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Structure of states in ^{208}Pb with major components of the neutron particle-hole configuration $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$

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Abstract. Using the $^{208}\text{Pb}(p,p')$ reaction via isobaric analog resonances in ^{209}Bi , a dozen states in the doubly magic nucleus ^{208}Pb with major strength of the neutron particle-hole configuration $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$ are investigated. Among them, two doublets at $E_x = 5.81, 6.01$ MeV are resolved by comparing the mean cross sections on the $d_{5/2}$ resonance. The 5.81 MeV doublet consists of two states with major configuration $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$ and spins 2^- and 3^- ; the 2^- state is newly identified. The distance between the two states is 0.4 ± 0.2 keV. The 6.01 MeV doublet consists of a 4^- state with dominant configuration $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$ and a 3^- state with a small admixture of $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$. The distance between the two states is 1.8 ± 0.3 keV. The 5886 state has been newly assigned the spin of 4^- . The spins of seven more states are verified. Almost the complete $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$ strength is identified in these twelve states. The center of gravity for the configuration $\nu(d_{5/2}^{+1}p_{1/2}^{-1})$ and $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$ is in line with the predictions by the shell model without residual interaction to within 10 keV.

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1. Introduction

States in the doubly magic nucleus ^{208}Pb are of several different types, collective states, pairing and octupole vibrations, as well as one-particle one-hole and two-particle two-hole states. Most states at $E_x < 6.5$ MeV are described as one-particle one-hole states. Studying them deserves special interest.

Seven low-lying states in ^{209}Pb are known to be rather pure single-particle states ($g_{9/2}$, $i_{11/2}$, $j_{15/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$). By coupling a neutron hole ($p_{1/2}$, $p_{3/2}$, $f_{5/2}$, $f_{7/2}$, $h_{9/2}$, $i_{13/2}$) with these states, particle-hole states in ^{208}Pb are created. The neutron pickup reaction on $^{209}\text{Pb}^*$ is capable of exciting these particle-hole states. An equivalent reaction is the inelastic proton scattering via isobaric analog resonances (IAR). One isobaric analog state in ^{209}Bi corresponds to each particle state in ^{209}Pb . The analog state can be created by adjusting the proton energy in the $^{208}\text{Pb}(p, p')$ reaction. The opportunity to determine from the angular distributions spectroscopic amplitudes of neutron holes coupled with the neutron particle is an advantage of the (p, p') reaction via IAR [1].

In this paper, in particular the configuration $\nu(d_{5/2}^{+1}p_{3/2}^{-1})$ located at $E_x \approx 5.9$ MeV in ^{208}Pb is discussed. Only a few of the other 42 neutron particle-hole configurations admix to the $d_{5/2}p_{3/2}$ states (namely $g_{9/2}f_{7/2}$, $g_{7/2}p_{1/2}$, $d_{3/2}p_{1/2}$, and $s_{1/2}f_{5/2}$), and admixtures of proton particle-hole configurations are negligible.

The shell model without residual interaction (SSM) describes the structure of many states in ^{208}Pb by particle-hole configurations. Ref. [2], henceforth called Paper S, describes the SSM in detail. The SSM predicts four states with the configuration $d_{5/2}p_{3/2}$. However, there are ten more states with the same spins (1^- , 2^- , 3^- , 4^-) predicted to have similar excitation energies. Hence, at least fourteen states are expected to share the major $d_{5/2}p_{3/2}$ fractions in the region at $E_x \approx 5.9$ MeV in ^{208}Pb .

In 1970, by essentially using only data from the $^{208}\text{Pb}(p, p')$ reaction via the lowest IAR in ^{209}Bi , the structure of twenty negative parity states at $E_x < 4.7$ MeV in ^{208}Pb was determined in a rather complete manner. The residual interaction among particle-hole configurations was deduced with the result that the mean value of an off-diagonal matrix element for unnatural parity is about 50 keV while for natural parity it is about 150 keV [3].

However, it is crucial to correctly identify the states, as well as to obtain firm spin and parity assignments. For the states at $E_x < 4.5$ MeV in ^{208}Pb this was realized in 1982 [4] and is now confirmed [5]. The complete identification, and spin and parity assignment for states at $E_x < 4.8$ MeV was carried out thirty years later [6], when the Q3D magnetic spectrograph at the Maier-Leibnitz-Laboratorium (Munich) [7, 8, 9] had matured to yield high linearity [10] and high particle resolution (Paper S).

More than a hundred states are observed in the range of $4.8 < E_x < 6.1$ MeV [5]. Therefore, many doublets with distances between two states of less than 4 keV are known [2, 5, 6, 11, 12, 13, 14, 15]. The main data discussed in this paper (twelve states at $5.75 < E_x < 6.05$ MeV) has been presented in Refs. [14, 15], henceforth called Paper P; Table 2 therein is called Table T.

The theoretical description of the (p, p') reaction via IAR is briefly described in Appendix A where further references are given. In this paper we essentially rely on relative mean cross sections derived from Table T. Determining spectroscopic amplitudes from cross sections depends strongly on the energy of the scattered protons and their angular momenta (the so-called penetrability). In addition, special

assumptions are made. Therefore, required formulas are given in Sec. 3.2.3.

An important experimental tool is the computer code *GASPAN* [16], see also Paper S. It allows spectra of the (p, p') and (d, p) reactions to be analyzed, using many different methods. Eventually, excitation energies for a limited energy region can be determined with a precision of 0.1 keV.

In this paper, two doublets at $E_x = 5.81$ and 6.01 MeV with distances of 0.4 keV and 1.8 keV are disentangled (Secs. 3.3, 4.4, and 4.6). The distribution of almost the complete $d_{5/2}p_{3/2}$ strength across twelve states has been identified (Sec. 4). The 5812 2^- state has been newly identified, spin of 4^- is newly assigned to the 5886 state. The identification of the 6012 state [5] and its tentative assignment of spin 4^- by Kulleck et al [17] is confirmed.

2. Model configurations

In ^{208}Pb at excitation energies $E_x < 6.4$ MeV, a dozen doublets with distances of less than 2 keV are known. Since the determination of excitation energies develops over time, each state identified is given a unique four-digit energy label $\tilde{E}_x \approx E_x$ (Paper S).

Most states in ^{208}Pb with spin and parity I^π and excitation energy $\tilde{E}_x$ are described by particle-hole configurations in the SSM,

$$|\tilde{E}_x I^\pi\rangle = \sum_{LJ, lj} c_{LJlj}^{\tilde{E}_x, I^\pi} |LJlj\rangle, \quad \text{with real amplitudes } c, \quad -1 < c_{LJlj}^{\tilde{E}_x, I^\pi} < +1. \quad (1)$$

The SSM is explained in detail in Paper S. The particles LJ and the holes lj move in orbitals with orbital momenta L, l and spins J, j , respectively; both the neutron particle-hole configurations $\nu(LJ^{+1}lj^{-1})$ and the proton configurations $\pi(LJ^{+1}lj^{-1})$ are abbreviated to $LJlj$. For brevity, the magnetic quantum numbers of the state and configuration are omitted.

In this paper, we discuss states containing major fractions of the configuration $d_{5/2}p_{3/2}$. They are predicted by the SSM at the excitation energy $E_x^{SSM} = 5.896$ MeV (see Table 1 in Paper P). The $d_{5/2}p_{3/2}$ states with spins 2^- and 4^- are not yet identified and some spin assignments listed in the Nuclear Data Sheets [5] (henceforth called NDS2007) are doubted. In this work, the fragmentation of the $d_{5/2}p_{3/2}$ strength over twelve states with spins 1^- to 4^- is determined. In these states, components of additional particle-hole configurations are observed, but not discussed in detail.

3. Experiments

We discuss measurements performed with the Q3D spectrograph at the 14 MV tandem accelerator in Munich for the $^{208}\text{Pb}(p, p')$ reaction on all known IAR and especially on top of the $d_{5/2}$ IAR; data for the $^{207}\text{Pb}(d, p)$ reaction is also considered (Paper P).

At excitation energies $4.8 < E_x < 6.1$ MeV in ^{208}Pb , the mean distance between states is about 10 keV. The resolution of about 3 keV is hardly sufficient to resolve all states. In general, it is difficult to identify states in a spectrum.

- The term “peak” is used to define a visible hump in the raw spectrum of some reaction such as $^{208}\text{Pb}(p, p')$.
- The term “level” defines the result of the peak fitting using computer codes such as *GASPAN* [16]. The result of a fit may depend on details such as the handling of secondary peaks produced by contamination reactions such as $^{14}\text{N}(p, p')$. The electron knockout reaction, $^{208}\text{Pb}(p, p' + z e^-) ^{208}\text{Pb}^{z+}$, $z = 0, \dots, 82$, enlarges each peak and

produces secondary peaks [18].

- The term “state” defines a physical state in a nucleus described by some model configurations such as Eq. (1). The correspondence between a level and a single state or an ensemble of states is not straightforward.
- Therefore the term “doublet” defines an unresolved or incompletely resolved peak in the spectrum of the reaction considered, produced by a pair of states (similarly “triplet” etc). Finally, a previously assumed doublet may be recognized as a single state.
- The term “multiplet” defines a group of states with a certain configuration $LJlj$ and spins $|J - j| \leq I \leq J + j$.

