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Dynamical Analysis of Compositions

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Abstract This paper analyzes musical opus from the point of view of two mathematical tools, namely the entropy and the multidimensional scaling (MDS). The Fourier analysis reveals a fractional dynamics, but the time rhythm variations are diluted along the spectrum. The combination of time-window entropy and MDS copes with the time characteristics and is well suited to treat a large volume of data. The experiments focus on a large number of compositions classified along three sets of musical styles, namely "Classical", "Jazz" and "Pop & Rock" compositions. Without lack of generality, the present study describes the application of the tools and the sets of musical compositions in a methodology leading to clear conclusions, but extensions to other possibilities are straightforward. The results reveal significant differences in the musical styles, demonstrating the feasibility of the proposed strategy and motivating further developments towards a dynamical analysis of musical compositions.

Keywords: Dynamics · Musical compositions · Fractional dynamics · Entropy · Multidimensional scaling.

1 Introduction

For many centuries, philosophers, music composers and mathematicians worked hard to find mathematical formulae that could explain the process of music creation. As a matter of fact, music and mathematics are intricately related: strings vibrate at certain frequencies and sound waves can be described by mathematical equations. Although it seems not possible to find some "equations" that will model all music works, it is true that there are certain inherent mathematical structures in all music types. Through the history of music, we have been faced with the proposal of formal techniques for melody composition, claiming that musical pieces could be created as a result of applying certain rules to some given initial material [1-12]. More recently, the exponential growth of computing power made it possible to generate music automatically. In this line of thought, knowing that the overall process involves not only the musical production itself, but also the corresponding "resonance" produced in the human listener, we can establish that methodologies well defined in the context of dynamical systems, may prove to be valuable tools for studying this phenomenon. While the process involves both physical and mental phenomena, the synergies of both can define new frontiers in "system dynamics". It is recognized that studying new phenomena, often involving conceptual variables not clearly defined, pose complex scientific challenges, requiring adequate methodologies for obtaining substantive quantitative conclusions. Therefore, in this paper are adopted three mathematical formalisms, namely Fractional Calculus (FC), entropy and Multidimensional Scaling (MDS).

FC goes back to the beginning of the theory of differential calculus and deals with the generalization of standard integrals and derivatives to a non-integer, or even complex order. Leibniz, in a letter to L`Hôpital (1695), raised the question "Can the meaning of derivatives with integer order be generalized to derivatives with non-integer orders?" and later noticed "It will lead to a paradox, from which one day useful consequences will be drawn". Many important mathematicians contributed to the FC theory over the years, among them Liouville, Riemann, Weyl, Fourier, Abel, Leibniz, Grünwald and Letnikov. A wide range of potential fields of application are possible by bringing to a broader paradigm concepts of physics and

engineering and FC emerged as a remarkable mathematical tool for analysing and modelling systems and signals with long range memory and power law behaviour [13-30].

The concept of entropy was introduced in the field of thermodynamics by Clausius (1862) and Boltzmann (1896) and was later applied by Shannon (1948) and Jaynes (1957) to information theory. However, recently more general entropy measures have being proposed, allowing the relaxation of the additivity axiom for application in several types of complex systems. The novel ideas are presently under a large development and open up promising perspectives [31-45].

MDS has its origins in psychometrics and psychophysics, where it is used as a tool for perceptual and cognitive modeling. From the beginning MDS has been applied in many fields, such as psychology, sociology, anthropology, economy, educational research, etc. In the last decades this technique has been applied also in others areas, including computational chemistry, machine learning, concept maps and wireless network sensors [46-49].

Bearing these facts in mind, the present study combines both concepts and is organized as follows. Section 2 introduces a brief description of the fundamental concepts and methods. Section 3 formulates and develops the musical study through a time window entropy calculation and multidimensional scaling analysis association. Finally, section 4 outlines the main conclusions.

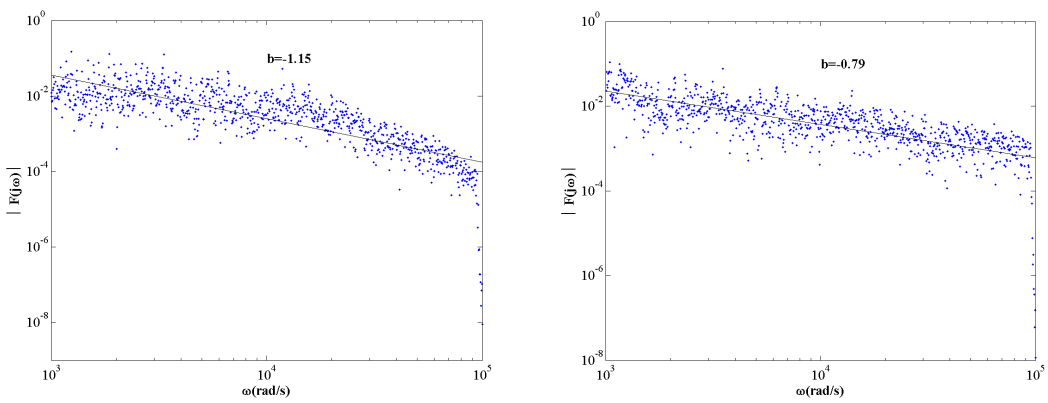
2 Concepts and Methods

This section presents the main tools adopted in this study, namely the musical signal and Fourier analysis, the entropy and the multidimensional scaling.

2.1 Signal, Fourier Transform and Fractional Dynamics

In the context of this study, a musical work is a set of one or more time sequenced digital data streams, representing a certain time sampling of the original musical source. For all musical works used, the original data streams resulted from sampling at 44 kHz and were subsequently converted to a single (mono) digital data stream, each sample being a 32 bit signed floating value.

These signals have a strong variability which makes difficult their direct comparison in the time domain. The Fourier spectrum reveals that they have characteristics close to those encountered in fractional systems. For example, Fig. 1 depicts the amplitude versus frequency, $|F(j\omega)|$, of seven musical compositions. For analyzing their dynamics the power law trendline $|F(j\omega)| = a\omega^b$, $a \in \Re^+$, $b \in \Re$, was superimposed for each case, leading to a good fit. We verify the non-integer values of b and the fractional behavior is, therefore, clear, but the Fourier transform poses some limitations since it is a tool that distributes the time signal variations through the frequency domain. Bearing these ideas in mind, in the sequel are adopted methods that establish a compromise between smoothing the high signal variability and handling the rhythm and style time evolution that are the essence of each composition.



