New belief functions based methods for multicriteria decision making

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ABSTRACT

Any decision is closely linked to the quality and availability of information. Innovative methodologies are proposed to help decisions based on imperfect information provided by more or less reliable and conflicting sources in uncertain context. These methods combine new uncertainty theories and multicriteria decision making.

The ER-MCDA methodology associates the Analytic Hierarchy Process (AHP), used as a conceptual analytical framework, fuzzy sets, possibility and belief function theories. DSmT-AHP replaces the initial AHP aggregation principles by a fusion process and introduces specific discounting factors able to make a difference between importance and reliability of criteria. Finally, COWA-ER is the newest method that allows multicriteria decision making under uncertainty. All these methods are fitting perfectly to the domain of expert assessment in the context of natural hazards in mountains. This paper proposes a synthesis of the principles of the methods developped and described in details in other specific papers.

Keywords: Multicriteria Decision making, Expert assessment, Information imperfection, Information fusion, belief function theory, Dempster-Shafer Theory, Dezert-Smarandache Theory, Fuzzy sets theory, Possibility theory,

1 INTRODUCTION

Risk decision context related to natural hazards in mountains

Rapid mass movement hazards such as snow avalanches put humans and property at risk with dramatic consequences (Figure 1).

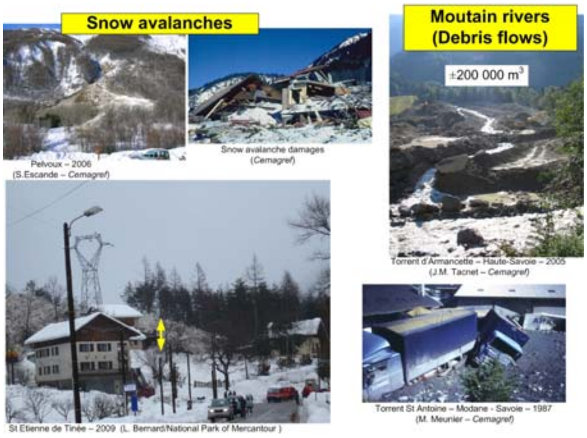


Figure 1: Decisions are needed to protect people and assets in mountains [16].

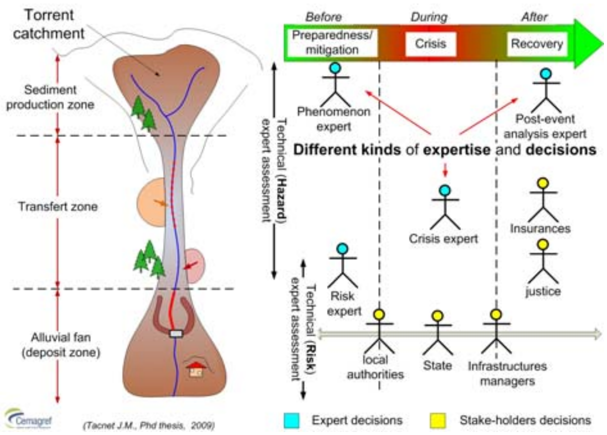


Figure 2: Managing risk implies a complex decision context [15].

In a context of insufficient knowledge on natural phenomena, expert assessments are required for multiple decision and risk management purposes using multidisciplinary quantitative or qualitative approaches (Figure 2). Those expert assessments are considered as decision processes. They depend on the availability, quality, and uncertainty of the available information resulting from measurements, historical analysis, eye witness accounts as well as subjective, possibly conflicting, assessments made by the experts themselves. As an example, the definition of risks zones is often based on the extrapolation of historical information known on particular points using morphology based analysis (Figure 3).

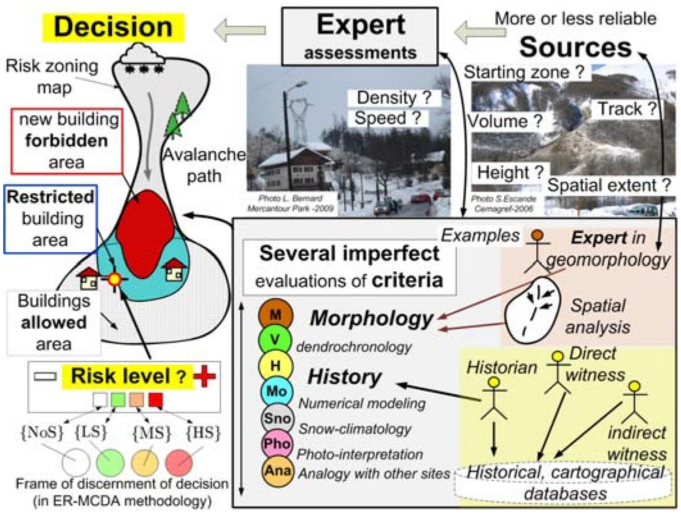


Figure 3: A multi-source context [17].

