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Transport and Traffic Engineering Laboratory (LICIT)

Technical Report on the Minimization of Potential Air Conflicts using Speed Control

Author:

David Rey*

Under the supervision of:

Christophe Rapine†

Nour-Eddin El-Faouzi‡

Rémy Fondacci§

*PhD candidate at the Traffic Engineering and Transportation laboratory LICIT, IFSTTAR/ENTPE, 25 Avenue François Mitterrand, 69675 Bron Cedex, France. E-mail: `david.rey@ifsttar.fr`

†Associate Professor at GSCOP laboratory, 46 rue Felix Viallet, 38000 Grenoble Cedex 1, France. E-mail: `christophe.rapine@gscop.fr`

‡Researcher and Head of the Traffic Engineering and Transportation laboratory LICIT, IFSTTAR/ENTPE, 25 Avenue François Mitterrand, 69675 Bron Cedex, France. E-mail: `nour-eddin.elfaouzi@ifsttar.fr`

§Head of security departement at DSAC-CE, BP 601 69125 Lyon Saint Exupéry Aéroport, France. E-mail: `remy.fondacci@aviation-civile.gouv.fr`

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Introduction

This report provides an insight into the mathematical methods developed to model the speed regulation problem in Air Traffic Flow Management (ATFM). The speed regulation problem aims at optimizing aircraft speeds along their flights in order to smooth the air traffic flow. To ensure safe flight conditions, Air Traffic Controllers (ATCos) solve every day a great amount of potential air conflicts. An air conflict occurs if two or more aircraft are flying too close to each other. By predicting flights trajectories, ATCos anticipate potential conflicts and provide clearances to pilots for solving the potential conflicts. The ultimate objective of speed regulation is to deliver an enhanced traffic to ATCos. This can be achieved in the light of the En-Route Air traffic Soft Management Ultimate System (ERASMUS) project [2, 3]. The techniques presented in this report aim at contributing to Conflict Detection and Resolution (CD&R) methods in Air Traffic Management (ATM). The reader can find more information on existing CD&R methods in ATM, in a complete review written by Kuchar and Yang in 2000 [4]. The document is divided into sections that focus on different parts of the speed regulation problem modelling. Although a chronological structure has been respected, each section can be read independently. We chose to develop a linear framework for the speed regulation problem therefore most of the techniques presented aim at providing a linear formulation of the model. In a first section, (1), some information about potential conflicts detection is provided and in a second section, (1), modelling choices are discussed. Sections (3)and (4) are dedicated to specific sub-models depending on the geometric configuration of the potential conflicts. A complete formulation of the model is then presented in (5).

1 Potential Air Conflicts

Air conflicts are defined according to the ICAO separation norms [6] which require that no aircraft should enter the protection zone of another. The protection zone consists of an horizontal separation of 5 Nautical Miles (NM) and 1000 feet, hence it can visualized as a cylinder centered on every aircraft (see figure 1). When two cylinders are predicted to intersect in the future, aircraft are in a potential conflict. If the loss of separation occurs, we say that both aircraft are in a conflict. Two types of potential conflicts can be identified according flights trajectories: if their trajectories are not parallel, we say aircraft are in a potential crossing conflict. When two flights share the same route, i.e. their trajectories overlap, we say aircraft are in a potential trailing conflict (see figure 2). When optimizing the air traffic flow, potential conflicts detection is a critical step of the process as it should accurately estimate aircraft 4-dimensional trajectories in the near future. This prediction is then used to perform the optimization process.

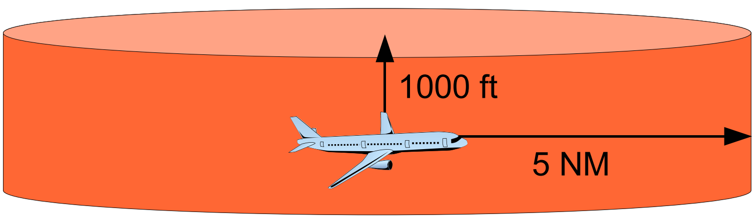
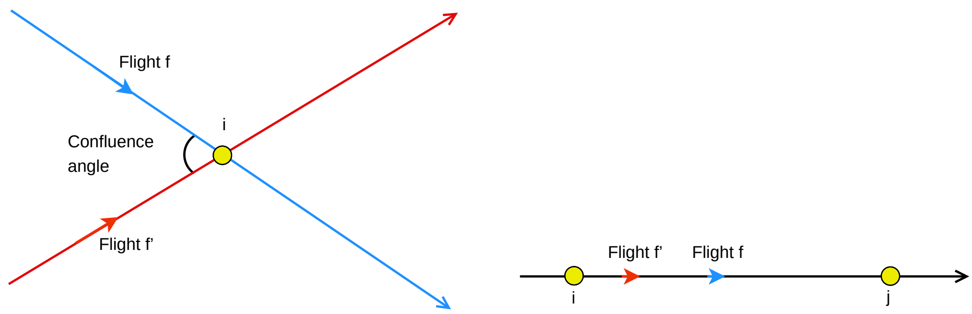


Figure 1: Separation norm



(a) Crossing Conflict (b) Trailing Conflict

Figure 2: Conflict Types

2 Modelling Choices

Modelling potential conflicts can be achieved in several ways depending on the targeted framework. We believe that all potential conflicts are not equivalent for ATCos. More precisely, we believe that the severity of potential conflicts can be related to their duration: the longer the potential conflict is predicted to last, the greater is ATCos' potential workload increase. In order to model this dependency we propose to turn spatial separation into time separation. Using the equations describing the motion of flights, we can convert their relative distance into a time interval which can be improved to provide the required separation between aircraft. This projection onto the time domain lead us to use crossing times of flights, i.e the time a flight flies over a given point of space, as decision variables in the model. By estimating potential conflict duration, we can estimate the global potential conflict duration and seek to minimize this quantity by moderately adjust aircraft speeds. In the next section the general case where aircraft trajectories are not parallel is investigated.

3 Crossing Conflicts

3.1 A geometric approach

Suppose aircraft are moving at constant speed in a 2-dimensional Euclidean plane. Assume flight f crosses point i at time zero and its trajectory coincides with the x-axis. Without any loss of generality, their cinematic equations in the plane formed by their trajectories can be expressed as:

$$\begin{cases} x_f(t) &= v_f t \\ y_f(t) &= 0 \\ x_{f'}(t) &= v_{f'}(t - \Delta T^i_{ff'}) \cos\theta \\ y_{f'}(t) &= \pm v_{f'}(t - \Delta T^i_{ff'}) \sin\theta \end{cases} \tag{1}$$

where θ is the confluence angle between flights trajectories and $\Delta T^i_{ff'}$ is the absolute value of the crossing time difference between aircraft f and f' at point i: $\Delta T^i_{ff'} = |t^i_f - t^i_{f'}|$. Note that if $\theta \equiv 0(\pi)$, that is aircraft trajectories overlap, the problem can be simplified. This case is investigated in section 4. Therefore suppose $\theta \not\equiv 0(\pi)$, the Euclidean distance between flights f and f' at time t is:

$$D(t) = \sqrt{\big(x_f(t) - x_{f'}(t)\big)^2 + \big(y_f(t) - y_{f'}(t)\big)^2} \tag{2}$$

Squarring equation 2 gives:

$$\begin{aligned}
D^2(t) &= \big(v_f t - v_{f'}(t-\Delta T^i_{ff'})\cos\theta\big)^2 + \big(\mp v_{f'}(t-\Delta T^i_{ff'})\sin\theta\big)^2 && (3)\\
&= v_f^2 t^2 + v_{f'}^2(t-\Delta T^i_{ff'})^2\cos^2\theta - 2v_f v_{f'} t(t-\Delta T^i_{ff'})\cos\theta + v_{f'}^2(t-\Delta T^i_{ff'})^2\sin^2\theta && (4)\\
&= v_f^2 t^2 + v_{f'}^2 t^2\cos^2\theta + v_{f'}^2(\Delta T^i_{ff'})^2\cos^2\theta - 2v_{f'}^2 t\Delta T^i_{ff'}\cos^2\theta - 2v_f v_{f'} t^2\cos\theta\\
&\quad + 2v_f v_{f'} t\Delta T^i_{ff'}\cos\theta + v_{f'}^2 t^2\sin^2\theta + v_{f'}^2(\Delta T^i_{ff'})^2\sin^2\theta - 2v_{f'}^2 t\Delta T^i_{ff'}\sin^2\theta && (5)\\
&= t^2(v_f^2 + v_{f'}^2 - 2v_f v_{f'}\cos\theta) - t(2v_{f'}^2\Delta T^i_{ff'} + 2v_f v_{f'}\Delta T^i_{ff'}\cos\theta) + v_{f'}^2(\Delta T^i_{ff'})^2 && (6)
\end{aligned}$$

In order to determine potential conflict duration, we need to solve equation: $D(t) = D$, where D is the radius of the protection zone projected on the plane where aircraft trajectories belong. This can be resumed to solve the following quadratic equation:

$$At^2 + Bt + C = 0 \tag{7}$$

with:

$$\begin{cases} A &= v_f^2 + v_{f'}^2 - 2v_f v_{f'}\cos\theta\\ B &= -2v_{f'}^2\Delta T^i_{ff'} - 2v_f v_{f'}\Delta T^i_{ff'}\cos\theta\\ C &= v_{f'}^2(\Delta T^i_{ff'})^2 - D^2\end{cases} \tag{8}$$