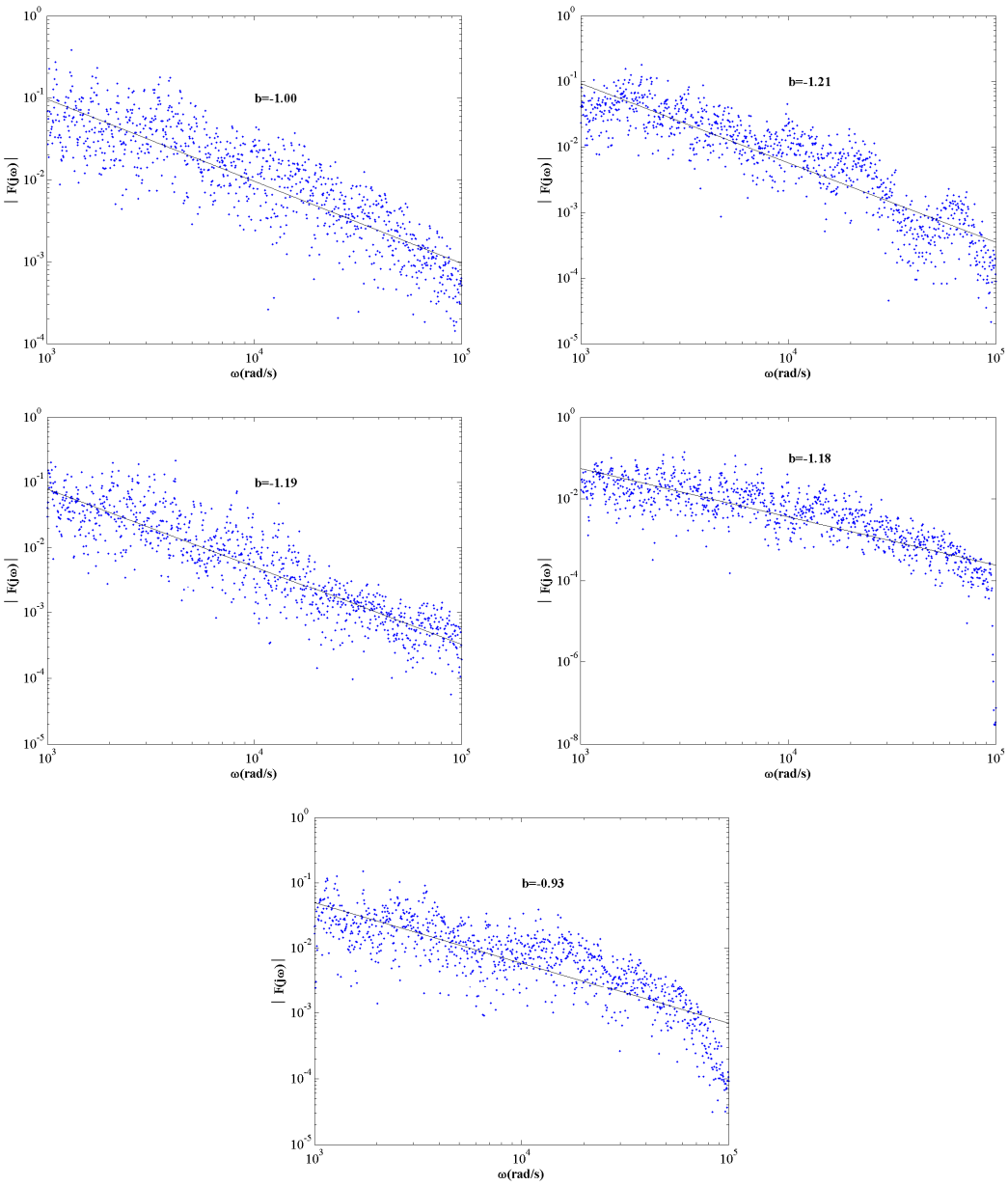


Fig. 1. Fourier spectrum (amplitude versus frequency) and power law trendline approximations of seven musical compositions: Albinoni: "*Adagio in G Minor*" $(b=-1.15)$, Bee Gees: "*Saturday Night Fever*" $(b=-0.79)$, Coldplay: "*Viva La Vida*" $(b=-1.00)$, Ella Fitzgerald: "*Night And Day*" $(b=-1.21)$, Stan Getz and Astrud Gilberto: "*Corcovado*" $(b=-1.19)$, Mozart: "*Kleine Nachtmusik KV525 Allegro"* $(b=-1.18)$ and The Beatles: "*Yellow Submarine*" $(b=-0.93)$

2.2 Entropy

Khinchin formulated four axioms for an information measure *H,* yielding an ordinary Boltzmann-Gibbs statistical mechanics. The axioms consist in:

$$H = H(p_1, \cdots, p_W) \tag{1}$$

$$H(W^{-1}, \cdots, W^{-1}) \geq H(p_1, \cdots, p_W) \tag{2}$$

$$H(p_1,\cdots,p_W)=H(p_1,\cdots,p_W,0) \tag{3}$$

$$H\left(\left\{p_{ij}^{A,B}\right\}\right)=H\left(\left\{p_i^A\right\}\right)+\sum_i p_i^A H\left(\left\{p_i^B(i\mid j)\right\}\right) \tag{4}$$

where W is the number of possible events and p_i is the probability that event i occurs, so that $\sum_{i=1}^{W} p_i = 1$.

Axiom 1 states that H only depends on the probabilities p_i, $i=1,\cdots,W$. Axiom 2 indicates that H takes a maximum for the uniform probability distribution (all probabilities are $p_i=\frac{1}{W}$). Axiom 3 says that H does not change if the sample set is enlarged by another event with zero probability. Axiom 4 postulates that given two systems A and B, not necessary independent, H should be independent of the way information is collected. When systems A and B are independent, it results $p_{ij}^{A,B}=p_i^A p_j^B$ and axiom 4 reduces to the rule of additivity of information:

$$H\left(\left\{p_{ij}^{A,B}\right\}\right)=H\left(\left\{p_i^A\right\}\right)+H\left(\left\{p_j^B\right\}\right) \tag{5}$$

This condition is less stringent than axiom (4) and states the entropy of independent systems should be additive.

The most celebrated entropy is the so-called Shannon entropy S defined by:

$$S=-\sum_{i=1}^{W} p_i \ln(p_i) \tag{6}$$

that satisfies the four Khinchin axioms (1)-(4).

Two of the most studied generalizations of the entropy are the Rényi and Tsallis entropies given by:

$$S_q^{(R)}=\frac{1}{1-q}\ln\left(\sum_{i=1}^{W} p_i^{\,q}\right), q>0 \tag{7}$$

$$S_q^{(T)}=\frac{1}{q-1}\left(1-\sum_{i=1}^{W} p_i^{\,q}\right) \tag{8}$$

that reduce to the Shannon entropy when $q\to 1$.

Recently M. Ubriaco [42] proposed the fractional entropy:

$$S_q^{(U)}=\sum_{i=1}^{W}(-\ln p_i)^q p_i \tag{9}$$

that has the same properties as the Shannon entropy, except additivity.

Several other measures were proposed such as the Landsberg-Vedral, Abel, Kaniagakis and Sharma-Mittal entropies [40].

2.3 Multidimensional Scaling

MDS is a generic name for a family of algorithms that construct a configuration of points in a low dimensional space from information about inter-point distances measured in high dimensional space. The new geometrical configuration of points, which preserves the proximities of the high dimensional space, facilitates the perception of data's underlying structure and often makes it much easier to understand. The problem addressed by MDS can be stated as follows: given n items in a m–dimensional space and an $n \times n$ matrix of proximity measures among the items, MDS produces a p-dimensional configuration X, $p \le m$, representing the items such that the distances among the points in the new space reflect, with some degree of fidelity, the proximities in the data. The proximity measures the (dis)similarities among the items, and in general, it is a distance measure: the more similar two items are, the smaller their distance is.

The Minkowski distance metric provides a general way to specify distance for quantitative data in a multidimensional space:

$$d_{ij}=\left(\sum_{k=1}^{l} w_k \left|x_{ik}-x_{jk}\right|^r\right)^{1/r} \tag{10}$$

where l is the number of dimensions, x_{ik} is the value of dimension k for object i and w_k is a weight. For $w_k = 1$, with $r = 2$, the metric equals the Euclidean distance metric, while $r = 1$ leads to the city-block (or Manhattan) metric. In practice, the Euclidean distance metric is generally used, but there are several other definitions that can be applied, including for binary data [50].

