Comparative study of the use of C1-continuous finite elements and splines for contact problems with large slidings

Benoit Magnain, Alain Batailly, Nicolas Chevaugeon, Patrice Cartraud

ECCM 2010
IV European Conference on Computational Mechanics
Palais des Congrès, Paris, France, May 16-21, 2010

Comparative study of the use of C^1-continuous finite elements and splines for contact problems with large slidings

B. Magnain[1], A. Batailly[2], N. Chevaugeon[3], P. Cartraud[3]

[1] *Institut PRISME, ENSI Bourges, France, benoit.magnain@ensi-bourges.fr*
[2] *Laboratoire de dynamique des structures et vibrations, Université McGill, Canada, alain.batailly@mcgill.ca*
[3] *GéM, École Centrale de Nantes, France, {nicolas.chevaugeon,patrice.cartraud}@ec-nantes.fr*

1 Introduction

The numerical simulation of contact problems is still a delicate matter especially when large transformations are involved. In that case, relative large slidings must be taken into account between the contact surfaces and the discretization error induced by linear finite elements may not be satisfactory. In particular, linear elements lead to a facetization of the contact surface, meaning an unavoidable discontinuity of the normal vector to this surface. Uncertainty over the precision of the results, irregularity of the speed on the contact nodes and even numerical oscillations are the consequences of such discontinuity. Among the different methods developed in order to fix this problem, one may be interested in studies carried out using mortar formulations [1, 2]. Other authors [3, 4] proposed to create a C^1-continuous surface based on the initial mesh in order to optimize the detection of penetrations. Finally, work published in the area of biomechanics about the numerical simulation of mammographies [5] considered C^1-continuous Hermitte finite elements.

$$\mathbf{u_i} = \{u_i, \frac{du_i}{dx}, \frac{du_i}{dy}, v_i, \frac{dv_i}{dx}, \frac{dv_i}{dy}\}$$

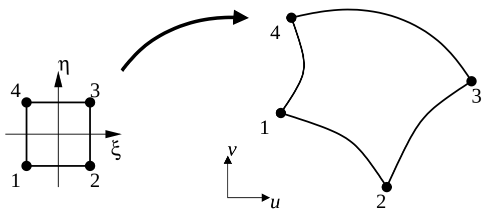


Figure 1: Hermitte element (dof and reference element)

In the present paper, we focus on these last two methods. They are combined with a finite element code using the bi-potential method to manage contact [6]. We restrict our study to 2D contact problems for which we use 3 or 4-noded linear finite elements.

2 Hermitte element and superposition of spline for contact problems

The first method consists in the implementation of a 4-noded Hermitte element with 6 degrees of freedom (dof) per node such as the one depicted in Fig. 1. For each node, in addition to the usual displacement dof u and v, coordinates of the two tangent vectors to each associated edge are taken into account. As a result, continuity of the tangent vector between two adjacent elements is ensured. The other

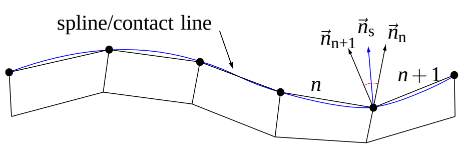
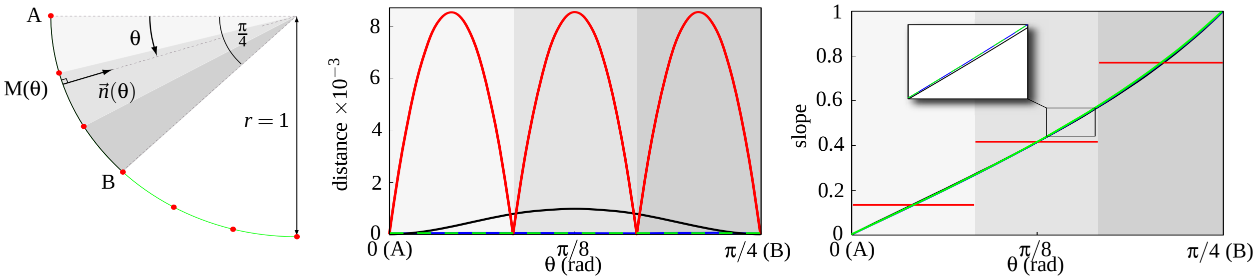


Figure 2: Spline on regular mesh

method proposed in our study relies on the computation of cubic spline passing by all the nodes of the contact line [3] as pictured in Fig. 2. Beside of the continuity of the normal vector along the contact surface, this method advantageously does not depend on the element type of the mesh. Consequently, it may be used for any kind of contact surface and can easily be extended to 3D contact through bi-cubic surface splines. The main challenge associated with the use of these methods essentially affects the computation of the gap function within the contact algorithm.

3 Results

In order to assess the relevance of the proposed developments, several numerical applications are presented. First, the results aim at validating the choices made and the implementation of the methods through contact detection and the spatial distribution of contact reactions. For the sake of brevity, results pictured in Figs. 3(a), 3(b) and 3(c) simply focus on the improvement of continuous methods for computing both the gap and the slope of the normal vector to the contact profile. Then, a comparison between the two methods and their contribution comparing to regular linear elements is explained in details. The



(a) circular contact profile (b) distances (c) slope, , direction of $\vec{n}(\theta)$

Figure 3: Distance to the exact circular profile (green line) and slope of the normal to this profile with a regular mesh with 3 elements (red line), a 4 points cubic B-spline (blue line) and one Hermitte element (black line) between points A and B.

results pictured in this study – in terms of contact reactions on finite element models – highlight the improvements due to the use of Hermitte elements and splines in terms of quality and stability of the results. The focus is made also on the capacity of each method to compute contact reactions and their CPU cost.

References

[1] M. A. Puso and T. A. Laursen. A mortar segment-to-segment frictional contact method for large deformation. *Computer methods in applied mechanics and engineering*, 193:4891–4913, 2004.

[2] K. A. Fischer and P. Wriggers. Mortar based frictional contact formulation for higher order interpolation using moving friction cone. *Computer methods in applied mechanics and engineering*, 195:5020–5036, 2006.

[3] M. Legrand. *Modèles de prediction de l'interaction rotor/stator dans un moteur d'avion*. Thèse de doctorat (phd thesis), École Centrale de Nantes, Nantes, France, 2005.

[4] J. J. Munoz. Modelling unilateral frictionless contact using the null-space method and cubic $b-$spline interpolation. *Computer methods in applied mechanics and engineering*, 197:979–993, 2008.

[5] J. H. Chung, V. Rajagopal, T. A. Laursen, P. M. F. Nielsen, and M. P. Nash. Frictional contact mechanics methods for soft materials: Application to tracking breast cancers. *Journal of biomechanics*, 2007.

[6] B. Magnain, Z. Q. Feng, and J. M. Cros. Modélisation des problèmes d'impact avec dissipation d'énérgie par frottement. *Compte rendu de mécanique*, 333, 2005.