Let $\alpha = \cos\theta$, computing the discriminant value gives:

$$\begin{aligned}
\Delta_1 &= B^2 - 4AC && (9)\\
&= (2v_f v_{f'}\Delta T^i_{ff'}\alpha - 2v_{f'}^2\Delta T^i_{ff'})^2 - 4(v_f^2 + v_{f'}^2 - 2v_f v_{f'}\alpha)(v_{f'}^2(\Delta T^i_{ff'})^2 - D^2) && (10)\\
&= 4\alpha^2 v_f^2 v_{f'}^2(\Delta T^i_{ff'})^2 + 4v_{f'}^4(\Delta T^i_{ff'})^2 - 8\alpha v_f v_{f'}^3(\Delta T^i_{ff'})^2\\
&\quad - 4(v_f^2 v_{f'}^2(\Delta T^i_{ff'})^2 - v_f^2 D^2 - 2\alpha v_f v_{f'}^3(\Delta T^i_{ff'})^2 + 2\alpha v_f v_{f'} D^2 + v_{f'}^4(\Delta T^i_{ff'})^4 - v_{f'}^2 D^2) && (11)\\
&= 4\alpha^2 v_f^2 v_{f'}^2(\Delta T^i_{ff'})^2 + 4v_{f'}^4(\Delta T^i_{ff'})^2 - 8\alpha v_f v_{f'}^3(\Delta T^i_{ff'})^2\\
&\quad - 4v_f^2 v_{f'}^2(\Delta T^i_{ff'})^2 - 4v_f^2 D^2 + 8\alpha v_f v_{f'}^3(\Delta T^i_{ff'})^2 - 8\alpha v_f v_{f'} D^2 - 4v_{f'}^4(\Delta T^i_{ff'})^4 + 4v_{f'}^2 D^2 && (12)\\
&= 4\alpha^2 v_f^2 v_{f'}^2(\Delta T^i_{ff'})^2 - 4v_f^2 v_{f'}^2(\Delta T^i_{ff'})^2 + 4v_f^2 D^2 - 8\alpha v_f v_{f'} D^2 + 4v_{f'}^2 D^2 && (13)\\
&= 4v_f^2 v_{f'}^2(\Delta T^i_{ff'})^2(\alpha^2 - 1) + 4D^2(v_f^2 - 2\alpha v_f v_{f'} + v_{f'}^2) && (14)
\end{aligned}$$

There is a potential conflict at i if $\Delta_1 > 0$, which is equivalent to:

$$\Delta T^i_{ff'} < D\sqrt{\frac{v_f^2 - 2\alpha v_f v_{f'} + v_{f'}^2}{v_f^2 v_{f'}^2(1-\alpha^2)}} = \Gamma(v_f, v_{f'}) \tag{15}$$

The right-hand side of equation 15 depends on aircraft speeds and on the geometric configuration of intersection i, namely the confluence angle θ. Note that if θ tends to zero, $\Gamma(v_f, v_{f'})$ tends to infinity, this case is investigated in section 4. Clearly $\Gamma(v_f, v_{f'})$ is a non-linear function with respect to aircraft speeds. To overcome this issue we propose to assign constants values to aircraft speeds. To do so, we have to establish a policy for no potential conflict underestimation to occur. Therefore we chose to focus on a worst-case scenario framework. $\Gamma(v_f, v_{f'})$ represents the potential conflict duration when flights cross intersection point i at the same time. Our objective is to overestimate potential conflict duration, that is to maximize $\Gamma(v_f, v_{f'})$ with respect to v_f and $v_{f'}$ (see appendix B for a numerical approach). For all flight $f \in \mathcal{F}$, we have $v_f \in [\underline{v}_f, \overline{v}_f]$. Bounds on v_f are calculated by considering aircraft aerodynamics and the speed regulation range applied to the fleet. Without any loss of generality, let $r = v_f/v_{f'}$ be the aircraft speed ratio. We can express $\Gamma(v_f, v_{f'})$ as:

$$\Gamma(v_f, r) = \frac{D}{v_f|\sin\theta|}\sqrt{r^2 - 2r\alpha + 1} \tag{16}$$

A fonction is convex if and only if its hessian matrix is semi-definite positive [1]. To determine whether $\Gamma(v_f, r)$ is convex or not, we have to determine its hessian matrix. The first order partial derivatives of $\Gamma(v_f, r)$ are:

$$\frac{\partial\Gamma(v_f, r)}{\partial v_f} = \frac{-D}{v_f^2|\sin\theta|}\sqrt{r^2 - 2r\alpha + 1} \tag{17}$$

$$\frac{\partial\Gamma(v_f, r)}{\partial r} = \frac{D}{v_f v_{f'}|\sin\theta|}\frac{r-\alpha}{\sqrt{r^2 - 2r\alpha + 1}} \tag{18}$$

The second order partial are derivates are then:

$$\frac{\partial^2\Gamma(v_f, r)}{\partial v_f^2} = \frac{2D}{v_f^3|\sin\theta|}\sqrt{r^2 - 2r\alpha + 1} \tag{19}$$

$$\frac{\partial^2\Gamma(v_f, r)}{\partial v_f \partial r} = \frac{-D}{v_f^2 v_{f'}|\sin\theta|}\frac{r-\alpha}{\sqrt{r^2 - 2r\alpha + 1}} \tag{20}$$

$$\frac{\partial^2\Gamma(v_f, r)}{\partial r^2} = \frac{D}{v_f v_{f'}^2|\sin\theta|}\frac{1-\alpha^2}{(r^2 - 2r\alpha + 1)^{3/2}} \tag{21}$$

and the hessian matrix $\mathbf{H}$ of function $\Gamma(v_f, r)$ is:

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2\Gamma(v_f,r)}{\partial v_f^2} & \frac{\partial^2\Gamma(v_f,r)}{\partial v_f \partial r} \\ \frac{\partial^2\Gamma(v_f,r)}{\partial v_f \partial r} & \frac{\partial^2\Gamma(v_f,r)}{\partial r^2} \end{pmatrix} = \begin{pmatrix} \frac{2D}{v_f^3|\sin\theta|}\sqrt{r^2 - 2r\alpha + 1} & \frac{-D}{v_f^2 v_{f'}|\sin\theta|}\frac{r-\alpha}{\sqrt{r^2-2r\alpha+1}} \\ \frac{-D}{v_f^2 v_{f'}|\sin\theta|}\frac{r-\alpha}{\sqrt{r^2-2r\alpha+1}} & \frac{D}{v_f v_{f'}^2|\sin\theta|}\frac{1-\alpha^2}{(r^2-2r\alpha+1)^{3/2}} \end{pmatrix}$$

A symmetric matrix is positive semidefinite if and only if its minors are positive. Since $\frac{\partial^2\Gamma(v_f,r)}{\partial v_f^2} \geq 0$, we need to compute the determinant of $\mathbf{H}$, that is:

$$|\mathbf{H}| = \frac{\partial^2\Gamma(v_f, r)}{\partial v_f^2} \cdot \frac{\partial^2\Gamma(v_f, r)}{\partial r^2} - \left(\frac{\partial^2\Gamma(v_f, r)}{\partial v_f \partial r}\right)^2 = \frac{D^2}{v_f^4 v_{f'}^2|\sin\theta|^2}\left(\frac{2 - 2\alpha^2 - (r-\alpha)^2}{r^2 - 2r\alpha + 1}\right) \tag{22}$$

The sign of $|\mathbf{H}|$ depends on: $\frac{2-2\alpha^2-(r-\alpha)^2}{r^2-2r\alpha+1}$. Solving the quadratic equation at the denominator $r^2 - 2r\alpha + 1 = 0$, we have:

$$\Delta_2 = 4\alpha^2 - 4 = 4(\alpha^2 - 1) < 0 \tag{23}$$

Thus $r^2 - 2r\alpha + 1 > 0$ and the sign of $|\mathbf{H}|$ depends on the numerator: $2 - 2\alpha^2 - (r-\alpha)^2 = -r^2 + 2\alpha r - 3\alpha^2 + 2$. Solving equation $-r^2 + 2\alpha r - 3\alpha^2 + 2 = 0$ we have:

$$\Delta_3 = 4\alpha^2 - 4(3\alpha^2 - 2) = 8(1 - \alpha^2) > 0 \tag{24}$$

Therefore $\mathbf{H}$ is not semi-definite positive and hence $\Gamma(v_f, r)$ is not convex. Let $\varphi(r)$ be the function $r \mapsto \sqrt{r^2 - 2r\alpha + 1}$. Clearly $\varphi(r)$ is convex, therefore maximum values of $\varphi(r)$ are achieved for minimum and maximum values of r, that is $\underline{r} = \underline{v}_f / \overline{v}_{f'}$ and $\overline{r} = \overline{v}_f / \underline{v}_{f'}$. Let $\overline{\varphi} \in \mathbb{R}$ be:

$$\overline{\varphi} = \max_{r \in [\underline{r}, \overline{r}]} \varphi(r) = \max\left(\varphi(\underline{r}), \varphi(\overline{r})\right) \tag{25}$$

