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# Information Propagation Speed in Bidirectional Vehicular Delay Tolerant Networks

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**Abstract**—In this paper, we provide an analysis of the information propagation speed in bidirectional vehicular delay tolerant networks on highways. We show that a phase transition occurs concerning the information propagation speed, with respect to the vehicle densities in each direction of the highway. We prove that under a certain threshold, information propagates on average at vehicle speed, while above this threshold, information propagates dramatically faster at a speed that increase exponentially when vehicle density increases. We provide the exact expressions of the threshold and of the average propagation speed near the threshold. We show that under the threshold, the information propagates on a distance which is bounded by a sub-linear power law with respect to the elapsed time, in the referential of the moving cars. On the other hand, we show that information propagation speed grows quasi-exponentially with respect to vehicle densities in each direction of the highway, when the densities become large, above the threshold. We confirm our analytical results using simulations carried out in several environments.

## I. INTRODUCTION

The limits of the performance of multi-hop packet radio networks have been studied for more than a decade, yielding fundamental results such as those of Gupta and Kumar [12] on the capacity of fixed ad hoc networks. Following early work such as [13] evaluating the potential of mobility to increase capacity, recent research studies focussed on the limits of the performance beyond the end-to-end hypothesis, *i.e.*, when end-to-end paths may not exist and communication routes may only be available through time and mobility. In this context nodes may carry packets for a while until a path becomes available. Such networks are generally referred as Delay Tolerant Networks (DTNs). Interest in DTN modeling and analysis has risen as novel network protocols and architectures are being elaborated to accommodate various forms of new, intermittently connected networks, which include vehicular ad hoc networks (VANETs), power-saving sensor networks, etc.

In this paper, we study the information propagation speed in the typical case of bidirectional vehicular DTNs, *e.g.* on highways. Our analysis shows that a phase transition occurs concerning information propagation speed, with respect to the vehicle density on both directions. We prove that under a certain threshold, information propagates on average at vehicle speed, while above this threshold, information propagates much faster. We provide the exact expressions of the threshold and of the average propagation speed near the threshold.

With applications such as safety, ad hoc vehicular networks are receiving increasing attention (see recent survey [4]). Delay tolerant architectures have thus been considered in this context,

and various analytical models have been proposed. In [9], the authors study vehicle traces and conclude that vehicles are very close to being exponentially distributed on highways. In [6], the authors provide a model for critical message dissemination in vehicular networks and derive results on the average delay in delivery of messages with respect to vehicle density. The authors of [11] propose an alternative model for vehicular DTNs and derived results on node connectivity. In [10], the authors model vehicles on a highway, and study message propagation among vehicles in the same direction, taking into account speed differences between vehicles, while in [8] authors study message dissemination among vehicles in opposing directions and conclude that using both directions increases dissemination significantly.

Studies [1], [2] introduce a model based on space discretization to derive upper and lower bounds in the highway model under the assumption that the radio propagation speed is finite. Their bounds, although not converging, clearly indicates the existence of a phase transition phenomenon for the information propagation speed. Comparatively, we introduce a model based on Poisson point process on continuous space, that allows both infinite and finite radio propagation speed, and derive fine-grained results. Using our model, we prove and explicitly characterize the phase transition.

In this context, our contributions are as follows: (1) we develop a new vehicule-to-vehicule model for information propagation in bidirectional vehicular DTNs in Section II; (2) we show the existence of a threshold (with respect to vehicle density), above which information speed increases dramatically over vehicle speed, and below which information propagation speed is on average equal to vehicle speed, and (3) we give the exact expression of this threshold, in Section III; (4) in Section V, we prove that, under the threshold, even though the average propagation speed equals the vehicle speed, DTN routing using cars moving on both directions provides a gain in the propagation distance, and this gain follows a simple power law with respect to vehicle density below this threshold, is bounded by a sub-linear power law with respect to the elapsed time, in the referential of the moving cars; (5) we characterize information propagation speed as increasing quasi-exponentially with the vehicle density when the latter becomes large above the threshold, in Section IV; (6) we cover both infinite radio propagation speed cases, then finite radio propagation speed cases in Section VI; (7) we validate the provided analysis with simulations in several environments,

