

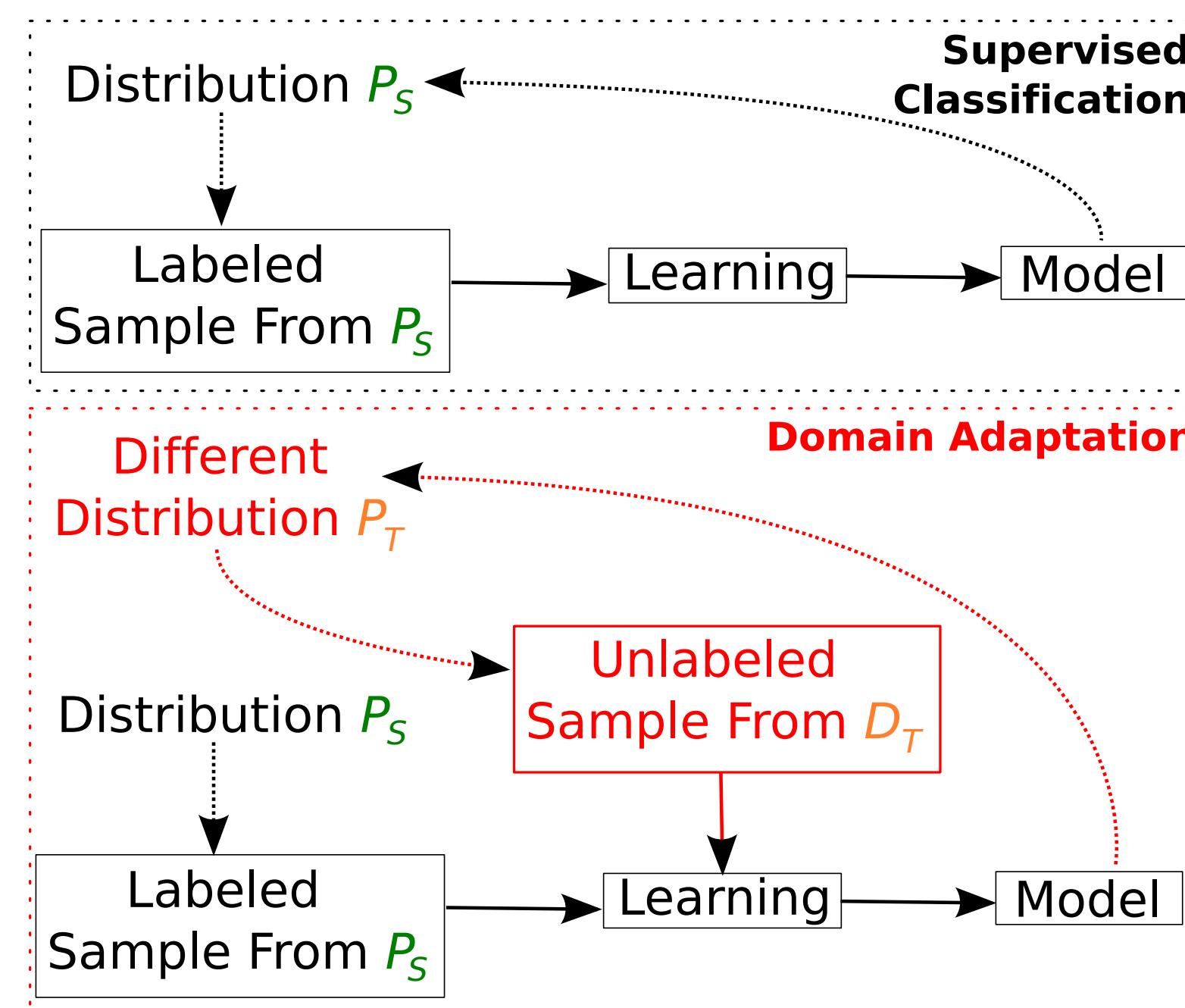
Sparse Domain Adaptation in a Good Similarity-Based Projection Space

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INTRODUCTION, NOTATIONS AND MOTIVATION

