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Foliations by closed geodesics of unbounded length

Pierre Mounoud

Abstract

We give examples of compact pseudo-Riemannian manifolds endowed with a foliation \mathcal{F} whose leaves are closed geodesics of unbounded (Riemannian) length. Our examples show that the set of lightlike leaves of \mathcal{F} can either be the whole manifold or a proper subset of it (it cannot be empty by a result of S. Suhr).

1 Introduction

In [5], Sullivan gave the first examples of compact manifolds endowed with a foliation by circles of unbounded length. Sullivan's article contains also a modification by Thurston of his construction. Thurston's example has the additional property of being real-analytic and therefore it can be explicitly given.

Otherwise, Wadsley proved in [6] that the leaves of a foliation by closed Riemannian geodesics have locally bounded length. It implies that there are no Riemannian metrics making Sullivan's or Thurston's examples geodesic. Of course, there exists connections making them geodesic (it is true for any foliation). But are there interesting ones? For example, we could wonder, as Epstein in [1], if there is a flat connection making any of them geodesic.

In this paper, we will focus on pseudo-Riemannian connections. S. Suhr recently proved in [4] that the pseudo-Riemannian analogous of Wadsley's theorem is true for geodesic foliations that do not contain any lightlike leaves. Hence, a geodesic foliation by circles of unbounded length has to have lightlike leaves. With the help of Thurston's example, we prove:

Theorem 1.1 *There exists a smooth foliation by circles of unbounded length, denoted \mathcal{F} , on a smooth compact pseudo-Riemannian manifold (M, g) such that the leaves of \mathcal{F} are geodesics of g . The foliation \mathcal{F} and the metric g can be chosen so that all the leaves of \mathcal{F} are lightlike or so that there exist leaves of \mathcal{F} of any type. Moreover, in the lightlike case \mathcal{F} and g can be chosen to be real-analytic.*

The type changing situation is perhaps the more interesting. To begin with, no example of type changing geodesic foliations by circles was known by us. Moreover, this case is related to the existence of pseudo-Riemannian manifolds all of whose geodesics are closed. Indeed, if (M, g) is a pseudo-Riemannian manifold, then there exists on its tangent bundle TM a pseudo-Riemannian metric \bar{g} (called the Sasaki metric) such that the orbits of the geodesic flow of (M, g) are geodesics of \bar{g} (see [1] p.47). If we were able to find a non-Riemannian manifold (M, g) so that all its geodesics are closed, then (TM, \bar{g}) would have a type changing geodesic foliation by circles.

Furthermore, it is interesting to note that type changing geodesic foliations satisfy what could be called an "anti Wadsley's theorem". We mean that a type changing geodesic foliation is never generated by an action of \mathbf{S}^1 (see proposition 5.1). Applying this result to geodesic flows gives:

Corollary 5.3 *A pseudo-Riemannian manifold having a periodic geodesic flow is Riemannian.*

Hence, if there exists a (non-Riemannian) pseudo-Riemannian manifold all of whose geodesics are closed it has to be quite complicated. Besides, S. Suhr proved in [4] that there are no Lorentzian surfaces all of whose geodesics are closed.

2 Thurston-Sullivan example

We start by recalling Thurston's example of a real-analytic foliation by circles of unbounded length on a compact manifold. The presentation we are giving is elementary but it has the drawback that the geometric ideas behind the construction are hidden. For a geometric description the reader is invited to look at [5] or [1] p. 222.

