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Degenerate determinant representation of solutions of the NLS equation, higher Peregrine breathers and multi-rogue waves.

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Abstract

We present a new representation of solutions of the focusing NLS equation as a quotient of two determinants. This work is based on a recent paper in which we have constructed a multi-parametric family of this equation in terms of Wronskians. This formulation was written in terms of a limit involving a parameter.
Here we give a very compact formulation without presence of a limit. This is a completely new result which gives a very efficient procedure to construct families of quasi-rational solutions of the NLS equation. With this method, we construct Peregrine breathers of orders $N = 4$ to 7 and multi-rogue waves associated by deformation of parameters.

1 Introduction

In 1972 Zakharov and Shabat solved the nonlinear Schrödinger equation (NLS) using the inverse scattering method. The case of periodic and almost periodic algebro-geometric solutions to the focusing NLS equation were first constructed in 1976 by Its and Kotlyarov [15]. The first quasi-rational solutions of NLS equation were constructed in 1983 by Peregrine, nowadays called worldwide Peregrine breathers. In 1986 Eleonski, Akhmediev and Kulagin obtained the two-phase almost periodic solution to the NLS equation

and obtained the first higher order analogue of the Peregrine breather[3]. Other families of higher order were constructed in a series of articles by Akhmediev et al. [1, 2] using Darboux transformations.
In 2010, it has been shown in [7] that rational solutions of the NLS equation can be written as a quotient of two Wronskians. With this formulation, it was possible to recover as a particular case, Akhmediev's quasi-rational solution of the NLS equation.
Recently, it has been constructed in [13] a new representation of the solutions of the NLS equation in terms of a ratio of two Wronskians determinants of even order $2N$ composed of elementary functions; the related solutions of NLS are called of order N. Quasi-rational solutions of the NLS equation were obtained by the passage to the limit when some parameter tended to 0.
Here we obtain a new representation of quasi-rational solutions of NLS in term of a quotient of two determinants different from preceding works which does not involve Wronskians.
With this method, we obtain very easily solutions of NLS equation with parameters of deformations until order 7. The following orders will be able to be the object of other publications.
These results can be compared with those obtain recently by Akhemediev et al. in [5] with Darboux dressing method and numerical approach.
This method appears to be very efficient without common measurement with the preceding one exposed in [13] and leads to new results about NLS equation.

2 Expression of solutions of NLS equation in terms of Wronskian determinant and quasi-rational limit

2.1 Solutions of NLS equation in terms of Wronskian determinant

We recall results obtained in [13]. We consider the focusing NLS equation

$$iv_t + v_{xx} + 2|v|^2 v = 0. \tag{1}$$

From [12], the solution of the NLS equation can be written in the form

$$v(x,t) = \frac{\det(I + A_3(x,t))}{\det(I + A_1(x,t))} \exp(2it - i\varphi). \tag{2}$$

In (2), the matrix $A_r = (a_{\nu\mu})_{1\leq\nu,\mu\leq 2N}$ $(r = 3,\, 1)$ is defined by

$$a_{\nu\mu} = (-1)^{\epsilon_\nu} \prod_{\lambda\neq\mu} \left| \frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu} \right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu} + e_\nu). \tag{3}$$

κ_ν, δ_ν, γ_ν are functions of the parameters λ_ν, $\nu = 1, \ldots, 2N$ satisfying the relations

$$0 < \lambda_j < 1, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \leq j \leq N. \tag{4}$$

They are given by the following equations,

$$\kappa_\nu = 2\sqrt{1 - \lambda_\nu^2}, \quad \delta_\nu = \kappa_\nu \lambda_\nu, \quad \gamma_\nu = \sqrt{\frac{1 - \lambda_\nu}{1 + \lambda_\nu}}, \tag{5}$$

and

$$\kappa_{N+j} = \kappa_j, \quad \delta_{N+j} = -\delta_j, \quad \gamma_{N+j} = 1/\gamma_j, \quad j = 1 \ldots N. \tag{6}$$

The terms $x_{r,\nu}$ $(r = 3,\, 1)$ are defined by

$$x_{r,\nu} = (r-1) \ln \frac{\gamma_\nu - i}{\gamma_\nu + i}, \quad 1 \leq j \leq 2N. \tag{7}$$

