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An Extension of a Fuzzy Ontology for Flexible Querying

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Abstract—In this paper, we propose a personalized approach for flexible querying of information systems. This approach consists in the combination of the reasoning capabilities of the fuzzy DLR-Lite ontology and the expressivity of the SQLf language. The interpretation of the gradual inclusion (subsumption) axioms of the ontology is based on the Gōdel fuzzy implication. Its generalization to a tree of inclusions is also proposed. This tree and its property of propagation of degrees are the basic theoretical elements of our application, which consists in querying of a multimodal transport information system which is embedded in a mobile terminal characterized by limited storage and processing capabilities.

Keywords—Ontology, flexible querying, personalization, fuzzy DLR-Lite, SQLf, fuzzy sets theory, multimodal transportation.

I. INTRODUCTION

The personalization of flexible querying of information systems dedicated to multimodal transportation networks is addressed in this paper. We propose an approach which consists in the development of a fuzzy semantic layer in order to extend the functionalities of information systems to reasoning. This layer is made of reasoning functions which combine domain application data with ontology axioms. Ontologies are able (i) to model the domain knowledge, (ii) to unify its vocabulary, (iii) to reason with available knowledge and (iv) to deduce implicit data from explicit data.

The proposed approach is developed in the framework of flexible querying of information systems which allows the user to express preferences in his/her queries. In such a system, the delivered answers are ranked from the most to the least preferred. In this context, user preferences can be expressed through the SQLf language to fuzzy predicates (conditions) and provides more expressive SQL-like statements.

From the reasoning point of view, several fuzzy ontologies (based on description logics [1]) have been developed (see [2], [8], [9], [10], [11], [12]) in which theoretical aspects have been studied (the definition of fuzzy concepts and relations, the definition and the interpretation of fuzzy inclusion or subsumption axioms, the fuzzy extension of subsumption algorithms, etc.). However, these ontologies are not suitable to address querying issue. Indeed, ontologies save axioms, individuals and related data in text files which are not appropriate for the needs of real applications. Ontology-based querying systems are faced with a large amount of data to such extent that the combination of ontologies and databases become an absolute necessity. In addition, the developed ontology-based querying systems, such as [15], [5], [10], [13], are based on limited description logics in terms of expressivity. To overcome this limitation, databases are combined with ontologies to provide DBMS functionalities such as data persistence and query evaluation. Ontology axioms are then used in the query rewriting process to take into account the logical relationships between concepts.

In this context, we consider the Description Logic extended to m-ary Relations [5] denoted DLR-Lite. More precisely, we are interested in the fuzzy extension of the DLR-Lite, called fuzzy DLR-Lite [7], [14], [15], which is made of a reduced set of fuzzy operators and abstraction rules which are connected to databases through SQL statements.

In this paper, we aim to design a flexible querying system that combines a fuzzy DLR-Lite knowledge base with the SQLf language, so as to take profit from the expressivity of the SQLf language and the reasoning mechanisms of the fuzzy DLR-Lite. Firstly, we show the relevance of the Gōdel fuzzy implication to the interpretation of gradual inclusion (subsumption) axioms of the fuzzy ontology. Then, we consider the enrichment of the fuzzy DLR-Lite knowledge base with the SQLf language. The resulting knowledge base is finally applied on a sample mobile application in the field of multimodal transportation. The developed application is embedded in a lightweight terminal in which the accessible data are limited to a subset of the knowledge base. This restriction is due to the limited storage and processing capabilities of such terminals.

The remainder of the paper is structured as follows. Section II introduces the fuzzy DLR-Lite knowledge base. In section III, the Gōdel fuzzy implication is used to define an interpretation to the gradual inclusion axiom; its contrapositive is also used to define a restricted form of the negation operator. We also show in section III the way in which our interpretation is extended to a tree of gradual inclusion axioms and its degree propagation property. Section IV introduces briefly the SQLf language and points out the relevance of the enrichment of
the fuzzy DLR-Lite knowledge base with this language. Our sample application is described in section V. Section VI recalls our contributions and draws some lines for future works.

II. THE FUZZY DLR-LITE ONTOLOGY

In this section, we introduce some elements of both the fuzzy sets theory and the fuzzy DLR-Lite knowledge base.