Typically MDS is used to transform the data into two or three dimensions for visualizing the result to uncover data's hidden structure, but any $p < m$ is also possible. A rule of thumb to determine the maximum number of m is to ensure that there are at least twice as many pairs of items then the number of parameters to be estimated, resulting in $m \geq 4p + 1$ [51]. The geometrical representation obtained with MDS is indeterminate with respect to translation, rotation, and reflection [52].

There are two forms of MDS, namely the metric MDS and the nonmetric MDS. The metric MDS uses the actual values of dissimilarities, while nonmetric MDS effectively uses only their ranks [53, 54]. Metric MDS assumes that the dissimilarities δ_{ij} calculated in the original m–dimensional data and distances d_{ij} in the p–dimensional space are related as follows:

$$d_{ij} \approx f\left(\delta_{ij}\right) \tag{11}$$

where f is a continuous monotonic function. Metric (scaling) refers to the type of transformation f of the dissimilarities and its form determines the MDS model. If $d_{ij} = \delta_{ij}$ (it means $f(x) = x$) and a Euclidean distance metric is used we obtain the classical (metric) MDS.

In metric MDS the dissimilarities between all objects are known numbers and they are approximated by distances. Thus objects are mapped into a low dimensional space, distances are calculated, and compared with the dissimilarities. Then objects are moved in such way that the fit becomes better, until an objective function (called stress function in the context of MDS) is minimized. In non-metric MDS, the metric properties of f are relaxed but the rank order of the dissimilarities must be preserved. The transformation function f must obey the monotonicity constraint $\delta_{ij} < \delta_{rs} \Rightarrow f(\delta_{ij}) \leq f(\delta_{rs})$ for all objects. The advantage of nonmetric MDS is that no assumptions need to be made about the underlying transformation function f. Therefore, it can be used in situations that only the rank order of dissimilarities is known (ordinal data). Additionally, it can be used in cases which there are incomplete information. In such cases, the configuration X is constructed from a subset of the distances, and, at the same time, the other (missing) distances are estimated by monotonic regression. In nonmetric MDS it is assumed that $d_{ij} \approx f(\delta_{ij})$, therefore $f(\delta_{ij})$ are often referred as the disparities [55-57] in contrast to the original dissimilarities δ_{ij}, on one hand, and the distances d_{ij} of the configuration space on the other hand. In this context, the disparity is a measure of how well the distance d_{ij} matches the dissimilarity δ_{ij}.

With further developments over the years, MDS techniques are commonly classified according to the type of data to analyze. From this point of view, the techniques are embedded into the following MDS categories: *i*) one–way *versus* multi–way: In K–way MDS each pair of objects has K dissimilarity measures from different replications (e.g., repeated measures); *ii*) one–mode *versus* multi–mode: Similar to *i*) but the K dissimilarities are qualitatively different (e.g., distinct experimental conditions).

There is no rigorous statistical method to evaluate the quality and the reliability of the results obtained by an MDS analysis. However, there are two methods often used for that purpose: The Shepard plot and the stress. The Shepard plot is a scatter plot of the dissimilarities and disparities against the distances, usually overlaid with a line having unitary slope. The plot provides a qualitative evaluation of the goodness of fit. On other hand, the stress value gives a quantitative evaluation. Additionally, the stress plotted as a function of dimensionality can be used to estimate the adequate p–dimension. When the curve ceases to decrease significantly the resulting "elbow" may correspond to a substantial improvement in fit.

Beyond the aspects referred before, there are other developments of MDS that includes Procrustean methods, individual differences models (also known as three-way models), and constrained configuration.

In the Procrustean methods the data is analyzed by scaling each replication separately and then comparing or aggregating the different MDS solutions. The individual differences models scale a set of K dissimilarity matrices into only one MDS solution. The procedure of constraints on the configuration (which Borg and Groenen called confirmatory MDS [58]) is used when the researcher has some substantive underlying theory regarding a decomposition of the dissimilarities and consequently tries to restrain the configuration space.

3 Musical Study

This section formulates and develops the musical study by means of entropy measures applied to a large sample of representative musical works and a subsequent multidimensional scaling analysis performed on the correlation of those entropy measures.

3.1 Entropy Analysis of Musical Compositions

Fig. 2 shows several "entropy curves" of the musical composition "*Adagio in G Minor*", erroneously attributed to Tomaso Albinoni but actually composed by Remo Giazotto. By using the entropy formulations of Shannon, Rényi, Tsallis and Ubriaco, and applying it to the musical signal (a stream of 32 bit data) with a predefined window duration T, a sequence of time-stamped real values is obtained, producing a particular entropy curve that depends on the musical composition, the entropy formula and the time sampling duration. Despite variations in the entropy formulas, it is perceptible that the curves are qualitatively similar. The $T = 1$ s rectangular window duration was used because it is a good compromise between the original signal's frequency (tenths of microseconds) and the musical piece's duration (hundreds of seconds).

A first set of experiments demonstrated that, as expected, the larger the value of T the smother the curve, at the expenses of loosing the instantaneous analysis in favor of an average characterization. Given the sensing capabilities of the human being it was considered that $T = 1$ s was a good compromise between the different criteria. A second set of experiments analyzed the effect of changing the order q in the Rényi, Tsallis and Ubriaco entropies. It was observed that the curves for $q < 1$ and $q > 1$ varied relatively to the plot for order $q = 1$ (equivalent to the Shannon formulation), but maintaining an identical pattern. Therefore, in the sequel are considered only the orders $q = \{1/2, 2\}$ for illustrating the characteristics of each entropy definition.

In Figures 1-3 all entropy curves consist in sequences of time-stamped real values. The horizontal axis represents time and is labeled at each 60 seconds. The vertical axis represents entropy.

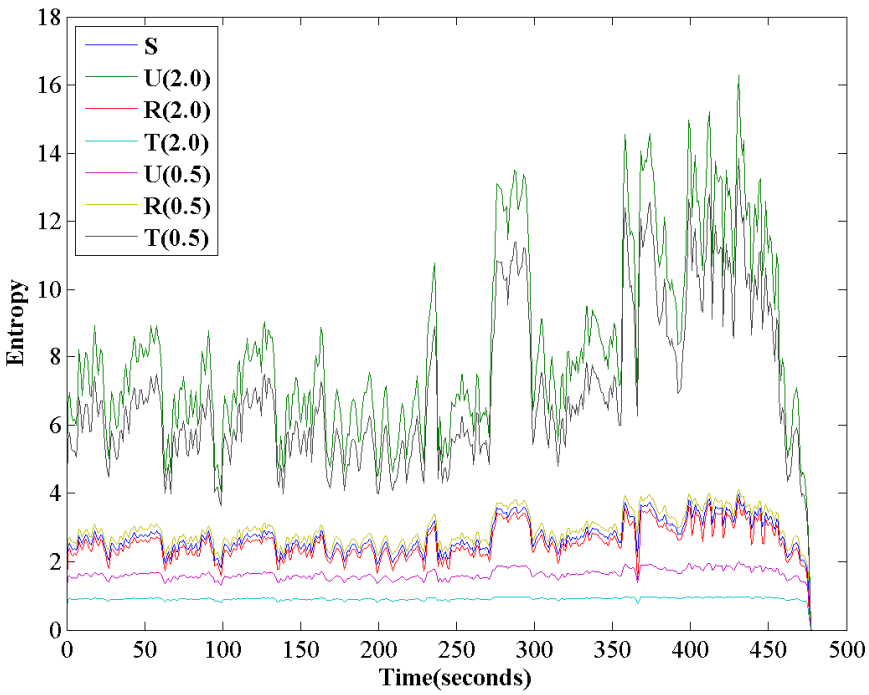


Fig. 2. Entropies S (blue trace) and $S_q^{(R)}, S_q^{(T)}, S_q^{(U)}$ for $q = \{1/2, 2\}$ versus time of Albinoni's *"Adagio in G Minor"* composition using a sampling rate of $T = 1$ second. S – Shannon; R(q) – Rényi for order q; T(q) – Tsallis for order q; U(q) – Ubriaco for order q

Several entropy curves of Wolfgang Mozart's composition "*Kleine Nachtmusik KV525 Allegro*" are shown in Fig. 3. The musical signal was subjected to entropy calculation with a $T = 1$ second sampling duration. Again it is perceptible that the curves are qualitatively similar.