The parameters e_ν are defined by

$$e_j = ia_j - b_j, \quad e_{N+j} = ia_j + b_j, \quad 1 \leq j \leq N, \tag{8}$$

where a_j and b_j, for $1 \leq j \leq N$ are arbitrary real numbers.
The terms ϵ_ν are defined by :

$$\begin{aligned} \epsilon_\nu &= 0, \quad 1 \leq \nu \leq N \\ \epsilon_\nu &= 1, \quad N+1 \leq \nu \leq 2N. \end{aligned}$$

We use the following notations :
$\Theta_{r,\nu} = \kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y - ie_\nu, \quad 1 \leq \nu \leq 2N.$
We consider the functions

$$\begin{aligned} \phi_{r,\nu}(y) &= \sin \Theta_{r,\nu}, \quad 1 \leq \nu \leq N, \\ \phi_{r,\nu}(y) &= \cos \Theta_{r,\nu}, \quad N+1 \leq \nu \leq 2N. \end{aligned} \tag{9}$$

$W_r(y) = W(\phi_{r,1}, \ldots, \phi_{r,2N})$ is the Wronskian

$$W_r(y) = \det[(\partial_y^{\mu-1}\phi_{r,\nu})_{\nu,\,\mu\in[1,\ldots,2N]}]. \tag{10}$$

Then we get the following link between Fredholm and Wronskian determinants [10]

Proposition 2.1

$$\det(I + A_r) = k_r(0) \times W_r(\phi_{r,1}, \ldots, \phi_{r,2N})(0), \tag{11}$$

where

$$k_r(y) = \frac{2^{2N}\exp(i\sum_{\nu=1}^{2N}\Theta_{r,\nu})}{\prod_{\nu=2}^{2N}\prod_{\mu=1}^{\nu-1}(\gamma_\nu - \gamma_\mu)}.$$

In (11), the matrix A_r is defined by (3).
It can be deduced the following result :

Proposition 2.2 *The function v defined by*

$$v(x,t) = \frac{W_3(0)}{W_1(0)}\exp(2it - i\varphi). \tag{12}$$

is solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0.$$

2.2 Quasi-rational solutions of NLS equation in terms of a limit of a ratio of Wronskian determinants

In the following, we take the limit when the parameters $\lambda_j \to 1$ for $1 \le j \le N$ and $\lambda_j \to -1$ for $N+1 \le j \le 2N$.
We consider the parameter λ_j written in the form

$$\lambda_j = 1 - 2j^2\epsilon^2, \quad 1 \le j \le N. \tag{13}$$

When ϵ goes to 0, we realize limited expansions at order p, for $1 \le j \le N$, of the terms
$\kappa_j = 4j\epsilon(1-\epsilon^2 j^2)^{1/2}$, $\delta_j = 4j\epsilon(1-2\epsilon^2 j^2)(1-\epsilon^2 j^2)^{1/2}$,
$\gamma_j = j\epsilon(1-\epsilon^2 j^2)^{-1/2}$, $x_{r,j} = (r-1)\ln\frac{1+i\epsilon j(1-\epsilon^2 j^2)^{-1/2}}{1-i\epsilon j(1-\epsilon^2 j^2)^{-1/2}}$,

$\kappa_{N+j} = 4j\epsilon(1-\epsilon^2 j^2)^{1/2}$, $\delta_{N+j} = -4j\epsilon(1-2\epsilon^2 j^2)(1-\epsilon^2 j^2)^{1/2}$,
$\gamma_{N+j} = 1/(j\epsilon)(1-\epsilon^2 j^2)^{1/2}$, $x_{r,N+j} = (r-1)\ln\frac{1-i\epsilon j(1-\epsilon^2 j^2)^{-1/2}}{1+i\epsilon j(1-\epsilon^2 j^2)^{-1/2}}$.
The parameters a_j and b_j, for $1 \leq N$ are chosen in the form

$$a_j = \hat{a}_j \epsilon^{2N-1}, \quad b_j = \hat{b}_j \epsilon^{2N-1}, \quad 1 \leq j \leq N. \tag{14}$$

We have the result given in [12] :

Proposition 2.3 *With the parameters λ_j defined by (13), a_j and b_j chosen as in (14), for $1 \leq j \leq N$, the function v defined by*

$$v(x,t) = \exp(2it - i\varphi) \lim_{\epsilon \to 0} \frac{W_3(0)}{W_1(0)}, \tag{15}$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0,$$

depending on $2N$ parameters $\tilde{a}_j$, $\tilde{b}_j$, $1 \leq j \leq N$.