A. The Fuzzy Sets Theory

The fuzzy sets theory is introduced by Zadeh [16] to express the gradual membership of an element to a set. Formally, a fuzzy set \( F \) with a couple of \((t\text{-norm}, t\text{-conorm})\) such as \( F \) is characterized by its membership function. The gradual membership of an element to a set. Formally, a fuzzy set \( F \) is characterized by its membership function.

\[ \mu_F(x) \]

The projection axioms allow to define new concepts and their attached facts by extracting one or more columns from other relations. Two kinds of projection axioms are defined in the ontology component: the simple projection and the restricted projection.

The simple projection is of the form \( \exists[i_1, \ldots, i_k]R \) that expresses the projection of the relation \( R \) on columns \( i_1, \ldots, i_k \), and the restricted projection is of the form \( \exists[i_1, \ldots, i_k]R. (\text{Cond}_1 \cap \ldots \cap \text{Cond}_h) \) which restricts the projection of \( \exists[i_1, \ldots, i_k]R \) according to the boolean conditions \( \text{Cond}_{j, l=1,\ldots,h} \) of the form \( (i \Theta \nu) \), where \( i \) corresponds to a column number of the relation \( R \), \( \Theta \in \{<,\leq,=,\neq,>,\geq\} \) and \( \nu \) is a value.

3) The Abstraction Component \( \mathcal{A} \): It forms the set of statements that connect concepts and relations, defined in the ontology component, to physical relational tables defined in a database. Simple and complex abstraction statements are defined as follows:

- **Simple abstraction statement** has the form:
  \[ R_1 \rightarrow R_2(c_1[t_1], \ldots, c_n[t_n])c_i \] which states that the \( n \)-ary relation \( R_1 \) of the ontology component is mapped into the projection on columns \( c_1, \ldots, c_n \) of the \( m \)-ary table \( R_2 \) \((n \leq m)\) each of which is of type \( t_i \). The score of the delivered tuples is provided by the column \( c_i \).
- **Complex abstraction statement** has the form:
  \[ R_1 \rightarrow (t_1, \ldots, t_n)[c_i].sql \] where \( R_1 \) is an \( n \)-ary relation of the ontology component and \( sql \) is an SQL query which returns the \( n \)-ary tuples \( (t_1, \ldots, t_n) \) ranked in decreasing order of their scores \( c_i \).

C. Interpretation of the Ontology Axioms \( \mathcal{O} \)

An interpretation of the fuzzy DLR-Lite ontology, denoted \( \mathcal{O} = \langle \Delta, \prec \rangle \), consists of a finite interpretation domain \( \Delta \) and of an interpretation function \( \mathcal{O} \) which associates for each relation \( R \) (of arity \( m \)) an interpretation \( R^\mathcal{O} : \Delta^m \rightarrow [0,1] \) where \( R^\mathcal{O}(c) \) denotes the grade of membership of the tuple \( c \) to the relation \( R \).

It is worth noticing that the interpretation function \( R^\mathcal{O} \) has the same meaning as the membership function \( \mu_R \) as defined in the fuzzy sets theory. Therefore, both \( R^\mathcal{O}(x) \) and \( \mu_R(x) \) notations are identical.

For sake of clarity, we only introduce the interpretation of the gradual inclusion axiom; see [14] for more details about the interpretation of simple and restricted projection axioms.

The interpretation of the gradual inclusion axiom is defined as follows:

\[ \mathcal{O} = \langle R_1 \cap \ldots \cap R_i \subseteq R_j \rangle[n] \iff \forall c \in \Delta^m, n \cap R_i^\mathcal{O}(c) \cap \ldots \cap R_j^\mathcal{O}(c) \leq R_i^\mathcal{O}(c) \] (1) where \( m \) is the arity of the relations \( R_{i,j=1,\ldots,m} \) and \( R_i \) and \( \cap \) is a t-norm.

The formula (1) states that if \( c \) is an instance of \( R_{i,j=1,\ldots,j} \) at the degree \( s_i = R_i^\mathcal{O}(c) \) then \( c \) is an instance of \( R_i \) at the minimal degree \( n \cap s_i \cap \ldots \cap s_j \).