Thurston's flow lives on the manifold $N \times \mathbf{S}^1 \times \mathbf{S}^1$ where N is the quotient of H the 3-dimensional Heisenberg group,

$$H = \left\{ \begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}, (x, y, z) \in \mathbb{R}^3 \right\},$$

by the left action of the lattice Γ given by

$$\Gamma = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}, (a, b, c) \in \mathbb{Z}^3 \right\}$$

and the circles \mathbf{S}^1 are identified with $\mathbb{R}/2\pi\mathbb{Z}$. We denote by p the projection from $H \times \mathbb{R}^2$ to $N \times \mathbf{S}^1 \times \mathbf{S}^1$. Let (t, u) be the coordinates on \mathbb{R}^2 (thus we have coordinates (x, y, z, t, u) on $H \times \mathbb{R}^2$). Let V_1 and V_2 be the vectorfields on $H \times \mathbb{R}^2$ given by

$$\begin{aligned} V_1 &= \cos(t)\partial_x + \sin(t)(\partial_y + x\partial_z), \\ V_2 &= -\sin(t)\partial_x + \cos(t)(\partial_y + x\partial_z). \end{aligned}$$

We note that the vectorfields $V_1, V_2, \partial_z, \partial_t$ and ∂_u are everywhere independent and that they are invariant by the action of $\Gamma' = \Gamma \times (2\pi\mathbb{Z})^2$. Thus, they define a moving frame on the manifold $N \times \mathbf{S}^1 \times \mathbf{S}^1$.

We will denote by X Thurston's vectorfield on $H \times \mathbb{R}^2$, it is defined by:

$$X = \sin(2u)V_1 + 2\sin^2(u)\partial_t - \cos^2(u)\partial_z.$$

We see that X is also Γ' -invariant.

In what follows, we will denote by the same letter a Γ' -invariant vectorfield on $H \times \mathbb{R}^2$ and its projection on $N \times \mathbf{S}^1 \times \mathbf{S}^1$.

There is an easy way to see that the foliation spanned by X on $N \times \mathbf{S}^1 \times \mathbf{S}^1$ has closed leaves of unbounded length. On $U = \{(x, y, z, t, u) \in H \times \mathbb{R}^2, u \not\equiv 0 \pmod{\pi}\}$ we define the vectorfield W by

$$W = \frac{1}{2\sin^2(u)}X = \frac{1}{\tan(u)}V_1 + \partial_t - \frac{1}{2\tan^2(u)}\partial_z.$$

We can easily compute φ , the flow of W :

$$\varphi_s(x, y, z, t, u) = \left(\frac{\sin(t+s) - \sin(t)}{\tan(u)} + x, \frac{\cos(t) - \cos(t+s)}{\tan(u)} + y, \right. \\ \left. z + \frac{\sin(2t) - \sin(2t+2s)}{4\tan^2(u)} + \frac{(\cos(t+s) - \cos(t))(\sin(t) + x\tan(u))}{\tan^2(u)}, t + s, u \right).$$

Hence, it is clear that the projection of W on $N \times \mathbf{S}^1 \times \mathbf{S}^1$ is a vectorfield defined on $p(U)$ which has a 2π -periodic flow. It follows that the integral curves of X on $p(U)$ are all closed and that their length is unbounded (if we choose a Riemannian metric α such that $\alpha(X, X) = 1$, their length is equal to $2\pi/\sin^2(u)$). To conclude, we need to check that the integral curves on $N \times \mathbf{S}^1 \times \mathbf{S}^1 \setminus p(U)$ are also closed. It is true because on this closed subset (known as the “bad set“) $X = \partial_z$ and because there is an element of Γ acting by translation along the coordinate z .

In what follows, we will principally use a smooth deformation of X , denoted X_ξ , instead of X itself. This vectorfield is defined by:

$$X_\xi = 2\xi(u) \cos(u)V_1 + 2\xi^2(u)\partial_t - \cos^2(u)\partial_z,$$

where $\xi : \mathbb{R} \rightarrow \mathbb{R}$ is the smooth, 2π -periodic function defined by $\xi(u) = e^{-1/\sin^2(u)}$ if $0 < u < \pi$ and by $\xi(u) = -e^{-1/\sin^2(u)}$ if $-\pi < u < 0$. We remark that $\xi^{(k)}(n\pi) = 0$ for any $k \in \mathbb{N}$ and any $n \in \mathbb{Z}$, consequently $|\xi|$ is also smooth.