3.1. Data evaluation

For some $\tilde{E}_x I^\pi$ state, the measured angular distribution for the $^{208}\text{Pb}(p, p')$ reaction via LJ IARs in ^{209}Bi is fitted by a series of even order Legendre polynomials P_K ,

$$\frac{d\sigma}{d\Omega}(\tilde{E}_x, I^\pi, \Theta, E_p, LJ) = \sum_{K=0,2,\dots}^{K_{max}} A_K(\tilde{E}_x, I^\pi, E_p, LJ) P_K(\Theta). \quad (2)$$

For natural parity states, a considerable direct (p, p') component often overwhelms the resonant part for $\Theta \lesssim 100^\circ$. Hence, if a direct (p, p') component is present, Eq. (2) should be used for scattering angles $\Theta \gtrsim 100^\circ$ only.

More precise angular distributions (often with $K_{max} = 6$) are obtained through experiments with semiconductor detectors at scattering angles up to $\Theta = 173^\circ$ [17, 19]; however, not all levels are observed and the resolution was only 18 and 15 keV, respectively.

The isotropic component A_0 defines the mean cross section of the state $\tilde{E}_x I^\pi$ on the LJ IAR. The dependence on the beam energy E_p is eliminated by applying the inverse Lorentzian factor a^{Lz} [Eq. (A.10)]. Hence, the angle-integrated (mean) cross section on top of the LJ IAR is determined by

$$A_0^{mean}(\tilde{E}_x, I^\pi, LJ) = \frac{1}{N} \sum_{i=1}^N A_0(\tilde{E}_x, I^\pi, E_p^i, LJ) / a^{Lz}(E_p^i, LJ). \quad (3)$$

The values shown in Tables 1 and 2 are determined from data taken on the $d_{5/2}$ IAR shown in Table T. Here the factor a^{Lz} varies by 0.6 – 1 for proton energies $16.4 < E_p < 16.6$ MeV. The anisotropy coefficient A_K/A_0 [Eq. (2)] is given for $K = 2$.

3.2. Determination of s.p. widths, spins, configuration strengths, and excitation energies

3.2.1. Basic assumptions. The basic assumption of an overwhelming resonant reaction is well founded for states with unnatural parity at scattering angles $\Theta \gtrsim 40^\circ$ and for states with natural parity at scattering angles $\Theta \gtrsim 100^\circ$ [2, 6, 12, 13, 14, 17, 22, 23].

The basic assumption of an isolated IAR is taken for granted for the states under discussion. The distance between the $d_{5/2}$ and the $s_{1/2}$ IARs is 465 keV while the widths of the IARs is about 300 keV [22]. The mean cross section of a state with a $d_{5/2}lj$ component is calculated to decrease by a factor of about 5 and 10, respectively, in relation to the proton energy corresponding to the $s_{1/2}$ IAR and the $g_{7/2}+d_{3/2}$ doublet IAR. For the 5.81 and 6.01 MeV doublets, the available excitation functions (Fig. 7 of Ref. [22]) agree with the calculations. An excitation on the $d_{3/2}$ IAR is

Table 1: For states in ^{208}Pb with excitation energies $5.7 < E_x < 6.4$ MeV, strongly excited by the $^{208}\text{Pb}(p, p')$ reaction via the $d_{5/2}$ IAR with $A_0^{mean} > 10 \mu\text{b/sr}$, the energy label, the spin I^π , the cross sections A_0^{mean} [Eq. (3)], A_0^{corr} , the fraction c^2 of the $d_{5/2}p_{3/2}$ strength, the calculated penetrability, and the anisotropy A_K/A_0 are shown. The state 5812 has been newly identified, the spins for the 5886 and 6012 states are new (printed in bold face).

$\tilde{E}_x$	I^π		Ref.	$LJlj = d_{5/2}p_{3/2}$			on $d_{5/2}$ IAR	
				A_0^{corr}	c_{LJlj}^2	a^{para}	A_0^{mean}	A_2/A_0
	[5]	this work		Eq.(4) $\mu\text{b/sr}$	Eq.(5) $\times 100^a$	Eq.(A.5)	[14] $\mu\text{b/sr}$	Eq.(2)
5778	$2^-, 3^-$	2^-	[13]	68	33 ± 2	0.953	75	$+0.2 \pm 0.1$
5812		2^-	b	c	$54 \pm 7 \pm 6^d$	0.973	c	c
5813	3^-	3^-	[5]	256^c	$50 \pm 5 \mp 5^d$	0.973	270^c	-0.7 ± 0.2
5874	3^-	3^-	[5]	96	33 ± 5	1.008	95	-0.2 ± 0.3
5886	3^-	4^-	e	108	29 ± 5	1.015	105	$+0.3 \pm 0.2$
5924	2^-	2^-	[5]	16	7 ± 3	1.020	15	-0.5 ± 0.3
5947	1^-	1^-	[5]	21	17 ± 4	1.027	20	$+0.7 \pm 0.3$
5969	4^-	4^-	[5]	26	7 ± 2	1.034	25	$+0.3 \pm 0.2$
6010	3^-	3^-	[5]	48	16 ± 8	1.095	40	-0.0 ± 0.5
6012		4^-	f	252	68 ± 16	1.096	210	$+0.3 \pm 0.2$
6087	1^-	2^-	[13]	< 13	< 6	1.148	$< 10^g$	
6191	3^-	3^-	[5]	< 8	< 3	1.228	$< 5^g$	
6264	1^-	1^-	[5]	33	26 ± 13	1.286	20	$+0.9 \pm 0.3$
6314	1^-	1^-	[5]	63	51 ± 25	1.773	20	-0.0 ± 0.3

(a) Instead of the uncertainty of the cross sections A_0^{corr} , A_0^{mean} , the corresponding uncertainty of the strength c^2 is shown.

(b) Secs. 4.6.1, 4.6.3.

(c) For the unresolved 5812 + 5813 doublet.

(d) See assumption H in Table 5 discussed in Sec. 4.6.1.

(e) Sec. 4.3.

(f) Sec. 4.4.

(g) Included for completeness (Sec. 4.1.1).

present (see also Table T); however, no significant enhancement on the $s_{1/2}$ IAR is observed.

Spectra for $5.4 < E_x < 6.2$ MeV on all IARs are shown in Fig. 1 of Paper P. The 5.81 MeV level is excited on the $g_{9/2}$, $d_{5/2}$ and $g_{7/2} + d_{3/2}$ IARs. Spectra taken on the $g_{9/2}$ and $d_{5/2}$ IARs for $5.75 < E_x < 5.90$ MeV are shown in Fig. 4 of Ref. [12]. The 5778 2^- level is strongly excited on the $d_{5/2}$ IAR only (Table T).

3.2.2. Mean cross sections. Should configurations $LJlj$ with different holes lj contribute to the state, then the strength of each configuration can only be determined with difficulty. The determination of the spin of the states, the configuration strengths in the states and the s.p. widths is highly correlated (Appendix A). However, for states with energies $E_x \approx 5.0$ and $E_x \approx 5.9$ MeV strongly excited on the $d_{5/2}$ IAR, the configurations $d_{5/2}p_{1/2}$ and $d_{5/2}p_{3/2}$ are assumed to contribute in each case mainly, in

Table 2: Data similar to Table 1 for excitation energies $4.2 < E_x < 5.3$ MeV. In addition, the strength $c_{d_{5/2}p_{1/2}}^2$ determined from the $^{207}\text{Pb}(d, p)$ reaction [with an estimated uncertainty of 10%, Eq. (10)] is shown. The non-vanishing value of A_2/A_0 indicates admixtures of $d_{5/2}f_{5/2}$ and $d_{5/2}p_{3/2}$.

$\tilde{E}_x$	I^π	Ref.	$LJlj = d_{5/2}p_{1/2}$					on $d_{5/2}$ IAR	
			$^{208}\text{Pb}(p, p')$			$^{207}\text{Pb}(d, p)$		A_2/A_0	Ref.
			A_0^{corr}	c_{LJlj}^2	a^{ptra}	$(c_{LJ}^{d,p})^2$	G^λ		
			Eq.(4) $\mu\text{b/sr}$	Eq.(5) $\times 100$	Eq.(A.5) $\times 1000$	Eq.(10) $\times 100$	[20, 21]		
4230	2^-	[5]	16	5 ± 2	0.724	123	5	+0.35	[19]
4698	3^-	[5]	117	25 ± 3	0.883	965	27	-0.9 ± 0.2	[17]
4974	3^-	[5]	207	44 ± 5	1.006	1683	48	-0.7 ± 0.2	[17]
5038	2^-	[13]	199	59 ± 6	1.038	1470	59	-0.5 ± 0.1	[17]
5127	2^-	[13]	129	38 ± 4	1.085	836	33	-0.6 ± 0.1	[17]
5245	3^-	[5]	140	30 ± 3	1.153	858	34	-0.50	[19]

comparison to the SSM (Table 1 in Paper P). We define a cross section corrected for the change in the penetrability of the dominant configuration $LJlj$ [Eq. (A.5)],

$$A_0^{corr}(\tilde{E}_x, I^\pi, LJlj) = \frac{1}{N} \sum_{i=1}^N \frac{A_0(\tilde{E}_x, I^\pi, E_p^i, LJ)}{\left[a_{LJlj}^{ptra} (E_p^i, \tilde{E}_x) \right]^2 a^{Lz}(E_p^i, LJ)}. \quad (4)$$

Tables 1 and 2 show penetrability-corrected cross sections for $E_x \approx 5.9$ and 5.0 MeV, respectively. They correspond to Table T where data for states in the wider range $4.6 < E_x < 6.4$ MeV is listed more completely: The excitation energies, the mean cross section A_0^{mean} [Eq. (3)] on *all* IARs, as well as spectroscopic factors for the $^{207}\text{Pb}(d, p)$ reaction [20] and the $^{209}\text{Bi}(d, ^3\text{He})$ reaction [24] are given.