D. The Query Language

A fuzzy DLR-Lite query consists in a conjunction query combined with a scoring function to rank answers. The query is of the following form:

\[ q(x)[s] \leftarrow \exists y R_1(z_1)[s_1] \ldots R_l(z_l)[s_l], \quad \text{OrderBy}(s = f(s_1, \ldots, s_l)), \]

where \( q \) is a \( k \)-ary relation, each \( R_i \) is a relation, \( x \) (resp. \( y \)) is a vector of distinguished (resp. non distinguished) variables, \( z_i \) is a tuple of constants or variables from \( x \) or \( y \), \( f \) is a scoring function.
Inclusion axioms of the fuzzy DLR-Lite ontology is based on A. Gödel Fuzzy Implication Based Interpretation of the Gradual Inclusion Axioms. We finally generalize the Gödel fuzzy implication to a tree of gradual inclusion axioms by substituting its left-hand side. Each axiom substitution generates a new query which is added to a disjunctive query. Each subquery of the resulting disjunctive query is finally translated into an SQL query to be evaluated over the facts comprised.

Example 1. Let Journey be a relational table of journeys defined from a multimodal transportation system. We define the fuzzy relation FastJourney from the Journey relation and the following inclusion (subsumption) axiom:

\[(\text{FastJourney} \sqsubseteq \text{FavoriteJourney})[0.7],\]

which states that each fast journey is also a favorite journey at the minimal degree of 0.7.

Any query involving favorite journeys, such as:

\[q(t)[s] \leftarrow \text{FavoriteJourney}(t), \text{OrderBy}(s)\]

is rewritten, based on the above gradual inclusion axiom, in terms of fast journeys (the t-norm used is min) as follows:

\[q'(t)[s] \leftarrow \text{FastJourney}(t)[s'], \text{OrderBy}(s = \text{min}(0.7, s'))\]

where \(s' = \text{fast}^g(t)\) is the grade which expresses in what extent the journey \(t\) is fast.

We obtain for each journey \(t\) a minimal grade of satisfaction to the query \(q\) equals to \(\text{min}(0.7, \text{fast}^g(t))\) since the interpretation of the gradual inclusion axiom means:

\[
\forall t. \text{Favorite}^g(t) \geq \text{min}(0.7, \text{fast}^g(t)).
\]

III. THE GRADUAL INCLUSION AXIOM AND THE GÖDEL FUZZY IMPLICATION

In this section, we show that the interpretation of the gradual inclusion axioms of the fuzzy DLR-Lite ontology is based on the Gödel fuzzy implication. We also show the interest of its contrapositive for the definition of a restricted form of the negation operator. We finally generalize the Gödel fuzzy implication based interpretation to a tree of gradual inclusion axioms.

A. Gödel Fuzzy Implication Based Interpretation of the Gradual Inclusion (Subsumption) Axiom

Let \((A \sqsubseteq B)[\alpha]\) be a gradual inclusion axiom, denoted by \(A \sqsubseteq^\alpha B\). It is interpreted in the framework of the fuzzy DLR-Lite [14] as follows:

\[
\forall x \in A, B^g(x) \geq \text{min}(\alpha, A^g(x))
\]

We can prove that this interpretation is based on a gradual inclusion defined by the Gödel fuzzy implication \((\mu_A(x) \Rightarrow^g \mu_B(x) = 1)\) if \(\mu_A(x) \leq \mu_B(x); \mu_B(x), \text{otherwise})\):

\[A \sqsubseteq^\alpha_G B \iff \text{min}_x(\mu_A(x) \Rightarrow^g \mu_B(x)) = \alpha\]

In the remainder of the paper, we use equally the notations \(A \sqsubseteq^\alpha B\) and \(A \sqsubseteq^\alpha_G B\).

Proof: The Gödel fuzzy implication is an R-implication defined by:

\[\mu_A(x) \Rightarrow^g \mu_B(x) = \sup_{[0,1]} \{u | \text{min}(\mu_A(x), u) \leq \mu_B(x)\}\]

We denote \((\mu_A(x) \Rightarrow^g \mu_B(x))\) by \(I(\mu_A(x), \mu_B(x))\).

As \(I(\mu_A(x), \mu_B(x))\) is a u value then:

\[
\forall x, \text{min}(\mu_A(x), I(\mu_A(x), \mu_B(x))) \leq \mu_B(x)
\]

having \(\alpha = \text{min}_x(\mu_A(x), I(\mu_A(x), \mu_B(x)))\) then

\[
\forall x, \alpha \leq I(\mu_A(x), \mu_B(x)) \Rightarrow
\forall x, \text{min}(\mu_A(x), \alpha) \leq \text{min}(\mu_A(x), I(\mu_A(x), \mu_B(x))).
\]

From the formula (4), we obtain: \(\forall x, \text{min}(\mu_A(x), \alpha) \leq \mu_B(x)\).