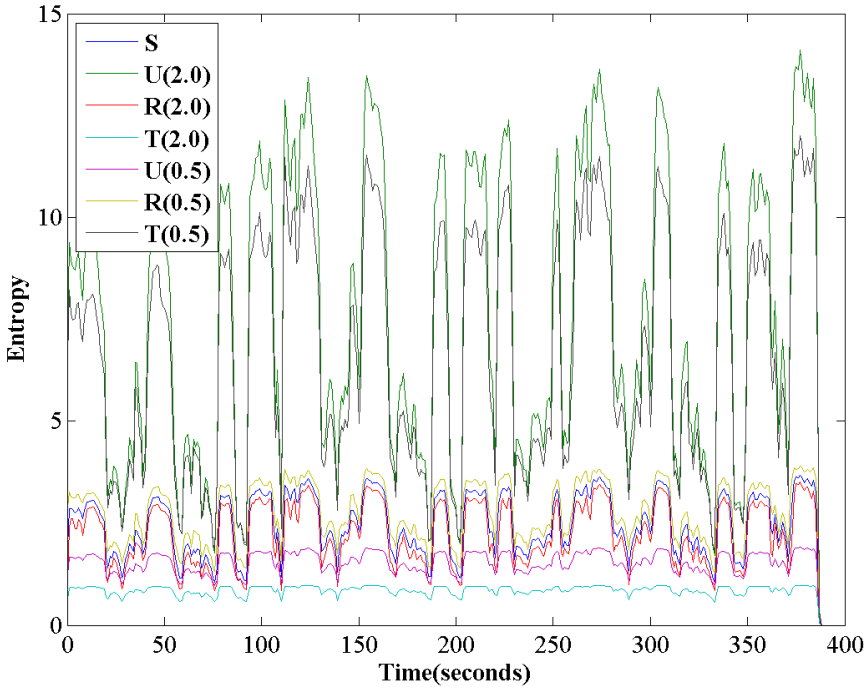


Fig. 3. Entropies S (blue trace) and $S_q^{(R)}, S_q^{(T)}, S_q^{(U)}$ for $q = \{1/2, 2\}$ versus time of Mozart's "*Kleine Nachtmusik KV525 Allegro*" composition using a sampling rate of $T = 1$ second. S – Shannon; R(q) – Rényi for order q; T(q) – Tsallis for order q; U(q) – Ubriaco for order q

In Fig. 4 several Shannon entropy curves are shown, corresponding to several musical signals subjected to entropy calculation with a $T = 1$ second sampling duration. The entropy curves represent seven different musical compositions, namely Albinoni: "*Adagio in G Minor*", Bee Gees: "*Saturday Night Fever*", Coldplay: "*Viva La Vida*", Ella Fitzgerald: "*Night And Day*" Stan Getz and Astrud Gilberto: "*Corcovado*", Mozart: "*Kleine Nachtmusik KV525 Allegro",* and The Beatles: "*Yellow Submarine*".

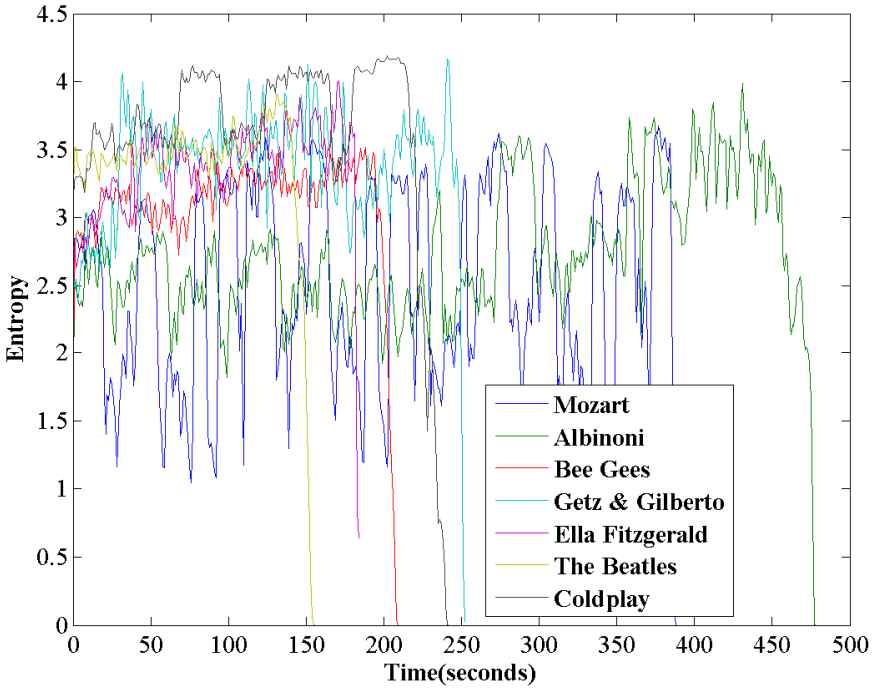


Fig. 4. Entropy S versus time of seven musical compositions using a sampling rate of T = 1 second. Musical compositions – Albinoni: "*Adagio in G Minor*", Bee Gees: "*Saturday Night Fever*", Coldplay: "*Viva La Vida*", Ella Fitzgerald: "*Night And Day*" Stan Getz and Astrud Gilberto: "*Corcovado*", Mozart: "*Kleine Nachtmusik KV525 Allegro",* and The Beatles: "*Yellow Submarine*"

3.2 Correlation of Entropy Curves from Musical Compositions

Having established the concept of time evolution of the entropy measure, $S(t)$, for musical compositions, the question of how the entropies of compositions with different "types" are correlated was investigated. Fifty musical titles from "Classical" music, fifty titles from "Jazz" music and fifty titles from "Pop & Rock" music were selected and $S(t)$ computed, using a sampling rate of $T = 1$ second. The list of musical titles is described in the Appendix 1. We considered a large number of compositions in order to avoid results valid only for some particular case (e.g., music, composer or style). Therefore, the statistical analysis in the perspective of the MDS scheme seems more adequate for revealing global characteristics and patterns.

After having computed the entropy data for the 150 musical titles, each musical composition was correlated against the remaining ones. The correlation factor r_{ij} between the entropies $S_i(t)$ and $S_j(t)$ was considered as:

$$r_{ij} = \frac{\int_0^{t_{\max}} S_i(t) S_j(t) dt}{\sqrt{\int_0^{t_{\max}} S_i^2(t) dt \int_0^{t_{\max}} S_j^2(t) dt}}, \quad i, j = 1, \cdots, 150 \tag{12}$$

where $t_{\max}$ is the music length.

In each correlation step, to make correlation independent of time duration and entropy scaling, the shortest composition was previously extended to reach the duration of the longest one (i.e., $t_{\max}$) and, for each entropy curve, the values were scaled to fit the range [0,1].