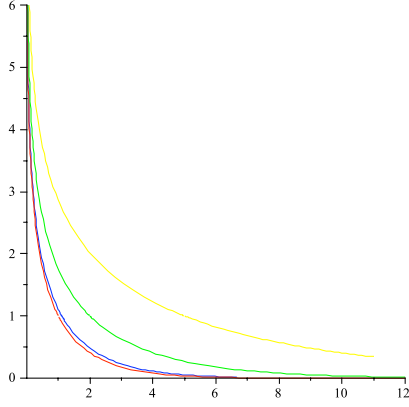


Fig. 1. Information propagation threshold with respect to  $(\lambda_e, \lambda_w)$  for infinite radio speed in red. In blue for radio speed  $v_r = 10v$ , in green  $v_r = 2v$ , in yellow  $v_r = 1.25v$ .

which confirm the results of the analysis, in Section VII.

## II. MODEL AND RESULTS

In the following, we consider a bidirectional vehicular network, such as a road or a highway, where vehicles move in two opposite directions (say east and west, respectively) at speed  $v$ . Let us consider eastbound vehicle density as Poisson with intensity  $\lambda_e$ , while westbound vehicle density is Poisson with intensity  $\lambda_w$ . Furthermore, we consider that the radio propagation speed  $v_r$  (including store and forward processing time) is infinite, and that the radio range of each transmission in each direction is equal to 1 unit length. Case for finite radio speed is investigated in a separate section.

The main result presented in this paper is that, concerning the information propagation speed in such an environment, a phase transition occurs when  $\lambda_e$  and  $\lambda_w$  are conjugate on the curve  $y = xe^{-x}$ , i.e., either  $\lambda_e \neq \lambda_w$  and

$$\lambda_e e^{-\lambda_e} = \lambda_w e^{-\lambda_w}, \quad (1)$$

or  $\lambda_e = \lambda_w = 1$ .

Figure 1 shows the threshold curve for  $v_r = \infty$  in red. We show that below this threshold, the average information propagation speed is blocked to vehicle speed, while above the curve, information propagates strictly faster on average. We focus on the propagation of information in the eastbound lane. As described in [1], the information beacon propagates in the following manner: it moves toward the east jumping from car to car until it stops because the next car is beyond radio range. The propagation is instantaneous, since we assume that radio routing speed is infinite. The beacon waits on the last eastbound car until the gap is filled by westbound cars, so that the beacon can move again to the next eastbound car.

We denote  $\mathbf{T}_i$  the duration the beacon waits when blocked for the  $i$ th time and  $\mathbf{D}_i$  the distance traveled by the beacon just after. The random variables  $\mathbf{T}_i$  and  $\mathbf{D}_i$  are dependent but, due to the Poisson nature of vehicle traffic, the tuples in the sequence  $(\mathbf{T}_i, \mathbf{D}_i)$  are *i.i.d.* as noticed in [2]. From now on, we denote  $(\mathbf{T}, \mathbf{D})$  the independent random variable.

We denote  $L(t)$  the distance traveled by the beacon during a time  $t$  on the eastbound lane. We consider the distance traveled with respect to the referential of the eastbound cars. We define the average information propagation speed  $v_p = \lim_{t \rightarrow \infty} \frac{\mathbf{E}(L(t))}{t}$ . By virtue of the renewal processes, we have  $v_p = \frac{\mathbf{E}(\mathbf{D})}{\mathbf{E}(\mathbf{T})}$ . For the remainder of the paper, for  $x > 0$ , we denote  $x^*$  the conjugate of  $x$  with respect to function  $xe^{-x}$ :  $x^*$  is the alternate solution of the equation  $x^*e^{-x^*} = xe^{-x}$ . We prove the following theorem.

*Theorem 1: For all  $(\lambda_e, \lambda_w)$ , the information propagation speed  $v_p$  with respect to the referential of the eastbound cars is  $v_p < \infty$ , and,*

$$\lambda_e < \lambda_w^* \Rightarrow v_p = 0, \quad (2)$$

$$\lambda_e > \lambda_w^* \Rightarrow v_p > 0. \quad (3)$$

*Theorem 2: When  $\lambda_w^* > \lambda_e$  (case  $v_p = 0$ ), when  $t \rightarrow \infty$ ,*

$$\mathbf{E}(L(t)) \leq B(\lambda_e, \lambda_w)(2vt)^{\frac{\lambda_e}{\lambda_w^*}}. \quad (4)$$

for some  $B(\lambda_e, \lambda_w)$ , explicit function of  $(\lambda_e, \lambda_w)$ .