The s.p. widths $\Gamma_{LJ}^{s.p.}$ for particles LJ and the s.p. widths $\Gamma_{lj}^{s.p.}$ for holes lj are not well known (see Table 2 in Paper S). The agreement of some products $\Gamma_{LJ}^{s.p.} \Gamma_{lj}^{s.p.}$ with earlier results is discussed in Sec. 4.1.2. However, in this paper we discuss only ratios of the cross section A_0^{corr} together with further simplifying assumptions (Secs. 4.4, 4.5, 4.6, and 4.7).

3.2.3. States with one major configuration. A weak admixture of a secondary configuration $LJl'j'$ does not have much effect upon the mean cross section A_0^{alc} [Eq. (A.13)] provided that $\Gamma_{l'j'}^{s.p.} \ll \Gamma_{lj}^{s.p.}$.

In this paper we discuss states with major $d_{5/2}p_{3/2}$ components. Here, weak admixtures of the configuration $d_{5/2}f_{5/2}$ may be neglected since $\Gamma_{f_{5/2}}^{s.p.} = 0.25 \Gamma_{p_{3/2}}^{s.p.}$ [6, 22, 23]; admixtures of $d_{5/2}p_{1/2}$ with $\Gamma_{p_{1/2}}^{s.p.} = 1.8 \Gamma_{p_{3/2}}^{s.p.}$ are small [14, 20, 21], see Table T. (For states with major $d_{5/2}p_{1/2}$ components, weak admixtures of the configurations $d_{5/2}f_{5/2}$, $d_{5/2}p_{3/2}$ may be similarly neglected.) The discussion in Sec. 4 is based on two assumptions.

(i) By assuming a **single** neutron particle-hole configuration $LJl\bar{j}$ to contribute to the cross section of a certain state $\tilde{E}_x I^\pi$, the calculated mean cross section [Eq. (A.13)] is described by a single term. Hence the penetrability-corrected cross section

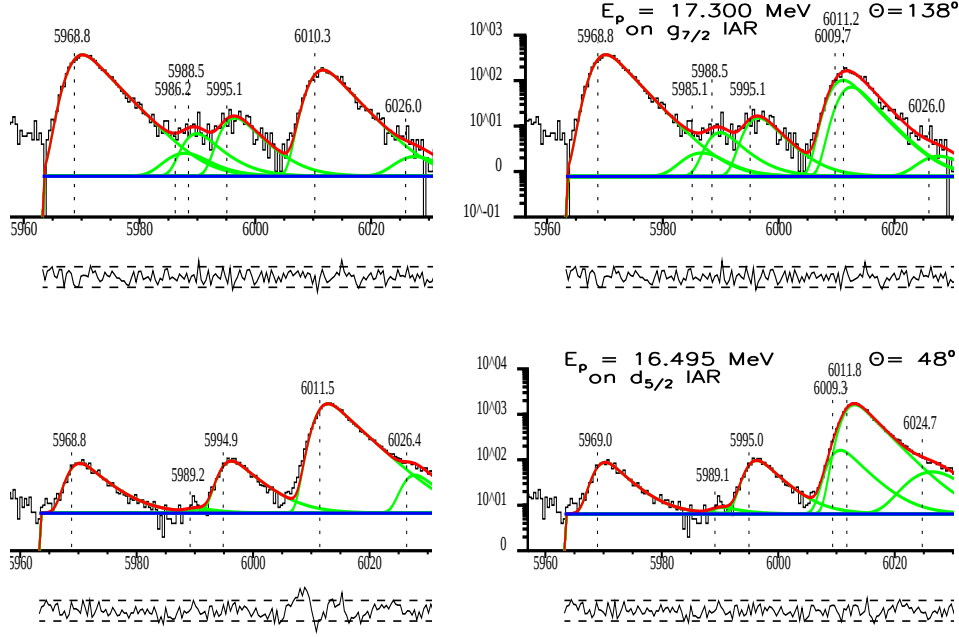


Figure 1: (online color) Spectra of the $^{208}\text{Pb}(p, p')$ reaction for $5960 < E_x < 6030$ keV taken on the $d_{5/2}$ IAR (bottom) and on the $g_{7/2}$ IAR (top) and fitted by *GASPAN* (online colors: blue for the fit of the smooth background, green for each peak, red for the whole spectrum). By fitting the 6.01 MeV peak with a single level (left), the residuum spectra exceed the $\pm 2\sigma$ limits (dashed lines) and the excitation energies determined from the $^{208}\text{Pb}(p, p')$ spectra on the two IAR are different in contrast to a fit of the 6.01 MeV peak with two levels (right). The peaks at $E_x \approx 5.986$ and 6.024 MeV are satellites from neighbouring peaks because L -electrons carry away about 14 keV from the proton energy [18] (see Sec. 3).

[Eq. (4)] may be used to determine the $d_{5/2}p_{3/2}$ strength in states at $E_x \approx 5.9$ MeV and their distribution across the four different spins 1^- to 4^- .

(ii) In the discussion (Sec. 4), another important assumption is the **completeness of the total strength** for the configuration $LJlj$ in a certain group of states with spins $|J - j| \leq I \leq J + j$. Then, by introducing relative cross sections

$$\sigma^{rel}(\tilde{E}_x, I^\pi, LJlj) = \frac{A_0^{corr}(\tilde{E}_x, I^\pi, LJlj)}{\sum_{\tilde{E}_x} \sum_I A_0^{corr}(\tilde{E}_x, I^\pi, LJlj)}, \quad (5)$$

the complete $LJlj$ strength for all states $\tilde{E}_x I^\pi$ with spin I yields

$$\sum_{\tilde{E}_x} \sigma^{rel}(\tilde{E}_x, I^\pi, LJlj) = (2I + 1), \quad (6)$$

meaning that the relative cross section σ^{rel} [Eq. 5] is proportional to the configuration strength $|c_{LJlj}|^2$ [Eq. (1)]. The total sum of all relative strengths yields

$$\sum_I \sum_{\tilde{E}_x} \sigma^{rel}(\tilde{E}_x, I^\pi, LJlj) = (2J + 1)(2j + 1), \quad (7)$$

since

$$\sum_{I=|J-j|}^{J+j} (2I+1) = (2J+1)(2j+1). \quad (8)$$

In the selected group of states at $E_x \approx 5.9\text{ MeV}$, the spin I of each state can be determined by observing the strength of each configuration $LJlj$, and especially $d_{5/2}p_{3/2}$, to be in the range $0 < c^2 < 1$ within the experimental uncertainties. Two states distinguished by different energy labels $\tilde{E}_x$ may build an unresolved doublet with the same excitation energy E_x . In such a doublet, the sum of the strengths c^2 weighted by the spin factors $2I+1$ can only be determined.

3.3. Determination of excitation energies

The line shape for $^{208}\text{Pb}(p, p')$ is asymmetric. The instrumental resolution is 3 keV, the energy loss in the target produces a tail to each peak, see Paper S. In addition, satellite peaks from the $^{208}\text{Pb}(p, p' + z e^-)^{208}\text{Pb}^{z+}$ reaction appear [18]. Some satellite peaks are comparable to peaks from physical states. Hence, determining excitation energies depends on several circumstances of the fit.

3.3.1. Determining energies using GASPAN. Excitation energies are determined by using GASPAN [16] and additional computer programs described in Paper S. The calibration of the excitation energies has been verified as correct to within better than 1 keV for $3.1 < E_x < 8.0\text{ MeV}$ by comparison with NDS2007.

Fig. 1 shows $^{208}\text{Pb}(p, p')$ spectra of only 70 keV length taken on the $d_{5/2}$ and $g_{7/2}$ IARs for $3.1 < E_x < 8.0\text{ MeV}$ on a logarithmic scale. The peak-to-valley ratio is 500 : 1. Besides the strong peaks at $E_x = 5.97$ and 6.01 MeV, several minor peaks are fitted. Some peaks are satellites from the electron knockout reaction [18].

On the $d_{5/2}$ IAR, by assuming a single level for the 6.01 MeV peak, the residuum spectra deviates by up to $\pm 4\sigma$ in a systematic manner (bottom left frame). Also, the excitation energies differ by 1.2 keV for the spectra taken on the $d_{5/2}$ and $g_{7/2}$ IARs (left frames). Clearly, the 6.01 MeV peak can be fitted only by assuming two levels having a distance of about 1.5 keV (right frames).