In order to set an upper bound on potential conflict duration, we can turn v_f into a decision variable of the model. Indeed, speeds of flights can be converted into travel time using crossing times at two consecutive waypoints and the corresponding distance. Let $t_f^{prec(i)}$ (resp. $t_{f'}^{prec(i)}$) be the crossing time of flight f (resp. f') at the predecessor of waypoint i on the path of flight f (resp. f'), the distance between i and its previous waypoint on can be denoted: d_f^i (resp. $d_{f'}^i$). We can now define $\Gamma_{ff'}^i \in \mathbb{R}$, as follows:

$$\Gamma_{ff'}^i = \max\left(\left(t_f^i - t_f^{prec(i)}\right)\frac{D\overline{\varphi}}{d_f^i|\sin\theta|}, \left(t_{f'}^i - t_{f'}^{prec(i)}\right)\frac{D\overline{\varphi}}{d_{f'}^i|\sin\theta|}\right) \tag{26}$$

$$= \max\left(\left(t_f^i - t_f^{prec(i)}\right)G_{ff'}^i, \left(t_{f'}^i - t_{f'}^{prec(i)}\right)G_{f'f}^i\right) \tag{27}$$

where $G_{ff'}^i = \frac{D\overline{\varphi}}{d_f^i|\sin\theta|}$ and $G_{f'f}^i = \frac{D\overline{\varphi}}{d_{f'}^i|\sin\theta|}$ are real constants. Here the max function is used to prevent the model from giving priority to a specific aircraft speed while solving the potential conflict. $\Gamma_{ff'}^i$ represents the potential conflict duration if aircraft are due to cross simultaneously intersection point i and it is an upper bound on $\Gamma(v_f, v_{f'})$, that is:

$$\forall v_f \in [\underline{v}_f, \overline{v}_f], v_{f'} \in [\underline{v}_{f'}, \overline{v}_{f'}], \quad \Gamma(v_f, v_{f'}) \leq \Gamma_{ff'}^i \tag{28}$$

Since we now have an upper bound on the potential conflict duration, we can estimate the potential duration according to the crossing time difference at intersection point i. Let $\omega_{ff'}^i \in \mathbb{R}$ be an over-estimation of the potential conflict duration between flights f and f' at i, $\omega_{ff'}^i$ can be defined as:

$$\omega_{ff'}^i = (\Gamma_{ff'}^i - \Delta T_{ff'}^i)^+ = (\Gamma_{ff'}^i - |t_f^i - t_{f'}^i|)^+ \tag{29}$$

where $(X)^+ = \max(X, 0)$. To linearize equation 29, we need to linearize the max function as well as the expression of $\Gamma_{ff'}^i$ and the crossing time difference: $\Delta T_{ff'}^i = |t_f^i - t_{f'}^i|$. This last quantity depends on the order in which the flights arrive at this particular point.

Remark 1 *Using a* max *function to approximate the value of* $\Gamma_{ff'}^i$ *is not the best approximation possible. Indeed, using a* min *yields a lower upper bound on the potential conflict duration when flights cross intersection point i simultaneously. However, when minimizing the potential conflict duration, that is* $\omega_{ff'}^i$*, it is difficult to express linearly* $\Gamma_{ff'}^i$ *as a minimum of two values.*

3.2 Modelling aircraft crossing order

Let $\mathcal{P}_c$ be the set of all potential crossing conflicts, for all $(f, f', i) \in \mathcal{P}_c$, let $y_{ff'}^i \in \{0, 1\}$ be a binary decision variable defined as:

$$y_{ff'}^i = \begin{cases} 1 & \text{if } t_f^i < t_{f'}^i \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

Note that for all triplet $(f, f', i) \in \mathcal{P}_c$ decision variables $y^i_{ff'}$ and $y^i_{f'f}$ are complementary:

$$y^i_{ff'} + y^i_{f'f} = 1 \tag{31}$$

We can express the crossing times (decision variables) according to the crossing order of flights:

$$t^i_{f'} \leq t^i_f + (\bar{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \tag{32}$$

When $y^i_{ff'} = 0$, constraint 32 becomes $t^i_{f'} \leq t^i_f$, modelling the constraint on flights crossing times. If $y^i_{ff'} = 1$, then constraint 32 becomes redundant for it is always satisfied. If one exchanges the roles of indices f and f' in constraint (32), the reciprocal constraint is obtained thus completing the modellization. In order to define $\Delta T^i_{ff'}$ as a continuous decision variable in the model, we can linearize its expression using the next constraints:

$$\Delta T^i_{ff'} \leq t^i_f - t^i_{f'} + 2(\bar{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \tag{33}$$

$$\Delta T^i_{ff'} \geq t^i_f - t^i_{f'} \tag{34}$$

Note that exchanging the roles of indices f and f' in constraints 33 and 34, and adding the constraint:

$$\Delta T^i_{ff'} = \Delta T^i_{f'f} \tag{35}$$

to the model yields the exact value of the crossing time difference: $\Delta T^i_{ff'} = |t^i_f - t^i_{f'}|$. Hence we can express the potential conflict duration $\omega^i_{ff'}$ using the expression of $\Gamma^i_{ff'}$. The objective function 29 can be linearized using lower bounds on quantity $\omega^i_{ff'}$:

$$\omega^i_{ff'} \geq \left(t^i_f - t^{prec(i)}_f\right) \cdot G^i_{ff'} - \Delta T^i_{ff'} \tag{36}$$

$$\omega^i_{ff'} \geq 0 \tag{37}$$

Once again, if one exchanges the roles of indices f and f' in the above constraints, we must beware of counting twice every potential conflict duration. Therefore we introduce another equality constraint in the model:

$$\omega^i_{ff'} = \omega^i_{f'f} \tag{38}$$

and we add a 1/2 factor in front of every term in the objective function. The model aiming at minimizing potential conflicts duration can then be expressed as:

Model 1 (Crossing Conflicts Model)

$$\boxed{\min \sum_{(f,f',i)\in\mathcal{P}_c} \frac{1}{2}\omega^i_{ff'}}$$

$$\forall (f,i) \in \mathcal{P} :$$
$$\mathcal{SM} = \Big\{ \quad \underline{t}^i_f \leq t^i_f \leq \overline{t}^i_f$$
$$\forall (f,f',i) \in \mathcal{P}_c :$$
$$\mathcal{CM} = \begin{cases} \omega^i_{ff'} \geq \left(t^i_f - t^{prec(i)}_f\right) \cdot G^i_{ff'} - \Delta T^i_{ff'} \\ \omega^i_{ff'} \geq 0 \\ \omega^i_{ff'} = \omega^i_{f'f} \\ \Delta T^i_{ff'} \leq t^i_f - t^i_{f'} + 2(\overline{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \\ \Delta T^i_{ff'} \geq t^i_f - t^i_{f'} \\ \Delta T^i_{ff'} = \Delta T^i_{f'f} \\ t^i_{f'} \leq t^i_f + (\overline{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \\ y^i_{ff'} + y^i_{f'f} = 1 \end{cases}$$
$$(t^i_f, \Delta T^i_{ff'}, \omega^i_{ff'}) \in \mathbb{R},\ y^i_{ff'} \in \{0,1\}.$$

Remark 2 *Values of $y^i_{ff'}$ can be settled by the instance properties: if speed regulation is not strong enough to permute aircraft crossing order at a given intersection point, then only one configuration holds and $y^i_{ff'}$ is fixed. This property can considerably diminish the number of remaining binary decision variables, thus improving the optimization algorithm performances.*

In the next section, we consider the special case where aircraft trajectories are parallel, i.e aircraft are moving on the same track. This research was motivated by the existence of airways, where flights are due to follow the same route over long distances.

4 Trailing Conflicts

4.1 Time Segments Modelling

Long distance flights are often merged with airways in order to provide an enhanced control environment. Indeed, when sharing long flights segments, aircraft are redirected toward air corridors thus enabling ATCos to focus on specific routes. However, when flights trajectories are parallel, $\Gamma(v_f, v_{f'})$ in formula 15 fails to provide the required crossing time difference for its denominator tends to zero. In order to solve potential conflicts occurring on airways an adapted framework is required. In this section we focus on the resolution of potential trailing conflicts through speed regulation. As speed regulation is a restricted conflict resolution method, all potential conflicts may not be solvable using this technique. Aircraft takeovers are naturally prohibited in a speed regulation context, therefore an intuitive policy to implement a speed regulation oriented conflict resolution method is to focus on a First In First Out (FIFO) discipline. To do so, we start by estimating the potential conflict duration on an infinite segment. In a second time a FIFO discipline is added to the modellization aiming at representing real air navigation conditions. We discard the case where aircraft are facing each other for such situation should not occur in a real air traffic scenario and moreover this case is not solvable using speed regulation methods only. Assume both aircraft are flying in the same direction, we can express the motion of two objects on a common segment using their cinematic equations:

$$\begin{cases} x_f(t) &= v_f t \\ x_{f'}(t) &= v_{f'}(t - \Delta T^i_{ff'}) \end{cases} \tag{40}$$

Recall that $\Delta T^i_{ff'}$ is the crossing time difference at intersection point i, which can here be assimilated to the segment initial extremity or the airway entry point. To determine the beginning and the end of the loss of separation between flights f and f' we have to solve the equation $D(t) = D$, that is:

$$|x_f(t) - x_{f'}(t)| = D \tag{41}$$

Assume flights speeds are different, $v_f \neq v_{f'}$, if $x_f(t) \geq x_{f'}(t)$ then we have:

$$v_f t - v_{f'}(t - \Delta T^i_{ff'}) = D \tag{42}$$

$$t(v_f - v_{f'}) + v_{f'}\Delta T^i_{ff'} = D \tag{43}$$

$$t = \frac{D - v_{f'}\Delta T^i_{ff'}}{v_f - v_{f'}} = \frac{-D + v_{f'}\Delta T^i_{ff'}}{v_{f'} - v_f} \tag{44}$$