## III. PHASE TRANSITION: PROOF OF THEOREM 1

### A. Proof Outline

We call cluster a maximal sequence of cars such that two consecutive cars are within radio range. A westbound (respectively, eastbound) cluster is a cluster made exclusively of westbound (respectively, eastbound) cars. A full cluster is made of westbound and eastbound cars. We define the length of the cluster as the distance between the first and last cars augmented by a radio range. We denote  $L_w$  a westbound cluster length. We start by computing in Section III-B the Laplace transform of  $L_w$ :  $f_w(\theta) = \mathbf{E}(e^{-\theta L_w})$ , thus proving that the exponential tail of the distribution of  $L_w$  is given by

$$P(L_w > x) = \Theta(e^{-\lambda_w^* x}). \quad (5)$$

To evaluate how information will propagate, we compute the distribution of the gap length  $G_e$  between the cluster of eastbound cars on which the beacon is blocked and the next cluster of eastbound cars. We show in Section III-D that the density  $p_e(x)$  of gap distribution length is  $\Theta(e^{-\lambda_e x})$ .

Now, let  $\mathbf{T}(x)$  be the time needed to meet a westbound cluster long enough to fill a gap of length  $x$  (i.e., a westbound cluster of length larger than  $x$ ). We show in Section III-C that:

$$\mathbf{E}(\mathbf{T}(x)) = \Theta\left(\frac{1}{vP(L_w > x)}\right) = \Theta(e^{\lambda_w^* x}). \quad (6)$$

The average time  $\mathbf{T}$  to get a bridge over a gap is

$$\begin{aligned} \mathbf{E}(\mathbf{T}) &= \int_1^\infty \mathbf{E}(\mathbf{T}(x))p_e(x)dx \\ &= \frac{1}{2v} \int_1^\infty \Theta(\exp((\lambda_w^* - \lambda_e)x))dx. \end{aligned} \quad (7)$$

As a result, the threshold with respect to  $(\lambda_w, \lambda_e)$  where  $\mathbf{E}(\mathbf{T})$  diverges is clearly when we have:  $\lambda_w^* = \lambda_e$ , or, in other words, since  $\lambda_w^*e^{-\lambda_w^*} = \lambda_w e^{-\lambda_w}$ , when we have:

$$\lambda_w e^{-\lambda_w} = \lambda_e e^{-\lambda_e}. \quad (8)$$

### B. Cluster Length Distribution

*Lemma 1:* The Laplace transform of a random westbound cluster length  $f_w(\theta) = \mathbf{E}(e^{-\theta L_w})$  satisfies:

$$f_w(\theta) = \frac{(\lambda_w + \theta)e^{-\lambda_w - \theta}}{\theta + \lambda_w e^{-\lambda_w - \theta}}. \quad (9)$$

*Proof:* This is a straightforward result borrowed from queueing theory. ■

*Lemma 2:* We have the asymptotic formula:

$$P(L_w > x) = \frac{(\lambda_w - \lambda_w^*)e^{\lambda_w^* - \lambda_w}}{(1 - \lambda_w^*)\lambda_w^*} e^{-\lambda_w^* x} (1 + o(1)) \quad (10)$$

*Proof:* The asymptotics on  $P(L_w > x)$  are given by inverse Laplace transform since  $f_w(\theta)$  has a main singularity on  $\theta = -\lambda_w^*$ . ■

### C. Road Length to Bridge a Gap

Now, let us assume that we want to fill a gap of length  $x$ . We want to know the average length of westbound road until the first cluster that has a length greater than  $x - 1$ . Figure 2 depicts a gap of length  $x$ , and the length of westbound road until a cluster is encountered which can bridge the gap. Let  $f_w(\theta, x) = \mathbf{E}(1_{(L_w < x)} e^{-\theta L_w})$ .

