Deriving a Hoare-Floyd logic for non-local jumps from a formulae-as-types notion of control

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Hoare-Floyd logics for non-local jumps are notoriously difficult to obtain, especially in the presence of local mutable variables [7]. As far as we know, the question of proving the correctness of imperative programs which combine local mutable *higher-order procedural variables* and non-local jumps has not even been addressed. On the other hand, we know since Griffin's pioneering work [3] how to prove the correctness of (higher-order) functional programs with control in direct style, thanks to the formulæ-as-types interpretation of classical logic.

In [1], Chapter 3, we have thus extended the formulæ-as-types notion of control to imperative programs with higher-order procedural mutable variables and non-local jumps. Our technique, which was inspired by Landin's seminal paper [4], consists in defining an imperative dependent type system **ID** by translation into a functional dependent type system (which is actually Leivant's **ML1P** [5]). This imperative language, called $\textsc{Loop}^\omega$, was defined by the authors in [2].

Similarly to **ML1P**, the imperative type system is parametrized by a first-order signature and an equational system $\mathscr{E}$ which defines a set of functions in the style of Herbrand-Gödel. The syntax of imperative types of **ID** (with dependent procedure types and dependent records) is the following:

$$\sigma, \tau : : = \mathbf{nat}(n) \mid \mathbf{proc} \; \forall \vec{\imath}(\mathbf{in} \; \vec{\tau}; \mathbf{out} \; \vec{\sigma}) \mid \exists \vec{\imath}(\sigma_1, \ldots, \sigma_n) \mid n = m$$

Typing judgements of **ID** have the form $\Gamma; \Omega \vdash e : \psi$ if e is an expression and $\Gamma; \Omega \vdash s \rhd \Omega'$ if s is a sequence, where environments Γ and Ω corresponds respectively to immutable and mutable variables. Note that our type system is *pseudo-dynamic* in the sense that the type of mutable variables can change in a sequence and the new types are given by Ω' (as in [8]). For instance, here is the typing rule of the **for** loop:

$$\frac{\Gamma; \Omega, \vec{x} : \vec{\sigma}[\mathbf{0}/i] \vdash e : \mathbf{nat}(n) \qquad \Gamma, y : \mathbf{nat}(i); \vec{x} : \vec{\sigma} \vdash s \rhd \vec{x} : \vec{\sigma}[\mathbf{s}(i)/i]}{\Gamma; \Omega, \vec{x} : \vec{\sigma}[\mathbf{0}/i] \vdash \mathbf{for} \; y := 0 \; \mathbf{until} \; e \; \{s\}_{\vec{x}} \rhd \vec{x} : \vec{\sigma}[n/i]}$$

Embedding a Hoare-Floyd logic

It is almost straightforward to embed a Hoare-Floyd logic into **ID**. Indeed, let us take a global mutable variable, dubbed *assert*, and let us assume that this global variable is simulated in the usual *state-passing style* (the variable is passed as an explicit **in** and **out** parameter to each procedure call). Consequently, any sequence shall be typed with a sequent of the form $\Gamma; \Omega, \textit{assert} : \varphi \vdash s \rhd \Omega', \textit{assert} : \psi$. If we now introduce the usual Hoare notation for triples (which hides the name of global variable *assert*), we obtain judgments of the form $\Gamma; \Omega \vdash \{\varphi\} s \rhd \Omega' \{\psi\}$. Rules very similar to Hoare rules are then derivable: for instance, the type of *assert* corresponds to the invariant in a loop, and to the type of *pre*

and *post* conditions in a procedure type. The only rule which is not directly derivable is the well-known *consequence rule*:

$$\frac{\Gamma, \Omega \vdash \varphi' \Rightarrow \varphi \qquad \Gamma; \Omega \vdash \{\varphi\}s \rhd \Omega'\{\psi\} \qquad \Gamma, \Omega \vdash \psi \Rightarrow \psi'}{\Gamma; \Omega \vdash \{\varphi'\}s \rhd \Omega'\{\psi'\}}$$

This rule deserves a specific treatment since no proof-term is required for the proof obligations. However, it is well-known that in intuitionistic logic the proof of some formulas have no computational content (they are called *data-mute* in [5]). The consequence rule is thus derivable if we restrict (without loss of generality) the set of assertions to data-mute formulas.

Non-local jumps

The imperative language was then extended in [1] with labels and non-local jumps. At the (dependent) type level, this extension (called $\mathbf{ID}^c$) corresponds to an extension from intuitionistic logic to classical logic. For instance, the following typing rules for labels and jumps are derivable (where first-class labels are typed by the negation):

$$\frac{\Gamma, k : \neg \vec{\sigma}; \vec{z} : \vec{\tau} \vdash s \rhd \vec{z} : \vec{\sigma} \qquad \Gamma; \Omega, \vec{z} : \vec{\sigma} \vdash s' \rhd \Omega'}{\Gamma; \Omega, \vec{z} : \vec{\tau} \vdash k : \{s\}_{\vec{z}}; \, s' \rhd \Omega'} \qquad \frac{\Gamma; \Omega, \vec{z} : \vec{\tau} \vdash k : \neg \vec{\sigma} \qquad \Gamma; \Omega, \vec{z} : \vec{\tau} \vdash \vec{e} : \vec{\sigma}}{\Gamma; \Omega, \vec{z} : \vec{\tau} \vdash \mathbf{jump}(k, \vec{e})_{\vec{z}} \rhd \vec{z} : \vec{\tau}'}$$

However, deriving a Hoare-Floyd logic for non-local jumps is not straightforward since there is no obvious notion of *data-mute* formula in classical logic (as noted also in [6]), and thus the consequence rule is in general not derivable. The problem comes from the fact that, in presence of control operators, the proof-terms corresponding to proof-obligations may interact with the program. We shall exhibit an example of such program and we shall present a general solution to this problem which relies on the distinction between purely functional terms and imperative procedures (possibly containing non-local jumps).

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