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# Reply to “Comment on ‘Modal expansion approach to optical-frequency-comb generation with monolithic whispering-gallery-mode resonators’ ”

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In a Comment [Deych, *Phys. Rev. A* **84**, 017801 (2011)] on our recent article [Chembo and Yu, *Phys. Rev. A* **82**, 033801 (2010)] on Kerr comb generation in monolithic whispering-gallery-mode (WGM) resonators, the author claims that even though our main results “remain intact,” the framework of our analysis contains some mathematical errors. In this Reply, we demonstrate that the author’s criticisms and his alternative theory are incorrect, and that all the disagreeing claims come from the author’s misunderstanding of the physical system under investigation. In particular, the main conceptual error in the Comment is the confusion between evanescent and radiative coupling mechanisms for WGM resonators (in terms of  $Q$  factor, this is in our case a confusion between  $\sim 10^9$  and  $\sim 10^{2000}$ , respectively). This essential misconception leads for example the author to introduce a nonphysical  $\sim 10^{2000}$  multiplicative correction factor for the laser pump power. We show in this Reply that our WGM resonators are radiatively closed (because of the quasi-infinite radiative  $Q$  factor), so that the Mie scattering formalism proposed by Deych, which relies on radiative coupling, can only lead to erroneous results. We also show that the modal expansion approach in our original paper is appropriate, and all our approximations are physically well justified. We therefore stand by our modeling results, which are moreover in excellent agreement with the experiments reported in [Chembo, Strekalov, and Yu, *Phys. Rev. Lett.* **104**, 103902 (2010)].

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## I. INTRODUCTION

In a Comment [1] on our recent article [2] on Kerr comb generation in whispering-gallery-mode (WGM) resonators, Deych claims that the framework of our analysis “contains some mathematical errors.” He went on supporting this assertion with an alternative theory that is claimed to be “mathematically rigorous,” but which, nevertheless, “shows that the main conclusions of that (our) paper remain intact.” In this Reply, we will show that our mathematical treatment is indeed correct and appropriate. Not surprisingly, our results are also correct, as supported by the excellent agreement with

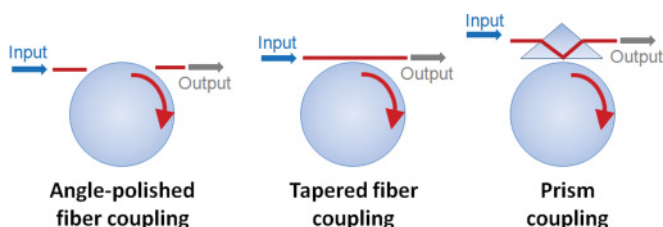


FIG. 1. (Color online) Common ways to couple laser light into monolithic WGM resonators. Such resonators are *radiatively* closed (quasi-infinite radiative  $Q$  factor). This is why the coupling is always performed evanescently, but never radiatively. The pertinent coupling coefficient is not the inverse radiative quality factor  $Q_{\text{rad}}^{-1}$  ( $\sim 0$ ), but rather the inverse evanescent quality factor  $Q_{\text{ev}}^{-1}$ . In this configuration the coupled WGM cavity is a  $1 \times 1$  nonlinear transmission system (two-port device), not a radiative scatterer. The Mie scattering formalism proposed by Deych, which assumes radiative coupling, is then irrelevant in our case and leads to erroneous results.

our experimental data [3]. A careful analysis of the Comment led us to conclude that the author’s objections stem primarily from an erroneous understanding for the physical conditions of typical WGM resonators under investigation.

Deych’s comment had two main objections. First, according to the author, the external drive term in Eq. (1) in the article cannot be a part of the modal expansion, which is therefore mathematically incorrect. The article has made it clear that this external term is not part of the resonator field in the modal expansion. It rather stands for the external *evanescent* pump field considered for the purpose of coupling. In all experimental realizations, such evanescent coupling is accomplished by mode- and phase-matching resonator and external fields through spatial overlap (see Fig. 1). Coupling in monolithic WGM resonators is *never* achieved radiatively as analyzed by Deych in the Comment. As we will further show, this essential misconception leads him to a flawed model for the modal equations.