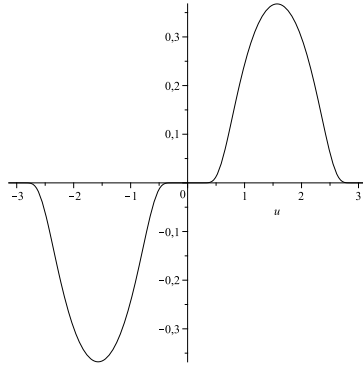


Figure 1: The graph of ξ

As above, we define W_ξ as $\frac{1}{2\xi^2}X_\xi$. The flow of W_ξ is very similar to the flow of W , in particular it is also periodic, therefore X_ξ also generates a foliation by circles with unbounded length. This foliation is no more real-analytic.

3 The lightlike construction

In this section, we construct a real-analytic Lorentzian metric g on $N \times \mathbf{S}^1 \times \mathbf{S}^1$ such that $D_X X = 0$ (where D is the Levi-Civita connection of g) and $g(X, X) = 0$ everywhere. First, we remark that the vectorfields X , V_1 , V_2 , $2\partial_t + \partial_z$ and ∂_u are everywhere independent and therefore they define a moving frame on $N \times \mathbf{S}^1 \times \mathbf{S}^1$. Then, we define g as the metric which is given in the moving frame $(X, \partial_u, V_1, V_2, 2\partial_t + \partial_z)$ by the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

This is clearly a real-analytic Lorentzian metric. The 1-form X^\flat (defined by $X^\flat(v) = g(X, v)$) is du and therefore is closed. For any vectorfield Z , we have:

$$\begin{aligned} 0 = dX^\flat(X, Z) &= X \cdot g(X, Z) - W \cdot g(X, X) - g(X, [X, Z]) \\ &= g(D_X X, Z) + g(D_Z X, X) = g(D_X X, Z) + \frac{1}{2} Z \cdot g(X, X) \\ &= g(D_X X, Z). \end{aligned}$$

Hence $D_X X = 0$.

Let us remark that the Riemannian metric on the distribution spanned by $\{V_1, V_2, 2\partial_t + \partial_z\}$ can be replaced by any pseudo-Riemannian metric. As there is so many metrics making X geodesic, we may wonder if there is a best one (as for example a flat one).

4 The type changing construction

In this section, we will provide a metric g such that $D_{X_\xi} X_\xi = 0$ and such that $g(X_\xi, X_\xi)$ changes signs. We start the construction in a neighborhood of the bad set i.e. for u close to $0 \pmod{\pi}$. As above we use a moving frame to write the metric. Let g_0 be the symmetric 2-tensorfield whose expression in the moving frame $(\partial_t, V_1, V_2, \partial_z, \partial_u)$ is given by the following matrix:

$$G_0 = \begin{pmatrix} \frac{1}{4} \cos^4(u) & \frac{|\xi(u)|}{2 \cos(u)} & 0 & -\frac{\xi(u)|\xi(u)|}{\cos^2(u)} + \frac{1}{2} \cos^2(u) \xi^2(u) & 0 \\ \frac{|\xi(u)|}{2 \cos(u)} & 0 & \frac{1}{2} & \frac{|\xi(u)| \xi^2(u)}{\cos^3(u)} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{\xi(u)}{\cos(u)} & 0 \\ -\frac{\xi(u)|\xi(u)|}{\cos^2(u)} + \frac{1}{2} \cos^2(u) \xi^2(u) & \frac{|\xi(u)| \xi^2(u)}{\cos^3(u)} & \frac{\xi(u)}{\cos(u)} & \xi^4(u) & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

For $u = 0 \pmod{\pi}$, it is easy to verify that this matrix defines a symmetric bilinear form of signature $(3, 2)$ (in particular non degenerate). It follows by continuity that there exists $\eta > 0$ such that g_0 is a well defined pseudo-Riemannian metric of signature $(3, 2)$ on $N \times \mathbf{S}^1 \times]-\eta, \eta[$ and on $N \times \mathbf{S}^1 \times]\pi - \eta, \pi + \eta[$.