3.3.2. Mean excitation energy and variance. The excitation energy $E_x^{GASPAN} \pm \delta E_x^{GASPAN}$ is determined from the fit by GASPAN for several runs of typically 30 minutes beam time. Typical individual uncertainties are between 0.04 keV for levels with high statistics and 0.30 keV for levels with low cross sections and low peak-to-valley ratios (e.g. $2\text{ }\mu\text{b/sr}$ and 3 : 1). Fig. 2 shows the values E_x^{GASPAN} and their uncertainties for the 5778 2^- state and the $E_x = 5.81\text{ MeV}$ doublet.

A study for the 6.01 MeV doublet reveals similar shifts of the values E_x^{GASPAN} with scattering angle and proton energy but with a spread of excitation energies within a range of about 2 keV. The fit by GASPAN is more difficult than for the 5.81 MeV doublet because of the presence of satellite peaks from L -electrons and several other close-lying physical states (Fig. 1).

The mean value

$$\langle E_x^{GASPAN} \rangle = \sqrt{\frac{\sum_{i=1}^n [E_x^{GASPAN}(i)/\delta E_x^{GASPAN}(i)]^2}{\sum_{i=1}^n [1/\delta E_x^{GASPAN}(i)]^2}} \quad (9)$$

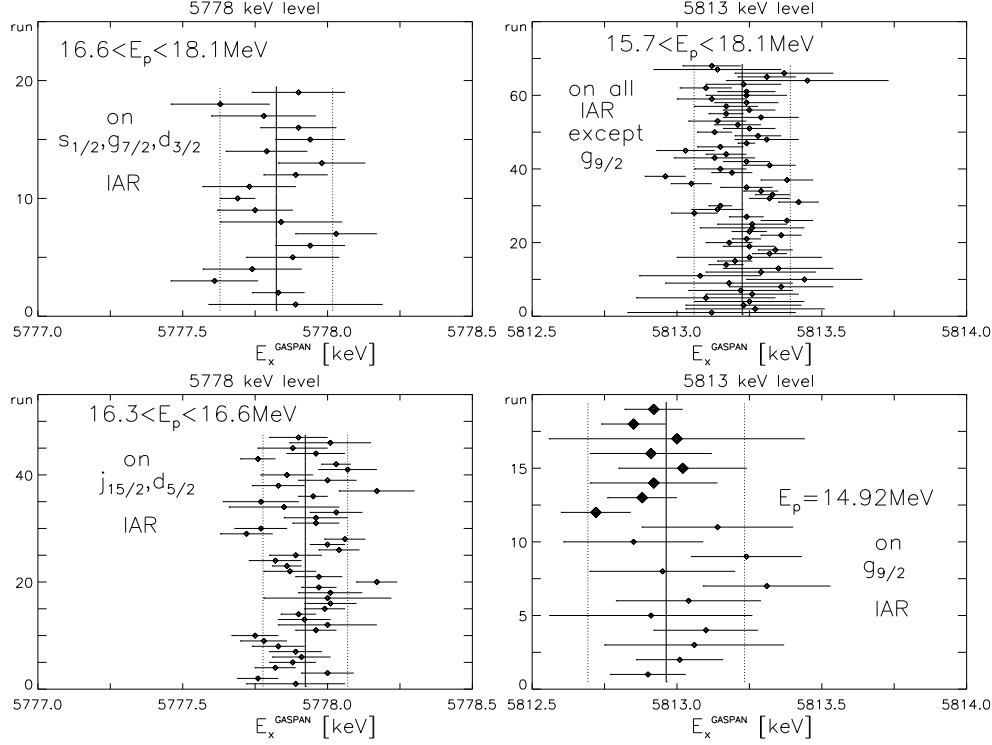


Figure 2: Excitation energies determined by using *GASPAN* (Sec. 3.3.1), (left) for the 5778 level, (right) for 5813 level. For each run, E_x^{GASPAN} is shown by a dot, the uncertainty δE_x^{GASPAN} by a horizontal bar. The runs are sorted according (i) to the scattering angle, (ii) to the proton energy. The vertical lines show the mean value $\langle E_x^{GASPAN} \rangle$ [Eq. (9)] and the uncertainty determined from the values E_x^{GASPAN} for all runs, (top left) 5777.86 ± 0.20 , (bottom left) 5777.92 ± 0.15 keV, (top right) 5813.23 ± 0.17 , (bottom right) 5812.96 ± 0.27 keV. On the $g_{9/2}$, the excitation energies for $80^\circ \leq |180^\circ - \Theta| \leq 90^\circ$ (large symbols) are lower than for $\Theta < 80^\circ$.

does not depend on the proton energy or scattering angle for the 5778 2^- state (Fig. 2, left frames). However for the $E_x = 5.81$ MeV doublet, the mean value on the $g_{9/2}$ IAR is different from that on the $d_{5/2}$ IAR (right frames). On the $g_{9/2}$ IAR, the values are less precise because of the much lower mean cross section ($15 \mu\text{b/sr}$ on the $g_{9/2}$ IAR vs. $270 \mu\text{b/sr}$ on the $d_{5/2}$ IAR, see Table T).

3.4. Data from the $^{207}\text{Pb}(d,p)$ reaction

The strength of the configuration $LJ p_{1/2}$ is derived from the spectroscopic factors G_{LJ}^λ determined by the $^{207}\text{Pb}(d,p)$ reaction [20, 21],

$$\left| c_{LJ}^{d,p}(\tilde{E}_x, I^\pi) \right|^2 = \frac{2}{2I+1} G_{LJ}^\lambda(\tilde{E}_x, I^\pi). \quad (10)$$

Table 2 shows values G_{LJ}^λ and $\left| c_{LJ}^{d,p}(\tilde{E}_x, I^\pi) \right|^2$, see also Table T.

4. Discussion

4.1. General remarks

NDS2007 discusses the 5.81 and 6.01 MeV doublets. The 5.81 MeV level is attributed to a single state[‡]. The 6.01 MeV doublet is recognized as possibly consisting of two states[§].

4.1.1. Assumption of completeness. The spin assignments are mainly based on the assumption that the configuration strengths for configurations in a certain subspace is complete. Then, as shown for the lowest negative parity states with $E_x < 4.7$ MeV [3], a sufficiently large gap in the model configuration space may yield unitarity relations for a group of states and configurations.

In this sense, for spins 1^- and 2^- , the gaps between the respective configurations $d_{3/2}p_{1/2}$ and $s_{1/2}p_{3/2}$ with $E_x^{SSM} = 6.033$ and 6.371 MeV, and for spins 3^- and 4^- , the gaps between the respective configurations $g_{7/2}p_{1/2}$ and $j_{15/2}i_{13/2}$ with $E_x^{SSM} = 5.922$ and 6.487 MeV are considered to be sufficiently large, see Table 1 in Paper P. For unnatural parity states the mean matrix element of the residual interaction is about 50 keV, for natural parity about 150 keV [3]. Hence at least for unnatural parity states, the unitarity relations may be assumed to hold for the states at $E_x \lesssim 6.3$ MeV.

At $5.7 < E_x < 6.3$ MeV (Table 1 of Paper P) a strong mixing of the $d_{5/2}p_{3/2}$ configuration with other configurations can be expected, since the SSM predicts three, four, four and three respective configurations for spins 1^- , 2^- , 3^- and 4^- within 300 keV.

Fig. 4 in Ref. [22] shows an excitation function of the 6087 2^- state. It is strongly excited on the $d_{3/2}$ IAR and weakly on the $s_{1/2}$ IAR. The cross section at $E_p = 16.45$ MeV is explained by the Lorentzian tails. A vanishing excitation on the $d_{5/2}$ IAR is assumed (Table 1). The 6191 3^- state [5] is weakly excited on all IARs (see Figs. 1 and 2 in Ref. [13]). The 6087 2^- and 6191 3^- states are not discussed any further below, but shown in Fig. 3.

For states containing only $d_{5/2}p_{1/2}$ components, the shape of the angular distribution on the $d_{5/2}$ IAR should be isotropic, $K_{max} = 0$. The non-vanishing anisotropy coefficients A_2/A_0 of the angular distribution indicate admixtures of $d_{5/2}f_{5/2}$ and $d_{5/2}p_{3/2}$ (Table 2). However, as is known [25], even small admixtures produce large, non-vanishing values A_K/A_0 , $K > 0$ [Eq. (A.11)] through the interference with the

[‡] NDS2007 adds the following footnote to the 5813.27 level observed by $^{208}\text{Pb}(n, n' \gamma)$. 2005YaZW report a transition with $E_\gamma = 2338.765 \pm 14$ with $I_\gamma = 1.30 \pm 6$ placed from the 4953 level. In (d,p γ), there is a 2338γ placed from the 5813 level. From $I_\gamma(2338\gamma)/I_\gamma(3199\gamma) = 0.85 \pm 14$, one expects $I_\gamma = 0.133 \pm 23$ for a component of the 2338γ placed from the 5813 level in (n,n' γ). This leaves $I_\gamma = 1.17 \pm 6$ for placement of the 2338γ from the 4953 level. E_γ for placement from the 5813 level is taken from the level energies.