3 Expression of solutions of NLS equation in terms of a ratio of two determinants

We construct here solutions of the NLS equation which does not involve Wronskian determinant and a passage to the limit, but which is expressed as a quotient of two determinants.
For this we need the following notations :

$$A_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{3,\nu}/2 - ie_\nu/2,$$
$$B_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{1,\nu}/2 - ie_\nu/2,$$

for $1 \leq \nu \leq 2N$, with κ_ν, δ_ν, $x_{r,\nu}$ defined in (5), (6) and (7).
The parameters e_ν are defined by (8). For simplicity of the reduction, we choose a_j and b_j in the form

$$a_j = \tilde{a_1} j^{2N-1} \epsilon^{2N-1}, \quad b_j = \tilde{b_1} j^{2N-1} \epsilon^{2N-1}, \quad 1 \leq j \leq N. \tag{16}$$

Below we use the following notations :

$$f_{4j+1,k} = \gamma_k^{4j-1} \sin A_k, \quad f_{4j+2,k} = \gamma_k^{4j} \cos A_k,$$
$$f_{4j+3,k} = -\gamma_k^{4j+1} \sin A_k, \quad f_{4j+4,k} = -\gamma_k^{4j+2} \cos A_k,$$

for $1 \leq k \leq N$, and

$$f_{4j+1,k} = \gamma_k^{2N-4j-2} \cos A_k, \quad f_{4j+2,k} = -\gamma_k^{2N-4j-3} \sin A_k,$$
$$f_{4j+3,k} = -\gamma_k^{2N-4j-4} \cos A_k, \quad f_{4j+4,k} = \gamma_k^{2N-4j-5} \sin A_k,$$

for $N+1 \leq k \leq 2N$.
We define the functions $g_{j,k}$ for $1 \leq j \leq 2N$, $1 \leq k \leq 2N$ in the same way, we replace only the term A_k by B_k.

$$g_{4j+1,k} = \gamma_k^{4j-1} \sin B_k, \quad g_{4j+2,k} = \gamma_k^{4j} \cos B_k,$$
$$g_{4j+3,k} = -\gamma_k^{4j+1} \sin B_k, \quad g_{4j+4,k} = -\gamma_k^{4j+2} \cos B_k,$$

for $1 \leq k \leq N$, and

$$g_{4j+1,k} = \gamma_k^{2N-4j-2} \cos B_k, \quad g_{4j+2,k} = -\gamma_k^{2N-4j-3} \sin B_k,$$
$$g_{4j+3,k} = -\gamma_k^{2N-4j-4} \cos B_k, \quad g_{4j+4,k} = \gamma_k^{2N-4j-5} \sin B_k,$$

for $N+1 \leq k \leq 2N$.
Then it is clear that

$$q(x,t) := \frac{W_3(0)}{W_1(0)}$$

can be written as

$$q(x,t) = \frac{\Delta_3}{\Delta_1} = \frac{\det(f_{j,k})_{j,\,k\in[1,2N]}}{\det(g_{j,k})_{j,\,k\in[1,2N]}}. \tag{17}$$

We recall that $\lambda_j = 1 - 2j\epsilon^2$. All the functions $f_{j,k}$ and $g_{j,k}$ depend on ϵ. We use the expansions

$$f_{j,k}(x,t,\epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} f_{j,1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad f_{j,1}[l] = \frac{\partial^{2l} f_{j,1}}{\partial \epsilon^{2l}}(x,t,0),$$

$$f_{j,1}[0] = f_{j,1}(x,t,0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad 1 \leq l \leq N-1,$$

$$f_{j,N+k}(x,t,\epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} f_{j,N+1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad f_{j,N+1}[l] = \frac{\partial^{2l} f_{j,N+1}}{\partial \epsilon^{2l}}(x,t,0),$$

$$f_{j,N+1}[0] = f_{j,N+1}(x,t,0), \quad 1 \le j \le 2N, \quad 1 \le k \le N, \quad 1 \le l \le N-1.$$