Applying G_0 to $(2\xi^2(u), 2\xi(u) \cos(u), 0, -\cos^2(u), 0)$, we get $(2\xi(u)|\xi(u)|, 0, 0, 0, \cos^2(u))$. It implies that $g(X_\xi, X_\xi) = 4\xi^3(u)|\xi(u)|$ (and therefore $g(W_\xi, W_\xi) = \frac{|\xi(u)|}{\xi(u)} = \pm 1$) and that

$$X_\xi^\perp = \text{span} \{V_1, V_2, \partial_z, -\cos^2(u) \partial_t + 2\xi(u)|\xi(u)| \partial_u\}.$$

At this time, the metric g_0 is only defined on a neighborhood of the bad set. In order to extend it, we write it in the moving frame $(W, V_1, V_2, \partial_z, -\cos^2(u) \partial_t + 2\xi(u)|\xi(u)| \partial_u)$. It reads:

$$\begin{pmatrix} \frac{|\xi(u)|}{\xi(u)} & 0 \\ 0 & L(u) \end{pmatrix},$$

where

$$L(u) = \begin{pmatrix} 0 & \frac{1}{2} & \frac{|\xi(u)| \xi^2(u)}{\cos^3(u)} & -\frac{1}{2} \cos(u) |\xi(u)| \\ \frac{1}{2} & 0 & \frac{\xi(u)}{\cos(u)} & 0 \\ \frac{|\xi(u)| \xi^2(u)}{\cos^3(u)} & \frac{\xi(u)}{\cos(u)} & \xi^4(u) & -\frac{1}{2} \cos^4(u) \xi^2(u) - 2\xi(u) |\xi(u)| \\ -\frac{1}{2} \cos(u) |\xi(u)| & 0 & -\frac{1}{2} \cos^4(u) \xi^2(u) - 2\xi(u) |\xi(u)| & \frac{1}{4} \cos^8(u) + 4\xi^4(u) \end{pmatrix}.$$

When $u \in]0, \eta[\cup]\pi - \eta, \pi[$, the matrix $L(u)$ lies in the space on bilinear forms of signature $(2, 2)$ i.e. in $\text{SL}(4, \mathbb{R})/\text{SO}(2, 2)$. As this space is a connected manifold there exists a smooth map $M :]0, \pi[\rightarrow \text{SL}(4, \mathbb{R})/\text{SO}(2, 2)$ such that M and L coincide on $]0, \eta/2[\cup]\pi - \eta/2, \pi[$. Let g_1 be the metric on $N \times \mathbf{S}^1 \times]0, \pi[$ whose matrix in the moving frame $(W, V_1, V_2, \partial_z, -\cos^2(u)\partial_t + 2\xi(u)|\xi(u)|\partial_u)$ is $\begin{pmatrix} 1 & 0 \\ 0 & M(u) \end{pmatrix}$.

When $u \in]-\pi, -\pi + \eta[\cup]-\eta, 0[$, the matrix $L(u)$ lies in the space on bilinear forms of signature $(3, 1)$ i.e. in $\text{SL}(4, \mathbb{R})/\text{SO}(3, 1)$. As $\text{SL}(4, \mathbb{R})/\text{SO}(3, 1)$ is a connected manifold there exists a smooth map $N :]-\pi, 0[\rightarrow \text{SL}(4, \mathbb{R})/\text{SO}(3, 1)$ such that N and L coincide on $] -\pi, -\pi + \eta/2[\cup]-\eta/2, 0[$. Let g_2 be the metric on $N \times \mathbf{S}^1 \times]-\pi, 0[$ whose matrix in the moving frame $(W, V_1, V_2, \partial_z, -\cos^2(u)\partial_t + 2\xi(u)|\xi(u)|\partial_u)$ is $\begin{pmatrix} -1 & 0 \\ 0 & N(u) \end{pmatrix}$.