This interpretation, based on the Gödel fuzzy implication, allows the definition of a minimal threshold of satisfaction to queries in which substitutions of the gradual inclusion axioms are performed.

In addition, the Gödel fuzzy implication provides effective means of computing the minimal grade of the gradual inclusion axioms (formula (1)).

B. The Gradual Inclusion Axiom and the Contrapositive of the Gödel Fuzzy Implication

The contrapositive of the Gödel fuzzy implication could be used to define a new type of fuzzy inclusion (subsumption) denoted \(\sqsubseteq^{\beta}_{\text{ContG}}\):

\[A \sqsubseteq^{\beta}_{\text{ContG}} B \iff \text{min}_x(1 - \mu_B(x) \Rightarrow^g 1 - \mu_A(x)) = \beta
\]

The interpretation of this form of fuzzy inclusion axioms is deduced from the interpretation based on the Gödel fuzzy implication as follows:

\[A \sqsubseteq^{\beta}_{\text{ContG}} B \Rightarrow \forall x, \neg A^g(x) \geq \text{min}(\beta, \neg B^g(x))
\]

The proof of this interpretation is similar to the previous proof. This form of gradual inclusion axioms allows to express a restricted negation operator.

Example 2. Let TouristJourney \(\sqsubseteq^6_{\text{ContG}}\) FavoriteJourney be a fuzzy inclusion (subsumption) axiom which means that the fuzzy set of touristic journeys is included in the fuzzy set of favorite journeys at the minimal degree of 0.6. In the case that the access is limited to favorite journeys, the system can answer queries about journeys which aren’t touristic, based on the above fuzzy inclusion axiom, as follows:

\[\neg \text{TouristJourney}(t) \geq \text{min}(1 - \text{FavoriteJourney}(t), 0.6)
\]

(formula (5)).

Please notice that this negation is only applicable to relations involved in gradual inclusion axioms.

C. Interpretation of a Tree of Gradual Inclusion Axioms

The proposed interpretation of the gradual inclusion axiom is generalized in the case of a tree of inclusion axioms. This tree structure and its property of degree propagation are the basis of the developed application in section 5.

Without loss of generality, we restrict the tree structure to two levels of inclusion as depicted in Fig. 1a.

Such a tree structure means: \(A \sqsubseteq^\alpha_{\text{Go}} B\) and \(B \sqsubseteq^\beta_{\text{Go}} C\), where \(\alpha, \beta \in [0,1]\).
By applying the formula (2), we obtain:

\[ \mu_B(x) \geq \min(\mu_A(x), \alpha) \]  

\[ \mu_C(x) \geq \min(\mu_B(x), \beta) \]  

The propagation property allows the deduction of the grade of inclusion of the concept \( A \) in the concept \( C \) (see Fig. 1b).

**Proof:** From the formula (7), we deduce:

\[ \mu_B(x) \geq \min(\mu_A(x), \alpha) \Rightarrow \]

\[ \min(\beta, \mu_B(x)) \geq \min(\beta, \min(\mu_A(x), \alpha)) = \min(\alpha, \beta, \mu_A(x)) = \min(\min(\alpha, \beta), \mu_A(x)). \]

From

\[ \begin{cases} 
\min(\beta, \mu_B(x)) \geq \min(\alpha, \beta, \mu_A(x)) \\
\mu_C(x) \geq \min(\mu_B(x), \beta) \text{ (cf. formula 8)} \\
\Rightarrow \mu_C(x) \geq \min(\min(\alpha, \beta), \mu_A(x)). 
\end{cases} \]

This property of degree propagation can be generalized to a series of gradual inclusion axioms of the form:

\[ A_1 \subseteq^{\alpha} A_2 \subseteq^{\beta} \ldots \subseteq^{\alpha_{i-1}} A_i, \]

where \( i < j \) can be interpreted as follows:

\[ \forall x: \mu_{A_j}(x) \geq \min(\mu_{A_i}(x), \alpha). \]

where \( \alpha = \min(\alpha, \alpha_{i+1}, \ldots, \alpha_{j-1}) \).