Using all the correlation results, a 150 × 150 symmetrical matrix of musical correlations $\mathbf{R} = [r_{ij}]$, $i,j = 1,\ldots,150$, was created with the main diagonal filled with "1s" (i.e., similarities in the MDS sense).

3.3 Multidimensional Scaling of Musical Compositions

The "Classical / Jazz / Pop & Rock" **R** entropy correlation matrix, in which each cell represents the entropic similarity between a pair of musical compositions, was subjected to a MDS analysis. The resulting Sheppard plot is depicted in Fig. 5, which shows a rather good distribution of points around the 45 degree line.

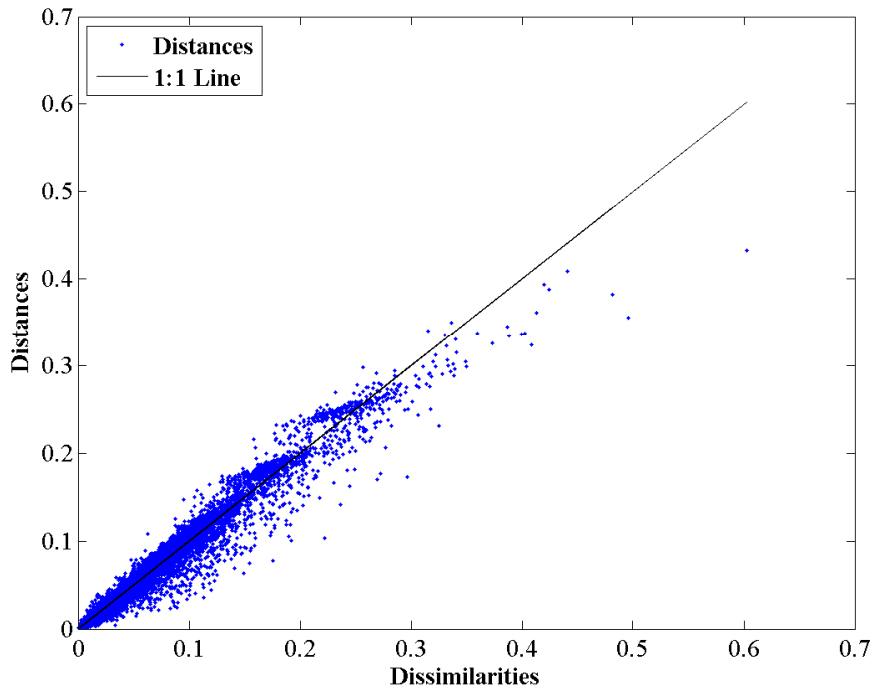


Fig. 5. Sheppard plot (Stress 0.16) for the "Classical / Jazz / Pop & Rock" three-dimensional scaling

Fig. 6 plots the Stress as a function of the dimension of the representation space, showing that a 5 to 7 dimensional space would probably describe slightly better the entropy correlations between the 150 musical compositions. However, a three dimensional representation was adopted because the graphical representation is easier to analyze and, on the other hand, yields a reasonable accuracy.

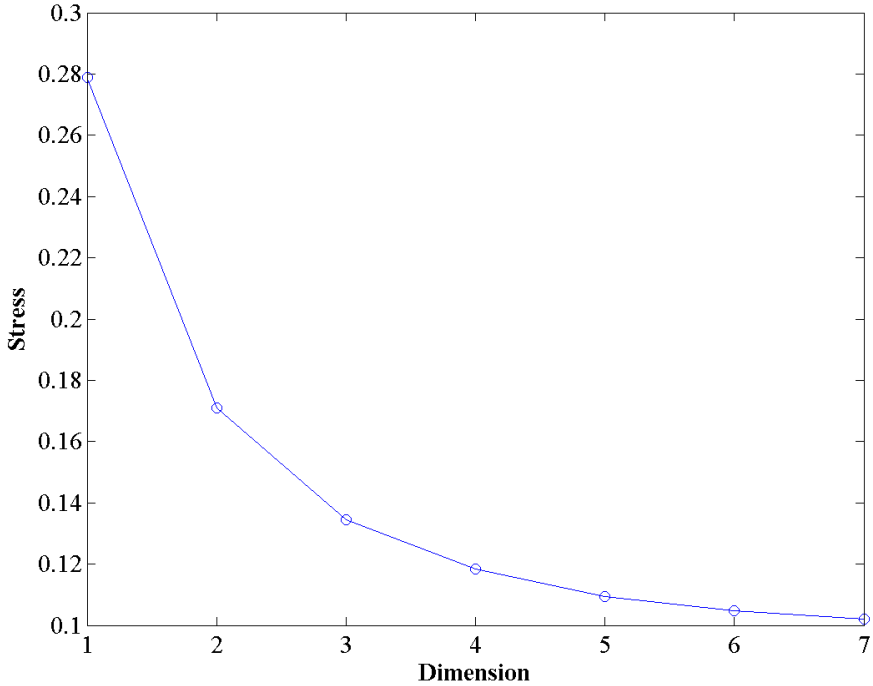


Fig. 6. Stress plot for the "Classical / Jazz / Pop & Rock" multidimensional scaling

For the three-dimensional scaling, an interactive visualization system was used to analyze how the different compositions would be spatially located and related between themselves. Figures 7-8 depict examples of the visual outputs created:

- The red '+' represent the "Classical" musical compositions;
- The blue points represent the "Jazz" musical compositions;
- The green 'x' represent the "Pop & Rock" musical compositions.

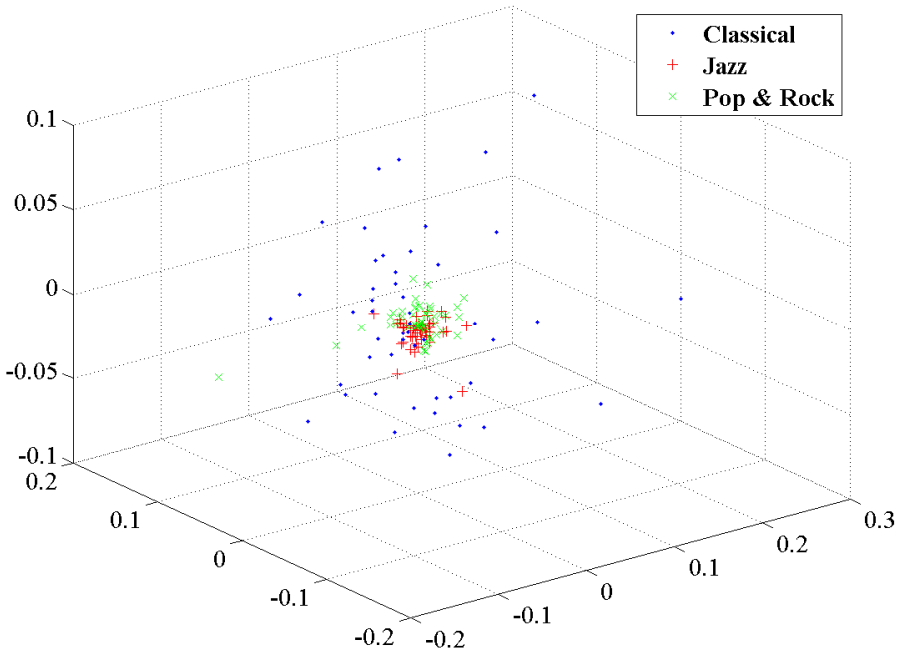


Fig. 7. Three-dimensional MDS visualization of "Classical / Jazz / Pop & Rock" entropy based correlations. Jazz – blue points, Pop & Rock – green 'x', Classical – red '+'

From the analysis of Fig. 7, which shows a global view, some conclusions can be stated:

- Almost all the "Jazz" compositions (blue points) are positioned in a first cluster, very near to each other;
- Almost all the "Pop & Rock" compositions (green 'x') are near to each other, grouped in a second cluster;
- The "Jazz" and "Pop & Rock" clusters, although near, occupy different spatial volumes;
- The "Classical" compositions (red '+') are the most spatially distributed and do not seem to cluster.