Similarly if $x_f(t) \leq x_{f'}(t)$ we obtain:

$$t = \frac{D + v_{f'}\Delta T^i_{ff'}}{v_{f'} - v_f} \tag{45}$$

Therefore the roots of equation (41) are $\frac{\pm D + v_{f'}\Delta T^i_{ff'}}{v_{f'} - v_f}$. The time corresponding to the beginning of the conflict t_b is the lowest root and the time corresponding to the end of the conflict t_e is the highest one. The conflict duration on an infinite segment T_∞ is then:

$$T_\infty = t_e - t_b = \frac{2D}{|v_f - v_{f'}|} \tag{46}$$

Note that if $v_f = v_{f'}$ conflict times are not well defined. In that case the distance between aircraft is constant and conflict duration depends on the relative positions of flights at the segment entry point. Let us suppose $v_f \neq v_{f'}$; to compute the conflict duration on segment $[i, j]$ we define the times t_i and t_j corresponding to the presence of both aircraft on the shared segment:

$$\begin{cases} t_i &= \max(t^i_f, t^i_{f'}) \\ t_j &= \min(t^j_f, t^j_{f'}) \end{cases} \tag{47}$$

For all $t \in [t_i, t_j]$ both aircraft are moving on segment $[i, j]$. The loss of separation with respect to segment $[i, j]$ depends on the relative position of $[t_b, t_e]$ and $[t_i, t_j]$. Therefore, conflict duration ρ on the time segment $[t_i, t_j]$ is obtained by truncating T_∞, as follows:

$$\rho = \left(T_\infty - (t_i - t_b)^+ - (t_e - t_j)^+\right)^+ \tag{48}$$

In order to integrate a FIFO policy in the model we have to estimate potential conflict duration assuming no takeovers between aircraft occurs. Let v_l be the speed of the leading aircraft, v_p be the speed of the follower and let ρ^S be the conflict duration on segment $S = [i, j]$ assuming a FIFO discipline. We can distinguish three different situations:

1. Aircraft fly at the same speed: $v_l - v_p = 0$. If $D/\overline{v}_p \geq \Delta T^i_{ff'}$, then aircraft are already in a potential conflict at segment entry point i and potential conflict duration is as long as the time segment $[t_i, t_j]$:
$$\rho^S = t_j - t_i$$
2. The leader is faster: $v_l - v_p > 0$. Then $t_b < t_i$ and the potential conflict beginning time, if it occurs on $[t_i, t_j]$, is t_i. Using equation 48 we have:
$$\rho^S = \big(t_e - t_b - t_i + t_b - (t_e - t_j)^+\big)^+ = \big(t_e - t_i - (t_e - t_j)^+\big)^+$$
which is equivalent to:
$$\rho^S = \big(\min(t_e - t_i, t_j - t_i)\big)^+ \tag{49}$$
3. The follower is faster: $v_l - v_p < 0$. Then the FIFO constraint imposes the potential conflict, if it occurs on $[t_i, t_j]$, to end at t_j, thus $t_e > t_j$. Using equation 48 we have:
$$\rho^S = \big(t_e - t_b - (t_i - t_b)^+ - t_e + t_j\big)^+ = \big(t_j - t_b - (t_i - t_b)^+\big)^+$$
which is equivalent to:
$$\rho^S = \big(\min(t_j - t_b, t_j - t_i)\big)^+ \tag{50}$$

The analytical expressions of times t_b and t_e are not linear with respect to the reciprocal of aircraft speeds. We already assumed aircraft speed were constants on shared segments, hence we need to decide which value to assign to each flight speed. In the FIFO paradigm, the worst-case scenario is achieved when the leader is flying at its minimum speed and the follower at its maximum one, therefore we can reformulate conflict beginning and ending times t_b and t_e: let τ^b_f, τ^e_f (resp. $\tau^b_{f'}, \tau^e_{f'}$) be the worst-case conflict beginning and ending dates when f (resp. f') is the leader, we have:

$$\text{if } f \text{ is the leader: } \begin{cases} \tau^b_f & = \min\left(\frac{-D+\overline{v}_{f'}(t^i_{f'}-t^i_f)}{\overline{v}_{f'}-\underline{v}_f}, \frac{+D+\overline{v}_{f'}(t^i_{f'}-t^i_f)}{\overline{v}_{f'}-\underline{v}_f}\right) \\ \tau^e_f & = \max\left(\frac{-D+\overline{v}_{f'}(t^i_{f'}-t^i_f)}{\overline{v}_{f'}-\underline{v}_f}, \frac{+D+\overline{v}_{f'}(t^i_{f'}-t^i_f)}{\overline{v}_{f'}-\underline{v}_f}\right) \end{cases} \tag{51}$$

$$\text{if } f' \text{ is the leader: } \begin{cases} \tau^b_{f'} & = \min\left(\frac{-D+\underline{v}_{f'}(t^i_f-t^i_{f'})}{\underline{v}_{f'}-\overline{v}_f}, \frac{+D+\underline{v}_{f'}(t^i_f-t^i_{f'})}{\underline{v}_{f'}-\overline{v}_f}\right) \\ \tau^e_{f'} & = \max\left(\frac{-D+\overline{v}_{f'}(t^i_f-t^i_{f'})}{\underline{v}_{f'}-\overline{v}_f}, \frac{+D+\overline{v}_{f'}(t^i_f-t^i_{f'})}{\underline{v}_{f'}-\overline{v}_f}\right) \end{cases} \tag{52}$$

where $\tau^b_f, \tau^b_{f'}$ are the lowest of these times and $\tau^e_f, \tau^e_{f'}$ are the greatest ones. Recall that $y^i_{ff'}$ is a binary decision variable which value is 1 if flight f flies over point i before flight f'. Let $(\tau_b, \tau_e) \in \mathbb{R}$ be defined as:

$$\tau_b = \begin{cases} \tau^b_f & \text{if } f \text{ is the leader} \\ \tau^b_{f'} & \text{otherwise} \end{cases} \qquad \tau_e = \begin{cases} \tau^e_f & \text{if } f \text{ is the leader} \\ \tau^b_{f'} & \text{otherwise} \end{cases} \tag{53}$$

To decide which potential conflict duration formula we must use according to flight speeds, we must consider the worst-case speed scenario as well, that is:

1. if $\underline{v}_l = \overline{v}_p \Rightarrow \rho^S = \begin{cases} t_j - t_i & \text{if } D/v_p \geq \Delta T^i_{ff'} \\ 0 & \text{otherwise} \end{cases}$

2. if $\underline{v}_l > \overline{v}_p \Rightarrow \rho^S = \Big(\min(\tau_e - t_i, t_j - t_i)\Big)^+$

3. if $\underline{v}_l < \overline{v}_p \Rightarrow \rho^S = \Big(\min(t_j - \tau_b, t_j - t_i)\Big)^+$

The next step is to express the potential trailing conflict duration in linear terms with respect to decision variables t^i_f, $t^i_{f'}$, t^j_f, $t^j_{f'}$ and $y^i_{ff'}$. We start by investigating case 1, where $\underline{v}_l = \overline{v}_p$, in section 4.2. Cases 2 and 3 are then treated together in section 4.3.

4.2 Modelling the separation condition at the segment entry point

When $\underline{v}_l = \overline{v}_p$, the worst-case potential conflict duration is equal to the time segment $[t_i, t_j]$ if flights are predicted to be under the separation norm at the segment entry point i. However if one manages to separate flights at i, then potential conflict duration is null. In order to model this discrete behavior, we propose to introduce a binary decision variable, $z^i_{ff'}$, that vanishes when separation at the segment entry point is achieved. Let $z^i_{ff'}$ be defined as:

$$z^i_{ff'} = \begin{cases} 1 & \text{if } D/\overline{v}_{f'} < t^i_{f'} - t^i_f \text{ and if } f \text{ is the leader} \\ 0 & \text{otherwise} \end{cases} \tag{54}$$

Decision variable $z^i_{ff'}$ (resp. $z^i_{f'f}$) is designed to model the separation of flights at point i when flight f (resp f') is the leader: if the crossing time difference $t^i_{f'} - t^i_f$ (resp. $t^i_f - t^i_{f'}$) is greater than the worst-case separation time interval, $D/\overline{v}_{f'}$ (resp. $D/\overline{v}_f$), then $z^i_{ff'}$ (resp. $z^i_{f'f}$) is set to 1. If the crossing time difference is not large enough to separate aircraft at point i, then decision variables $z^i_{ff'}$ and $z^i_{f'f}$ are set to 0.