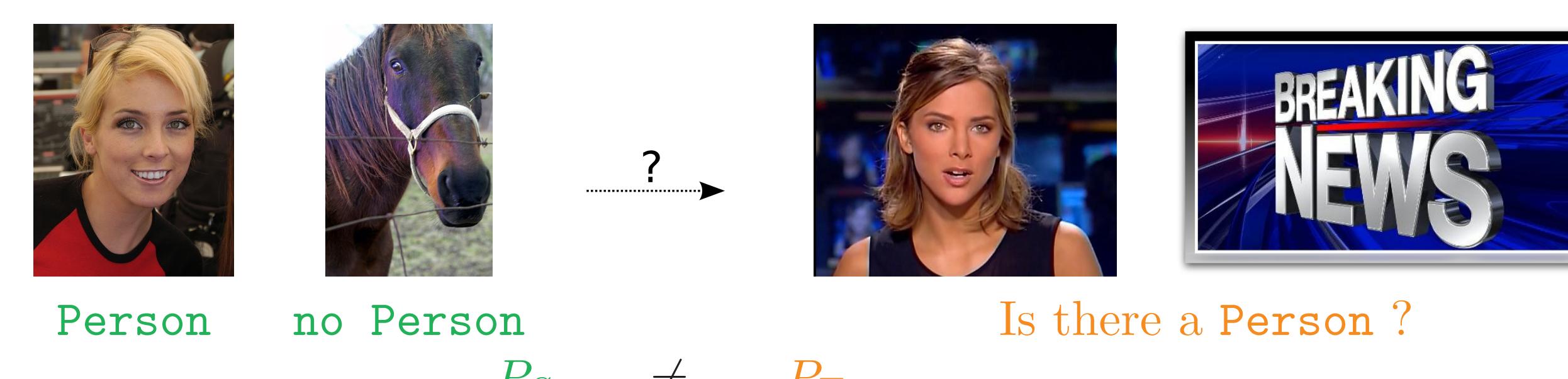
We consider binary classification task:

- X input space, $Y = \{-1, 1\}$ label set
- P_S source domain: distribution over $X \times Y$
 D_S marginal distribution over X
- P_T target domain: different distribution over $X \times Y$
 D_T marginal distribution over X
- errors of a hypothesis $h : X \rightarrow Y$
 - $\text{err}_S(h)$, $\widehat{\text{err}}_S(h)$ source domain errors
 - $\text{err}_T(h)$, $\widehat{\text{err}}_T(h)$ target domain errors
- Supervised Classification objective:
 $\cdot h \in \mathcal{H}$ with a low $\text{err}_S(h)$
- Domain Adaptation objective:
 $\cdot h \in \mathcal{H}$ with a low $\text{err}_T(h)$



For example:

- We have **labeled** images from a **Web image corpus**, i.e. $\sim P_S$
- Is there a Person in **unlabeled** images from a **Video corpus**, i.e. $\sim D_T$?



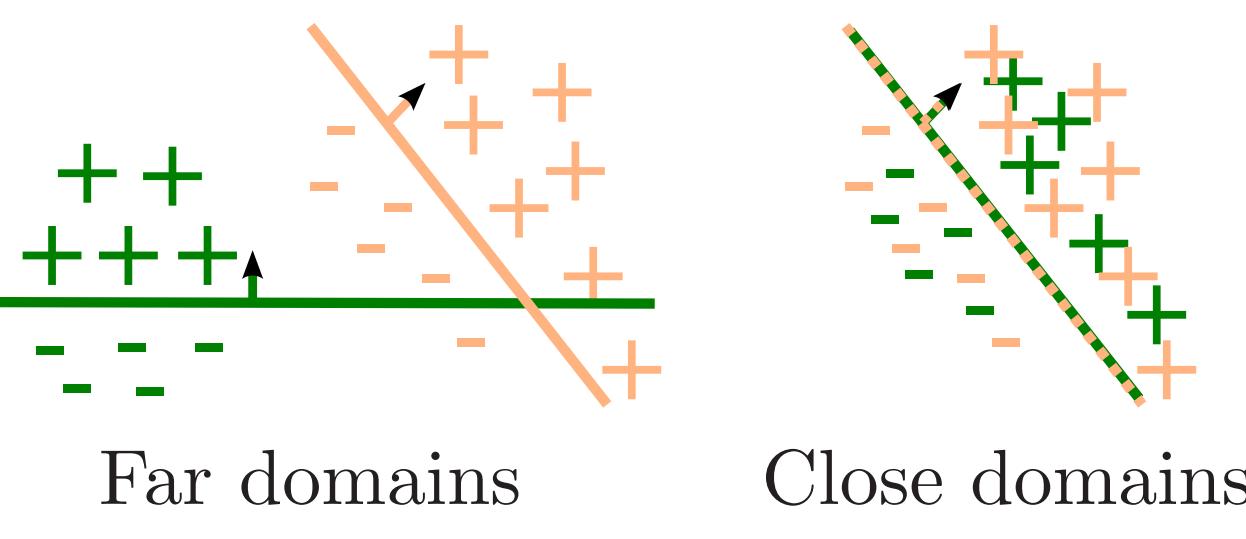
\Rightarrow The **Learning** distribution is **different** from the **Testing** distribution
 \Rightarrow How can we learn, from the **source domain**, a low-error classifier on the **target domain**?