[§] NDS2007 adds the following footnote to the 6009.77 level observed by $^{208}\text{Pb}(n, n' \gamma)$. 2005YaZW report levels at 6010, deexciting via $E_\gamma = 1924$ and 2535, and at 6011 deexciting via $E_\gamma = 1687$ and 3397. For the 6011 level, the two transitions give discrepant energies, the 1687 γ giving $E(\text{level}) = 6010.72 \pm 15$ and the 3397 γ giving 6011.64 ± 6 . For the 6010 level, the two transitions give consistent energies of 6009.78 ± 11 and 6009.78 ± 3 , respectively. The 1687 γ thus gives an $E(\text{level})$ lying 1 keV below that given by the 3398 γ and 1 keV above that given by the 1924 and 2535 γ 's. In (d,p γ), the 3397 γ is not reported and a 1686 γ is placed from the 6010 level. The branchings of the 1686, 1924, and 2535 γ 's in (d,p γ) and (n,n' γ) agree well. Thus, in spite of the energy discrepancy, the (d,p γ) scheme seems correct and is adopted by the evaluator. The 1687 γ is thus included with the 6010 level and removed from the 6011 level. E_γ is taken from the level energies.

major configuration. Hence, a dominant $d_{5/2}p_{1/2}$ strength may be still assumed in these states.

For states containing only $d_{5/2}p_{3/2}$ components, the shape of the angular distribution on the $d_{5/2}$ IAR is described by Legendre polynomials with $K_{max} = 2$, see Eq. (A.12). For a $d_{5/2}f_{5/2}$ admixture, $K_{max} = 4$ is needed to describe the shape [Eq. (A.11)]. Kulleck et al [17] found the anisotropy coefficients A_K/A_0 for $K = 4, 6$ to be small for all levels with $4.6 < E_x < 6.3$ MeV which do *not* contain a natural parity state. Hence, a dominant $d_{5/2}p_{3/2}$ strength may be still assumed in these states.

Table 3: Product of s.p. widths $\Gamma_{LJ}^{s.p.} \Gamma_{lj}^{s.p.}$.

LJ	lj	$\Gamma_{LJ}^{s.p.} \Gamma_{lj}^{s.p.}$			
		keV ²	Ref.	keV ²	Section
$d_{5/2}$	$p_{1/2}$	1290 ± 180	[6, 26]	1510 ± 150	4.1.2
$d_{5/2}$	$p_{3/2}$	700 ± 80	[6, 26]	920 ± 90	4.1.2

4.1.2. *Product of the s.p. widths $\Gamma_{d_{5/2} p_{1/2}}^{s.p.}$ and $\Gamma_{d_{5/2} p_{3/2}}^{s.p.}$.* The SSM predicts four states at $E_x = 5.896$ MeV with the $d_{5/2}p_{3/2}$ configuration and two states at $E_x = 4.998$ MeV with the $d_{5/2}p_{1/2}$ configuration. Fourteen states at $E_x \approx 5.9$ MeV may be assumed to contain most of the $d_{5/2}p_{3/2}$ strength (Table 1). Six states in the region $4.2 < E_x < 5.3$ MeV contain almost the complete $d_{5/2}p_{1/2}$ strength (Table 2); both the $^{208}\text{Pb}(p, p')$ reaction and the $^{207}\text{Pb}(d, p)$ reaction yield similar results for the distribution of the $d_{5/2}p_{1/2}$ strength.

With the assumptions (i) and (ii) declared in Sec. 3.2.3, the product of the decay widths can be determined from the sum of the penetrability-corrected cross sections A_0^{corr} [Eqs. (4, 8, and A.13)],

$$\Gamma_{LJ}^{s.p.} \Gamma_{lj}^{s.p.} = \frac{1}{(2j+1)N_{LJ}(E_{LJ}^{res})} \sum_{\tilde{E}_x} \sum_I A_0^{corr}(\tilde{E}_x, I^\pi, LJlj). \quad (11)$$

Table 3 shows the values for $LJ = d_{5/2}$ and $lj = p_{1/2}, p_{3/2}$. They differ from previous values by 10 – 30% [6, 26]. The exact value of $\Gamma_{d_{5/2} p_{3/2}}^{s.p.} \Gamma_{d_{5/2} p_{1/2}}^{s.p.}$ is not important for the following discussion (Secs. 4.2, 4.4, 4.5, 4.6, and 4.7).

4.1.3. *Center of gravity.* The center of gravity of states assumed to contain the complete $LJlj$ strength for a certain spin is calculated from the penetrability-corrected cross sections A_0^{corr} [Eq. (4)],

$$E_x^{CofG}(LJlj, I^\pi) = \frac{\sum_{i=1}^N \tilde{E}_x^i A_0^{corr}(\tilde{E}_x^i, I^\pi, LJlj)}{\sum_{i=1}^N A_0^{corr}(\tilde{E}_x^i, I^\pi, LJlj)}. \quad (12)$$

The center of gravity for *all* states assumed to contain the complete $LJlj$ strength is given by

$$\langle E_x^{CofG} \rangle(LJlj) = \frac{\sum_I (2I+1) E_x^{CofG}(LJlj, I^\pi)}{\sum_I (2I+1)}. \quad (13)$$

For the states with major $d_{5/2}p_{3/2}$ fractions, the common center of gravity agrees well with the SSM prediction (Table 4).

The difference between the center of gravity for all states assumed to contain the complete $LJlj$ strength [Eq. (13)] and that for a certain spin [Eq. (12)] defines the multiplet splitting. For the $d_{5/2}p_{1/2}$ and $d_{5/2}p_{3/2}$ configurations, Table 4 shows the center of gravity for each spin E_x^{CofG} [Eq. (12)], the center of gravity for the complete configuration $\langle E_x^{CofG} \rangle$ [Eq. (13)], and the multiplet splitting $E_x^{CofG} - \langle E_x^{CofG} \rangle$. Fig. 3 displays the values for the $d_{5/2}p_{3/2}$ configuration.

In a similar way to the states with dominant $d_{5/2}p_{3/2}$ strength, the center of gravity for all states with major $d_{5/2}p_{1/2}$ fractions (Sec. 4.1.2) is in line with the SSM prediction (Table 4). The multiplet splitting is much smaller than for $d_{5/2}p_{3/2}$.

Table 4: Center of gravity for states in ^{208}Pb with major parts of the particle-hole configurations $d_{5/2}p_{1/2}$ and $d_{5/2}p_{3/2}$.

$LJlj$	Reaction	E_x^{SSM} [2] MeV	I^π	$\langle E_x^{CofG} \rangle$ [Eq. (13)] MeV	E_x^{CofG} [Eq. (12)] MeV	$E_x^{CofG} - \langle E_x^{CofG} \rangle$ MeV
$d_{5/2}p_{1/2}$	$^{207}\text{Pb}(d, p)$	4.998	$2^-, 3^-$	5.00 ± 0.01		
			2^-		5.03	$+0.03 \pm 0.01$
			3^-		4.97	-0.03 ± 0.01
	$^{208}\text{Pb}(p, p')$		$2^-, 3^-$	5.01 ± 0.01		
			2^-		5.02	$+0.02 \pm 0.01$
			3^-		4.97	-0.02 ± 0.01
$d_{5/2}p_{3/2}$	$^{208}\text{Pb}(p, p')$	5.896	$1^- - 4^-$	5.90 ± 0.01		
			1^-		6.18	$+0.28 \pm 0.06$
			2^-		5.81	-0.09 ± 0.01
			3^-		5.83	-0.07 ± 0.02
			4^-		5.97	$+0.07 \pm 0.01$

4.2. Uneven distribution of the $d_{5/2}p_{3/2}$ strength

From the SSM, most of the $d_{5/2}p_{3/2}$ strength is expected in the region $E_x \approx 5.9$ MeV. By assuming the spin assignments of NDS2007 and by assuming that the complete $d_{5/2}p_{3/2}$ strength [Eq. (7)] has been identified, no apparent combination of the measured cross sections A_0^{orr} can reproduce the expected values $(2I + 1)$ [Eq. (6)]. With any assumed spin for the 6012 state, the sum of the $d_{5/2}p_{3/2}$ strength largely differs from the expected values; even the sum for some combinations of spins disagrees by factors two to five, see Table 5 under assumptions A-D.

In Secs. 4.3 and 4.7, the 5886 state is assigned the spin of 4^- . In Sec. 4.4 the 6012 state is shown to have the spin of 4^- . In Secs. 4.5 and 4.6.1, the missing 1^- and 2^- strengths are located. In Secs. 4.6.1, the 5.81 MeV level is shown to be a doublet of two states with spins 2^- and 3^- . The energy spacing is less than 1 keV (Sec. 4.6.3).

Tentatively, Kulleck et al [17] already assigned a spin of 4^- to the 6.01 MeV level. Yet the doublet was not resolved, and the even more strongly excited 5.81 MeV level was not analyzed because of contaminations from the $^{208}\text{Pb}(p, d)$ reaction.