Remark 3 *Unlike variables $y^i_{ff'}$ and $y^i_{f'f}$, variables $z^i_{ff'}$ and $z^i_{f'f}$ are not complementary, i.e. their sum is not necessarily equal to* 1. *This models the fact that aircraft separation at point i may occur independently of the leadership of the potential conflict.*

The role of the leadership in potential trailing conflicts requires that the optimization process considers both scenarios. Let $\rho^S_{ff'}$ and $\rho^S_{f'f}$ be the potential conflict durations according to crossing order of flights at i, that is:

$$\rho^S = \begin{cases} \rho^S_{ff'} & \text{if } f \text{ is the leader} \\ \rho^S_{f'f} & \text{if } f' \text{ is the leader} \end{cases} \tag{55}$$

Note that since the leadership on segment S is defined according to the value of decision variable $y^i_{ff'}$, only one potential conflict duration holds: $\rho^S_{ff'}$ or $\rho^S_{f'f}$. Therefore ρ^S can naturally be defined as the sum of these quantities. Separating aircraft at the segment entry point when flights speeds are equal solves the potential trailing conflict over the whole shared flight segment. In order to reproduce this discrete behavior in the model, we first introduce the following constraints:

$$\rho^S_{ff'} \leq z^i_{ff'} \cdot \overline{\rho}^S_{ff'} \tag{56}$$

where $\overline{\rho}^S_{ff'} = \overline{t}^j_f - \underline{t}^i_{f'}$ is an upper bound on the potential conflict durations according to the leadership of the conflict. Potential trailing conflict durations can be determined using the next constraint:

$$\rho^S_{ff'} \geq (t^j_f - t^i_{f'}) - z^i_{ff'} \cdot \overline{\rho}^S_{ff'} \tag{57}$$

When f is the leader and separation at the segment entry point is achieved, i.e. $z^i_{ff'} = 1$, then constraint 57 becomes redundant for its right-hand side becomes negative. If $z^i_{ff'} = 0$, potential conflict duration is lower bounded by $t^j_f - t^i_{f'}$, which is the target quantity when f is the leader. Finally, we have to introduce the separation condition at i according to $z^i_{ff'}$:

$$t^i_{f'} - t^i_f \geq (1 - z^i_{ff'}) \cdot D/\overline{v}_{f'} - (\overline{t}^i_f - \underline{t}^i_{f'} + D/\overline{v}_{f'}) \cdot (1 - y^i_{ff'}) \tag{58}$$

Since we chose to estimate the potential conflict duration according to the leadership of the conflict, we need to guarantee that every constraint is symmetric with respect to the leadership. When f is the leader, that is $y^i_{ff'} = 1$, constraint 58 becomes:

$$t^i_{f'} - t^i_f \geq (1 - z^i_{ff'}) \cdot D/\overline{v}_{f'} \tag{59}$$

Constraint 59 models the separation condition when f is the leader. If $y^i_{ff'} = 0$, that is $t^i_{f'} \leq t^i_f$, constraint 58 becomes:

$$t^i_{f'} - t^i_f - (\underline{t}^i_{f'} - \overline{t}^i_f) \geq (1 - z^i_{ff'}) \cdot D/\overline{v}_{f'} - D/\overline{v}_{f'} \tag{60}$$

which left-hand side is positive and right-hand side is negative, thus the constraint becomes redundant. Exchanging the roles of the indices of flights, constraint 58 can also be written as:

$$t^i_f - t^i_{f'} \geq (1 - z^i_{f'f}) \cdot D/\overline{v}_f - (\overline{t}^i_{f'} - \underline{t}^i_f + D/\overline{v}_f) \cdot y^i_{ff'} \tag{61}$$

which gives the correct constraint related to the separation condition when f' is the leader. The same mechanism holds for constraints 56 and 57. Finally, in order to increase dependency between binary decision variables, one can take advantage of the definition of decision variable $z^i_{ff'}$ and include the next constraint to the model:

$$z^i_{ff'} \leq y^i_{ff'} \tag{62}$$

Indeed, if f is *not* the leader, then by definition, $z^i_{ff'} = 0$. Moreover, constraint 56 states that potential conflict duration vanishes if the aircraft separation is achieved at i. Since potential trailing conflict duration depends on the leadership of the conflict, combining constraints 62 and 56 reduces the expression of the potential trailing conflict duration: $\rho^S = \rho^S_{ff'} + \rho^S_{f'f}$ to one term. We can now present a complete set of constraints aiming at minimizing the potential trailing conflict duration when aircraft worst-case speeds are equal:

$\forall (f, f', S) \in \mathcal{P}_t$ such that $\underline{v}_f = \overline{v}_{f'}$:

$$\rho^S_{ff'} \leq z^i_{ff'} \cdot \overline{\rho}^S_{ff'} \tag{63}$$

$$\rho^S_{ff'} \geq (t^j_f - t^i_{f'}) - (1 - z^i_{ff'}) \cdot \overline{\rho}^S_{ff'} \tag{64}$$

$$\rho^S_{ff'} \geq 0 \tag{65}$$

$$z^i_{ff'} \leq y^i_{ff'} \tag{66}$$

$$t^i_{f'} - t^i_f \geq (1 - z^i_{ff'}) \cdot D/\overline{v}_{f'} - (\overline{t}^i_f - \underline{t}^i_{f'} + D/\overline{v}_{f'}) \cdot (1 - y^i_{ff'}) \tag{67}$$

4.3 Flight overtaking and distancing

Assume $\underline{v}_l \neq \overline{v}_p$, in that case the potential conflict duration is independent of the separation condition at i, indeed, it depends on the relation between the leader minimum speed and the follower maximum speed. Two cases are thus required to model the potential conflict duration: the distancing case, where $\underline{v}_l > \overline{v}_p$, and the overtaking case, where $\underline{v}_l < \overline{v}_p$. Let $\psi^S \in \mathbb{R}$ be an auxiliary decision variable, we can linearize the max and min functions in equations 49,50 using the following constraints set:

$$\psi^S \geq \tau_e - t_j \quad \text{if} \quad \underline{v}_l > \overline{v}_p \tag{68}$$

$$\psi^S \geq t_i - \tau_b \quad \text{if} \quad \underline{v}_l < \overline{v}_p \tag{69}$$

$$\psi^S \geq 0 \tag{70}$$

$$\rho^S \geq t_j - t_i + \psi^S \tag{71}$$

$$\rho^S \geq 0 \tag{72}$$

ψ^S adjusts the potential conflict duration according to the relative positions of τ_e and t_j, or τ_b and t_i. Since the maximum conflict duration is $t_j - t_i$, we need only to crop the ends of the time segment if the worst-cases conflict beginning or ending time belongs to $[t_i, t_j]$. To pursue the linearization, we have to identify which flight is the leader and which is the follower. Using decision variable $y^i_{ff'}$, we can express, $t_j - \tau_e$ and $\tau_b - t_i$ as:

$$\begin{aligned} \tau_e - t_j &= (\tau^e_f - t^j_f) y^i_{ff'} + (\tau^e_{f'} - t^j_{f'})(1 - y^i_{ff'}) \\ &= \left(\frac{\pm D + \overline{v}_{f'}(t^i_{f'} - t^i_f)}{\overline{v}_{f'} - \underline{v}_f} - t^j_f \right) y^i_{ff'} + \left(\frac{\pm D + \underline{v}_{f'}(t^i_f - t^i_{f'})}{\underline{v}_{f'} - \overline{v}_f} - t^j_{f'} \right)(1 - y^i_{ff'}) \end{aligned} \tag{73}$$

$$\begin{aligned} t_i - \tau_b &= (t^i_{f'} - \tau^b_f) y^i_{ff'} + (t^i_f - \tau^b_{f'})(1 - y^i_{ff'}) \\ &= \left(\frac{t^i_{f'} - \pm D + \overline{v}_{f'}(t^i_{f'} - t^i_f)}{\overline{v}_{f'} - \underline{v}_f} \right) y^i_{ff'} + \left(t^i_f - \frac{\pm D + \underline{v}_{f'}(t^i_f - t^i_{f'})}{\underline{v}_{f'} - \overline{v}_f} \right)(1 - y^i_{ff'}) \end{aligned} \tag{74}$$

Developing the above expressions yields:

$$\tau_e - t_j = \frac{y^i_{ff'}}{\overline{v}_{f'} - \underline{v}_f}\left(\pm D + \overline{v}_{f'}t^i_{f'} - \overline{v}_{f'}t^i_f\right) + (t^j_{f'} - t^j_f)y^i_{ff'} - t^j_{f'} + \frac{(1-y^i_{ff'})}{\underline{v}_{f'} - \overline{v}_f}\left(\pm D + \underline{v}_{f'}t^i_{f'} - \underline{v}_{f'}t^i_f\right) \tag{75}$$

$$t_i - \tau_b = \frac{y^i_{ff'}}{\overline{v}_{f'} - \underline{v}_f}\left(\mp D - \overline{v}_{f'}t^i_f + \underline{v}_f t^i_{f'}\right) + \frac{(1-y^i_{ff'})}{\underline{v}_{f'} - \overline{v}_f}\left(\mp D - \underline{v}_{f'}t^i_{f'} + \overline{v}_f t^i_f\right) \tag{76}$$

Remark 4 *The sign before quantity D depends on the sign of the denominator of the fraction factorizing the terms in parenthesis. Since the expressions of the denominators are differences of aircraft speed bounds, i.e. constant values, they can be determined before the optimization.*

To achieve a linear formulation, products of bounded continuous and binary decision variables can be linearized using auxiliary variables, this technique (presented in appendix C) is applied to linearize decision variables products: $t^i_f y^i_{ff'}$, $t^i_{f'}y^i_{ff'}$, $t^j_f y^i_{ff'}$, $t^j_{f'}y^i_{ff'}$. Operator $\mathcal{L}$ is used to express the linearization of two decision variables, we define auxiliary variables β^i_f, $\beta^i_{f'}$, β^j_f and $\beta^j_{f'}$ as:

$$\beta^i_f = \mathcal{L}(y^i_{ff'}, t^i_f), \quad \beta^i_{f'} = \mathcal{L}(y^i_{ff'}, t^i_{f'}) \tag{77}$$

$$\beta^j_f = \mathcal{L}(y^i_{ff'}, t^j_f), \quad \beta^j_{f'} = \mathcal{L}(y^i_{ff'}, t^j_{f'}) \tag{78}$$