DOMAIN ADAPTATION

Theorem 1 ([2]). Let \mathcal{H} an hypothesis space. If D_S and D_T are respectively the marginal distributions of source and target instances, then for all $\delta \in [0, 1]$, with probability at least $1 - \delta$, for every $h \in \mathcal{H}$:

$$\text{err}_T(h) \leq \text{err}_S(h) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \nu,$$

where $d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T)$ is the $\mathcal{H}\Delta\mathcal{H}$ -distance between D_S and D_T and $\nu = \text{err}_S(h^*) + \text{err}_T(h^*)$, with $h^* = \operatorname{argmin}_{h \in \mathcal{H}} (\text{err}_S(h) + \text{err}_T(h))$.



Idea

Build a new projection space
to move closer the domains.

LEARNING WITH GOOD SIMILARITY FUNCTIONS

Definition 1 ([1]). $K : X \times X \rightarrow [-1; 1]$ is an (ϵ, γ, τ) -good similarity function for a binary classification problem P if

- (i) A $1 - \epsilon$ probability mass of examples (x, y) satisfy

$$\mathbb{E}_{(x', y') \sim P} [yy' K(x, x') | R(x')] \geq \gamma,$$

- (ii) $\Pr_{x'}[R(x')] \geq \tau$ (Notation: R set of reasonable points).

Properties

- Generalization of kernels: K may be **not** symmetric and **not** PSD
- A low-error linear classifier can be learned by minimizing the Pb. (**SF**) in the explicit projection space defined by $R = \{\mathbf{x}'_j\}_{j=1}^{d_u}$:

$$\phi^R(\cdot) = \langle K(\cdot, \mathbf{x}'_1), \dots, K(\cdot, \mathbf{x}'_{d_u}) \rangle$$

(Notation: \mathcal{H}_{SF} the hypothesis space of such classifiers)

DOMAIN ADAPTATION OF LINEAR CLASSIFIERS BASED ON GOOD SIMILARITY FUNCTIONS

Building a new projection $\phi_{new}^{R'}$ to move closer D_S and D_T with \mathcal{C}_{ST} a pair set $(\mathbf{x}_s, \mathbf{x}_t) \in U_S \sim D_S \times U_T \sim D_T$ such that the deviation of losses of \mathbf{x}_s and \mathbf{x}_t is small

$$\left| \left[1 - y \sum_{j=1}^{d_u} \alpha_j K(\mathbf{x}_s, \mathbf{x}'_j) \right]_+ - \left[1 - y \sum_{j=1}^{d_u} \alpha_j K(\mathbf{x}_t, \mathbf{x}'_j) \right]_+ \right| \leq \frac{\| ({}^t \phi^{R'}(\mathbf{x}_s) - {}^t \phi^{R'}(\mathbf{x}_t)) \operatorname{diag}(\boldsymbol{\alpha}) \|_1}{\| {}^t \phi_{new}^{R'}(\mathbf{x}_s) - {}^t \phi_{new}^{R'}(\mathbf{x}_t) \|_1} \implies \phi_{new}^{R'}(\cdot) = \langle \underbrace{\alpha_1 K(\cdot, x'_1)}_{K_{new}(\cdot, x'_1)}, \dots, \underbrace{\alpha_{d_u} K(\cdot, x'_{d_u})}_{K_{new}(\cdot, x'_{d_u})} \rangle$$

Our global optimization problem

With $\{(\mathbf{x}_i, y_i)\}_{i=1}^{d_l}$ i.i.d. from P_S , $R' = \{\mathbf{x}'_j\}_{j=1}^{d_u}$ and \mathcal{C}_{ST} a set of pairs $(\mathbf{x}_s, \mathbf{x}_t) \sim D_S \times D_T$.