We can now give the metric, we are looking for. By construction the metrics g_0, g_1 and g_2 glue together in a smooth metric g on $M \times \mathbf{S}^1 \times \mathbf{S}^1$. Moreover on $p(U) = N \times \mathbf{S}^1 \times (]-\pi, 0[\cup]0, \pi[)$, we have:

$$g(W_\xi, W_\xi) = \pm 1$$

$$X_\xi^\perp = \text{span}\{V_1, V_2, \partial_z, -\cos^2(u)\partial_t + 2\xi(u)|\xi(u)|\partial_u\}$$

We have to prove now that the foliation generated by X_ξ is geodesic. It will be done with the help of the following facts:

Fact 4.1 $D_{X_\xi}X_\xi = 0$ on $N \times \mathbf{S}^1 \times \mathbf{S}^1$ if and only if $D_{W_\xi}W_\xi = 0$ on $p(U)$.

Proof As $X_\xi = 2\xi^2(u)W$ and $X_\xi \cdot \xi^2(u) = 0$, we have $D_{X_\xi}X_\xi = 4\xi^4(u)D_{W_\xi}W_\xi$ therefore the equivalence is clear on $p(U)$. But $p(U)$ is dense in $N \times \mathbf{S}^1 \times \mathbf{S}^1$ and $D_{X_\xi}X_\xi = 0$ on $p(U)$ is also equivalent to $D_{X_\xi}X_\xi = 0$ on the whole manifold. \square

Koszul's formula entails easily the next fact, the proof of which is left to the reader.

Fact 4.2 Let Z be a vectorfield on a pseudo-Riemannian manifold (M, g) such that $g(Z, Z) = \pm 1$. We have $D_Z Z = 0$ if and only if Z preserves its orthogonal distribution.

As W_ξ^\perp is spanned by $V_1, V_2, \partial_z, -\cos^2(u)\partial_t + 2\xi(u)|\xi(u)|\partial_u$, facts 4.1 and 4.2 imply:

Fact 4.3 $D_{X_\xi}X_\xi = 0$ if and only if

$$g(W_\xi, [W_\xi, V_1]) = g(W_\xi, [W_\xi, V_2]) = g(W_\xi, [W_\xi, \partial_z]) = g(W_\xi, [W_\xi, -\cos^2(u)\partial_t + 2\xi(u)|\xi(u)|\partial_u]) = 0.$$

Computing the brackets above, we find:

$$\begin{aligned} [W_\xi, V_1] &= V_2, \\ [W_\xi, V_2] &= \frac{\cos(u)}{\xi(u)}\partial_z - V_1, \\ [W_\xi, \partial_z] &= 0, \\ [W_\xi, Y] &= -2\xi(u)|\xi(u)|\partial_u \cdot \frac{\cos(u)}{\xi(u)}V_1 + \frac{\cos^3(u)}{\xi(u)}V_2 + \xi(u)|\xi(u)|\partial_u \cdot \frac{\cos^2(u)}{\xi^2(u)}\partial_z, \end{aligned}$$

where $Y = -\cos^2(u)\partial_t + 2\xi(u)|\xi(u)|\partial_u$. All these brackets are tangent to W_ξ^\perp therefore, by fact 4.3, $D_{X_\xi}X_\xi = 0$.

It is not difficult to modify the metric g in order to have $D_X X = 0$ (i.e. to make Thurston's flow geodesic and not a smooth modification of it). At the beginning of the construction, we just have to replace the function ξ by the function sinus and to remove the absolute values. The metric becomes real-analytic in the neighborhood of the bad set but is only smooth globally (the gluing process is unchanged). Unfortunately, in that case we have $g(X, X) \geq 0$, the vectorfield X is nowhere timelike.

To conclude this section, we prove that X_ξ preserves the volume form of g i.e. that $\operatorname{div} X_\xi = 0$. We do it because the geodesic flow of any pseudo-Riemannian is divergence free.