We now can reformulate equations (75,76) by including the above reformulations:

$$\begin{aligned}\tau_e - t_j = &\frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(\pm D y^i_{ff'} + \overline{v}_{f'}\beta^i_{f'} - \overline{v}_{f'}\beta^i_f\right) + \beta^j_{f'} - \beta^j_f - t^j_{f'}\\ &+ \frac{1}{\underline{v}_{f'} - \overline{v}_f}\left(\pm D(1-y^i_{ff'}) + \underline{v}_{f'}(t^i_{f'} - t^i_f - \beta^i_{f'} + \beta^i_f)\right)\end{aligned} \tag{79}$$

$$\begin{aligned}t_i - \tau_b = &\frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(\mp D y^i_{ff'} - \overline{v}_{f'}\beta^i_f + \underline{v}_f\beta^i_{f'}\right)\\ &+ \frac{1}{\underline{v}_{f'} - \overline{v}_f}\left(\mp D(1-y^i_{ff'}) + \underline{v}_{f'}(\beta^i_{f'} - t^i_{f'}) + \overline{v}_f(t^i_f - \beta^i_f)\right)\end{aligned} \tag{80}$$

In order to include constraints (79,80) in the model, we can split them according to the value of decision variable $y^i_{ff'}$: let $\psi^S_{ff'}, \psi^S_{f'f} \in R$ be two auxiliary decision variables defined as:

$$\psi^S = \begin{cases}\psi^S_{ff'} & \text{if } f \text{ is the leader}\\ \psi^S_{f'f} & \text{otherwise}\end{cases} \tag{81}$$

Reformulating equations (68,69) and (71) gives:

$$\begin{cases}\psi^S_{ff'} & \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(-D y^i_{ff'} + \overline{v}_{f'}\beta^i_{f'} - \overline{v}_{f'}\beta^i_f\right) - \beta^j_f & \text{if: } \underline{v}_f > \overline{v}_{f'}\\ \psi^S_{ff'} & \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(D y^i_{ff'} - \overline{v}_{f'}\beta^i_f + \underline{v}_f\beta^i_{f'}\right) & \text{if: } \underline{v}_f < \overline{v}_{f'}\\ \rho^S_{ff'} & \geq \beta^j_f - \beta^i_{f'} + \psi^S_{ff'} & \end{cases} \tag{82}$$

$$\begin{cases}\psi^S_{f'f} & \geq \frac{1}{\underline{v}_{f'} - \overline{v}_f}\left(D(1-y^i_{ff'}) + \underline{v}_{f'}(t^i_{f'} - t^i_f - \beta^i_{f'} + \beta^i_f)\right) + \beta^j_{f'} - t^j_{f'} & \text{if: } \underline{v}_{f'} > \overline{v}_f\\ \psi^S_{f'f} & \geq \frac{1}{\underline{v}_{f'} - \overline{v}_f}\left(-D(1-y^i_{ff'}) + \underline{v}_{f'}(\beta^i_{f'} - t^i_{f'}) + \overline{v}_f(t^i_f - \beta^i_f)\right) & \text{if: } \underline{v}_{f'} < \overline{v}_f\\ \rho^S_{f'f} & \geq t^j_{f'} - \beta^j_{f'} - t^i_f + \beta^i_f + \psi^S_{f'f} & \end{cases} \tag{83}$$

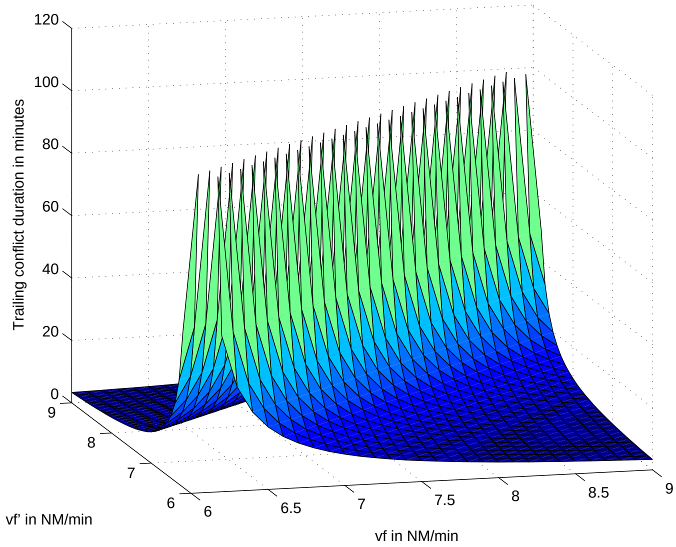


Figure 3: Potential trailing conflict duration according to aircraft speeds

Finally, in order to complete the formulation, we must include the FIFO constraint to the model:

$$y^i_{ff'} = y^j_{ff'} \tag{84}$$

Equation 84 states that the crossing order between flights f and f' must be the same at intersection points i and j, that is, no overtaking may occur along the shared segment.

4.4 Singular behaviour of the potential conflict duration

Computing values of τ_b and τ_e can lead to numerical instabilities if the leader's minimum speed is too close to the follower's maximum speed. Indeed, in that case $\underline{v}_l \neq \overline{v}_p$ thus we have to estimate τ_b and τ_e, which may tend to infinity if the concerned aircraft speeds are too close (see figure 4.4). To overcome this issue we propose to perform a test on the worst-case conflict beginning and ending times before solving the optimization problem. When aircraft worst-case speeds are different, they will be considered identical if:

$$\underline{v}_f \neq \overline{v}_{f'} \quad \text{and} \quad \frac{2D}{|\underline{v}_f - \overline{v}_{f'}|} \geq \overline{t}^j_f - \underline{t}^i_{f'} \tag{85}$$

or if:

$$\underline{v}_{f'} \neq \overline{v}_f \quad \text{and} \quad \frac{2D}{|\underline{v}_{f'} - \overline{v}_f|} \geq \overline{t}^j_{f'} - \underline{t}^i_f \tag{86}$$

In other terms, this test states that if the worst-case conflict duration is greater than the maximum conflict duration according to the time segment $[t_i, t_j]$, then potential conflict duration should be approximated using the close speeds case.

4.5 A linear formulation for the FIFO paradigm

Let $\mathcal{P}_t$ be the set of all potential trailing conflicts, the model aiming at minimizing potential trailing conflicts duration can be expressed as:

Model 2 (Trailing Conflicts Model)

$$\boxed{\min \sum_{(f,f',S)\in\mathcal{P}_t} \rho^S_{ff'} + \rho^S_{f'f}}$$

subject to:

$$\forall (f,i) \in \mathcal{P}:$$
$$\mathcal{SM} = \left\{ \quad \underline{t}^i_f \leq t^i_f \leq \overline{t}^i_f \right.$$
$$\forall (f,f',S) \in \mathcal{P}_t:$$
$$\mathcal{TM} = \left\{ \begin{array}{ll} \left.\begin{array}{ll} \rho^S_{ff'} & \leq z^i_{ff'} \cdot \overline{\rho}^S_{ff'} \\ \rho^S_{ff'} & \geq (t^j_f - t^i_{f'}) - (1 - z^i_{ff'}) \cdot \overline{\rho}^S_{ff'} \\ t^i_{f'} - t^i_f & \geq (1 - z^i_{ff'}) \cdot D/\overline{v}_{f'} - (\overline{t}^i_f - \underline{t}^i_{f'} + D/\overline{v}_{f'}) \cdot (1 - y^i_{ff'}) \\ z^i_{ff'} & \leq y^i_{ff'} \end{array}\right\} & \text{if: } \underline{v}_f = \overline{v}_{f'} \\ \left.\rho^S_{ff'} \geq \beta^j_f - \beta^i_{f'} + \psi^S_{ff'} \right\} \text{if: } \underline{v}_f \neq \overline{v}_{f'} & \\ \left.\psi^S_{ff'} \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(-Dy^i_{ff'} + \overline{v}_{f'}\beta^i_{f'} - \overline{v}_{f'}\beta^i_f\right) - \beta^j_f \right\} \text{if: } \underline{v}_f > \overline{v}_{f'} & \\ \left.\psi^S_{ff'} \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(Dy^i_{ff'} - \overline{v}_{f'}\beta^i_f + \underline{v}_f\beta^i_{f'}\right) \right\} \text{if: } \underline{v}_f < \overline{v}_{f'} & \\ t^i_{f'} \leq t^i_f + (\overline{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} & \\ y^i_{ff'} = y^j_{ff'} & \\ y^i_{ff'} + y^i_{f'f} = 1 & \\ \rho^S_{ff'}, \psi^S_{ff'} \geq 0 & \end{array} \right.$$
$$\beta^i_f \in [\underline{t}^i_f, \overline{t}^i_f], \beta^i_{f'} \in [\underline{t}^i_{f'}, \overline{t}^i_{f'}], \beta^j_f \in [\underline{t}^j_f, \overline{t}^j_f], \beta^j_{f'} \in [\underline{t}^j_{f'}, \overline{t}^j_{f'}],$$
$$(t^i_f, \Delta T^i_{ff'}, \rho^S_{ff'}, \psi^S_{ff'}) \in \mathbb{R}, \ (y^i_{ff'}, z^i_{ff'}) \in \{0,1\}.$$

Note that using this formulation, not all constraints are designed to be active simultaneously, thus allowing every possible configuration of aircraft speeds to be correctly modelled. Suppose for instance that $\underline{v}_f < \overline{v}_{f'}$ and that $\underline{v}_{f'} = \overline{v}_f$. First consider the triplet $(f, f', S) \in \mathcal{P}_t$, then according to condition $\underline{v}_f < \overline{v}_{f'}$, the following constraints are active:

$$\rho^S_{ff'} \geq \beta^j_f - \beta^i_{f'} + \psi^S_{ff'} \tag{88}$$
$$\psi^S_{ff'} \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(Dy^i_{ff'} - \overline{v}_{f'}\beta^i_f + \underline{v}_f\beta^i_{f'}\right) \tag{89}$$