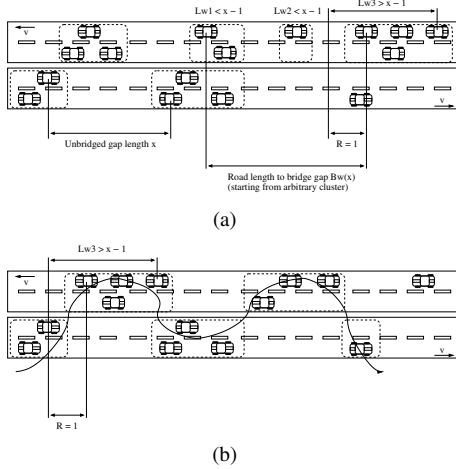


Fig. 2. Illustration of the road length  $B_w(x)$  until a gap  $x$  is bridged: (a) smaller clusters cannot bridge the gap, (b) until a westbound cluster of length at least  $x - 1$  is encountered.

We denote  $B_w(x)$  the westbound road length to bridge a gap of length  $x$ , starting from the beginning of an arbitrary westbound cluster. We denote  $\beta_w(\theta, x) = \mathbf{E}(\exp(-\theta B_w(x)))$ .

*Lemma 3:* We have

$$\beta_w(\theta, x) = \frac{P(L_w > x - 1)}{1 - \frac{\lambda_w}{\lambda_w + \theta} f_w(\theta, x - 1)}, \quad (11)$$

$$\mathbf{E}(B_w(x)) = (1 + O(e^{-\varepsilon x})) \frac{e^{\lambda_w}}{\lambda_w} \frac{(1 - \lambda_w^*)\lambda_w^*}{(\lambda_w - \lambda_w^*)e^{\lambda_w^* - \lambda_w}} e^{(x-1)\lambda_w^*}. \quad (12)$$

*Proof:* The identity (11) comes from renewal theory since the clusters and inter-cluster are i.i.d., quantity  $\frac{\lambda_w}{\lambda_w + \theta} f_w(\theta, x - 1)$  is the Laplace transform of the road length made of a

random inter-cluster and a cluster of length smaller than  $x - 1$ . A gap of length  $x$  will be filled if and only if it is filled by a cluster of length greater than  $x - 1$ . Thus, the average is

$$\begin{aligned} \mathbf{E}(B_w(x)) &= -\frac{\partial}{\partial \theta} \beta_w(0, x) \\ &= -\left( \frac{\partial}{\partial \theta} f_w(0, x - 1) - \frac{1}{\lambda_w} f_w(0, x - 1) \right) \\ &\quad \times \frac{1}{P(L_w > x - 1)} \\ &= \left( \frac{e^{\lambda_w}}{\lambda_w} + O(e^{-(x-1)\lambda_w^*}) \right) \frac{1}{P(L_w > x - 1)} \end{aligned}$$

### D. Gap Distribution

Let us call  $G_e$  an eastbound gap which is not bridged (see Figure 3). As illustrated in Figure 4,  $G_e$  can be decomposed into a westbound cluster length  $L_w^*$  without eastbound cars, plus a random exponentially distributed distance  $I_e$ .

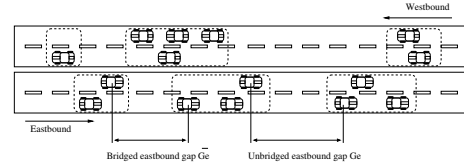


Fig. 3. Illustration of a bridged gap  $\tilde{G}_e$ , and an unbridged gap  $G_e$ .

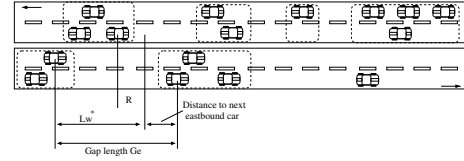


Fig. 4. Unbridged gap  $G_e$  model;  $L_w^*$  corresponds to a westbound cluster length without eastbound cars.

*Lemma 4:* The distribution of  $G_e$  satisfies

$$\mathbf{E}(e^{-\theta G_e}) = \frac{f_w(\theta + \lambda_e)}{f_w(\lambda_e)} \frac{\lambda_e}{\lambda_e + \theta}, \quad (13)$$

which is defined for all  $\Re(\theta) > -\lambda_e$ , and

$$\mathbf{E}(G_e) = -\frac{f_w'(\lambda_e)}{f_w(\lambda_e)} + \frac{1}{\lambda_e}. \quad (14)$$

*Proof:* We have  $\mathbf{E}(e^{-\theta L_w^*}) = \frac{\mathbf{E}(e^{-(\theta + \lambda_e)L_w})}{\mathbf{E}(e^{-\lambda_e L_w})}$ . ■

