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# Deriving a Hoare-Floyd logic for non-local jumps from a formulæ-as-types notion of control

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## Abstract

We derive a Hoare-Floyd logic for non-local jumps and mutable higher-order procedural variables from a formulæ-as-types notion of control for classical logic. The main contribution of this work is the design of an imperative dependent type system for non-local jumps which corresponds to classical logic but where the famous *consequence rule* is still derivable.

Hoare-Floyd logics for non-local jumps are notoriously difficult to obtain, especially in the presence of local mutable variables [7]. As far as we know, the question of proving the correctness of imperative programs which combine local mutable *higher-order procedural variables* and non-local jumps has not even been addressed. On the other hand, we know since Griffin’s pioneering work [3] how to prove the correctness of (higher-order) functional programs with control in direct style, thanks to the formulæ-as-types interpretation of classical logic.

In [1], Chapter 3, we have thus extended the formulæ-as-types notion of control to imperative programs with higher-order procedural mutable variables and non-local jumps. Our technique, which was inspired by Landin’s seminal paper [4], consists in defining an imperative dependent type system **ID** by translation into a functional dependent type system (which is actually Leivant’s **ML1P** [5]). This imperative language, called **LOOP<sup>ω</sup>**, was defined by the authors in [2].

Similarly to **ML1P**, the imperative type system is parametrized by a first-order signature and an equational system  $\mathcal{E}$  which defines a set of functions in the style of Herbrand-Gödel. The syntax of imperative types of **ID** (with dependent procedure types and dependent records) is the following:

$$\sigma, \tau ::= \mathbf{nat}(n) \mid \mathbf{proc} \forall i(\mathbf{in} \vec{\tau}; \mathbf{out} \vec{\sigma}) \mid \exists i(\sigma_1, \dots, \sigma_n) \mid n = m$$

Typing judgements of **ID** have the form  $\Gamma; \Omega \vdash e : \psi$  if  $e$  is an expression and  $\Gamma; \Omega \vdash s \triangleright \Omega'$  if  $s$  is a sequence, where environments  $\Gamma$  and  $\Omega$  corresponds respectively to immutable and mutable variables. Note that our type system is *pseudo-dynamic* in the sense that the type of mutable variables can change in a sequence and the new types are given by  $\Omega'$  (as in [8]). For instance, here is the typing rule of the **for** loop:

$$\frac{\Gamma; \Omega, \vec{x} : \vec{\sigma}[\mathbf{0}/i] \vdash e : \mathbf{nat}(n) \quad \Gamma, y : \mathbf{nat}(i); \vec{x} : \vec{\sigma} \vdash s \triangleright \vec{x} : \vec{\sigma}[\mathbf{s}(i)/i]}{\Gamma; \Omega, \vec{x} : \vec{\sigma}[\mathbf{0}/i] \vdash \mathbf{for} \ y := 0 \ \mathbf{until} \ e \ \{s\}_{\vec{x}} \triangleright \vec{x} : \vec{\sigma}[\mathbf{n}/i]}$$

## Embedding a Hoare-Floyd logic

It is almost straightforward to embed a Hoare-Floyd logic into **ID**. Indeed, let us take a global mutable variable, dubbed *assert*, and let us assume that this global variable is simulated in the usual *state-passing style* (the variable is passed as an explicit **in** and **out** parameter to each procedure call). Consequently, any sequence shall be typed with a sequent of the form  $\Gamma; \Omega, \mathbf{assert} : \varphi \vdash s \triangleright \Omega', \mathbf{assert} : \psi$ . If we now introduce the usual Hoare notation for triples (which hides the name of global variable *assert*), we obtain judgments of the form  $\Gamma; \Omega \vdash \{\varphi\}s \triangleright \Omega' \{\psi\}$ . Rules very similar to Hoare rules are then derivable: for instance, the type of *assert* corresponds to the invariant in a loop, and to the type of *pre*

and *post* conditions in a procedure type. The only rule which is not directly derivable is the well-known *consequence rule*:

$$\frac{\Gamma, \Omega \vdash \varphi' \Rightarrow \varphi \quad \Gamma; \Omega \vdash \{\varphi\}_s \triangleright \Omega' \{\psi\} \quad \Gamma, \Omega \vdash \psi \Rightarrow \psi'}{\Gamma; \Omega \vdash \{\varphi'\}_s \triangleright \Omega' \{\psi'\}}$$

This rule deserves a specific treatment since no proof-term is required for the proof obligations. However, it is well-known that in intuitionistic logic the proof of some formulas have no computational content (they are called *data-mute* in [5]). The consequence rule is thus derivable if we restrict (without loss of generality) the set of assertions to data-mute formulas.

## Non-local jumps

The imperative language was then extended in [1] with labels and non-local jumps. At the (dependent) type level, this extension (called  $\mathbf{ID}^c$ ) corresponds to an extension from intuitionistic logic to classical logic. For instance, the following typing rules for labels and jumps are derivable (where first-class labels are typed by the negation):

$$\frac{\Gamma, k : \neg \vec{\sigma}; \vec{z} : \vec{\tau} \vdash s \triangleright \vec{z} : \vec{\sigma} \quad \Gamma; \Omega, \vec{z} : \vec{\sigma} \vdash s' \triangleright \Omega'}{\Gamma; \Omega, \vec{z} : \vec{\tau} \vdash k : \{s\}_{\vec{z}}; s' \triangleright \Omega'} \quad \frac{\Gamma; \Omega, \vec{z} : \vec{\tau} \vdash k : \neg \vec{\sigma} \quad \Gamma; \Omega, \vec{z} : \vec{\tau} \vdash \vec{e} : \vec{\sigma}}{\Gamma; \Omega, \vec{z} : \vec{\tau} \vdash \mathbf{jump}(k, \vec{e})_{\vec{z}} \triangleright \vec{z} : \vec{\tau}'}$$

However, deriving a Hoare-Floyd logic for non-local jumps is not straightforward since there is no obvious notion of *data-mute* formula in classical logic (as noted also in [6]), and thus the consequence rule is in general not derivable. The problem comes from the fact that, in presence of control operators, the proof-terms corresponding to proof-obligations may interact with the program. We shall exhibit an example of such program and we shall present a general solution to this problem which relies on the distinction between purely functional terms and imperative procedures (possibly containing non-local jumps).

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