Consider now the triplet $(f', f, S) \in \mathcal{P}_t$, after exchanging the roles of indices f and f' in model 2 and according to condition $\underline{v}_{f'} = \overline{v}_f$, the following constraints are active:

$$\rho^S_{f'f} \leq z^i_{f'f} \cdot \overline{\rho}^S_{f'f} \tag{90}$$
$$\rho^S_{f'f} \geq (t^j_{f'} - t^i_f) - (1 - z^i_{f'f}) \cdot \overline{\rho}^S_{f'f} \tag{91}$$
$$t^i_f - t^i_{f'} \geq (1 - z^i_{f'f}) \cdot D/\overline{v}_f - (\overline{t}^i_{f'} - \underline{t}^i_f + D/\overline{v}_f) \cdot y^i_{ff'} \tag{92}$$
$$z^i_{f'f} \leq y^i_{f'f} \tag{93}$$

Assume flights f and f' are the only flights in a potential trailing conflict, then the model to solve is:

Model 3 (Example)

$$\boxed{\min \rho^S_{ff'} + \rho^S_{f'f}}$$

subject to:

$$\forall (f,i) \in \mathcal{P}:$$
$$\mathcal{SM} = \left\{ \quad \underline{t}^i_f \leq t^i_f \leq \overline{t}^i_f \right.$$
$$\forall (f,f',S) \in \mathcal{P}_t:$$
$$\mathcal{TM} = \left\{ \begin{array}{l} \left.\begin{array}{ll} \rho^S_{f'f} & \leq z^i_{f'f} \cdot \overline{\rho}^S_{f'f} \\ \rho^S_{f'f} & \geq (t^j_{f'} - t^i_f) - (1 - z^i_{f'f}) \cdot \overline{\rho}^S_{f'f} \\ t^i_f - t^i_{f'} & \geq (1 - z^i_{f'f}) \cdot D/\overline{v}_f - (\overline{t}^i_{f'} - \underline{t}^i_f + D/\overline{v}_f) \cdot y^i_{ff'} \\ z^i_{f'f} & \leq y^i_{f'f} \end{array}\right\} \textit{Constraints on } \rho^S_{f'f} \\ \left.\begin{array}{ll} \rho^S_{ff'} & \geq \beta^j_f - \beta^i_{f'} + \psi^S_{ff'} \\ \psi^S_{ff'} & \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f}\left(Dy^i_{ff'} - \overline{v}_{f'}\beta^i_f + \underline{v}_f\beta^i_{f'}\right) \end{array}\right\} \textit{Constraints on } \rho^S_{ff'} \\ t^i_{f'} \leq t^i_f + (\overline{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \\ t^i_f \leq t^i_{f'} + (\overline{t}^i_f - \underline{t}^i_{f'}) \cdot y^i_{f'f} \\ y^i_{ff'} = y^j_{ff'} \\ y^i_{ff'} + y^i_{f'f} = 1 \\ \rho^S_{ff'}, \psi^S_{ff'} \geq 0 \end{array} \right.$$

If f' is the leader (see model 4) that is, $y^i_{ff'} = 0$, then $y^i_{f'f} = 1$ and all β-reformulations vanish. In model 4, $\rho^S_{ff'}$ is only constrained to be positive and thus is set to 0 during the minimization. $\Delta T^i_{f'f}$ is now equal to the correct crossing time difference and the value of $z^i_{f'f}$ depends on the bounds of decision variables t^i_f and $t^i_{f'}$. If f is the leader (see model 5), that is $y^i_{ff'} = 1$, then $y^i_{f'f} = 0$ and all β-reformulations are equal to the corresponding crossing time. In model 5, $\rho^S_{f'f}$ is automatically set to 0 for $z^i_{f'f} = 0$ and $\rho^S_{ff'}$ is correctly lower bounded.

Model 4 (f' leader)

$$\boxed{\min \rho^S_{ff'} + \rho^S_{f'f}}$$

subject to:

$$\forall (f,i) \in \mathcal{P}:$$
$$\mathcal{SM} = \left\{ \quad \underline{t}^i_f \quad \leq t^i_f \leq \overline{t}^i_f \right.$$
$$\forall (f,f',S) \in \mathcal{P}_t:$$
$$\mathcal{TM} = \left\{ \begin{array}{l} \left. \begin{array}{ll} \rho^S_{f'f} & \leq z^i_{f'f} \cdot \overline{\rho}^S_{f'f} \\ \rho^S_{f'f} & \geq (t^j_{f'} - t^i_f) - (1 - z^i_{f'f}) \cdot \overline{\rho}^S_{f'f} \\ t^i_f - t^i_{f'} & \geq (1 - z^i_{f'f}) \cdot D/\overline{v}_f \\ z^i_{f'f} & \leq 1 \end{array} \right\} \text{Constraints on } \rho^S_{f'f} \\ \left. \begin{array}{ll} \rho^S_{ff'} & \geq \psi^S_{ff'} \\ \psi^S_{ff'} & \geq 0 \end{array} \right\} \text{Constraints on } \rho^S_{ff'} \\ t^i_f \leq t^i_{f'} + \overline{t}^i_f - \underline{t}^i_{f'} \\ t^i_{f'} \leq t^i_f \\ \rho^S_{ff'}, \psi^S_{ff'} \geq 0 \end{array} \right.$$

Model 5 (f leader)

$$\boxed{\min \rho^S_{ff'} + \rho^S_{f'f}}$$

subject to:

$$\forall (f,i) \in \mathcal{P}:$$
$$\mathcal{SM} = \left\{ \quad \underline{t}^i_f \quad \leq t^i_f \leq \overline{t}^i_f \right.$$
$$\forall (f,f',S) \in \mathcal{P}_t:$$
$$\mathcal{TM} = \left\{ \begin{array}{l} \left. \begin{array}{ll} \rho^S_{f'f} & \leq 0 \\ \rho^S_{f'f} & \geq (t^j_{f'} - t^i_f) - \overline{\rho}^S_{f'f} \\ t^i_f - t^i_{f'} & \geq \underline{t}^i_f - \overline{t}^i_{f'} \\ z^i_{f'f} & \leq 0 \end{array} \right\} \text{Constraints on } \rho^S_{f'f} \\ \left. \begin{array}{ll} \rho^S_{ff'} & \geq t^j_f - t^i_{f'} + \psi^S_{ff'} \\ \psi^S_{ff'} & \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f} \left(D - \overline{v}_{f'} t^i_f + \underline{v}_f t^i_{f'} \right) \end{array} \right\} \text{Constraints on } \rho^S_{ff'} \\ t^i_f \leq t^i_{f'} \\ t^i_{f'} \leq t^i_f + \overline{t}^i_{f'} - \underline{t}^i_f \\ \rho^S_{ff'}, \psi^S_{ff'} \geq 0 \end{array} \right.$$

5 A Complete Speed Regulation Model

Model 6 (Crossing and Trailing Conflicts Model)

$$\boxed{\min \sum_{(f,f',i)\in\mathcal{P}_c} \frac{1}{2}\omega^i_{ff'} + \sum_{(f,f',S)\in\mathcal{P}_t} \rho^S_{ff'} + \rho^S_{f'f}}$$

subject to:

$$\forall (f,i) \in \mathcal{P}:$$

$$\mathcal{SM} = \left\{ \quad \underline{t}^i_f \leq t^i_f \leq \overline{t}^i_f \right.$$

$$\forall (f,f',i) \in \mathcal{P}_c:$$

$$\mathcal{CM} = \left\{ \begin{array}{l} \omega^i_{ff'} \geq \left(t^i_f - t^{prec(i)}_f\right) \cdot G^i_{ff'} - \Delta T^i_{ff'} \\ \omega^i_{ff'} \geq 0 \\ \omega^i_{ff'} = \omega^i_{f'f} \\ \Delta T^i_{ff'} \leq t^i_f - t^i_{f'} + 2(\overline{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \\ \Delta T^i_{ff'} \geq t^i_f - t^i_{f'} \\ \Delta T^i_{ff'} = \Delta T^i_{f'f} \\ t^i_{f'} \leq t^i_f + (\overline{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \\ y^i_{ff'} + y^i_{f'f} = 1 \end{array} \right.$$

$$\forall (f,f',S) \in \mathcal{P}_t:$$

$$\mathcal{TM} = \left\{ \begin{array}{l} \left. \begin{array}{ll} \rho^S_{ff'} & \leq z^i_{ff'} \cdot \overline{\rho}^S_{ff'} \\ \rho^S_{ff'} & \geq (t^j_f - t^i_{f'}) - (1 - z^i_{ff'}) \cdot \overline{\rho}^S_{ff'} \\ t^i_{f'} - t^i_f & \geq (1 - z^i_{ff'}) \cdot D/\overline{v}_{f'} - (\overline{t}^i_f - \underline{t}^i_{f'} + D/\overline{v}_{f'}) \cdot (1 - y^i_{ff'}) \\ z^i_{ff'} & \leq y^i_{ff'} \end{array} \right\} \textit{if: } \underline{v}_f = \overline{v}_{f'} \\ \left. \rho^S_{ff'} \geq \beta^j_f - \beta^i_{f'} + \psi^S_{ff'} \right\} \textit{if: } \underline{v}_f \neq \overline{v}_{f'} \\ \left. \psi^S_{ff'} \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f} \left(-D y^i_{ff'} + \overline{v}_{f'} \beta^i_{f'} - \overline{v}_{f'} \beta^i_f\right) - \beta^j_f \right\} \textit{if: } \underline{v}_f > \overline{v}_{f'} \\ \left. \psi^S_{ff'} \geq \frac{1}{\overline{v}_{f'} - \underline{v}_f} \left(D y^i_{ff'} - \overline{v}_{f'} \beta^i_f + \underline{v}_f \beta^i_{f'}\right) \right\} \textit{if: } \underline{v}_f \neq \overline{v}_{f'} \\ t^i_{f'} \leq t^i_f + (\overline{t}^i_{f'} - \underline{t}^i_f) \cdot y^i_{ff'} \\ y^i_{ff'} = y^j_{ff'} \\ y^i_{ff'} + y^i_{f'f} = 1 \\ \rho^S_{ff'}, \psi^S_{ff'} \geq 0 \end{array} \right.$$

$$\beta^i_f \in [\underline{t}^i_f, \overline{t}^i_f], \beta^i_{f'} \in [\underline{t}^i_{f'}, \overline{t}^i_{f'}], \beta^j_f \in [\underline{t}^j_f, \overline{t}^j_f], \beta^j_{f'} \in [\underline{t}^j_{f'}, \overline{t}^j_{f'}],$$

$$(t^i_f, \Delta T^i_{ff'}, \omega^i_{ff'}, \rho^S_{ff'}, \psi^S_{ff'}) \in \mathbb{R}, \ (y^i_{ff'}, z^i_{ff'}) \in \{0,1\}.$$