We have the same expansions for the functions $g_{j,k}$.

$$g_{j,k}(x,t,\epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} g_{j,1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad g_{j,1}[l] = \frac{\partial^{2l} g_{j,1}}{\partial \epsilon^{2l}}(x,t,0),$$

$$g_{j,1}[0] = g_{j,1}(x,t,0), \quad 1 \le j \le 2N, \quad 1 \le k \le N, \quad 1 \le l \le N-1,$$

$$g_{j,N+k}(x,t,\epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} g_{j,N+1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad g_{j,N+1}[l] = \frac{\partial^{2l} g_{j,N+1}}{\partial \epsilon^{2l}}(x,t,0),$$

$$g_{j,N+1}[0] = g_{j,N+1}(x,t,0), \quad 1 \le j \le 2N, \quad 1 \le k \le N, \quad N+1 \le k \le 2N..$$

Combining the columns of the determinants appearing in $q(x,t)$ successively to eliminate in each column k (or $N+k$) of them the powers of ϵ lower than $2(k-1)$, and factorizing and simplifying each common terms, $q(x,t)$ can be replaced by $Q(x,t)$

$$Q(x,t) := \frac{\begin{vmatrix} f_{1,1}[0] & \dots & f_{1,1}[N-1] & f_{1,N+1}[0] & \dots & f_{1,N+1}[N-1] \\ f_{2,1}[0] & \dots & f_{2,1}[N-1] & f_{2,N+1}[0] & \dots & f_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{2N,1}[0] & \dots & f_{2N,1}[N-1] & f_{2N,N+1}[0] & \dots & f_{2N,N+1}[N-1] \end{vmatrix}}{\begin{vmatrix} g_{1,1}[0] & \dots & g_{1,1}[N-1] & g_{1,N+1}[0] & \dots & g_{1,N+1}[N-1] \\ g_{2,1}[0] & \dots & g_{2,1}[N-1] & g_{2,N+1}[0] & \dots & g_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{2N,1}[0] & \dots & g_{2N,1}[N-1] & g_{2N,N+1}[0] & \dots & g_{2N,N+1}[N-1] \end{vmatrix}} \tag{18}$$

Then we get the following result :

Proposition 3.1 *The function v defined by*

$$v(x,t) = \exp(2it - i\varphi) \times Q(x,t) \tag{19}$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0,$$

where $Q(x,t)$ is defined in (18).

4 Quasi-rational solutions of order N

Wa have already constructed in [13] solutions for the cases $N = 1, 2, 3$, and this method gives the same results. In all the following, for simplicity we present only the cases $\tilde{a}_1 = 0$ and $\tilde{b}_1 = 0$. It gives exactly Peregrine breathers.
Here, we recall the case $N = 3$ and give new solutions of NLS equation in the cases from $N = 4$ to 7.
Because of the length of the expressions, in the cases $N = 4$ to 7, for the polynomials n and d in the solutions v of the NLS equation defined by

$$v_N(x,t) = \frac{n(x,t)}{d(x,t)} \exp(2it - i\varphi) = (1 - \alpha_N \frac{G_N(2x,4t) + iH_N(2x,4t)}{Q_N(2x,4t)})e^{2it-i\varphi}$$

with

$$\begin{aligned} G_N(X,T) &= \textstyle\sum_{k=0}^{N(N+1)} \mathbf{g}_k(T)X^k \\ H_N(X,T) &= \textstyle\sum_{k=0}^{N(N+1)} \mathbf{h}_k(T)X^k \\ Q_N(X,T) &= \textstyle\sum_{k=0}^{N(N+1)} \mathbf{q}_k(T)X^k \end{aligned}$$

we only give in the appendix.