Deych’s second objection is that the WGM resonator is an *open* system with radiation far-field  $\exp[ikr]/r$  filling infinite space. Therefore, the eigenmodes used in the article are not Hermitian, and this causes divergence for the normalization integrals. The author complains that we artificially “hide” this problem by allowing the integral only within the resonator and its vicinity. Once again, Deych did not accurately consider the actual physical system under investigation. The light fields inside a WGM resonator are trapped in the resonator through total internal reflection. As we shall show, for the resonators we are considering (millimeter size), the total internal reflection is as perfect as *physically* possible. The radiative  $Q$  factor is around  $Q_{\text{rad}} \sim 10^{2000}$ , meaning that the radiative losses are around  $10^{2000}$  times smaller than any relevant loss mechanism (bulk absorption, etc.). The field outside the resonator is therefore *purely* evanescent. Hence, our WGM resonators are

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*radiatively closed* systems with exponentially decaying fields outside the resonator. This exponential decay explains why the normalization integrals are indeed converging. On the other hand, for such radiatively closed systems, the scattering theory proposed by the Comment is *not* an appropriate mathematical framework. The misunderstanding of the insignificance of radiation coupling and losses has led the author of the Comment to an erroneous theory for Kerr comb generation in WGM resonators.

Since the main results of the article are not challenged by the Comment, our detailed reply will thus focus only on the methodology and interpretations. The rest of the reply is organized as follows. We will first show that the external field coupling in the article is properly treated. Then, we will demonstrate that our approach to eigenmode determination of the resonator under investigation is appropriate. Finally, we will show that the theory proposed by Deych is flawed in the context of the study in our paper.

## II. COUPLING BY EXTERNAL FIELDS

The author of the Comment argues that our modal expansion

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\mu} \frac{1}{2} \mathcal{E}_{\mu}(t) e^{i\omega_{\mu}t} \Upsilon_{\mu}(\mathbf{r}) + \frac{1}{2} \mathcal{E}_{\text{ext}} e^{i\Omega_0 t} \mathbf{e}_0 + \text{c.c.} \quad (1)$$

is not correct because the external pumping term is not an eigenmode. In our paper,  $\mathbf{E}(\mathbf{r}, t)$  is not just the resonator mode field of the resonator, but the *total electric field* inside and outside of the resonator. It is therefore a superposition of *external evanescent field* (the second term) and of the *uncoupled resonator field* which is a linear combination of the eigenmodes  $\Upsilon_{\mu}$  (the first term). In fact, the external electric field in our case is not a spatially large field superimposed to the intracavity fields. It is a quasipoint source evanescent excitation, very localized in space, as shown in Fig. 1. For the sake of simplicity, this external laser field has been assumed constant (or a  $\delta$  function) in a small area where the evanescent field of the resonator and the pump field overlap. Indeed, we can replace  $\mathcal{E}_{\text{ext}} \mathbf{e}_0$  by an ideal point source field  $\mathcal{E}_{\text{ext}} \mathbf{e}_0 \cdot \delta(\mathbf{r} - \mathbf{r}_0)$ , where  $\delta$  is the Dirac function and  $\mathbf{r}_0$  is a point located at the edge of the resonator. The overlap integral  $\int_{\infty} \Upsilon_{\eta}^*(\mathbf{r}) \cdot \mathbf{e}_0 dV$  in Eq. (34) of the article stands for the coupling strength. This external term is obviously phenomenological, and is designed to avoid a rigorous calculation of the coupling which is not the focus of the article nor is necessary to study the nonlinear process in the resonator. In fact, all subsequent analysis will not change at all if we opt to remove the external field term in Eq. (1) and simply add a phenomenological pump term  $\mathcal{F}_{\eta}$  in Eq. (27) of the article.