Fig. 8 depicts a zoom of Fig. 7 near the center, where the clusters of "Jazz" and "Pop & Rock" compositions are more easily perceptible. It is visible that most of "Jazz" and "Pop & Rock" compositions are essentially distributed in the *XZ* plane, meaning that two dimensions are sufficient to differentiate between them, and that the additional dimension is only necessary for the "Classical" compositions, which are more scattered in the three dimensions.

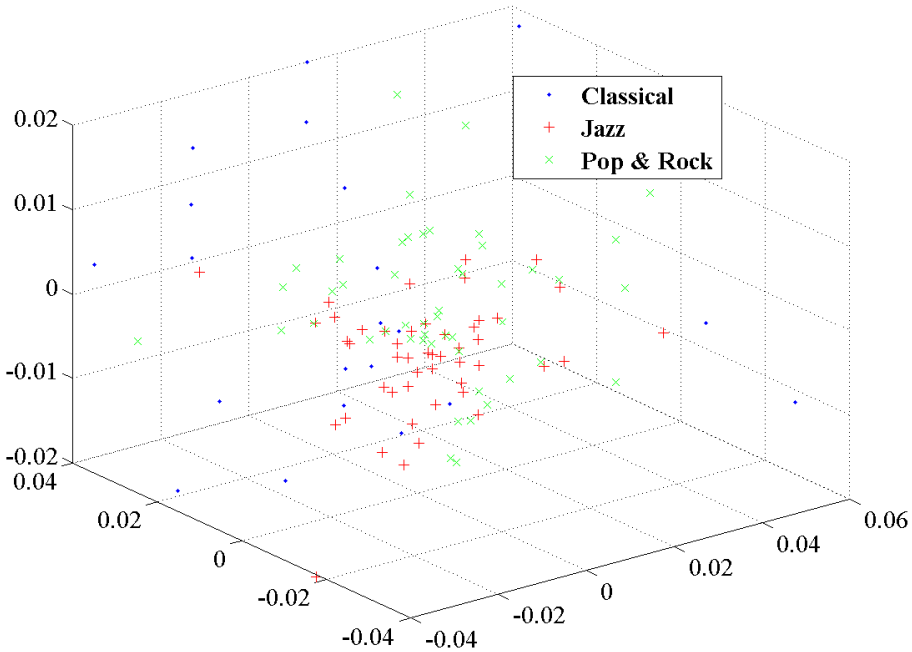


Fig. 8. Zoom of figure 7 with three-dimensional (MDS) visualization of "Classical / Jazz / Pop & Rock" entropy based correlations. Jazz – blue points, Pop & Rock – green 'x', Classical – red '+'

Fig. 9, which shows the median, upper and lower quartile, upper and lower adjacent values (whiskers), and the outlier individual points along the X/Y/Z axis of the MDS representation for the three types of musical compositions, confirms that the coordinate variations of three-dimensional scaling shown in Figures 7-8 are larger in the "Classic" set and that the "Jazz" and "Pop & Rock" sets are disjoint. The greater scattering of the "Classic" musical compositions is probably due to a larger variability in terms of musical contents.

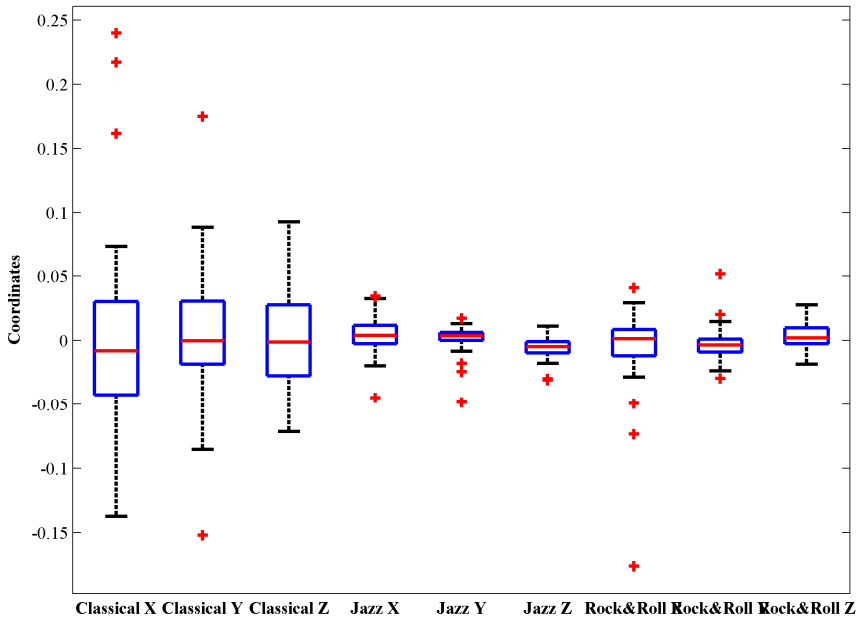


Fig. 9. Coordinate variations (X/Y/Z), showing the median, upper and lower quartile, upper and lower adjacent values (whiskers), and the outlier individual points, for the "Classical / Jazz / Pop & Rock" three-dimensional scaling.

4 Conclusions

Musical algorithm composition is an interesting and vast subject of research. Algorithmic and mathematical study of compositions refers to a methodology of using some formal processes to analyze and to compose music. This attempt is as old as music composition and we find it in the ancient Greeks in a rudimentary form which achieves an extra level of abstraction. The term "algorithm" has been adopted from the fields of computer and information science by the 1st half of the 20th century. Algorithm analysis and synthesis of compositions raises some questions, still to be answered, like: What about creativity and who is responsible for the creativity?

This paper addressed the analysis of musical compositions from a mathematical viewpoint and leads to a step towards understanding the internal mechanisms of composition itself. The proposed tools, namely entropy and multidimensional scaling, proved to be assertive methods to analyze music. Moreover, the mathematical tools allow further development by exploring several complementary implementations of the calculation algorithms. The fractional dynamics was addressed in the Fourier perspective. Due to the limitations imposed by this tool, the time-window entropy associated with the MDS statistical analysis was adopted. On one hand, this choice revealed to cope with the large signal variability and the large volume of data, but, on the other hand, the fractional dynamics was overlooked. Therefore, further work is necessary for tacking proper advantage of the fractional characteristics. Moreover, future research may also progress in other direction because music is, as every art, a matter of aesthetical evaluation and a result of cultural patterns.

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References

1. Grout, D. J., Palisca, C. V.: A History of Western Music. W.W. Norton & Company, 6th edition (2001), ISBN 0-393-97527-4.