4.1 Case N=3

The expressions of the coefficients of the polynomials G, H and Q are given by :

$$\alpha_3 = 4, \quad \mathbf{g}_{12} = 0, \quad \mathbf{g}_{11} = 0, \quad \mathbf{g}_{10} = 6, \quad \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 90\,T^2 + 90, \quad \mathbf{g}_7 = 0,$$
$$\mathbf{g}_6 = 300\,T^4 - 360\,T^2 + 1260,$$
$$\mathbf{g}_5 = 0,$$
$$\mathbf{g}_4 = 420\,T^6 - 900\,T^4 + 2700\,T^2 - 2700,$$
$$\mathbf{g}_3 = 0,$$
$$\mathbf{g}_2 = 270\,T^8 + 2520\,T^6 + 40500\,T^4 - 81000\,T^2 + 180\,Tb - 4050,$$
$$\mathbf{g}_1 = 0,$$
$$\mathbf{g}_0 = 66\,T^{10} + 2970\,T^8 + 13140\,T^6 - 45900\,T^4 - 12150\,T^2 + 4050$$

$$\mathbf{h}_{12} = 0, \quad \mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 6\,T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 30\,T^3 - 90\,T, \quad \mathbf{h}_7 = 0,$$
$$\mathbf{h}_6 = 60\,T^5 - 840\,T^3 - 900\,T,$$
$$\mathbf{h}_5 = 0,$$
$$\mathbf{h}_4 = 60\,T^7 - 1260\,T^5 - 2700\,T^3 - 8100\,T,$$
$$\mathbf{h}_3 = 0,$$
$$\mathbf{h}_2 = 30\,T^9 - 360\,T^7 + 10260\,T^5 - 37800\,T^3 + 28350\,T,$$
$$\mathbf{h}_1 = 0,$$
$$\mathbf{h}_0 = 6\,T^{11} + 150\,T^9 - 5220\,T^7 - 57780\,T^5 - 14850\,T^3 + 28350T$$

$$\mathbf{q}_{12} = 1, \quad \mathbf{q}_{11} = 0, \quad \mathbf{q}_{10} = 6\,T^2 + 6, \quad \mathbf{q}_9 = 0, \quad \mathbf{q}_8 = 15\,T^4 - 90\,T^2 + 135, \quad \mathbf{q}_7 = 0,$$
$$\mathbf{q}_6 = 20\,T^6 - 180\,T^4 + 540\,T^2 + 2340,$$
$$\mathbf{q}_5 = 0,$$
$$\mathbf{q}_4 = 15\,T^8 + 60\,T^6 - 1350\,T^4 + 13500\,T^2 + 3375,$$
$$\mathbf{q}_3 = 0,$$
$$\mathbf{q}_2 = 6\,T^{10} + 270\,T^8 + 13500\,T^6 + 78300\,T^4 - 36450\,T^2 + 12150,$$
$$\mathbf{q}_1 = 0,$$
$$\mathbf{q}_0 = T^{12} + 126\,T^{10} + 3735\,T^8 + 15300\,T^6 + 143775\,T^4 + 93150T^2 + 2025$$

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain the classical Akhmediev's breather :

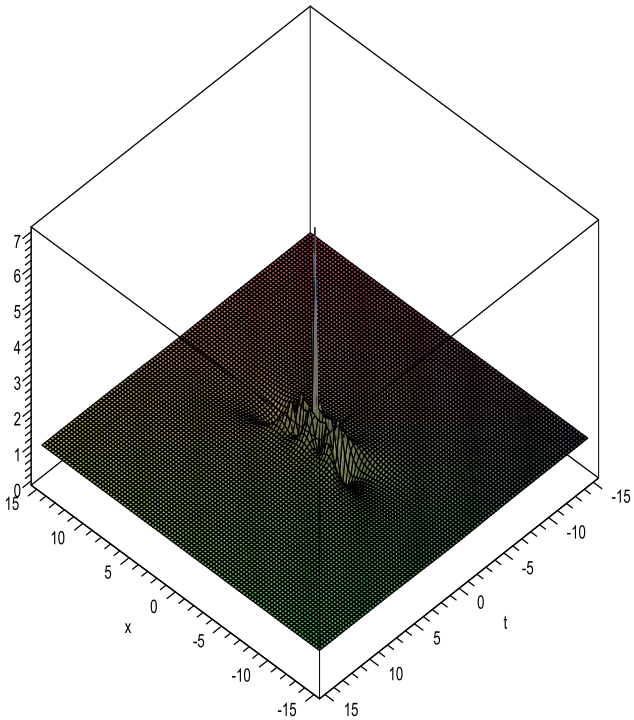

Figure 1: Solution of NLS, N=3, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$, we obtain the plot :