Our approach for modeling the coupling of the disk resonator is inspired by the transmission filter theory proposed by Yariv in [4], and further developed into a dynamical framework by Féron and co-workers in Ref. [5]. These authors have shown that adding phenomenologically an external field to the intracavity fields is a simple and efficient way to account for the input *evanescent* coupling. In Ref. [5] this theoretical approach was in excellent agreement with experimental measurements. From this standpoint, the excellent agreement we have achieved ourselves in Ref. [3] is logical and proves

that this phenomenological approach is correct. This theory even enables the accurate determination of the evanescent coupling  $Q$  factor ( $Q_{\text{ev}}$ , the key element missing in Deych's theory). Authors of Ref. [5] have proven the validity of this method for both linear cavities (WGM resonators) and nonlinear cavities (fiber-ring lasers). It is in fact easy to show that this coupling model is also valid for nonlinear WGM cavities, since the nonlinearity can be neglected in this case (as a small perturbation of the linear bulk refraction index; see [6]).

In an attempt to pursue mathematical rigor, the author of the Comment instead used the standard Mie scattering theory to model the coupling between an incident field and the resonator field. This is misdirected in several ways. First, this is not how a WGM resonator coupling is implemented at all in general and certainly not in our system under study (see Fig. 1). The practical approach to coupling is via the evanescent fields with phase- and mode-matching condition. The physical process is transmission through a filter, similar to light transmission through a Fabry-Pérot resonator, as explained in Refs. [4,5]. No one would treat the Fabry-Pérot resonator coupling with the Mie scattering theory. Second, as we will show in the next section, the radiation field of our system is infinitesimally small. The WGM resonator is a closed system in that regard, and therefore the scattering theory is not appropriate and leads to unphysical conclusions.

## III. EIGENMODES IN A RADIATIVELY CLOSED RESONATOR

The author of the Comment argues that our eigenmodes are ill-defined, and that our normalization condition  $\int_{\infty} \Upsilon_{\ell p}^*(\mathbf{r}) \cdot \Upsilon_{\ell' p'}(\mathbf{r}) dV = \delta_{\ell \ell'} \delta_{pp'}$  diverges because of the openness of the system due to nonnegligible radiation fields outside the resonator. The normalization integral spanning only around the resonator is arbitrary and “mathematically regrettable.”

The integral divergence claimed by Deych is in our case misleading. It does not occur in practice because the radiation loss responsible for this mathematical divergence is physically nonexistent in millimeter-size monolithic WGM resonators like ours. Let us consider the intrinsic quality factor  $Q_{\text{in}}$  of the unloaded resonator, which depends on bulk absorption loss, surface scattering loss, and radiation loss according to  $Q_{\text{in}}^{-1} = Q_{\text{bulk}}^{-1} + Q_{\text{surf}}^{-1} + Q_{\text{rad}}^{-1}$ . The radiation loss  $Q$  factor is given by

$$Q_{\text{rad}} \simeq x \exp\{2(\ell + 1/2) \times g[x/(\ell + 1/2)]\}, \quad (2)$$

where  $x = 2\pi a/\lambda_0$  is the spectral parameter and  $g$  is a nonlinear function obeying  $g(y) = -\sqrt{1 - y^2} + \text{acosh}(1/y)$  [7]. For example, for the WGM resonator used in our article,  $a = 2.5$  mm,  $\lambda_0 = 1560$  nm,  $\ell \simeq 2\pi a n_0/\lambda_0 \sim 14\,400$ , and  $x \simeq 2\pi a/\lambda_0 \sim 10\,000$ , then we have  $Q_{\text{rad}} \sim 10^{2000}$  ! This nonphysically large  $Q$  factor should be compared to  $Q_{\text{in}} \sim 10^9$  realized in practice, limited by other more mundane losses. In other words, there is simply *no* radiation field outside the resonator. This can easily be understood from basic principles of optics. Perfect internal reflection occurs when light is reflected by a plane surface (infinite radius of curvature): the radius of curvature  $a$  of our resonators (few millimeter) appears as infinity for a laser radiation at  $\lambda_0 = 1.55$   $\mu\text{m}$ ,

because the ratio  $a/\lambda_0 \sim \ell$  is huge. Moreover, Eq. (2) shows that convergence toward perfect internal reflection (infinite  $Q_{\text{rad}}$ ) is exponential in  $\ell$ . This phenomenology has been clearly pointed out by Oraevsky in a comprehensive treatment of WGM resonators [8].