Proposition 4.4 *The vectorfield X_ξ is divergence free.*

Proof. We denote the volume form associated to g by ω_g . We want to compute $\mathcal{L}_{X_\xi} \omega_g = d(i_{X_\xi} \omega_g)$. We see that, if S and T are two vectorfields chosen among $\partial_t, V_1, V_2, \partial_z$ and ∂_u , then $[S, T]$ belongs to $\operatorname{span}\{V_1, V_2, \partial_z\}$. It means that all the "brackets terms" in the computation of $d(i_{X_\xi} \omega)$ ($\partial_t, V_1, V_2, \partial_z, \partial_u$) vanish. Moreover, the function $\omega_g(\partial_t, V_1, V_2, \partial_z, \partial_u)$ is clearly constant on the submanifolds $N \times \mathbf{S}^1 \times \{u\}$. Therefore:

$$d(i_{X_\xi} \omega)(\partial_t, V_1, V_2, \partial_z, \partial_u) = \partial_u \cdot \omega_g(X_\xi, \partial_t, V_1, V_2, \partial_z) = 0.$$

5 Geodesic foliations with leaves of bounded length

As we said in the introduction, not only the leaves of a type changing geodesic foliation by circles can have unbounded length but they have to. It is actually a consequence of the results of Wadsley ([6]) and Suhr ([4]).

Proposition 5.1 *Let \mathcal{F} be an oriented 1-dimensional geodesic foliation on a pseudo-Riemannian manifold (M, g) . If the leaves of \mathcal{F} are circles with locally bounded length then they all have the same type.*

Proof. For any $x \in M$ we denote by F_x the leaf of \mathcal{F} containing x . According to Wadsley's theorem (see [6]), there exists a vectorfield Z tangent to \mathcal{F} such that the flow of Z is 2π -periodic (i.e. the leaves of \mathcal{F} are the orbits of an action of \mathbf{S}^1). We endow M with an auxiliary Riemannian metric α , such that $\alpha(Z, Z) = 1$. Hence, for any $x \in M$, the length of F_x divide 2π .

Let $U = \{x \in M, g(Z, Z) \neq 0\}$, it is an open *saturated* (i.e. a union of leaves of \mathcal{F}) subset of M . We can suppose that U is not empty (otherwise there is nothing to prove). Let Z_0 be the vectorfield on U defined by $Z_0 = 1/\sqrt{|g(Z, Z)|} Z$. Let U_0 be a connected component of U . According to [4], there exists $T > 0$ such that ϕ , the flow of Z_0 , is T -periodic on U_0 . We define a function ν on U_0 by

$$\nu(x) = \int_0^T \sqrt{\alpha_{\phi(x,t)}(Z_0, Z_0)} dt.$$

For any $x \in U_0$, $\nu(x)$ is a multiple of the length of F_x , therefore there exist $n(x) \in \mathbb{Q}$ such that $\nu(x) = 2\pi n(x)$. The function ν being continuous it follows that $n(x)$ is constant on U_0 . But when x tends to a point of ∂U_0 , $g(Z, Z)$ tends to 0 uniformly on F_x and therefore $\nu(x)$ tends to $+\infty$. Hence $\partial U_0 = \emptyset$ i.e. $U = M$ and \mathcal{F} does not change type. \square

Hence, the foliation generated by X_ξ is probably not far from being the simplest type changing geodesic foliation by circles on a compact manifold. However, it is perhaps possible to find a 4-dimensional example using the example of Epstein and Vogt (see [3]). For lower dimensions, we have:

Corollary 5.2 *If (M, g) is a 2 or 3 dimensional compact pseudo-Riemannian manifold and if \mathcal{F} is a geodesic foliation by circles then the leaves of \mathcal{F} do not change type.*

Proof. It is an immediate consequence of proposition 5.1 and Epstein's theorem (see [2]) saying that the leaves of a foliation by circles on a compact 3 dimensional manifold have bounded length.

□

As announced in the introduction, proposition 5.1 implies that

Corollary 5.3 *A pseudo-Riemannian manifold having a periodic geodesic flow is Riemannian.*

Proof. It is well known that the geodesic flow of a pseudo-Riemannian manifold (M, g) is a geodesic foliation of TM endowed with the Sasaki metric (see [1], [4]). It follows from proposition 5.1 that if the geodesic flow of (M, g) is periodic then it does not change type. But as it takes all the possible types, it means that g is Riemannian. □

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