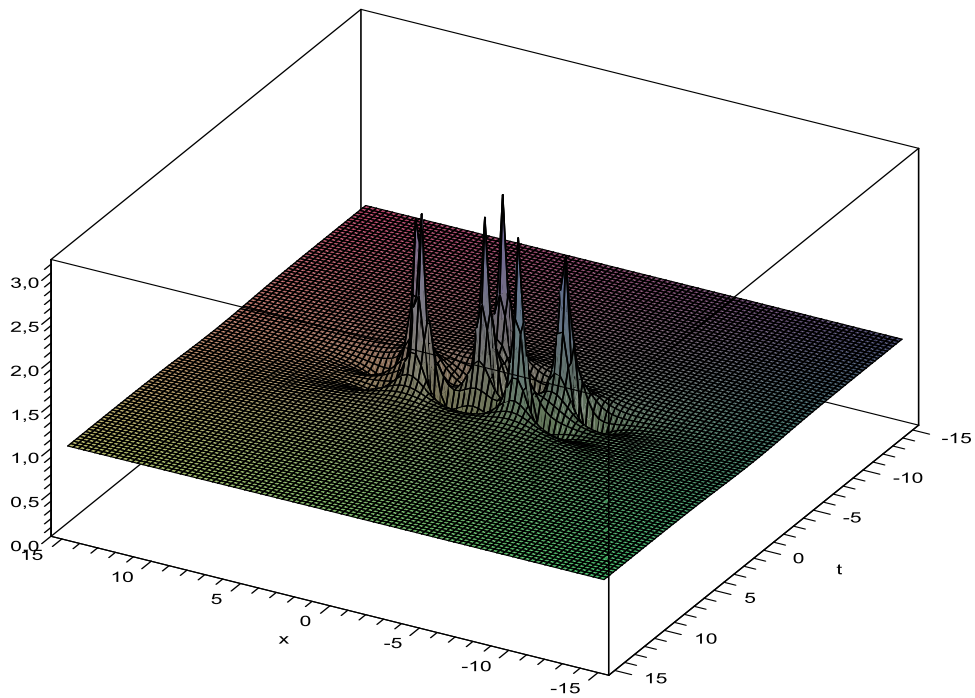

Figure 2: Solution of NLS, N=3, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$.

If we choose $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$, we obtain the regular plot :