In the end, phenomenon scenarios and decisions may rely on very uncertain and conflicting information without being able to fully determine what actually occurred, with imprecise, conflicting, or simply unknown information used in the hypotheses attempting to explain the result. Shared decision-aid tools are expected to produce, use expert assessments in an integrated risk management system able to consider the technical, environmental and social aspects of a decision [15].

Needs for new decision making methods

Multicriteria decision methods aim to choose, rank or sort alternatives on the basis of quantitative or qualitative criteria and preferences expressed by the decision-makers. As shown in the previous examples, specific decision making methods are expected to help decision making based on imperfect information provided by heterogeneous more or less reliable and conflicting sources. Considering uncertainty in multicriteria decision making remains an important issue [3,8,14] in multicriteria decision methods. This paper presents an overview and a synthesis of recent methods[1] that extends and improves classical multicriteria decision based both on multicriteria decision making methods and new uncertainty theories such as fuzzy sets, possibility and belief function theories. The first section introduces the problem and needs. The following sections describe three new methodologies that mix uncertainty theories, with a special interest in belief function theory, and multicriteria decision making. Second section describes the ER-MCDA methodology[2]. The third section describes DSmT-AHP[3] which proposes an evolution of AHP but also a very new discounting technique for importance. The fourth section is dedicated to COWA-ER[4] which is an interesting evolution of existing OWA method in the context of decision under uncertainty. The conclusion proposes a synthesis and cross-comparison of these methods and perspectives.

2 ER-MCDA : MULTICRITERIA DECISION ANALYSIS AND EVIDENTIAL REASONING

Types of information imperfection and related theories

A decision is closely related to information quality. Uncertainty, as often used in common language, is indeed only one of all the various types of information imperfection which include inconsistency, imprecision, incompleteness and uncertainty (Figure 4).

[1] See the bibliography of each presented method for calculation principles and developments about theoretical backgrounds.
[2] Evidential Reasoning and Multi Criteria Decision Analysis
[3] Dezert-Smarandache Theory- Analytic hierarchy Process
[4] Cautious Ordered Weighted Averaging-Evidential Reasoning

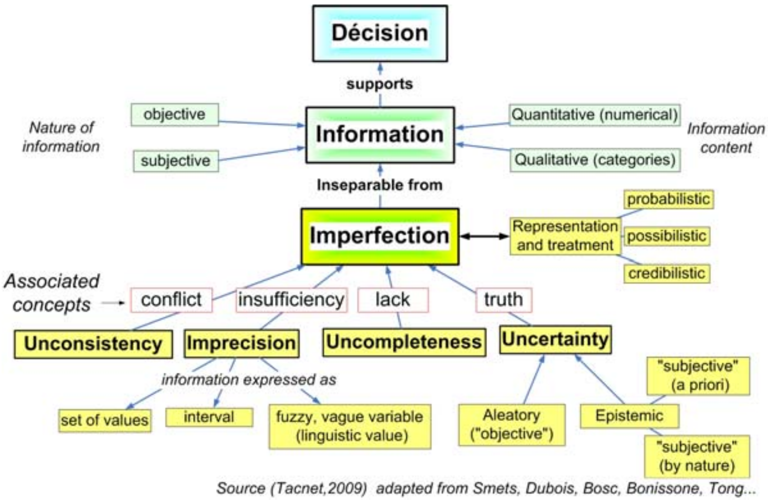


Figure 4: Different kinds of information imperfection [15].

In addition to the classical framework of probability, new uncertainty theories have been proposed to handle those different types of imperfect information such as evaluations provided through natural hazards expert assessment: fuzzy sets theory for vague information [21], possibility theory for uncertain and imprecise information [6,22]. Evidence or belief function theory allows one to represent and fuse information evaluation provided by more or less reliable and conflicting sources on the same hypotheses of a set called the frame of discernment. Each source (e.g., an expert) defines basic belief assignments (BBAs). In the classical Dempster-Shafer theory (DST), all the hypotheses are exhaustive and exclusive (Figure 5). A new theory called Dezert-Smarandache theory (DSmT) [11] provides a more versatile framework to represent uncertain, imprecise but also vague concepts. Information fusion consists in “conjoining or merging information that stems from several sources and exploits that conjoined or merged information in various tasks such as answering questions, making decisions, numerical estimation”[2]. Sources can be discounted with regard to their reliability [13].