### Example 3.

The extended complex abstraction statements allow to define concepts and relations corresponding to complex preferences, such as MyFastJourney that corresponds to "journeys having most of steps are fast", which is a fuzzy relation that can be defined by the following extended abstraction statement using an SQLf statement:

\[ \text{MyFastJourneys} \leftarrow (t_1, \ldots, t_k)[c_3, sqlf], \text{ where } R_1 \text{ is a concept from the ontology and } sqlf \text{ is an SQL query which delivers a fuzzy relation.} \]

### V. Illustrative Example

In this section, a sample application is developed in the field of multimodal transportation, in order to show the interests of our approach, and to emphasize the benefit gained from the interpretation of the gradual inclusion axiom, using the Gödel fuzzy implication and the t-norm \( \min \).

#### A. The Context of the application

The considered application is developed in the context of mobile applications. The user terminal is a lightweight terminal which is limited in terms of main storage and processing capabilities. Therefore, the user terminal only embeds a subset of the knowledge base that consists of (i) fuzzy relations defined by the abstraction statements, (ii) fuzzy predicates and (iii) gradual inclusion axioms.

The user can query the embedded application with conjunctive queries which will be evaluated over the available data, based on the gradual inclusion axioms.
We assume that a user interface is provided to define membership functions corresponding to fuzzy concepts. The embedded system can deduce, from the gradual inclusion axioms, the minimal grade of inclusion of the already defined fuzzy concepts in the user-defined fuzzy concepts.

B. An Embedded Multimodal Transport Information System

We design a mobile application in the field of multimodal transportation. The conceptual data model of the system is consistent with the Transmodel standard [6]. Transmodel is an European standard defining a unified conceptual data model of multimodal transport networks which combine bus, tramway, trolley bus and underground networks. Transmodel provides operating data model for network management activities such as timetables, staff, fleet and rolling stock management, real time monitoring and user information circulation.

In practice, travelers are more interested in information about available networks, timetables and services. So, a subset of the database is enough, from the user point of view, to answer his/her queries which are generally about journeys.

For the sake of clarity, the knowledge base of our application is limited to one level of gradual inclusion enriched after that with a user defined fuzzy predicate.

We introduce below the 3 steps for the design of the embedded application: (i) data preprocessing, (ii) querying of the embedded system and (iii) the enrichment of the system.

1) Step 1. Data Preprocessing: Let the following relational tables be a subset of a database from a multimodal transport information system:

\[
\text{Network, Line, Station, Link and LinkLine.}
\]

We define the Journey relation as follows:

\[
\text{Journey} \left(\text{#journey, #link, departure, arrival, source, destination, DepLink, ArrLink, modeLink, durLink, costLink, comfLink, waitLink}\right).
\]

It is assumed that journeys from source \(s\) to destination \(d\) are preprocessed.

Based on the SQLf queries, the following fuzzy relations are defined from the Journey relation:

1) \(\text{TouristJourney} \rightarrow \left[\#\text{Journey}\right]\). Select \#Journey From Journey Group By \#Journey Having (most \#Link are Touristic);

2) \(\text{PleasantJourney} \rightarrow \left[\#\text{Journey}\right]\). Select \#Journey From Journey Group By \#Journey Having (most \#Link are Short(waitLink));

The above extended complex abstraction statements state respectively that a touristic journey is a journey in which most of its steps (named link in the statements) cut across touristic sites and a pleasant journey is a journey in which the waiting time of most of its steps is short. Both touristic and short are fuzzy predicates.

The fuzzy predicate \(\text{MorningDepartureJourney}\) is also defined by the membership function described in Fig. 2.

The following gradual inclusion axioms could be obtained by a processing using the formula (2) over the database:

- TouristJourney \(\subseteq_{G^0.7} \text{MorningDepartureJourney}\)
- PleasantJourney \(\subseteq_{G^0.8} \text{MorningDepartureJourney}\)

2) Step 2. Querying the Embedded System: The data embedded in the user terminal are illustrated in Fig. 3. The fuzzy relations \(R_{i,j=1,2}(\#\text{Journey})\) (in the sense of the SQLf language) are resulting from SQLf queries of the extended complex abstraction statements and mapped respectively to fuzzy concepts (relations in the sense of the fuzzy DLR-Lite) \(\text{TouristJourney} \text{et PleasantJourney}\).