*Lemma 5:* The probability density  $p_e(x)$  of  $G_e$  satisfies:

$$p_e(x) = \frac{\lambda_e}{f_w(\lambda_e)} e^{-\lambda_e x} (1 + O(e^{-\varepsilon x})). \quad (15)$$

*Proof:* The proof comes from a straightforward singularity analysis on the inverse Laplace transform. ■

### E. Distribution of Waiting Time $\mathbf{T}$

*Lemma 6:* We have  $2v\mathbf{T} = L_w^* + I_w + B_w - 1$ , where  $I_w$  is the random distance to a next westbound car, and  $B_w$  the length of westbound road before the cluster that fill the gap starting from an arbitrary westbound cluster. And

$$2v\mathbf{E}(\mathbf{T}) = \mathbf{E}(L_w^*) - 1 + \frac{1}{\lambda_w} + \int_1^\infty \mathbf{E}(B_w(x))p_e(x)dx. \quad (16)$$

*Proof:* The total length of westbound road to bridge a gap of length  $x$  equals the distance to the beginning of the first westbound cluster ( $L_w^* + I_w$ ) plus the road length to bridge a gap starting from this cluster, namely  $B_w - 1$ . Since the relative speed of cars moving in opposite directions is  $2v$ , we have  $2vT = L_w^* + I_w + B_w - 1$ . We complete the proof by taking the expectations, and averaging on all possible gap lengths  $x$ . ■

*Corollary 1:* The quantity  $\mathbf{E}(\mathbf{T})$  converges when  $\lambda_e > \lambda_w^*$  and diverges when  $\lambda_e < \lambda_w^*$ .

*Proof:* The proof comes from the leading terms of  $\mathbf{E}(B_w(x))$  and  $p_e(x)$ . ■

### F. Distance $\mathbf{D}$ Traveled after Waiting Time $\mathbf{T}$

We denote  $C_e$  the distance traveled in the eastbound road referential beyond the gap after it has been bridged and before the next gap. As depicted in Figure 5, we have  $\mathbf{D} = G_e + C_e$ .

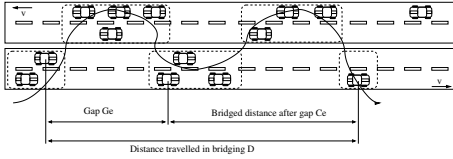


Fig. 5. Total distance  $\mathbf{D}$  traveled when a bridge is created  $\mathbf{D} = G_e + C_e$ .

*Lemma 7:* The Laplace transform  $\mathbf{E}(e^{-\theta C_e})$  is defined for all  $\Re(\theta) > -(\lambda_e + \lambda_w)^*$ .

*Proof:* The random variable  $C_e$  is smaller in probability than a full cluster. ■

*Lemma 8:* The average value of  $C_e$  satisfies:

$$\mathbf{E}(C_e) = \frac{1}{\lambda_e} \frac{1 - f_w(\lambda_e)}{f_w(\lambda_e)} + \frac{f'_w(\lambda_e)}{f_w(\lambda_e)}. \quad (17)$$

*Proof:* The probability that an eastbound car is not connected or bridged to the next eastbound car equals  $f_w(\lambda_e)$ . The average inter eastbound car distance is  $\frac{1}{\lambda_e}$ . We define  $\bar{G}_e$  such a random distance under the condition that it is bridged or smaller than 1 (see Figure 3). It satisfies:

$$f_w(\lambda_e)\mathbf{E}(G_e) + (1 - f_w(\lambda_e))\mathbf{E}(\bar{G}_e) = \frac{1}{\lambda_e}, \quad (18)$$

which gives  $\mathbf{E}(\bar{G}_e) = \frac{1}{\lambda_e} + \frac{f'_w(\lambda_e)}{1 - f_w(\lambda_e)}$ .