Model 6 is a 01-Mixed Integer Linear Program (MILP) which is a combination of sub-models 1 and 2. Notice that these sub-models are independent and can thus be deactivated if required. Model 6 can be presented in a reduced form as:

Model 7 (MILP for the speed regulation problem)

$$\boxed{\min \sum_{(f,f',i)\in\mathcal{P}_c} \frac{1}{2}\omega^i_{ff'} + \sum_{(f,f',S)\in\mathcal{P}_t} \rho^S_{ff'} + \rho^S_{f'f}}$$

subject to:

$$\forall (f,i) \in \mathcal{P} : \mathcal{SM}$$

$$\forall (f,f',i) \in \mathcal{P}_c : \mathcal{CM}$$

$$\forall (f,f',S) \in \mathcal{P}_t : \mathcal{TM}$$

$$(\omega^i_{ff'}, \rho^S_{ff'}, \rho^S_{f'f}) \in \mathbb{R}.$$

Conclusion

In this report a complete modellization of the speed regulation problem is proposed. The final model is a combination of two sub-models, 1 and 2, that are designed to minimize the total potential conflicts duration for both crossing and trailing conflicts respectively. All the models are developed using worst-case bounds on aircraft speeds in order to provide a robust framework where no potential conflict can be leftover. The models are formulated with linear constraints and objective functions, hence they can be solved using industrial-strength optimization solvers.

A Notations and glossary

Quantity	Definition	Qualification in the model
$\mathcal{P}$	set of potential conflicts	set
$\mathcal{P}_c$	set of potential crossing conflicts	set
$\mathcal{P}_t$	set of potential trailing conflicts	set
D	horizontal separation norm	parameter
v_f	speed of flight f	none
$\underline{v}_f, \overline{v}_f$	lower and upper bounds on flight f speed	parameters
t_f^i	crossing time of flight f at point i	real decision variable
$\underline{t}_f^i, \overline{t}_f^i$	lower and upper bounds on t_f^i	parameters
$\Delta T_{ff'}^i$	crossing time difference between flights f and f' at point i: $\Delta T_{ff'}^i = \lvert t_f^i - t_{f'}^i \rvert$	real decision variable
$\omega_{ff'}^i$	crossing conflict duration at point i	real decision variable
$y_{ff'}^i$	crossing order of flights f and f' at point i: $y_{ff'}^i = 1$ if $t_f^i \leq t_{f'}^i$, $y_{ff'}^i = 0$ otherwise	binary decision variable
t_i	date after which both aircraft are on the shared segment	none
t_j	date after which one aircraft has left the shared segment	none
T_∞	trailing conflict duration on an infinite segment	none
t_b, t_e	trailing conflict beginning and ending dates	none
τ_b, τ_e	worst-case FIFO-trailing conflict beginning and ending dates	none
τ_f^b, τ_f^e	worst-case FIFO-trailing conflict beginning and ending dates if f is the leader	none
$\tau_{f'}^b, \tau_{f'}^e$	worst-case FIFO-trailing conflict beginning and ending dates if f' is the leader	none
v_l, v_p	speeds of the leader and the follower	none
$z_{ff'}^i$	flights f and f' separation at point i if f is the leader: $z_{ff'}^i = 0$ if $\Delta T_{ff'}^i \geq D/\overline{v}_{f'}$, $z_{ff'}^i = 1$ otherwise	binary decision variable
$z_{f'f}^i$	flights f and f' separation at point i if f' is the leader: $z_{f'f}^i = 0$ if $\Delta T_{ff'}^i \geq D/\overline{v}_f$, $z_{f'f}^i = 1$ otherwise	binary decision variable
ρ^S	trailing conflict duration on segment S	none
$\rho_{ff'}^S$	trailing conflict duration on segment S if f is the leader	real decision variable
$\rho_{f'f}^S$	trailing conflict duration on segment S if f' is the leader	real decision variable
$\overline{\rho}_{ff'}^S, \overline{\rho}_{f'f}^S$	upper bounds on $\rho_{ff'}^S$ and $\rho_{f'f}^S$	parameters
ψ^S	trailing conflict duration adjustment if $\underline{v}_l \neq \overline{v}_p$	none
$\psi_{ff'}^S$	trailing conflict duration adjustment if $\underline{v}_f \neq \overline{v}_{f'}$ and if f is the leader	real decision variable
$\psi_{f'f}^S$	trailing conflict duration adjustment if $\underline{v}_{f'} \neq \overline{v}_f$ and if f' is the leader	real decision variable
β_f^i	linearization of decision variable product: $\beta_f^i = t_f^i \cdot y_{ff'}^i$	real decision variable

B Crossing Conflicts and the Gamma Function

The next figures represent the surfaces generated by function $\Gamma(v_f, v_{f'})$ for a fixed value of θ, the confluence angle. The surfaces are generated by considering aircraft speeds as bounded variables. The planar plots and the color map on the right-hand side of the figures show that maximum values of $\Gamma(v_f, v_{f'})$ are achieved when aircraft speeds are both maximal or minimal according to θ. For low values of θ, maximum values of $\Gamma(v_f, v_{f'})$ are reached when aircraft fly at their maximal speed whereas for $\theta \geq 34°$, maximum values of $\Gamma(v_f, v_{f'})$ are achieved for minimal aircraft speeds.

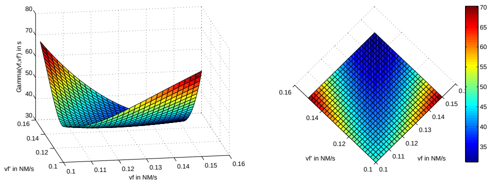


Figure 4: $\theta = 15°$

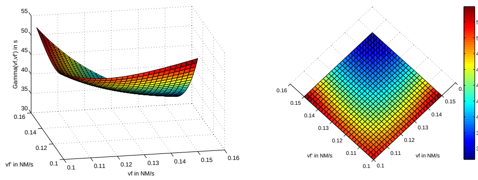


Figure 5: $\theta = 30°$

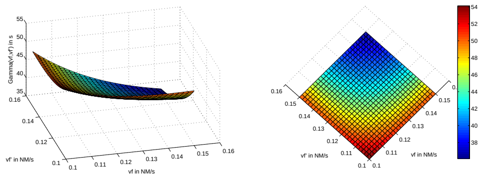


Figure 6: $\theta = 45°$

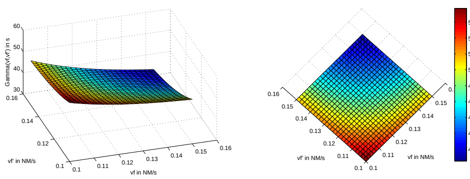


Figure 7: $\theta = 60°$

C Linearization through reformulations

Along the linearization process that is required to provide a linear framework for the speed regulation problem, several decision variables products are encountered and need to be refined. Most of the time these products appear to model a binary decision. However, according to this decision another decision ought to be taken on a continuous decision variable. In order to linearize constraints where a binary and a continuous decision variable are expressed as a product, we need to introduce an auxiliary decision variable. This can be achieved if the continuous decision variable is bounded [5], which is often the case in the models. Let b be a binary decision variable and $c \in [\underline{c}, \overline{c}]$ be a continuous decision variable. To linearize the decision variable product $p = bc$, we can introduce $p \in \mathbb{R}$ as an auxiliary decision variable in the model by including the following constraints set to the model:

$$p \geq b\underline{c} \tag{99}$$
$$p \leq b\overline{c} \tag{100}$$
$$p \geq c - (1-b)\overline{c} \tag{101}$$
$$p \leq c - (1-b)\underline{c} \tag{102}$$

Clearly, if $b = 0$ then $0 \leq p \leq 0 \iff p = 0$ and constraints (101,102) become redundant. If $b = 1$ then $c \leq p \leq c \iff p = c$ and constraints (99,100) become bounds on p. This reformulation of the decision variable product bc is linear and thus provides an efficient method to linearize products of binary and bounded continuous decision variables. To simplify the models description, constraint $p = \mathcal{L}(b, c)$ will be used instead of the full constraints set linearizing the decision variables product.

$$\forall b \in \{0,1\}, c \in [\underline{c}, \overline{c}], p \in \mathbb{R} \qquad p = \mathcal{L}(b,c) \iff \begin{cases} p & \geq b\underline{c} \\ p & \leq b\overline{c} \\ p & \geq c - (1-b)\overline{c} \\ p & \leq c - (1-b)\underline{c} \end{cases} \tag{103}$$

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