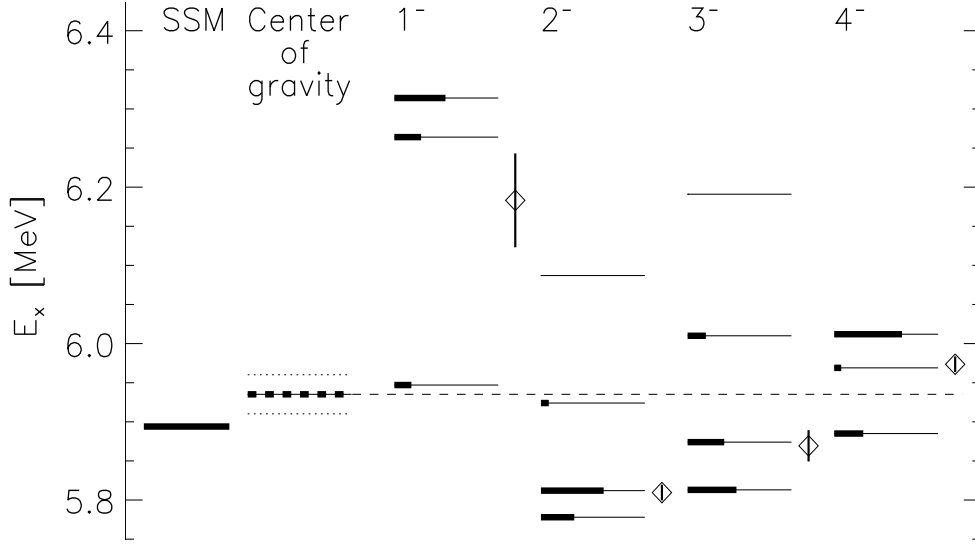


Figure 3: Excitation energy for states with major fragments of the configuration $\nu(d_{5/2}^{+1}p_{3/2}^{-1})I^\pi$ with spins $I^\pi = 1^-, 2^-, 3^-,$ and 4^- (Table 1). The thin bar shows the full strength $c^2 = 1$ while the thick bar shows the $d_{5/2}p_{3/2}$ strength determined by Eq. (5). The global center of gravity [dashed line continued across all spins, Eq. (13)] is in line with the SSM value shown on the left. The center of gravity for spins 1^- and 4^- is higher than the global center of gravity, for spins 2^- and 3^- it is lower (diamonds on the right).

4.3. The 5886 state

According to several arguments, the 5886 state is assigned a spin of 4^- ; only three are mentioned here. In Sec. 4.4 an additional argument is given. (In Fig. 1 of Paper P, the 5886 state is prematurely already assigned a spin of 4^- .)

■ Among over a hundred identified states below $E_x = 6.1$ MeV, the lowest distance of any pair of states with equal spin and parity is determined as being larger than 25 keV [2, 6, 12, 13, 14]. Therefore, the 5874 and 5886 states have different spins.

■ The shape of an angular distribution for the pure $d_{5/2}p_{3/2}$ states with spins 3^- and 4^- differs greatly, $A_K/A_0 = -0.629$ and $+0.289$, respectively [Eq. (A.12)]. If they are assigned spins 3^- and 4^- , respectively, the angular distributions of the 5874 and 5886 states have a shape corresponding to the configuration $d_{5/2}p_{3/2}$, but not in the case of the reverse assignment.

■ For the 5874 state, the angular distribution of the $^{207}\text{Pb}(d,p)$ reaction is well described by $LJ = d_{5/2}$ [21] whereas for the 5886 state two components are needed. Valnion [21] proposed a mixture of $LJ = d_{5/2}$ and $g_{7/2}$ with $J = L + \frac{1}{2}$ and $L - \frac{1}{2}$, respectively, to explain the angular distribution for the polarization asymmetry. However, an equally good fit is obtained with $LJ = g_{7/2}$ and $g_{9/2}$ and spectroscopic factors $G_{g_{7/2}}^\lambda \approx 0.10$, $G_{g_{9/2}}^\lambda \approx 0.03$ [Eq. (10)] signifying a spin of 4^- .

Table 5: Discussion of spin assignments to states in ^{208}Pb with excitation energies $5.7 < E_x < 6.4$ MeV (Secs. 4.2, 4.4, 4.6.1, and 4.7). For each assumption A, B, $\dots$, the sum of the relative cross sections σ^{rel} for different spin assignments ($I_A^\pi, I_B^\pi, \dots$) is evaluated below the respective heading A, B, $\dots$. If the spins are correctly assigned (as finally achieved by assumption H), the sum should correspond to $2I + 1$ [Eq. (6)]. The question mark hints at assumed assignments, the exclamation mark new assignments. Assumptions are ruled out if the sum rule for any spin or any pair of spins deviates significantly from the expected value of $2I + 1$. Values which are too low are printed in italics, values which are too high in bold face.

$\tilde{E}_x$	σ^{rel}	I_A^π	I_B^π	I_C^π	I_D^π	I_E^π	σ^{rel}	I_F^π	I_G^π	σ^{rel}	I_H^π
	Eq. (5) ^a						Eq. (5) ^b			Eq. (5) ^c	
5778	1.8	2 ⁻	2 ⁻	2 ⁻	2 ⁻	2 ⁻	1.7	2 ⁻	2 ⁻	1.7	2 ⁻
5812							3.1	1 ^{-?}	4 ^{-?}	2.8 $\pm$ 0.3	2 ^{-!}
5813	6.9	3 ⁻	3 ⁻	3 ⁻	3 ⁻	3 ⁻	3.1	3 ⁻	3 ⁻	3.4 $\mp$ 0.3	3 ⁻
5874	2.6	3 ⁻	3 ⁻	3 ⁻	3 ⁻	3 ⁻	2.3	3 ⁻	3 ⁻	2.3	3 ⁻
5886	2.9	3 ^{-?}	3 ^{-?}	3 ^{-?}	3 ^{-?}	4 ^{-!}	2.6	4 ^{-!}	4 ^{-!}	2.6	4 ^{-!}
5924	0.4	2 ⁻	2 ⁻	2 ⁻	2 ⁻	2 ⁻	0.4	2 ⁻	2 ⁻	0.4	2 ⁻
5947	0.6	1 ⁻	1 ⁻	1 ⁻	1 ⁻	1 ⁻	0.5	1 ⁻		0.5	1 ⁻
5969	0.7	4 ⁻	4 ⁻	4 ⁻	4 ⁻	4 ⁻	0.6	4 ⁻		0.6	4 ⁻
6010	1.3	3 ⁻	3 ⁻	3 ⁻	3 ⁻	3 ⁻	1.3	3 ⁻	3 ⁻	1.3	3 ⁻
6012	6.8	1 ^{-?}	2 ^{-?}	3 ^{-?}	4 ^{-?}	4 ^{-!}	6.1	4 ^{-!}	4 ^{-!}	6.1	4 ^{-!}
6264							0.8	1 ⁻		0.8	1 ⁻
6314							1.5	1 ⁻		1.5	1 ⁻
$\sum \tilde{E}_x$	24						24			24	
I^π	2I+1	Configuration strength $\sum_I \sum_{\tilde{E}_x} \sigma^{rel}$ [Eq. (6)] for assumption A-H									
		A	B	C	D	E		F	G	H(accepted)	
1 ⁻	3	7.4	0.6	0.6	0.6	0.6		5.9	2.8		2.8
2 ⁻	5	2.2	9.0	2.2	2.2	2.2		2.1	2.1		4.9 $\pm$ 0.3
3 ⁻	7	13.7	13.7	20.5	13.7	10.8		6.7	6.7		7.0 $\mp$ 0.3
4 ⁻	9	0.7	0.7	0.7	7.5	10.4		9.3	12.4		9.3
1 ⁻ +3 ⁻	3+7	21.1	14.3	21.1	14.3	11.4		12.6	9.5		9.8 $\mp$ 0.3
2 ⁻ +3 ⁻	5+7	15.9	22.7	22.7	15.9	13.0		8.8	8.8		11.9

(a) Relative cross sections of levels investigated by assumptions A-E.

(b) Relative cross sections of levels investigated by assumptions F-G.

(c) Relative cross sections of levels investigated by assumption H.

4.4. Resolving the 6.01 MeV doublet

The 6012 state is essentially excited only on the $d_{5/2}$ IAR while the 6010 state is excited on all IAR (at proton energies $14.8 < E_p < 18.2$ MeV). The 6.01 MeV doublet is resolved by the $^{208}\text{Pb}(p, p')$ reaction (Sec. 3.3.1 and Fig. 1) yielding $\langle E_x^{GASPAN} \rangle = 6009.6 \pm 0.2$ and 6011.4 ± 0.3 keV from about 60 runs (Table T). The $^{207}\text{Pb}(d, p)$ reaction excites only the 6010 3⁻ state yielding $\langle E_x^{GASPAN} \rangle = 6009.5 \pm 0.1$ keV [Table T, Eq. (9)]. The 6010 3⁻ state contains 29% $g_{7/2}p_{1/2}$ strength [20]. The difference between the excitation energies for the 6010 and 6012 states is determined as 1.8 ± 0.3 keV. NDS2007 declares a distance of 1.9 ± 0.1 keV.

On the $d_{5/2}$ IAR for scattering angles of $\Theta > 100^\circ$, the 6010 3^- state is much more weakly excited than the 6012 state, on all other IAR much more strongly. The mean cross section A_0^{mean} [Eq. (3)] of the 6.01 MeV doublet agrees with the values from Ref. [17] (Table T).

The 6012 state is assigned the spin of 4^- by observing the mean cross sections A_0^{corr} [Table 1, Eq. (4)]. Namely, the large cross section for the 6012 state excludes spins of 1^- , 2^- , and 3^- (see assumptions A-C in Table 5).