Distance  $C_e$  traveled in bridging (beyond the first gap and extended to the next cluster, which is eventually bridged) is

$$\mathbf{E}(C_e) = (1 - f_w(\lambda_e)) (\mathbf{E}(\bar{G}_e) + \mathbf{E}(C_e)) \quad (19)$$

$$= \frac{1}{\lambda_e} \frac{1 - f_w(\lambda_e)}{f_w(\lambda_e)} + \frac{f'_w(\lambda_e)}{f_w(\lambda_e)}. \quad (20)$$

*Corollary 2:* The total distance  $D_e$  traveled including the first gap satisfies  $\mathbf{E}(D_e) = \mathbf{E}(G_e) + \mathbf{E}(C_e) = \frac{1}{\lambda_e f_w(\lambda_e)}$ , which remains finite for all vehicle densities. ■

Since  $\mathbf{E}(D_e)$  is finite (Corollary 2) and:  $\mathbf{E}(\mathbf{T})$  converges when  $\lambda_e > \lambda_w^*$ , and diverges when  $\lambda_e < \lambda_w^*$  (Corollary 1), we obtain the proof of Theorem 1.

## IV. ASYMPTOTIC ESTIMATES

### A. Near the Threshold

First, we investigate the case where  $(\lambda_e, \lambda_w)$  is close to the threshold boundary. In this case we have

$$2v\mathbf{E}(\mathbf{T}) = -\frac{f'_w(\lambda_e)}{f_w(\lambda_e)} + \int_1^\infty \mathbf{E}(B_w(x))p_e(x)dx$$

This leads to:

$$v_p \sim 2v \frac{(\lambda_w - \lambda_w^*)\lambda_w}{\lambda_e^2(1 - \lambda_w^*)\lambda_w^*} (\lambda_e - \lambda_w^*) e^{\lambda_w^* + \lambda_e - 2\lambda_w}. \quad (21)$$

### B. Large Densities

Now, we investigate the case where the vehicle densities become large, i.e.,  $\lambda_e, \lambda_w \rightarrow \infty$ . In this case, according to Lemma 4, we have:  $\mathbf{E}(L_w^*) = 1 + \frac{\lambda_w}{\lambda_e(\lambda_w + \lambda_e)}$ , and the expected gap length tends to 1. Therefore, the information propagation speed  $v_p = \frac{\mathbf{E}(\mathbf{D})}{\mathbf{E}(\mathbf{T})}$  grows quasi-exponentially with respect to the total vehicle density, i.e.,

$$v_p \sim 2v \frac{e^{\lambda_e + \lambda_w}}{1 + \frac{\lambda_w}{\lambda_e} + \frac{\lambda_e}{\lambda_w}}. \quad (22)$$

## V. POWER LAWS, PROOF OF THEOREM 2

Due to space limitation, we just hint the results in this section (a detailed proof can be found in [5]).

*Lemma 9:* When  $y$  tends to infinity,

$$P(B_w > y) = A(\lambda_e, \lambda_w) y^{-\frac{\lambda_e}{\lambda_w^*}} (1 + o(1)),$$

where  $A(\lambda_e, \lambda_w)$  is some explicit function.

Since  $B_w$  is the main contributor in  $\mathbf{T}$  we have

$$P(\mathbf{T} > t) = A(\lambda_e, \lambda_w) (t2v)^{-\frac{\lambda_e}{\lambda_w^*}} (1 + o(1)). \quad (23)$$

Let  $n(t)$  the number of waiting intervals the beacon has to experience before time  $t$ , we have the inequality

$$P(n(t) \geq n) \leq (P(\mathbf{T} \leq t))^n, \quad (24)$$

and,

$$\mathbf{E}(L(t)) = \mathbf{E}(n(t))\mathbf{E}(\mathbf{D}). \quad (25)$$

the last equality is the consequence of renewal theory and prove theorem 2.

## VI. FINITE RADIO PROPAGATION SPEED

In this short section we assume that the radio propagation speed  $v_r$  is finite and constant with  $v_r > v$  (in the static referential). The main change is that to fill an eastbound gap of length  $x$  one need a westbound cluster of length at least  $x \frac{1+\gamma}{1-\gamma}$  with  $\gamma = \frac{v}{v_r}$ . Therefore the threshold condition becomes  $\lambda_w^* = \frac{1-\gamma}{1+\gamma} \lambda_e$  as shown on Figure 1.

Similarly below the threshold we have  $\mathbf{E}(L(t)) = O(t^{\frac{1-\gamma}{1+\gamma} \frac{\lambda_e}{\lambda_w^*}})$ .