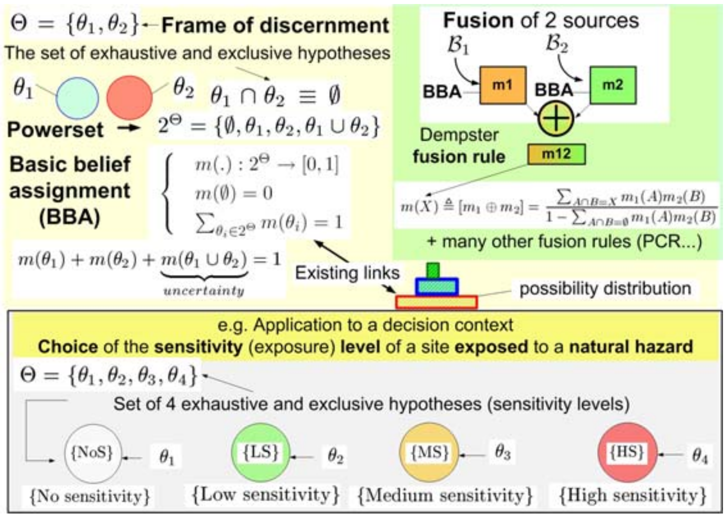


Figure 5: Basics of belief function theory and information fusion [18].

Principle of ER-MCDA methodology

ER-MCDA (Evidential Reasoning – Multicriteria Decision Analysis) [15,16,17] is a methodology that combines the Analytic Hierarchy Process (AHP)[9], a multi-criteria decision analysis method, and information fusion using Belief Function (or Evidence) Theory to represent, fuse and propagate information imperfections. Experts, considered more or less reliable, provide imprecise and uncertain evaluations of quantitative and qualitative criteria that are combined through information fusion. Fuzzy Sets and Possibility theories are used to transform quantitative and qualitative criteria into a common frame of discernment for decision in *Dempster-Shafer Theory* (DST) and *Dezert-Smarandache Theory* (DSmT) contexts.

A simplified version of an existing method, developed to assess the sensitivity of a snow avalanche site [15,16,17,18] is used as an example (Figure 6). The principle is to evaluate the sensitivity of an avalanche site according to the main criteria denoted as hazard (morphology, history, and snow climatology) and vulnerability (permanent winter occupants, dwellings, and infrastructures).

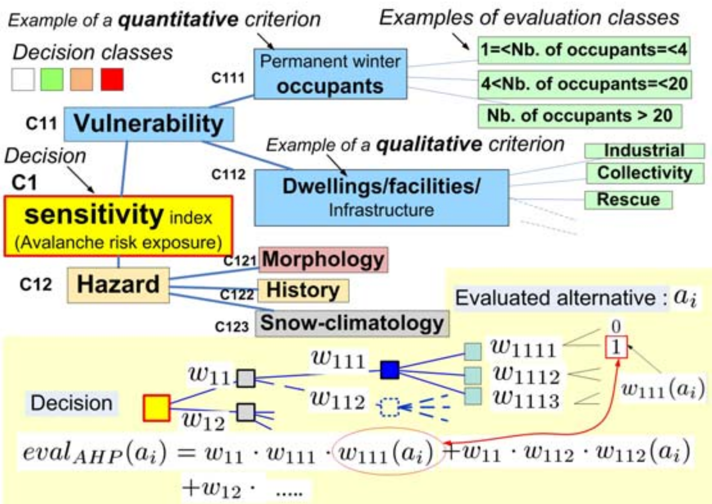


Figure 6: Principle of hierarchic multicriteria decision analysis (AHP) applied to the analysis of the sensitivity of a snow avalanche site [15,16,17,18].

A dissociated process that considers both imprecision, reliability and importance of sources

The ER-MCDA process consists in four dissociated steps: problem analysis, imperfect evaluation of criteria, mapping of imperfect evaluation into basic belief assignments and finally fusion of criteria to produce the decision (Figure 7).

The first step of the ER-MCDA process consists in describing the decision-making problem including identifying qualitative and quantitative decision criteria and assessing the dimensions of the event. The decision hypotheses (e.g., a site's sensitivity levels) are used to define the common frame of discernment that will be used for information fusion: low, medium, and high sensitivity (Figures 5 and 6).