Due to the storage limitation of the user terminal, both the relations \(R_{i,j=1,2}\) only contain the \#Journey and the score columns. The membership function of the fuzzy predicate \(\text{MorningDepartureJourney}\) is also embedded.

The user can query the system based on the embedded fuzzy predicates. The grade of satisfaction to both fuzzy conditions \(\text{Tourist} \text{and pleasant}\) are embedded within the relations \(R_{i,j=1,2}\). About the fuzzy relation \(\text{MorningDepartureJourney}\), a minimal grade of satisfaction can be obtained using the formula (2). For example:

\[
\forall t, \mu_{\text{Morning}}(t) \geq min(\mu_{\text{Touristic}}; 0.7) = min(\mu_{R_1}; 0.7),
\]

and

\[
\forall t, \mu_{\text{Morning}}(t) \geq min(\mu_{\text{Pleasant}}; 0.8) = min(\mu_{R_2}; 0.8).
\]

Finally, we can write:

\[
\forall t, \mu_{\text{Morning}}(t) \geq max(min(\mu_{R_1}; 0.7), min(\mu_{R_2}; 0.8)).
\]

3) Step 3. Enrichment of the Embedded System: The user can define his/her own fuzzy concepts to express his/her preferences. These concepts could be connected to the previously defined tree structure of concepts in order to personalize the embedded system. The query evaluation process is then made more accurate and close to user expectations.

Example 4. Through the fuzzy concept definition interface, the user can define \(\text{FavoriteDepartureJourney}\) as a fuzzy concept based on the favorite departure time (see Fig. 4). \(\text{FavoriteDepartureJourney}\) is then bound to \(\text{MorningDepartureJourney}\) fuzzy concept as follows:

\[
\text{MorningDepartureJourney} \subseteq_{G^0.8} \text{FavoriteDepartureJourney}.
\]
Indeed, if the embedded system, by applying the propagation property, the value of $\alpha$ in the above gradual inclusion axiom is $\alpha = \min(t_\text{Morning}(t) \Rightarrow G\alpha t_\text{Favorite}(t))$ and can be computed from the membership functions of both $\text{Morning}$ and $\text{Favorite}$ fuzzy predicates as depicted in Fig. 5.

Once the value of $\alpha$ is obtained ($\alpha = 0.5$ in this example), the embedded system, by applying the propagation property, deduces the grade of gradual inclusion links that bind the added concept to the existing ones as described in Fig. 6.

Any query which is about the fuzzy predicate $\text{FavoriteDepartureJourney}$ is evaluated over the available data which covers pleasant and touristic journeys such that:

$$\forall t, \mu_{\text{Favorite}}(t) \geq \max(\min(0.5, \mu_R(t)), \min(0.5, \mu_R(t))) \cdot$$

**Remark.** The deduction of the minimal grade of inclusion of a fuzzy concept in a user-defined fuzzy concept is based on both their respective membership function and the formula:

$$A \subseteq_{G\alpha} B \iff \min(t, \mu_A(x) \Rightarrow G\alpha \mu_{\text{B}}(x)) = \alpha.$$

This computation is only significant if the concept $B$ generalizes the concept $A$ (in the same way as concept hierarchies in ontologies) which means that $\text{Support}(A) \subseteq \text{Support}(B)$. Indeed, if $\text{Support}(A) \nsubseteq \text{Support}(B)$ then the grade of inclusion is equal to 0; which is in fact worthless.

**VI. CONCLUSION AND FUTURE WORKS**

The flexible querying approach developed in this paper is based on the combination of a fuzzy description logic (denoted fuzzy DLR-Lite) and the SQLf language that allows the expression of more complex fuzzy relations than those could be expressed by the SQL language. An interpretation of the gradual inclusion axiom based on the Gödel fuzzy implication is introduced. This interpretation provides a formal framework in which a restricted form of the negation operator is defined and a generalization to a tree of fuzzy relations is also proposed.

An implementation, based on algorithms for query evaluation introduced in [14], is in progress. It remains the development of an effective algorithm to compute the minimum grade of inclusion of fuzzy concepts added to the knowledge base. We also plan to extend the enrichment process to complex fuzzy predicates as those defined by division involving fuzzy relations.

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