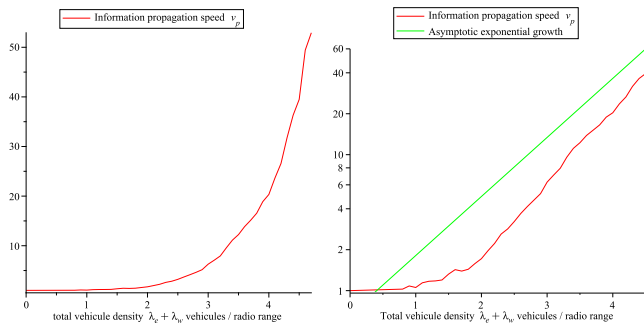


Fig. 6. Maple simulations. Information propagation speed  $v_p$  for  $\lambda_e = \lambda_w$ , versus  $\lambda_e + \lambda_w$ , in linear and semi-log scale, respectively.

## VII. SIMULATIONS

We first compare the theoretical analysis with measurements performed using Maple. In this case, the simulations follow precisely the bidirectional highway model described in Section II: we generate Poisson traffic of eastbound and westbound traffic on two opposite lanes moving at constant speed, which is set to  $v = 1m/s$ . The radio propagation range is  $R = 1m$ , and radio transmissions are instantaneous; the length of the highway is sufficiently large to provide a large number of bridging operations. We measure the information propagation speed which is achieved using optimal DTN routing, by selecting a source and destination pairs at large distances, taking the ratio of the propagation distance over the corresponding delay, and averaging over multiple iterations of randomly generated traffic. We vary the total traffic density, and we plot the resulting information propagation speed. Figure 6 shows the evolution of the information propagation speed near the threshold versus the total vehicle density, when  $\lambda_e = \lambda_w$ , in linear and semilogarithmic plots, respectively. We can observe the threshold at  $\lambda_e + \lambda_w = 2$  in Figure 6, which confirms the analysis presented previously in Section III, and corresponds to  $\lambda_e = \lambda_w = 1$  in Figure 1). In semilogarithmic scale, the simulation measurements quickly approach a straight line, and are close to the theoretically predicted exponential growth above the phase transition threshold, in Section IV.

We then depart from the exact Poisson model simulations in Maple, and we present simulation results obtained with the Opportunistic Network Environment (ONE [7]). Vehicles are distributed uniformly on both lanes of a road, and move at a constant unit speed. The total number of vehicles varies from 1000 to 5000. Again, we measure the fastest possible information propagation speed achieved using epidemic broadcast, assuming that radio transmissions are instantaneous and that there are no buffering or congestion delays, with radio range  $R = 10m$ . We vary the vehicle densities  $\lambda_e$  and  $\lambda_w$ , which are given in vehicles per radio range, and we perform several simulation iterations of randomly generated traffic. In Figure 7, we observe the threshold phenomenon at  $\lambda_e = \lambda_w = 1$ : the information propagation speed remains almost constant below the threshold but increases dramatically beyond it, similarly to our analysis and Maple simulation results.

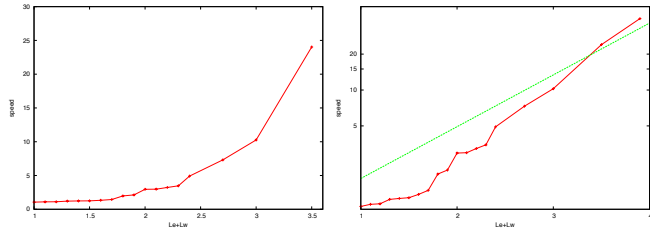


Fig. 7. ONE simulations. Information propagation speed for  $\lambda_e = \lambda_w$ , with respect to  $\lambda_e + \lambda_w$  in linear and semi-log scale, respectively.

## VIII. CONCLUDING REMARKS

This paper provided a detailed analysis for information propagation in bidirectional vehicular DTNs. We proved the existence of a threshold, concerning vehicle density, above which information speed increases dramatically over vehicle speed, and below which information propagation speed is on average equal to vehicle speed. We computed the exact expression of this threshold, and characterized the information propagation speed below and above this threshold. Combining all these different situations, we obtain an image of the way information propagates in vehicular networks on roads and highways, which is useful in designing appropriate routing protocols for VANETs. All our results were validated with simulations in several environments (The One and Maple).

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