2. Reitman, B.: History of Mathematical Approaches to Western Music. http://www.mlahanas.de/Greeks/PDF/math_music_hist.pdf, June 2003.
3. Fauvel, J., Flood, R., Wilson, R.: Music and Mathematics: From Pythagoras to Fractals. Oxford University Press (2003), ISBN 0-19-851187-6.
4. Kepler, J.: Mysterium cosmographicum. Tübingen (1596).
5. Kelley, R.: The Relationship Between Contrapuntal and Serial Composition Techniques As Seen in Works of Webern and Stravinsky. Furman University, Greenville, SC, April (1999).
6. Schoenberg, A.: Composition with Twelve Tones. Style and Idea (ed. L. Stein, tr. L. Black), Faber & Faber, London (revised 1984).
7. Richards, S.: John Cage As … Oxford: Amber Lane Press, (1996). ISBN 1-872868-17-7
8. Griffiths, P.: Modern Music and After – Directions since 1945. Oxford University Press (1995), ISBN 0-19-816511-0.
9. Corbett, J.: Extended Play – Sounding off from John Cage to Dr. Funkenstein. Duke University Press (1994). ISBN 0 822 31473 8
10. Maurer, J.: A brief history of Algorithm Composition Stanford University Center for Computer Research in Music and Acoustics (1999). https://ccrma.stanford.edu/~blackrse/algorithm.html
11. Bowles, E: Musicke's Handmaiden: Or Technology in the Service of the Arts. Ithaca, New York: Cornell University Press (1970).
12. Roads, C.: The Computer Music Tutorial. The MIT Press (1996). ISBN 0-262-68082-3.
13. Oldham, K.B., Spanier, J., The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order, Academic Press (1974).
14. Nigmatullin, R. R., A fractional integral and its physical interpretation. Theoret. and Math. Phys., **90**(3), 242-251 (1992).
15. Kenneth S. Miller and Bertram Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley and Sons (1993).
16. Stefan G. Samko, Anatoly A. Kilbas, Oleg I. Marichev, Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach Science Publishers, (1993).
17. Mainardi, F., Fractional relaxation-oscillation and fractional diffusion-wave phenomena. Chaos, Solitons & Fractals, **7**(9), 1461-1477 (1996).
6. Tenreiro Machado, J., Analysis and design of fractional-order digital control systems. Journal Systems Analysis, Modelling, Simulation, **27**(2-3), 107-122 (1997).
18. Podlubny, I.: Fractional Differential Equations, Volume 198: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution (Mathematics in Science and Engineering), Academic Press (1998).
19. Podlubny, I.: Fractional-order systems and $PI^{\lambda}D^{\mu}$-controllers. IEEE Transactions on Automatic Control, **44**(1), 208–213 (1999).
20. Chen, Y.Q., Moore, K.L.: Discretization schemes for fractional-order differentiators and integrators. IEEE Trans. Circuits and Systems-I: Fundam. Theory Appl. **49**(3), 363–367 (2002).
21. Vinagre, B. M., Chen, Y. Q., Petras, I.: Two direct Tustin discretization methods for fractional-order differentiator/integrator. J. Franklin Institute, **340**(5), 349–362 (2003).
22. Kilbas, A. A., Srivastava, H. M., Trujillo, J. J.: Theory and Applications of Fractional Differential Equations, Volume 204 (North-Holland Mathematics Studies), Elsevier (2006).
23. Richard L. Magin, Fractional Calculus in Bioengineering, Begell House Publishers, 2006.
24. George M. Zaslavsky, Hamiltonian Chaos and Fractional Dynamics, Oxford University Press, (2008).
25. Tenreiro Machado, J., Fractional Derivatives: Probability Interpretation and Frequency Response of Rational Approximations, Communications in Nonlinear Science and Numerical Simulations, Elsevier, **14**(9-10), 3492-3497 (2009).
26. Baleanu, D.: About fractional quantization and fractional variational principles. Communications in Nonlinear Science and Numerical Simulation, **14**(6), 2520–2523 (2009).
27. Francesco Mainardi, Fractional Calculus and Waves in Linear Viscoelasticity: An Introduction to Mathematical Models, Imperial College Press, (2010).
28. Riccardo Caponetto, Giovanni Dongola, Luigi Fortuna, Ivo Petráš, Fractional Order Systems: Modeling and Control Applications, World Scientific Publishing Company, (2010).
29. Concepcion Alicia Monje, YangQuan Chen, Blas Manuel Vinagre, Dingyu Xue, Vicente Feliu, Fractional Order Systems and Controls: Fundamentals and Applications, Series: Advances in Industrial Control, Springer, (2010).

30. Kai Diethelm: The Analysis of Fractional Differential Equations, Lecture Notes in Mathematics Series, Springer, (2010).
31. Shannon, C.E.: A Mathematical Theory of Communication, Bell System Technical Journal, 27, 379–423 & 623–656, July & October (1948).
32. Jaynes, E. T.: Information Theory and Statistical Mechanics, Phys. Rev. 106:620, (1957).
33. Khinchin, A. I.: Mathematical Foundations of Information Theory. New York: Dover, (1957).
34. Plastino, A., Plastino, A. R.: Tsallis entropy and Jaynes' Information Theory formalism, Brazilian Journal of Physics, 29(1), 50–60, (1999).
35. Li, X., Essex, C., Davison, M., Hoffmann, K. H., Schulzky, C.: Fractional Diffusion, Irreversibility and Entropy, Journal of Non-Equilibrium Thermodynamics. **28**(3), 279–291, (2003).
36. Haubold, H. J., Mathai, A.M., Saxena, R.K.: Boltzmann-Gibbs Entropy Versus Tsallis Entropy: Recent Contributions to Resolving the Argument of Einstein Concerning "Neither Herr Boltzmann nor Herr Planck has Given a Definition of W"?, Astrophysics and Space Science, **290**(3–4), 241–245, (2004).
37. Mathai, A.M., Haubold, H.J.: Pathway model, superstatistics, Tsallis statistics, and a generalized measure of entropy, Physica A: Statistical Mechanics and its Applications, **375**(1), 110–122, (2007).
38. Carter, T.: An introduction to information theory and entropy, Complex Systems Summer School, Santa Fe, June (2007).
39. Rathie, P., Da Silva, S.: Shannon, Levy, and Tsallis: A Note, Applied Mathematical Sciences, **2**(28), 1359–1363 (2008).
40. Beck, C.: Generalised information and entropy measures in physics, Contemporary Physics, **50**(4), 495–510, (2009).
41. Gray, R. M.: Entropy and Information Theory, Springer-Verlag (2009).
42. Ubriaco, M. R.: Entropies based on fractional calculus, Physics Letters A, **373**(30), 2516–2519, (2009).
43. Taneja, I.G., On measures of information and inaccuracy, J. Statist. Physics, **14**, 203-270 (1976).
44. Sharma, B.D., Taneja, R.K., Three generalized additive measures of entropy, Electr. Inform. Kybern, 13, 419-433 (1977).
45. Wehrl, A., General properties of entropy. Rev. Mod. Phys., **50**, 221-260 (1978).
46. Glunt, W., Hayden, T. L., & Raydan, M.: Molecular conformation from distance matrices. J. Computational Chemistry, 14, 114-120 (1993).
47. Tenenbaum, J., de Silva, V., & Langford, J.: A global geometric framework for nonlinear dimensionality reduction. Science, **290**(5500), 2319-2323 (2000).
48. Martinez-Torres, M., Barrero Garcia, F., Toral Marin, S.; Gallardo, S.: A Digital Signal Processing Teaching Methodology Using Concept-Mapping Techniques, IEEE Transactions on Education, **48**(3), 422 - 429 (2005).
49. Mao, Guoqiang & Fidan, B: Localization Algorithms and Strategies for Wireless Sensor Networks, Igi-Global (2009), ISBN 978-1-60566-397-5 (ebook).
50. Cox, T. & Cox, M.: Multidimensional scaling, 2nd edition, Chapman & Hall/CRC, (2001). ISBN 1584880945.
51. Carreira-Perpinan, M.: A review of dimension reduction techniques. Technical report CS-96-09, Department of Computer Science, University of Sheffield (1997).
52. Fodor, I.: A survey of dimension reduction techniques, Technical Report, Center for Applied Scientific Computing, Lawrence Livermore National Laboratory (2002
53. Shepard, R.: The analysis of proximities: multidimensional scaling with an unknown distance function," I and II, Psychometrika, **27**, 219-246 and 219-246 (1962).
54. Kruskal, J.: Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. Psychometrika, **29**, 1-27 (1964).
55. Kruskal, J. & Wish, M.: Multidimensional Scaling, Newbury Park, CA: Sage Publications, Inc (1978).
56. Martinez, W. & Martinez, A.: Exploratory Data Analysis with MATLAB, Chapman & Hall/CRC Press UK (2005). ISBN 1-58488-366-9.
57. Leeuw, J. & Patrick, M.: Multidimensional Scaling Using Majorization: SMACOF in R. Journal of Statistical Software, 31(3), (2009).
58. Borg, I. & Groenen, P.: Modern Multidimensional Scaling-Theory and Applications, 2nd edition, Springer-Verlag, New York (2005).