Neglecting the radiation loss and the related mathematical divergence in the eigenmode determination corresponds to disregard a loss term that is  $10^{2000}$  times smaller than the leading loss mechanisms. This is obviously well justified and a logical procedure based on a comprehensive mathematical treatment of monolithic WGM resonators with large  $\ell$ . In our case, the field outside the resonator can safely be considered as purely evanescent up to  $D_{\text{ev}} \sim Q_{\text{rad}}\lambda_0$ , which is infinity since  $D_{\text{ev}} \sim 10^{2000}$  m is even larger than the size of known Universe. It can be accurately demonstrated (see Ref. [8]) that outside the WGM resonator, this evanescent electric field decays exponentially as  $\exp(-r/r_{\text{ev}})$  with an evanescence length  $r_{\text{ev}} = \lambda_0/2\pi\sqrt{n_0^2 - 1} \sim \lambda_0$ . Owing to this exponential decay, there is *no* need to perform the normalization integral up to infinity, since exponential convergence is numerically achieved within a few- $\lambda_0$  radial span outside the resonator. This conclusion applies whenever  $a \gg \lambda_0$ , which is practically always the case for *monolithic* WGM resonators used for Kerr comb generation.

#### IV. ISSUES WITH SCATTERING MODEL

In the Comment, the author treats the resonator as an isolated resonator, which is radiatively coupled to incident light and scatters it to infinity. The resonator therefore only interacts with the incident light through its radiative field, characterized by its quality factor  $Q_{\text{rad}}$ . This is not only a wrong coupling process to consider, it is also practically unfeasible because monolithic WGM resonators are radiatively closed, as shown in the previous section. As a consequence of this misunderstanding of the underlying physics involved in the coupling mechanism of WGM resonators, Deych incorrectly concluded in Eq. (23) of the Comment that we should multiply our original pumping term  $\mathcal{F}_0$  by the factor  $\Delta\omega_{\text{in}}/\Delta\omega_{\text{rad}}$ , which is exactly equal to  $Q_{\text{rad}}/Q_{\text{in}} \sim 10^{2000}$ . This correction factor is clearly nonphysical, as pumping the cavity under these conditions concretely requires far more energy than available in our galaxy. However, this error makes sense in Deych’s framework: if light is to be coupled *radiatively* into a system with quasi-infinite  $Q_{\text{rad}}$  (that is, quasi-null radiative openness), then quasi-infinite optical power is needed [9]. This explains precisely why coupling of WGM resonators is performed evanescently instead of radiatively. As earlier

emphasized, correct modeling should consider the coupled WGM resonator as a two-port transmission system, but not as a radiative scatterer as assumed by Deych.

Interestingly, the review article [9] highlights the inefficiency of radiative coupling, *precisely* because of the correction term introduced by Deych. In our evanescent coupling,  $Q_{\text{rad}}$  has to be replaced by  $Q_{\text{ev}}$ , where  $Q_{\text{ev}}$  is related to the evanescent coupling strength of Eq. (27) in the article (labeled as  $Q_{\text{ext}}$ ). This yields a multiplicative correction term  $Q_{\text{ev}}/Q_{\text{in}} = 1$  in the case of critical coupling. No correction is then needed in our model.

It is also worthwhile pointing out that the simulation presented in the Comment in the attempt to prove its point is done with parameters that are several orders of magnitude away from those in the article. Figure 1 of the Comment illustrates the effect of the imaginary part  $n_a$  of the refraction index (i.e., the loss) in the range of  $10^{-3}$  while we are working with ultra-high  $Q$  resonators with  $n_a \sim n_0/Q \sim 10^{-9}$ . Figure 2 of the Comment shows eigenvalue derivatives up to  $\ell \sim 200$ , while we are working with  $\ell \sim 14400$ . Clearly these simulations are irrelevant with respect to our work presented in the article.

We would also like to add that contrarily to what is stated in the Comment, no parameter or correction has been added “by hands” in Eq. (27), which can be transparently obtained by following all the intermediary steps starting from Eq. (1). We regret that that no equivalent of our original evolution Eq. (27) has been proposed in the Comment, nor any numerical simulation with the parameters we had used. This would have helped the author of the Comment to identify unambiguously the intrinsic difference between evanescent and radiative couplings.