Assumption D excludes spin 3^- for the 5886 state and the spin of 4^- is assigned by assumption E. The assignment deduced in Sec. 4.3 is thus confirmed. The 5886 4^- state contains the remaining $d_{5/2}p_{3/2}$ strength, while the 5969 4^- state is only weakly excited on the $d_{5/2}$ IAR (Table 1). The sum of the $d_{5/2}p_{3/2}$ strength in the 5886 4^- , 5969 4^- , and 6012 4^- states agrees with the expectation within 15% while for spins 3^- , 2^- , and 1^- there are still discrepancies of 50% and factors two and five, respectively.

The shape of the angular distribution of the 6012 4^- state is well described by assuming a pure $d_{5/2}p_{3/2}$ component (Eq. (A.12) and Table 1). From experiments with semiconductor detectors [17], the minimum at $\Theta \approx 90^\circ$ predicted for the $d_{5/2}p_{3/2}$ angular distribution is verified by our data for the 6.01 MeV doublet. As suggested by Kulleck et al [17], the direct (p, p') contribution to the 6010 3^- state (starting already at $\Theta \approx 120^\circ$) partially explains the expressive A_4/A_0 contribution near $\Theta = 90^\circ$. Yet in addition, a weak $d_{5/2}f_{5/2}$ admixture to the 6012 state may also explain the large value A_4/A_0 [Eq. (A.11)].

4.5. The $d_{5/2}p_{3/2}$ 1^- states

Only one 1^- state is located within $5.7 < E_x < 6.1$ MeV; much $d_{5/2}p_{3/2}$ strength for the spin of 1^- is missing. Indeed, the 6264 1^- and 6314 1^- states contain the missing fractions (Table 1), but the penetrability is low, namely $a_{d_{5/2}p_{3/2}}^{para} = 0.60$ and 0.32 , respectively. Fig. 2 of Ref. [13]) shows spectra with the 6264 1^- and 6314 1^- states.

An open question is the spin and structure assignment to the five states at $5.638 < E_x < 5.650$ MeV where the highest member is a $(9)^+$ state (Paper S); one state may have a spin of 1^- [11]. Apparently, the configuration mixing among the 1^- states is large. Therefore the uncertainty of the center of gravity [Eq. (13)] may be even greater than shown in Fig. 3 and Table 4.

4.6. Resolving the 5.81 MeV doublet

4.6.1. Spin of the 5812 state. The 5.81 MeV level certainly contains a 3^- state [5]. A large fraction of the configuration $g_{7/2}p_{1/2}$ is derived from the $^{207}\text{Pb}(d, p)$ reaction (Table T). The cross sections A_0^{corr} of the 5.81 MeV level and the 6012 4^- state are similar (Table 1). Since the 5813, 5874 and 6010 states contain some fraction of the $d_{5/2}p_{3/2}$ 3^- strength and the 5886 and 6012 states most of the $d_{5/2}p_{3/2}$ 4^- strength, another state with a spin of either 1^- or 2^- must be present. We are assigning the energy labels 5812 and 5813 to the members of the doublet.

Spins of 1^- and 4^- are excluded since the sum of the cross sections for all states with spins of 1^- and 4^- in the region $5.7 < E_x < 6.4$ MeV would exceed expectations (see assumptions F and G in Table 5). Hence the 5812 state has a spin of 2^- .

The distribution of the $d_{5/2}p_{3/2}$ strength across the four spins (1^- , 2^- , 3^- , and 4^-) is in full agreement with this finding. The cross section of the 5.81 MeV

doublet can be divided up between the 5812 2^- state and the 5813 3^- state by about $45 \pm 5 : 55 \mp 5\%$, see assumption H in Table 5.

4.6.2. Particle-hole configurations in the 5812 and 5813 states. The 5.81 MeV doublet consists of two states with spins of 2^- and 3^- (Sec. 4.6.1). In the Q3D experiments, the doublet at $E_x = 5.81$ MeV is *not* resolved, see however Secs. 3.3.2 and 4.6.3. By assuming the cross section to be shared equally by both the 5812 2^- state and the 5813 3^- state, the angular distribution of the 5.81 MeV doublet is well described. A superposition of $d_{5/2}p_{3/2}$ components with spins of 2^- and 3^- yields an angular distribution with a high maximum near $\Theta = 90^\circ$. It is derived from the 3^- state, while the angular distribution for the 2^- state is rather flat [Eq. (A.12)].

A 12% admixture of the configuration $g_{7/2}p_{1/2}$ to the 3^- state in the 5.81 MeV level is determined from the $^{207}\text{Pb}(d, p)$ reaction [20]; a $d_{3/2}p_{1/2}$ admixture to the 2^- state is estimated to be much less.

4.6.3. Excitation energies of the 5812 and 5813 states. The excitation energy derived from $^{207}\text{Pb}(d, p)$ is higher than the mean value derived from the $^{208}\text{Pb}(p, p')$ reaction (Table T). Since only the 3^- state is excited by the $^{207}\text{Pb}(d, p)$ reaction ($LJ = g_{7/2}$ [20, 21]), the 5813 3^- state has a higher excitation energy than the 5812 2^- state.

From the SSM, another major component is assumed to be $g_{9/2}f_{7/2}$; the weak cross section on the $g_{9/2}$ IAR is in line with such an interpretation (Table T). The mean excitation energy determined on the $g_{9/2}$ IAR is lower than on all other IARs (Fig. 2),

$$\begin{aligned} \langle E_x^{GASPAN} \rangle(E_p = 14.92 \text{ MeV}) &= 5812.96 \pm 0.27 \text{ keV}, \\ \langle E_x^{GASPAN} \rangle(15.7 < E_p < 18.1 \text{ MeV}) &= 5813.23 \pm 0.17 \text{ keV}. \end{aligned} \quad (14)$$

The difference is explained by the different angular distributions of the $g_{9/2}f_{7/2}$ and $d_{5/2}p_{3/2}$ components on the respective IARs. Similar as for the 5686, 5694, 5836 states with spins of 6^- , 7^- , and 8^- [12], respectively, the 5812, 5813 may be described by pure $g_{9/2}f_{7/2}$ configurations. The excitation of the 5812 2^- is enhanced on the $g_{9/2}$ IAR at scattering angles $|\pi - \Theta| \approx 90^\circ$ and, hence, the mean excitation energy is lower (large symbols in Fig. 2).

As a common result from the $^{208}\text{Pb}(p, p')$ and the $^{207}\text{Pb}(d, p)$ reactions, we obtain

$$\begin{aligned} E_x(5812 \ 2^-) &= 5812.8 \pm 0.2 \text{ keV}, \quad E_x(5813 \ 3^-) = 5813.2 \pm 0.2 \text{ keV}, \\ E_x(5813 \ 3^-) - E_x(5812 \ 2^-) &= 0.4 \pm 0.2 \text{ keV}. \end{aligned} \quad (15)$$

4.7. The complete $d_{5/2}p_{3/2}$ strength

The sum of the $d_{5/2}p_{3/2}$ strength [Eq. (7)] located in twelve states at $5.7 < E_x < 6.4$ MeV is complete within about 10% (assumption H in Table 5).

For spin 1^- a strength of 2.8 ± 0.7 is determined while $2I + 1 = 3$ [Eq. (6)] is expected, for spin 2^- a strength of $4.8 \pm 0.5 \pm 0.3$ compared to $2I + 1 = 5$, for spin 3^- a strength of $6.9 \pm 0.7 \mp 0.3$ compared to $2I + 1 = 7$, for spin 4^- a strength of 9.4 ± 1.5 compared to $2I + 1 = 9$.

Fig. 3 displays the distribution of the $d_{5/2}p_{3/2}$ strength.

The 5812 2^- and 6012 4^- states contain about 60% and 70% of the $d_{5/2}p_{3/2}$ strength, respectively. Each of the three 3^- states consists of at least three configurations having less than 60% strength. While the 5947 1^- state contains 90% of the

$d_{3/2}p_{1/2}$ strength [20] and the 6314 1^- state 60 – 80% of the $s_{1/2}p_{3/2}$ strength [13], the 6264 1^- state has a complex structure.

The finding of the $d_{5/2}p_{3/2}$ strength to be complete within 10% verifies the assumption made in Sec. 4.1.1 that only minor $d_{5/2}p_{3/2}$ fractions contribute to the states at $E_x < 5.7$ MeV.

5. Summary

In the doubly magic nucleus ^{208}Pb , the SSM predicts eighty negative parity particle-hole states for $E_x < 6.1$ MeV. Only the structure of all negative parity states at $E_x \lesssim 4.8$ MeV and a few more higher-lying states is known up to now. In order to determine the residual interaction among particle-hole configurations, the structure of all eighty states must be known, however.

Twelve states with spins of 1^- , 2^- , 3^- , and 4^- are studied in detail at excitation energies $5.75 < E_x < 6.35$ MeV. They are identified as containing the complete strength of the neutron particle-hole configuration $d_{5/2}p_{3/2}$ within at least 90% for each spin. The center of gravity for the configuration $d_{5/2}p_{1/2}$ and $d_{5/2}p_{3/2}$ is in line with the predictions by the shell model without residual interaction (SSM) within 10 keV. The centroid energies of the $d_{5/2}p_{3/2}$ configuration for spins of 1^- to 4^- follow a parabolic shape with deviations from the global centroid between -90 keV (2^-) and $+280$ keV (1^-).