Appendix 1

Classical Titles

Bach: Brandenburg Concerto 2 Op1047
Bach: Concerto 3 BWV1043
Bach: Orchestral Suite BWV1067
Beethoven: Piano Concerto 2 Op19
Beethoven: Piano Sonata Op13 Pathetique
Beethoven: Sonata Op27 Adagio
Bizet: Carmen Suite 1 Intermezzo
Bizet: Carmen Suite 2 Nocturne
Bizet: Suite 2 Arlesienne
Brahms: Hungarian Dance 3
Brahms: Piano Concert 2 Mov 1
Brahms: Piano Concert 2 Mov 2
Chopin: Mazurka Op68
Chopin: Nocturne Op9
Chopin: Piano Concerto 2 Op21
Dvorak: Slavonic Dance Op72
Dvorak: Symphony 8 Op99 Allegretto
Dvorak: Symphony 9 Op95 New World
Grieg: Gynt Suite Op55 Solveig
Haendel: Concerto Grosso Op6
Haendel: Water Music Suite 1
Haydn: Oboe Concerto
Haydn: String Quartet Op64 The Lark
Liszt: Consolation 172
Liszt: Liebestraum
Mahler: Symphony 4
Mahler: Symphony 5
Massenet: Meditation de Thais
Mendelssohn: Midsummer Nights Dream Op61
Mendelssohn: Symphony 4 Op90 Italian
Mendelssohn: Symphony 5 Op107
Mozart: Concerto 3 KV447 Larghetto
Mozart: Kleine Nachtmusik KV525 Allegro
Mozart: KV527 Menuet Don Giovani
Ravel: Bolero
Ravel: Pavane Infante Defunte
Ravel: Rhapsodie Espagnole
Schubert: Impromptu Op90
Schubert: String Quartet 13
Schubert: Symphony 5 Op485 Andante
Schumann: Piano Concerto Op54 Andantino
Schumann: Symphony 4 Op120
Smetana: Bartered Bride Ouverture
Smetana: Moldava
Tchaikovsky: Nutcracker Op71 Danse Fee Dragee
Tchaikovsky: Nutcracker Op71 Danse Mirlitons
Tchaikovsky: Sleeping Beauty Adagio
Telemann: Suite 1 Table Music
Vivaldi: Cello Concerto RV415
Vivaldi: Four Seasons Winter Largo

Jazz Titles

Chet Baker: Let's Get Lost
Dave Brubeck: Quartet Take Five
Ella Fitzgerald: Dream a Little Dream of Me
Ella Fitzgerald: Every Time We Say Goodbye
Ella Fitzgerald: How Long Has This Been Going On
Ella Fitzgerald: I Have Got A Crush On You
Ella Fitzgerald: I Have Got the World in a String
Ella Fitzgerald: I Have Got You Under My Skin
Ella Fitzgerald: Let's Do It
Ella Fitzgerald: Looking For A Boy
Ella Fitzgerald: Manhattan
Ella Fitzgerald: My Funny Valentine
Ella Fitzgerald: My One And Only Love
Ella Fitzgerald: Night And Day
Ella Fitzgerald: Someone To Watch Me Over
Ella Fitzgerald: Taking a Chance in Love
Ella Fitzgerald: The Lady is a Tramp
Ella Fitzgerald: You Would Be Satisfied
Freddie Hubbard: Sandu
John Coltrane: Blue Train
Louis Armstrong: A Kiss To Build A Dream On
Louis Armstrong: Blueberry Hill
Louis Armstrong: Cest Si Bon
Louis Armstrong: Dream A Little Dream Of Me
Louis Armstrong: La Vie En Rose
Louis Armstrong: Let's Call The Whole Thing Off
Louis Armstrong: Let's Do It
Louis Armstrong: Mack The Knife
Louis Armstrong: Makin Whoopee
Louis Armstrong: Stompin At The Savoy
Louis Armstrong: Summertime
Louis Armstrong: We Will Be Together Again
Miles Davis: Black Satin
Miles Davis: ESP
Miles Davis: Little Church
Miles Davis: Miles Runs The Voodoo Down
Miles Davis: My Funny Valentine
Miles Davis: Nefertiti
Miles Davis: New Rhumba
Miles Davis: Petits Machins
Miles Davis: Portia
Miles Davis: Round Midnight
Miles Davis: Someday My Prince Will Come
Miles Davis: Summertime
Miles Davis: Time After Time
Peggy Lee: Fever
Modern Jazz Quartet: Bags Groove
Modern Jazz Quartet: Django
Modern Jazz Quartet: There Will Never Be Another You
US3: Cantaloop

Pop Rock Titles

Blur: Beetlebum
Blur: Girls And Boys
Blur: Song 2
Blur: There's No Other Way
Blur: To The End
Coldplay: 42
Coldplay: A Spell A Rebel Yell
Coldplay: Chinese Sleep Chant
Coldplay: In My Place
Coldplay: Lovers In Japan
Coldplay: Shiver
Coldplay: Square One
Coldplay: Talk
Coldplay: Trouble
Coldplay: Viva La Vida
Guns N Roses: Civil War
Guns N Roses: Knockin On Heavens Door
Guns N Roses: November Rain
Guns N Roses: Since I Don't Have You
Guns N Roses: Welcome to the Jungle
MIKA: Big Girl
MIKA: Grace Kelly
MIKA: Lollipop
MIKA: Love Today
MIKA: Relax Take It Easy
Oasis: Better Day
Oasis: Champagne Supernova
Oasis: Don't Look Back In Anger
Oasis: Go Let It Out
Oasis: Stand By Me
Oasis: Wonderwall
Radiohead: 2+2=5
Radiohead: High And Dry
Radiohead: Just
Radiohead: Lucky
Radiohead: No Surprises
Radiohead: Pyramid Song
Radiohead: The Bends
Sting: All This Time
Sting: Fields of Gold
Sting: Inside
Sting: Shape Of My Heart Live
Sting: Whenever I Say Your Name
Stone Roses: Dont Stop
Stone Roses: Elizabeth My Dear
Stone Roses: Made Of Stone
Stone Roses: Shoot You Down
Stone Roses: This Is The One
The Verve: Bittersweet Symphony
The Verve: Drugs Don't Work