#### V. CONCLUSION

In conclusion, we have shown that the Comment has not established any mathematical flaw in our theoretical analysis. All the disagreeing claims in the Comment come from the author’s misunderstanding of the physical system under investigation and confusion between the evanescent and radiative coupling mechanisms. This has led him to propose a theory which is physically incorrect in the context of our study. We stand by our modeling results which are also in excellent agreement with the experiments in Ref. [3].

This work was performed at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with NASA.

[1] L. Deych, *Phys. Rev. A* **84**, 017801 (2011).  
 [2] Y. K. Chembo and N. Yu, *Phys. Rev. A* **82**, 033801 (2010).  
 [3] Y. K. Chembo, D. V. Strekalov, and N. Yu, *Phys. Rev. Lett.* **104**, 103902 (2010).  
 [4] A. Yariv, *Electron. Lett.* **36**, 321 (2000).  
 [5] Y. Dumeige, S. Trebaol, L. Ghisa, T. K. N. Nguyen, H. Tavernier, and P. Féron, *J. Opt. Soc. Am. B* **25**, 2073 (2008).

[6] The coupling is accounted for in the last term of Eq. (6) of the article. This coupling term is proportional to  $\epsilon(\mathbf{r}, \omega, |\mathbf{E}|^2) \simeq n_0^2 + 2n_0\Delta n$  inside the cavity, where  $\Delta n$  stands for the excess index induced by the bulk absorption ( $n_a$ ) and the Kerr nonlinearity ( $n_2$ ) following  $\Delta n = -in_a + n_2n_0\epsilon_0c|\mathbf{E}|^2/2$ . It clearly appears that the effect of absorption and nonlinearity in the *coupling* can be disregarded since  $\Delta n \ll n_0$ . In other words, the efficiency

of the coupling essentially depends on the value of the linear bulk refraction index ( $n_0$ ). Therefore, the linear coupling theory proposed by Yariv and further developed in Ref. [5] is still valid in our case of nonlinear WGM cavity.

- [7] A. Chiasera, Y. Dumeige, P. Féron, M. Ferrari, Y. Jestin, G. Nunzi Conti, S. Pelli, S. Soria, and G. C. Righini, *Laser Photon. Rev.* **4**, 457 (2010).
- [8] A. N. Oraevsky, *Quantum Electron.* **32**, 377 (2002). On page 386: "... the  $Q$ -factor calculated above corresponds to a fantastically huge radiative lifetime. This means that, although the internal reflection is not total, it is close to the total reflection for sufficiently large values of  $n$  (labeled as  $\ell$  in our article). Therefore, WGMs can be treated approximately, but with very high accuracy, as TIR (total internal reflection) waves. The accuracy of this approximation *increases* (emphasis is ours) with  $n$ . For large values of  $n$ , the *nonradiative* (emphasis is ours) losses determine in fact the  $Q$ -factor of the WGM."

Note that Oraevsky derived this conclusion with  $\ell = 2000$ . We typically work with significantly higher values ( $\ell \sim 14\,400$ ), and the accuracy of the aforementioned TIR wave approximation increases *exponentially* with  $\ell$  [see Eq. (2)].

- [9] A. B. Matsko and V. S. Ilchenko, *IEEE J. Sel. Top. Quantum Electron.* **12**, 3 (2006). On page 4: "Radiative coupling is negligible in vast majority of experiments and applications with resonators exceeding few tens of micron in diameter. For example, direct calculation of the radiative  $Q$ -factor yields an enormous value  $10^{73}$  at  $\lambda = 600$  nm for a water droplet with radius  $50\ \mu\text{m}$ . This value is more than 60 orders of magnitude higher than the limit imposed on  $Q$  by light scattering and absorption in best transparent materials. This explains additional restriction on direct radiative coupling: on top of the above cross section limitation, efficiency of such coupling will be further reduced by the ratio  $Q_{\text{att}}/Q_{\text{rad}}$  (attenuation versus radiation  $Q$ s), turning it virtually into zero."