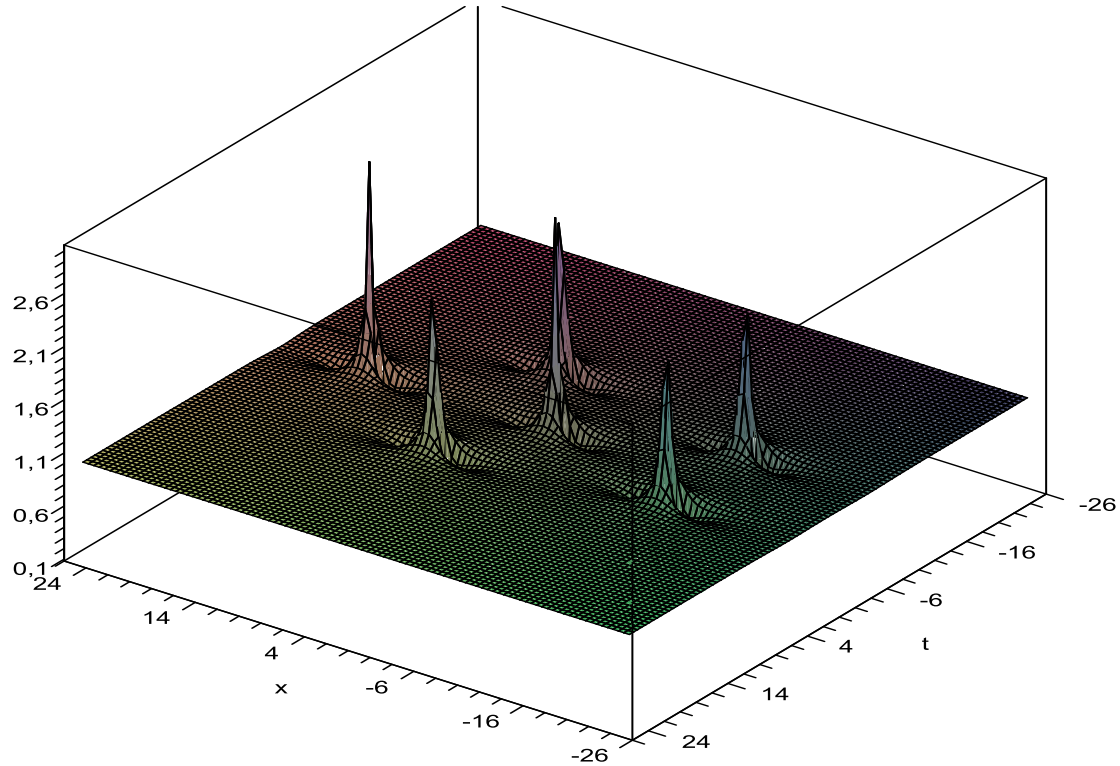

Figure 3: Solution of NLS, N=3, $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$.

4.2 Case N=4

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain the classical Akhmediev's breather :

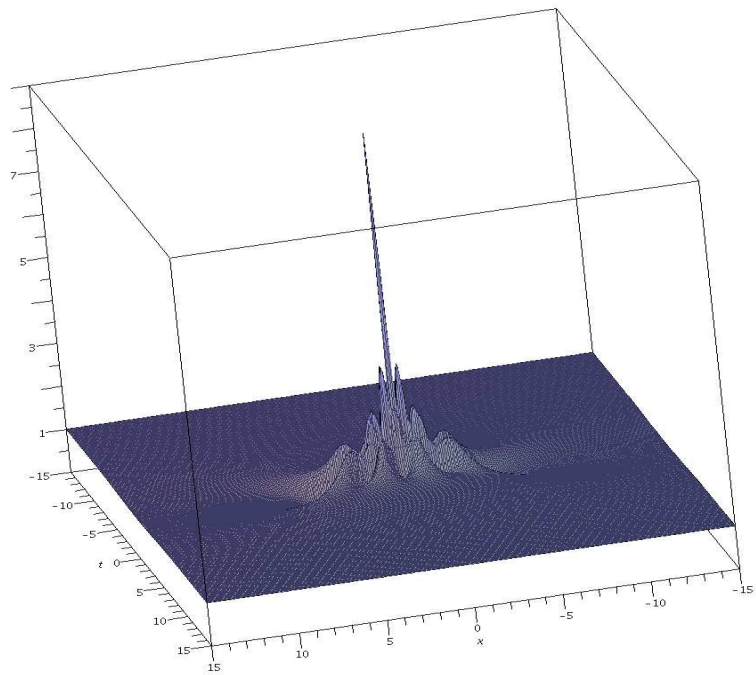

Figure 4: Solution of NLS, N=4, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$, we obtain the plot :