Quantitative criteria are evaluated through possibility distributions representing both imprecision and uncertainty (step1, Figure 8). A mapping model defined as a set of fuzzy intervals *L-R* links a criterion evaluation and the decision classes: it plays more or less the same role than the utility function in a total aggregation based multi-criteria decision method. For each evaluation of a criterion by one source, each interval of the possibility distribution is mapped to the common frame of discernment of decision according to surface ratios. At the end of the mapping process, all the criteria evaluations provided by each source are transformed in basic belief assignments (bba's) according the common frame of discernment of decision (steps 2,3 and 4, figure 8) : these bba's are then fused in a two-step process (Figure 8). The first step consists in the fusion of bba's corresponding, for each criterion, to the different evaluations provided by different sources (step5, Figure 8). The second step consists in the fusion of the bba's corresponding to each criterion and resulting from the first step of fusion. In this second step, each criterion is considered as a source which is discounted according to its importance in the decision process with a specific discounting method (step 6, Figure 8) (see section DSmT-AHP and [5,12]).

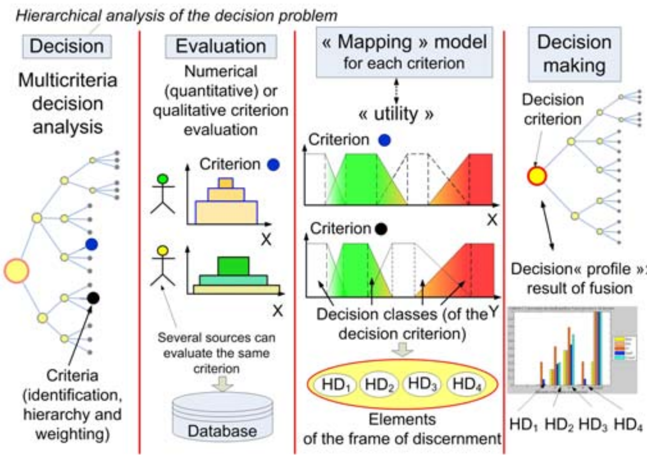


Figure 7: The ER-MCDA methodology is based on four dissociated steps from decision analysis to decision making based on fusion results [17].

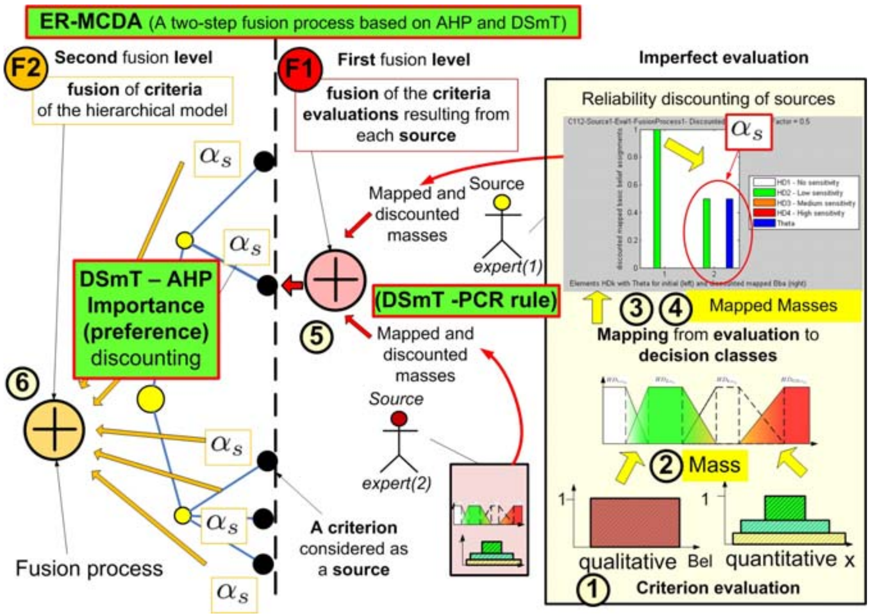


Figure 8: The classical AHP principle is replaced by a two-step fusion process able to consider both evaluation imperfection, reliability and importance of criteria and sources [15].

Main results and inputs of ER-MCDA methodology

The main inputs of this methodology are basic belief assignments elicitation, conflict identification and management, integration of different theoretical frameworks, choice and implementation of efficient fusion rule and new discounting techniques managing importance ,reliability and uncertainty in the fusion process.