At iteration l , we build the $\phi_{l+1}^{R'}$ space with the help of $\boldsymbol{\alpha}^l$ inferred by

$$\begin{aligned} & \text{(SF)} \\ & \min_{\boldsymbol{\alpha}^l} \frac{1}{d_l} \sum_{i=1}^{d_l} \left[1 - y_i \sum_{j=1}^{d_u} \alpha_j^l K_l(\mathbf{x}_i, \mathbf{x}'_j) \right]_+ + \lambda \|\boldsymbol{\alpha}^l\|_1 \\ & + \beta \sum_{(\mathbf{x}_s, \mathbf{x}_t) \in \mathcal{C}_{ST}} \|({}^t \phi_l^{R'}(\mathbf{x}_s) - {}^t \phi_l^{R'}(\mathbf{x}_t)) \operatorname{diag}(\boldsymbol{\alpha}^l)\|_1 \quad (\text{DASF}) \end{aligned}$$

Some little results

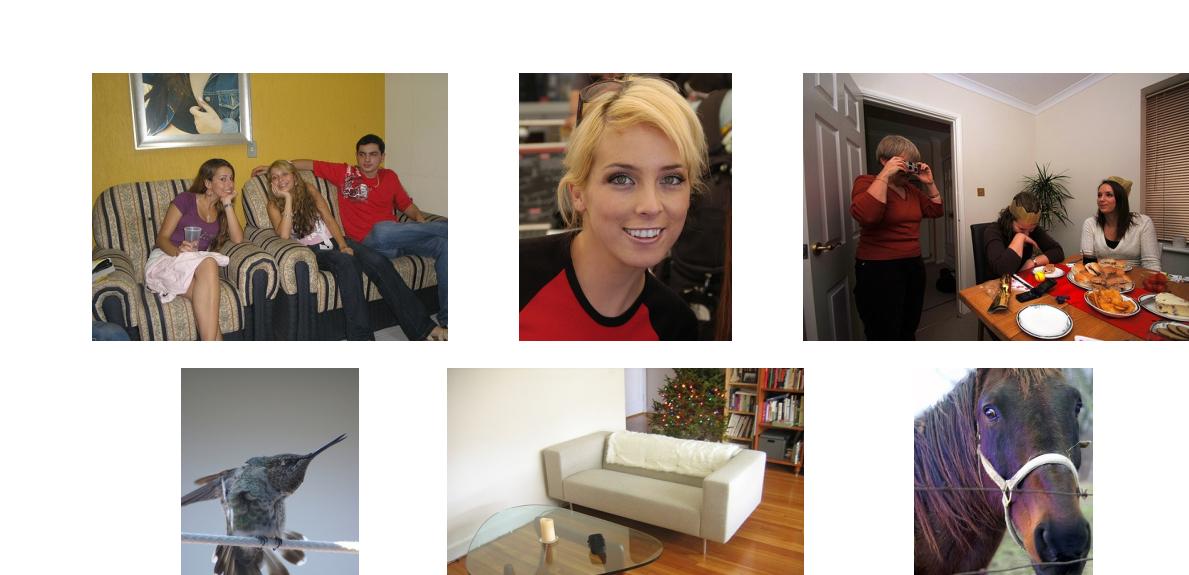
• Sparsity Analysis. With $B_R = \min_{\mathbf{x}'_j \in R} \max_{(\mathbf{x}_s, \mathbf{x}_t) \in \mathcal{C}_{ST}} |K(\mathbf{x}_s, \mathbf{x}'_j) - K(\mathbf{x}_t, \mathbf{x}'_j)|$,

$$\|\boldsymbol{\alpha}^*\|_1 \leq \frac{1}{\beta B_R + \lambda}$$

• Generalization bound. Following the notion of robustness of Xu&Mannor [3], the Problem (**DASF**) is $(2M_\eta, \frac{N_\eta}{\beta B_R + \lambda})$ robust on the source domain P_S , with $\eta > 0$, M_η is the η -covering number of X and $N_\eta = \max_{\substack{\mathbf{x}_a, \mathbf{x}_b \in D_S \\ \rho(\mathbf{x}_a, \mathbf{x}_b) \leq \eta}} \|{}^t \phi^R(\mathbf{x}_a) - {}^t \phi^R(\mathbf{x}_b)\|_\infty$. Thus for any $h \in \mathcal{H}_{SF}$, for any $\delta > 0$, with probability at least $1 - \delta$,

$$\text{err}_T(h) \leq \widehat{\text{err}}_S(h) + \frac{N_\eta}{\beta B_R + \lambda} + \sqrt{\frac{4M_\eta \ln 2 + 2 \ln \frac{1}{\delta}}{d_l}} + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \nu.$$

The reasonable points for Person:

