated on Matlab, with the following numerical values: $\omega_r = 7039.88$, $T_c = 0.00029$, $\alpha = 1$, $T_e = 0.000025$, $T_d = 0.0018$, $w_k \approx N(0, q_k)$, $q_k = 0.25$, $v_k \approx N(0, r_k)$, $r_k = 0.1$, for all $k \geq 0$, $X_0 \approx N(\bar{X}_0, Q_0)$, $P_0 = Q_0 = \text{diag}(0, 0.25)$, $\bar{X}_0 = [10 \ 0]^T$, X_0 , w_k and v_k are assumed independents, N and M are fixed at 5 and $\mathfrak{S} = \{0.125 \ 37 \ 410 \ 1010 \ 607\}$, $P_{tr} = [995 \ 3 \ 2 \ 0.1 \ 0.1; 1 \ 994 \ 3 \ 2 \ 0.1; 0.1 \ 1 \ 995 \ 3 \ 1; 0.1 \ 1 \ 1 \ 995 \ 3; 0.1 \ 0.1 \ 1 \ 3 \ 996] * 1e-3$.

Case 1: the fault is assumed constant.

We fix the real fault at an arbitrary value U in the interval $[0.01, 10000]$. The algorithm converges to the nearest value in \mathfrak{S} after only 15 iterations.

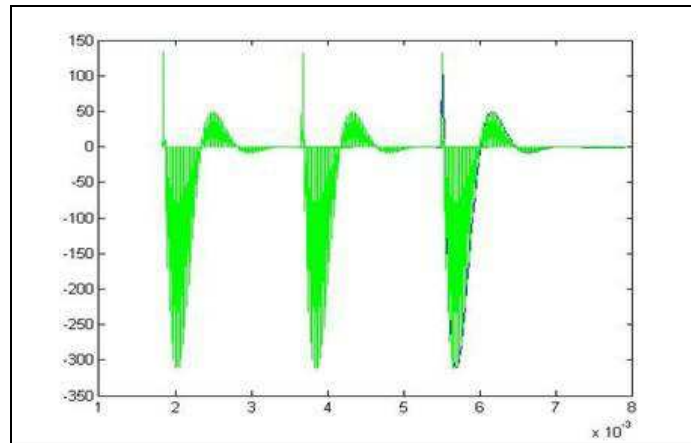


Fig. 1. Succession of shocks for a faulty roller bearing, real and estimated speed.

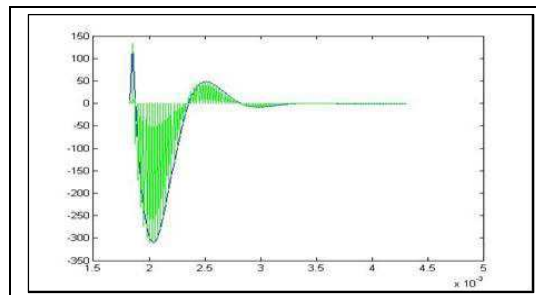


Fig. 2. Real (blue) and estimated (green) speed, Zoom of Fig 1.

Case 2: the fault amplitude takes value in the set $\mathfrak{S}_{\text{real}} = \{0.1 \ 45 \ 450 \ 570 \ 950\}$ different from $\mathfrak{S} = \{0.125 \ 37 \ 410 \ 1010 \ 607\}$, the set of U particles values.

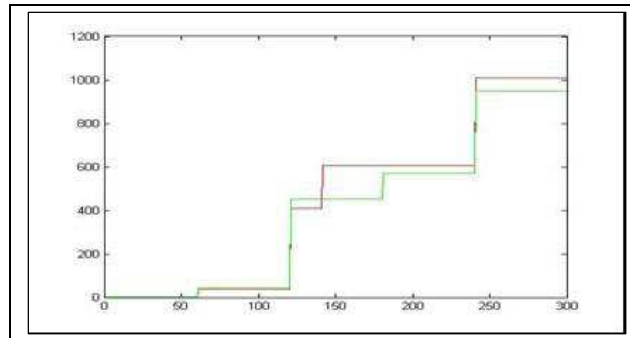


Fig. 3. Real (green) and estimated (red) default value.

7 Conclusion

In this study, the linear model proposed for the transmission of a vibratory signal to the sensor has made possible the representation of the roller bearing diagnosis by a hybrid system. The discrete parameter U represents exactly here the level of the suspected deterioration. This leaves a non linear filtering problem that we solve using deterministic particle filtering in two situations: 1°) the fault is constant, 2°) the deterioration is time varying in a discrete set of values.

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