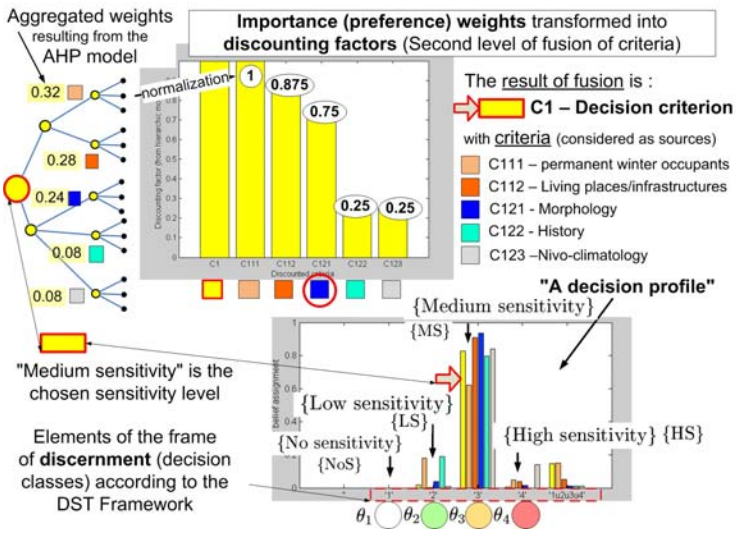


Figure 9: ER-MCDA produces a “decision profile” showing the best decision but also the confidence in the result due the quality of information, heterogeneity and reliability of sources.

The results of fusion are interpreted to decide which sensitivity level will be chosen (no sensitivity, NoS; low sensitivity, LS; medium sensitivity, MS; high sensitivity; HS) according either to the maximum basic belief assignments, credibility (pessimistic decision), plausibility (optimistic decision), or pignistic probability (compromise). In comparison with classical decision-aid methods, the ER-MCDA methodology therefore produces a comparative decision profile in which decision classes (elements of the frame of discernment) can be compared to each other (Figure 9). The quality of information leading to the decision is related to the decision itself.

3 DSmT-AHP : NEW FUSION RULES AND DISCOUNTING TECHNIQUES

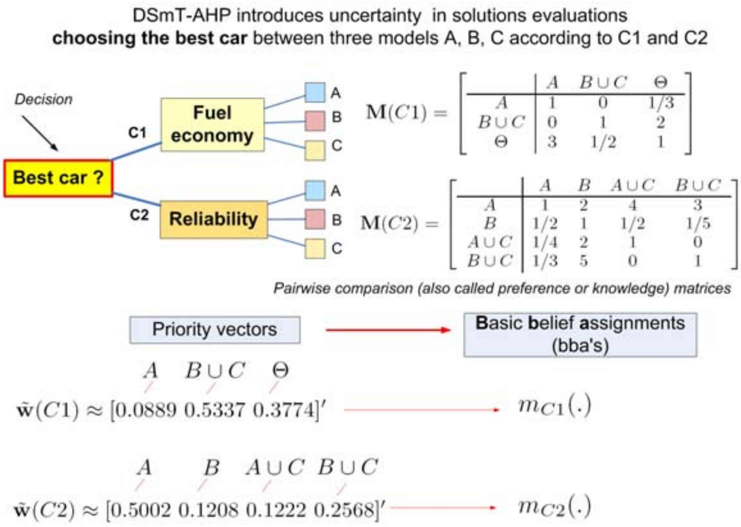
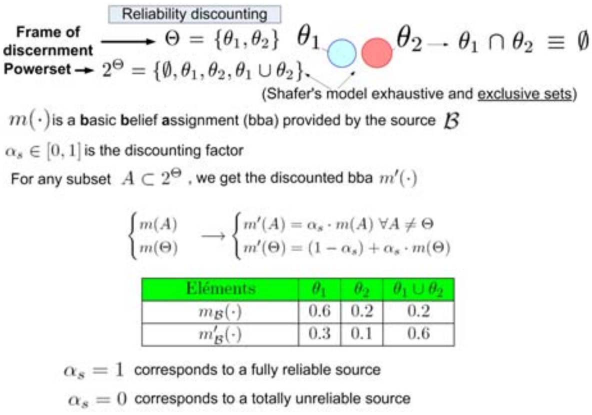


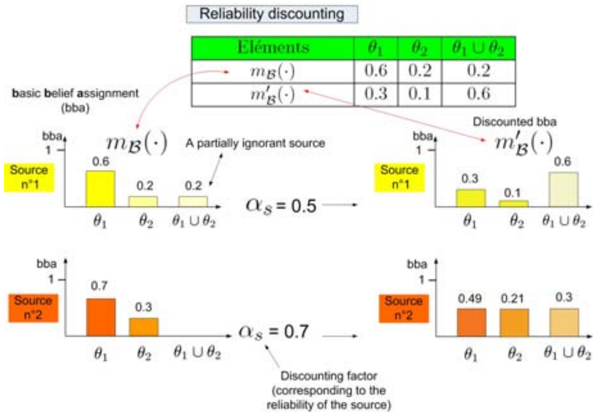
Figure 10: Comparison of sets of alternatives in DSmT-AHP context.