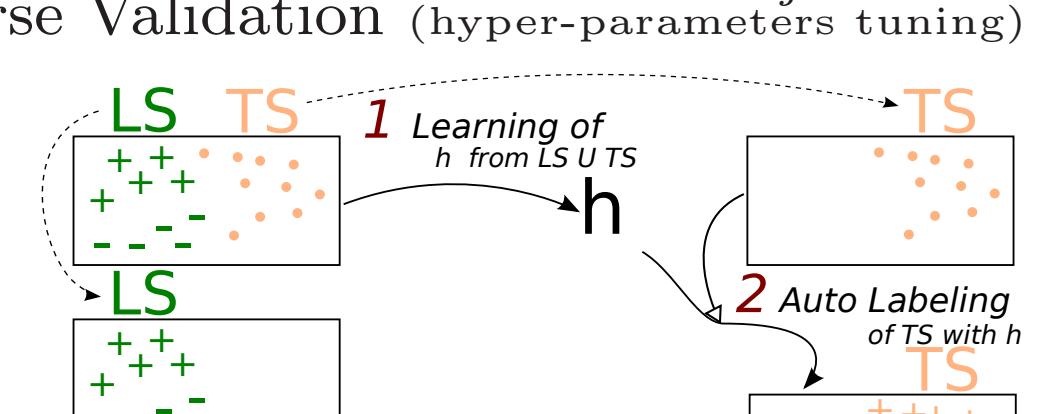


EXPERIMENTS ON MULTIMEDIA INDEXING

Experimental Setup

- Similarity function:
 K Gaussian kernel, $K(\cdot, \mathbf{x}'_j) = \frac{K(\cdot, \mathbf{x}'_j) - \mu_{\mathbf{x}'_j}}{\sigma_{\mathbf{x}'_j}} \in [-1, 1]$

- Reverse Validation (hyper-parameters tuning)

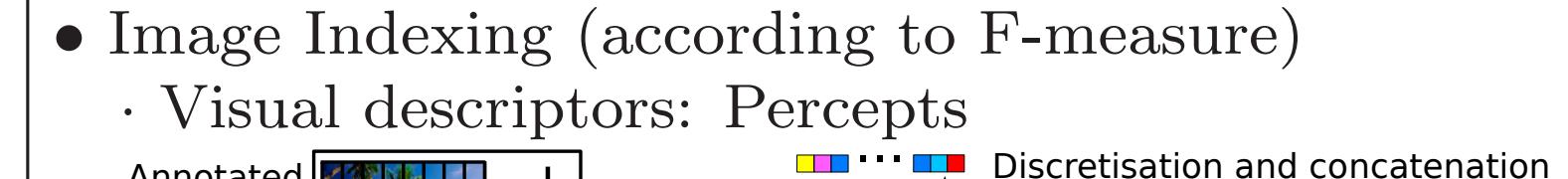


- Toy problem "inter-twining moons"

- 1 P_S , 8 P_T according to 8 rotations

- Image Indexing (according to F-measure)

- Visual descriptors: Percepts



- P_S : PascalVOC'07 ratio $+/- = 1/3$

- P_T : $+/- \neq 1/3$: PascalVOC'07 Test

- $+/- = 1/3$: TrecVid'07

Image Indexing: PascalVOC'07 Vs PascalVOC'07

Conc.	bird	boat	bottle	*	bus	car	cat	chair	cycle	cow	dining table	dog	*	horse	monitor	motor	person	plane	plants	sheep	sofa	train
SVM	0.18	0.29	0.01	0.16	0.28	0.230	0.24	0.10	0.15	0.15	0.24	0.31	0.16	0.17	0.56	0.34	0.12	0.16	0.16	0.36		
SV	867	351	587	476	1096	882	1195	392	681	534	436	761	698	670	951	428	426	631	510			
SF	0.18	0.27	0.11	0.12	0.34	0.20	0.21	0.10	0.11	0.10	0.18	0.24	0.12	0.17	0.46	0.34	0.13	0.12	0.13	0.20		
Reas.	237	203	233	212	185	178	241	139	239	253	200	247	203	243	226	178	236	128	224	202		
TSVM	0.14	0.14	0.11	0.16	0.37	0.14	0.22	0.13	0.12	0.13	0.22	0.17	0.12	0.12	0.44	0.18	0.10	0.12	0.15	0.19		
SV	814	704	718	445	631	779	864	390	888	515	704	828	861	861	1111	585	406	474	866	652		
DASVM	0.16	0.22	0.11	0.14	0.37	0.20	0.23	0.14	0.11	0.15	0.22	0.23	0.12	0.14	0.55	0.30	0.12	0.13	0.17	0.28		
SV	922	223	295	421	866	1011	1418	706	335	536	180	802	668	841	303	356	1434	246	486	407		
DASF	0.200	0.320	0.120	0.170	0.380	0.230	0.260	0.160	0.16	0.16	0.250	0.32	0.16	0.18	0.58	0.350	0.150	0.200	0.180	0.42		
Reas.	50	184	78	94	51	378	229	192	203	372	391	384	287	239	6	181	293	153	167	75		