The doublet at $E_x = 6.01$ MeV is resolved into two states having a distance of 1.8 keV, $E_x = 6010$ and 6012 keV. In line with the tentative assignment by Kulleck et al [17], the spin and parity of the 6012 state is assigned as 4^- ; the excitation energy tentatively determined by NDS2007 is confirmed. The 5886 state has been newly assigned the spin of 4^- . The $d_{5/2}p_{3/2}$ 4^- strength is almost completely located in the 5886 and 6012 states. The 6010 3^- state has a complex structure similar to the 5813 3^- state.

The doublet at $E_x = 5.81$ MeV is resolved into two states having a distance of 0.4 keV, $E_x = 5812$ and 5813 keV. The 5812 state has been newly identified; it has the spin of 2^- and consists of the two configurations $d_{5/2}p_{3/2}$ and $g_{9/2}f_{7/2}$ with a strength of about 50% each. The 5813 3^- state contains about 10% $g_{7/2}p_{1/2}$ strength besides about half of the $d_{5/2}p_{3/2}$ strength.

The mean cross section of many states with major $d_{5/2}f_{5/2}$ strength in the region $5.3 < E_x < 5.7$ MeV is greatly increased by weak $d_{5/2}p_{1/2}$, $d_{5/2}p_{3/2}$ admixtures. However, the knowledge of the distribution of almost the complete $d_{5/2}p_{1/2}$ and $d_{5/2}p_{3/2}$ strengths presented in this paper restricts such admixtures to be less than 10%. By this means, the structure of most negative parity particle-hole states up to $E_x \approx 6.1$ MeV can be determined in future. States with dominant proton particle-hole configurations may also be studied in more detail. Ultimately, the residual interaction among all eighty particle-hole configurations with negative parity at $E_x < 6.1$ MeV in ^{208}Pb can be determined with high reliability.

Appendix A. Inelastic proton scattering via isobaric analog resonances

Appendix A.1. Reaction mechanism and decay widths.

We consider the reaction where protons with an energy E_p are scattered on the nucleus ^{208}Pb . Following Bohr & Mottelson [27], the $^{208}\text{Pb}(p, p')$ reaction via IAR in ^{209}Bi

may be viewed as a two-stage process $1 \rightarrow r \rightarrow 2$. Starting from the ground state $E_x^0 0^+$ in ^{208}Pb , in the initial stage a LJ IAR is created. In the subsequent proton decay of the IAR, neutron particle-hole states in ^{208}Pb with excitation energy E_x^I , and final spin and parity I^π are excited and a proton lj is emitted,

$$E_x^0 0^+ \otimes LJ \rightarrow E_p LJ \rightarrow E_x^I I^\pi \otimes lj. \quad (\text{A.1})$$

In the $E_x^I I^\pi$ state, the particle LJ and the hole lj couple to the final spin I .

The decay amplitude $\gamma(E_p LJ \rightarrow E_x^I I^\pi \otimes lj)$ is normalized in such a way that the absolute square is the partial width (see Eq. (3F-1) in Ref. [27])

$$\Gamma(E_p LJ \rightarrow E_x^I I^\pi \otimes lj) = |\gamma(E_p LJ \rightarrow E_x^I I^\pi \otimes lj)|^2. \quad (\text{A.2})$$

This gives the probability per unit time (multiplied by $\hbar$) for the decay into the channel considered. The time reversal invariance implies $\Gamma(E_p LJ \rightarrow E_x^I I^\pi \otimes lj) = \Gamma(E_x^I I^\pi \otimes lj \rightarrow E_p LJ)$. Since the decay involves the transmission through a large barrier associated with centrifugal and Coulomb forces, the decay amplitudes strongly depend on the energy of the proton.

In the $^{208}\text{Pb}(p, p')$ reaction via IAR, the protons enter with the beam energy E_p and leave the two-stage process with the energy given by the difference between the beam energy E_p and the excitation energy of the $E_x^I I^\pi$ state,

$$E_{p'}(E_x^I, E_p) = E_p - E_x^I. \quad (\text{A.3})$$

In the SSM, the energy for populating a model configuration does not depend on the spin I ,

$$E_{p'}^{SSM}(LJ lj) = E_p - E_x^{SSM}(LJ lj). \quad (\text{A.4})$$

To state the case briefly, the decay width $\Gamma(E_p LJ \rightarrow E_x^I I^\pi \otimes lj)$ [Eq. (A.2)] is written as $\Gamma_{lj}(E_{p'})$ where $E_{p'}$ is the energy of the scattered proton [Eq. (A.3)]. The decay width Γ_{lj} depends strongly on the proton energy [28], in fact on the penetrability of the outgoing proton through the Coulomb barrier and the additional centrifugal barrier. We define the relative penetrability as

$$\left(a_{LJ lj}^{ptra}(E_p, E_x^I)\right)^2 = \frac{\Gamma_{lj}(E_{p'}(E_x^I, E_p))}{\Gamma_{lj}(E_p - E_x^{SSM}(LJ lj))} = \frac{\Gamma_{lj}(E_p - E_x^I)}{\Gamma_{lj}(E_p - E_x^{SSM}(LJ lj))}. \quad (\text{A.5})$$

This becomes unity on top of the LJ IAR ($E_p = E_{LJ}^{res}$) at the excitation energy $E_x^{SSM}(LJ lj)$ of the SSM configuration [Eq. (A.4)]. The relative penetrability $a_{LJ lj}^{ptra}$ depends only slightly on the orbital momentum l . The ratio of the penetrabilities for the spin $j = l + \frac{1}{2}$ to $j = l - \frac{1}{2}$ increases, the higher the angular momentum l becomes. Typically, the relative penetrability amplitude $a_{LJ lj}^{ptra}$ changes by a factor of two for a 1 MeV difference in the proton energy [28].

The decay width Γ_{LJ} for the entrance channel is calculated to vary little in the vicinity of the IAR, provided that the beam energy does not change by more than a few percent [28]. This holds for the data considered in this work.

For convenience, single particle (s.p.) widths are introduced by defining

$$\Gamma_{LJ}^{s.p.} = \Gamma[LJ \rightarrow E_x^0 0^+] \quad \text{at the resonance energy } E_{LJ}^{res}, \quad (\text{A.6})$$

$$\Gamma_{lj}^{s.p.} = \Gamma[g_{9/2} \rightarrow E_x^{SSM}(g_{9/2} lj)], \quad lj = p_{1/2}, p_{3/2}, f_{5/2}, f_{7/2}, h_{9/2}, i_{13/2}. \quad (\text{A.7})$$

Appendix A.2. Cross section.

In this paper we mainly discuss angle integrated (mean) cross sections. For details, we refer the reader to more complete descriptions of the reaction mechanism in Sec. III of Ref. [23], Sec. 2.2 of Ref. [1], and Appendix 3F of Ref. [27].

The differential cross section is given by [1]

$$\frac{d\sigma}{d\Omega}(E_x, I^\pi, E_p, LJ l j, \Theta) = N_{LJ}(E_p) a^{Lz}(E_p, LJ) \sum_{l_j} \sum_{l' j'} \sum_{K=0,2,\dots}^{K_{max}} a_K(E_x, I^\pi, E_p, LJ l j, LJ l' j') P_K(\cos(\Theta)). \quad (\text{A.8})$$

The normalization constant is given by

$$N_{LJ}(E_p) = \frac{\pi \hbar^2}{\frac{208}{209} m_p E_p [\Gamma_{LJ}^{tot}(E_p)]^2}. \quad (\text{A.9})$$

Near the resonance, the shape of the excitation function is Lorentzian,

$$a^{Lz}(E_p, LJ) = \frac{[\Gamma_{LJ}^{tot}(E_p)]^2}{4(E_p - E_{LJ}^{res})^2 + [\Gamma_{LJ}^{tot}(E_p)]^2}. \quad (\text{A.10})$$

In general however, the energy dependence of the total width Γ_{LJ}^{tot} produces an asymmetry of the excitation function.

The geometrical anisotropy coefficients a_K/a_0 defines the shape of the angular distribution. The shape is described by Legendre polynomials of maximum order

$$K_{max} \leq \min(2L, 2J, \max(l, l'), \max(j, j')). \quad (\text{A.11})$$

For a pure $d_{5/2}p_{1/2}$ configuration, the angular distribution should be isotropic. For a pure $d_{5/2}p_{3/2}$ configuration, the anisotropy coefficients are geometrical values,

$$\begin{aligned} \frac{A_2}{A_0} &= +0.800, -0.114, -0.629, +0.286 \\ \text{for } I^\pi &= 1^-, 2^-, 3^-, 4^-, \text{ respectively.} \end{aligned} \quad (\text{A.12})$$

The average cross section on top of the LJ IAR is calculated as

$$A_0^{calc}(\tilde{E}_x, I^\pi, E_{LJ}^{res}) = N_{LJ}(E_{LJ}^{res}) \frac{2I+1}{2J+1} \sum_{l_j} \left| c_{LJ l_j}^{\tilde{E}_x, I^\pi} \right|^2 \Gamma_{LJ}^{s.p.} \Gamma_{l_j}^{s.p.}, \quad (\text{A.13})$$

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