New fusion rules such as the PCR5[5] rule [4] are used to combine the different criteria. They improve and develop existing methods [1] based on the Dempster fusion rule which fails in cases where conflict between sources is high. In comparison with the original AHP method [9], DSmT-AHP introduces a new principle for alternatives comparison allowing to consider sets of alternatives (Figure 10). Weights are derived into bba's (basic belief assignments) and combined with the PCR5 fusion rule instead of the weighted sum based aggregation.

[5] Proportional Conflict Redistribution Rule



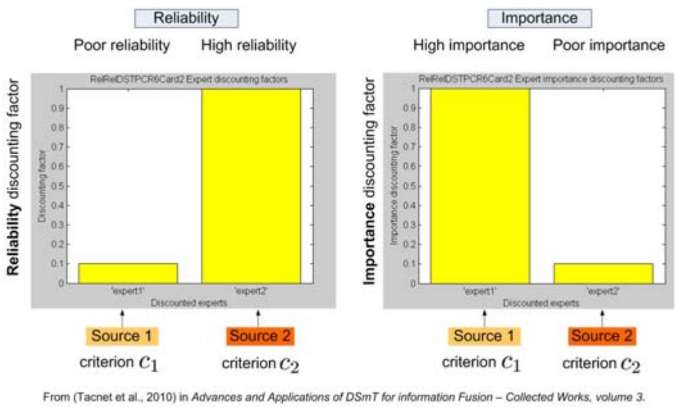
(a): Classical reliability discounting technique.



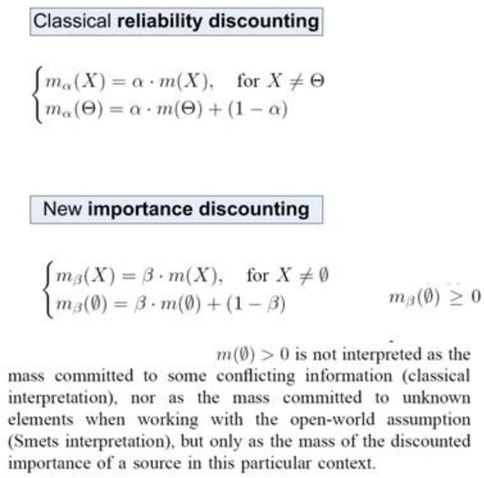
(b): Example of classical reliability discounting.

Figure 11: Principles of reliability discounting

In the classical discounting method [13], the mass is transferred from elements to the total ignorance when the source is not totally reliable (Figure 10a). After reliability discounting, the initial mass on the elements of the frame of discernment has been reduced (or discounted)(Figure 10b). An other main input of this new methodology is a new discounting technique making a difference between importance and reliability (Figures 12 a and 12b).



(a): Discounting techniques must make a difference between important and/or reliable sources (and or criteria).



(b): A new discounting technique is proposed to be used with PCR rule (DSmT framework).

Figure 12: New discounting method for importance

4 COWA-ER[6]: MULTICRITERIA DECISION MAKING UNDER UNCERTAINTY

[6] Cautious Ordered Weighted Averaging-Evidential Reasoning

When decision is done under uncertainty: choosing alternatives can have different consequences depending on the external context (or states of the world) (Figure 13).

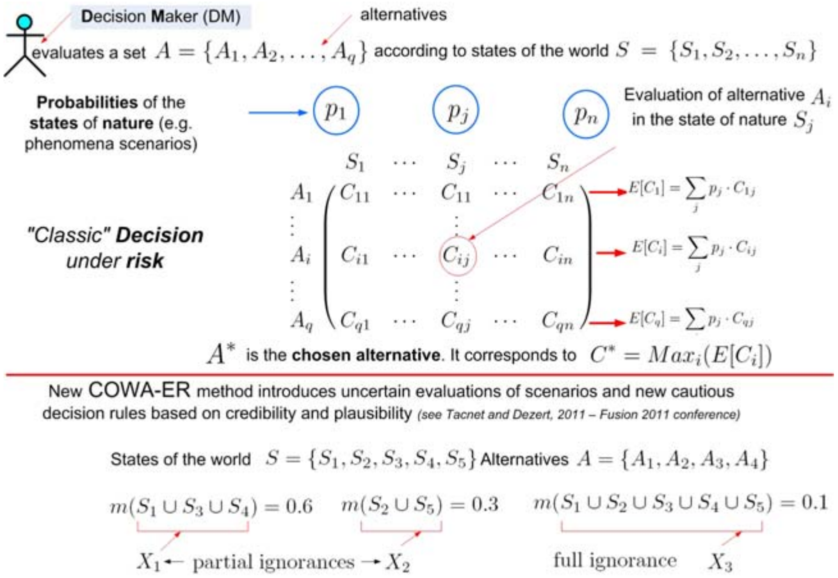


Figure 13: Context of decision under uncertainty and basic belief assignments.

