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PLANAR GRAPH HOMOLOGIES AND THE 4-COLOUR THEOREM.

J PILLAY

ABSTRACT. We present herein an alternate representation of Planar Graphs and make use of this to offer insights into the 4-Colour Theorem.

1. INTRODUCTION.

Planar graph theory needs very little in the way of introduction. The importance of a planar graph representation stems from its ability to simplify complex information. In a sense, by virtue of the nature of the representation, these dispose of unnecessary information and expose only the bare necessary on a platform capable of allowing one to make observations or inferences. Many algebras have been developed on the infrastructure provided by matrix algebra providing a platform for use in investigations surrounding chromatic numbers[B]. Similar representations such as the Kneser Graph are further investigated for studying Fractional-Chromatic-Numbers[S-U], here the study extends to general non-planar graphs and hyper Graphs. Recent trends in Graph Theory[1][V] extend into areas as diverse as Engineering, Electronics and even Management Sciences where the study of work delegation is of interest. At this point, an impression is that all available algebra has been exhausted on this one homology. We believe that the time is ripe for the study into the relation (which we believe exists) between a homology and the associated platform it generates. We propose alterations to the existing Planar-Graph-representation that enables one to see symmetry on a grand scale for specific use in chromatic number analysis.

¹Dedicated to my manager Ekaterina. Stakova for much support. I have never been an easy human to manage, however neither is Satyagraha (the principles by which we live.)

Key words and phrases. Graph Theory, Planar Graphs, 4-Colour Theorem.

2. THE THEORY

A planar Graph is a set (V, E) , we offer the morphing $(V, E) \rightarrow (B, L)$ where B is a set of blocks and L a set of conditioned edges simply lines in this system.

Where a set of edges E_n exist between the set of vertices $e_\alpha, \{e_1, \dots, e_n\}$, in our system, we collapse $E_n \mapsto l$, for a single line $l \in L$.

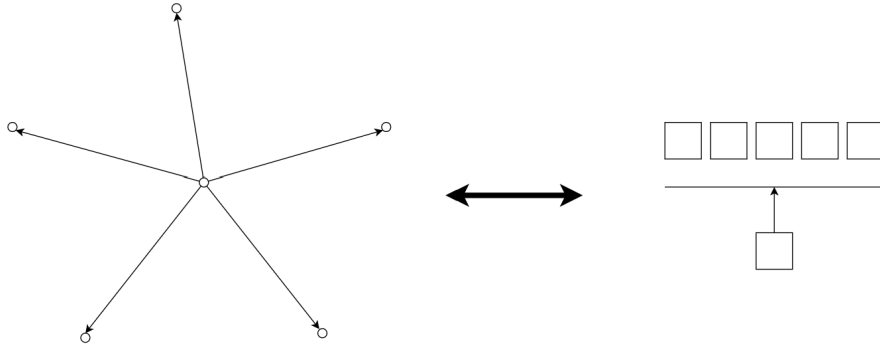


FIGURE 1. Morphism .

We introduce a concept of orientation on L in $S = (B, L)$. Four orientations are made available to $S_L, \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$. These simply signify whether the arrow extends above, below or on either side of a series of blocks b_i .

Definition

Each series of blocks within a single row R_j are allowed at most two line originator elements and orientations $E_{1j} \uparrow, E_{2j} \downarrow$ over a set of elements $b_i \in B$, which of course can be the same element. A V-System is defined to be any system of the form:

$$V_S := \bigcup_{\forall j} (b_{ij} \in B, E_{1j}, E_{2j})$$

Definition

A strategy is defined per block/originator-block as a choice of a set of three surrounding-lines of arbitrary orientations $\cup l$, per block b_i limiting its choice, with the remaining orientation acting as an inhibitor

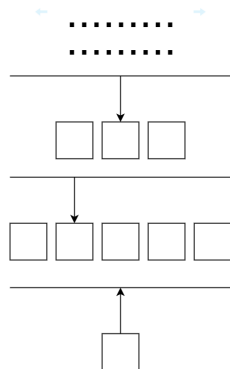


FIGURE 2. V-System .

of choice for an adjacent block instead. Making the choice for every $E_{1j} \uparrow$ per row $R_j \in V_S$ to be an inhibitor, with V_S surrounded by at most four lines (internal or external to V_S), one for each orientation of V_S when treated as a single block $\{\uparrow, \downarrow, \leftarrow, \rightarrow\}$, is defined to be a $(L, T), (R, T)$, strategy depending on whether we make the choice of blocks to be influenced left to right or the other way.

One can easily see that multiple lines extending to a block can be replaced with a single line extending from the block through to the originating elements from which the lines initially extended.

Theorem

Each V-System V_S has one of four *colouring strategies* $C : \{(L, T), (L, B), (R, T), (R, B)\}$ that solve them.

Proof

This easily follows from our preceding discussion.

Lemma

Each V-System V_S can be collapsed to a block b_V having the same conditioning as $b_i \in B \in S$. When composed in this manner. Here again the strategies $\{(L, T), (L, B), (R, T), (R, B)\}$ are availed, and when made use of again enable the further collapsing of $\cup V_S$ to a block. The ability to continue the process tells us that a 4-colour strategy always exists for any planar-graph.

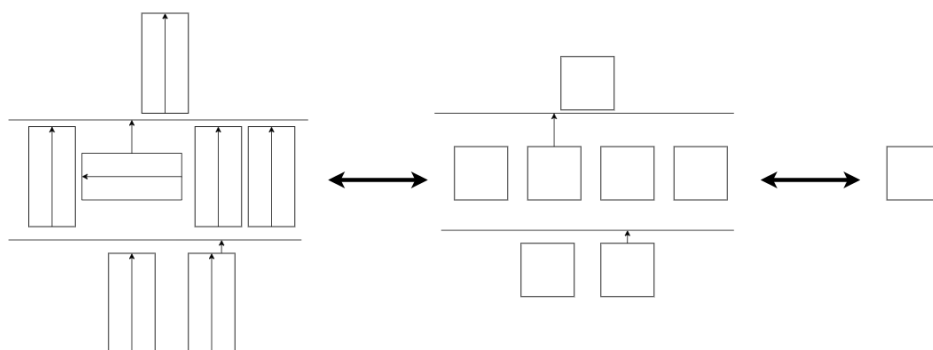


FIGURE 3. Collapsing of V-Systems .

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