A new methodology called COWA-ER (Cautious Ordered Weighted Averaging with Evidential Reasoning) [19] has been proposed for decision making under uncertainty to take into account imperfect evaluations of the alternatives and unknown beliefs about groups of the possible states of the world (scenarios) (Figure 13).

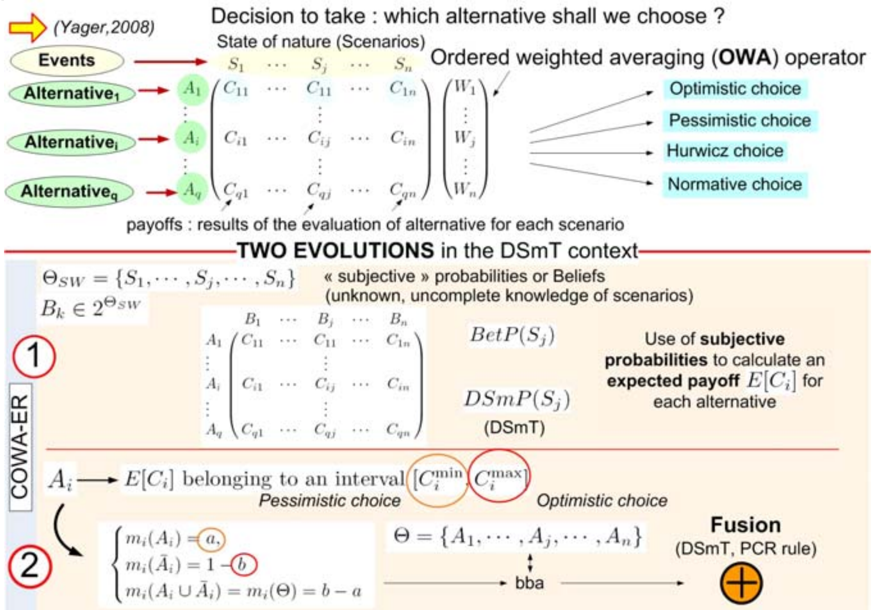


Figure 14: COWA-ER proposes two evolutions of Yager's OWA method [19].

COWA-ER mixes cautiously the principle of Yager's Ordered Weighted Averaging (OWA) [20,21] approach with the efficient fusion of belief functions proposed in Dezert-Smarandache Theory (DSmT) [11]. The original OWA approach considers several alternatives A_i evaluated in the context of different uncertain scenarii S_i and includes several ways (pessimistic, optimistic, hurwicz, normative) to interpret and aggregate the evaluations with respect to a given scenario. COWA-ER uses simultaneously the two extreme pessimistic and optimistic decision attitudes combined with an efficient fusion rule (PCR5) as shown on Figure 14.

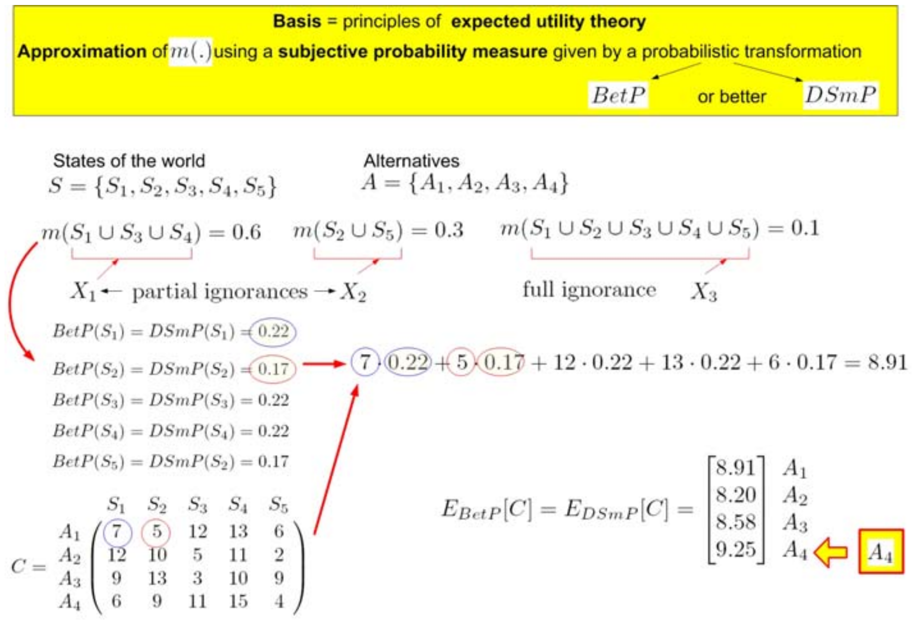


Figure 15: "Expected utility like method" based on subjective probability calculated from basic belief assignments.

In order to save computational resources (if required), we also have proposed a less efficient OWA approach using the classical concept of expected utility based on DSmP or BetP (Pignistic Probability). BetP and DSmP are methods to transform (approximate) any general bba (basic bleif assignment) into a subjective probability measure (called also a Bayesian bba) (Figure 15)[11].

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5 SYNTHESIS-CONCLUSION

Uncertainty is an important issue in decision making methods. A new framework, composed of ER-MCDA, DSmT-AHP and COWA-ER, is proposed on the basis of both new uncertainty theories such as belief function theory and multicriteria decision analysis methods. It considers both information imprecision, uncertainty and unconsistency and also reliability of heterogeneous sources. It uses recent and efficient fusion rules and discounting techniques. DSmT-AHP introduces imprecise evaluations of subsets and new discounting techniques. COWA-ER proposes a decision method based on expected DSmP and a framework for multicriteria decision in an uncertain context (Figure 16). ER-MCDA is an integrated approach able to consider multiple heterogeneous sources providing imperfect evaluations of criteria.

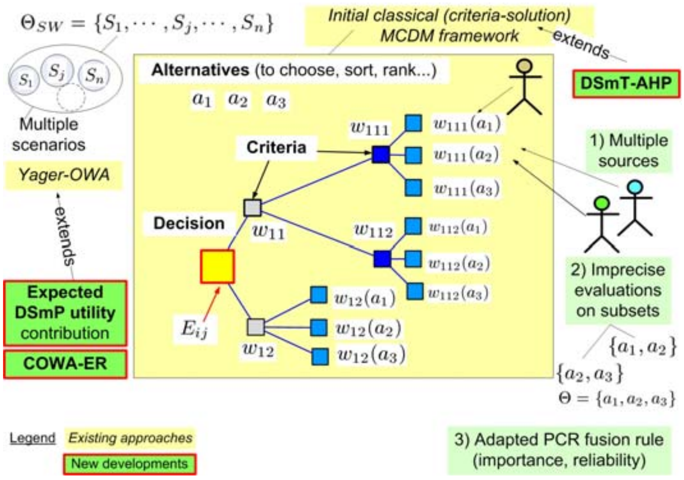

Figure 16: Main inputs of the DSmT and COWA-ER methodologies.

These methods are able to cope with the different contexts of decision under certainty, risk or uncertainty. They are parts of a framework dedicated to decision based on imperfect information (Figure 17). Developments and extensions of evidential reasoning to outranking methods (Electre TRI) are now under progress. From a practical point of view, those methods are tested and implemented on simple examples (see references below) but also on real application cases related to natural hazards management: they correspond exactly to the context of decision making related to expert assessment in this domain.

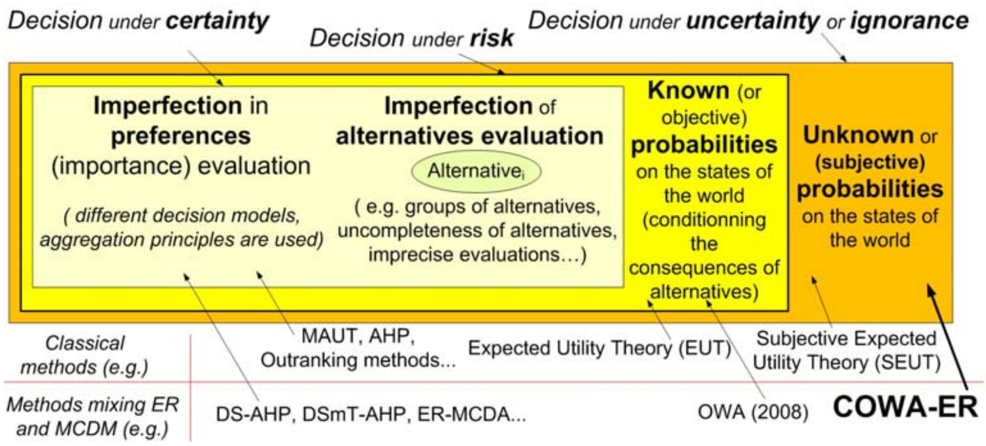

Figure 17: Synthesis of the new methods for multicriteria decision making in context of decisions under information imperfection (including uncertainty) [19].

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[7] http://www.paramount-project.eu

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