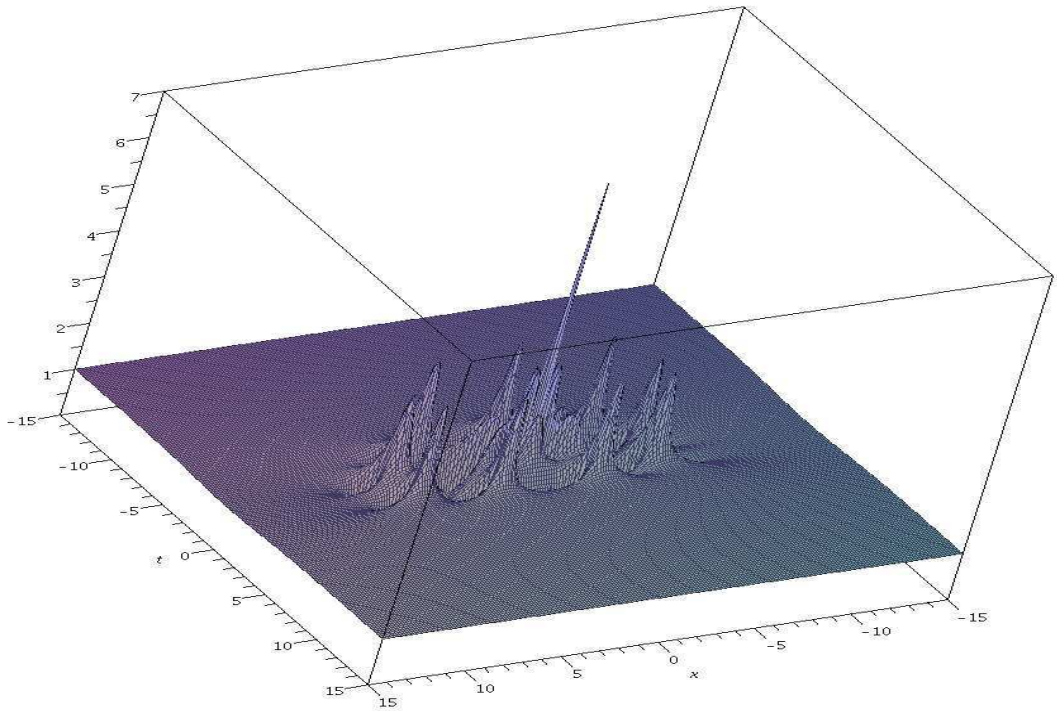

Figure 5: Solution of NLS, N=4, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$.

If we choose $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$, we obtain the regular plot :

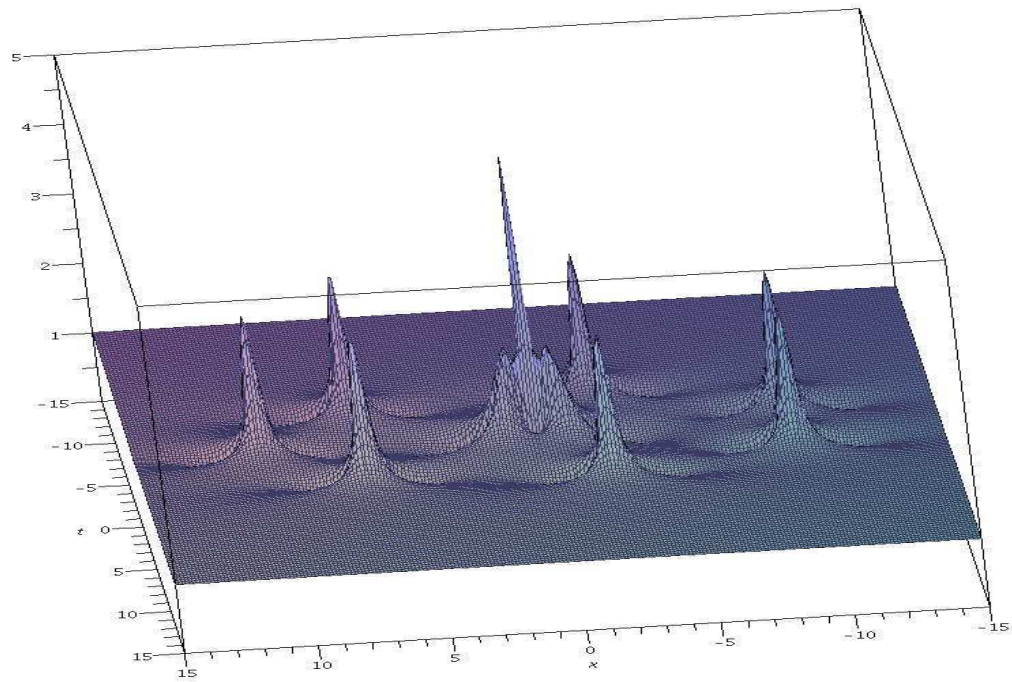

Figure 6: Solution of NLS, N=4, $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$.

4.3 Case N=5

For $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain Akhmediev's breather.

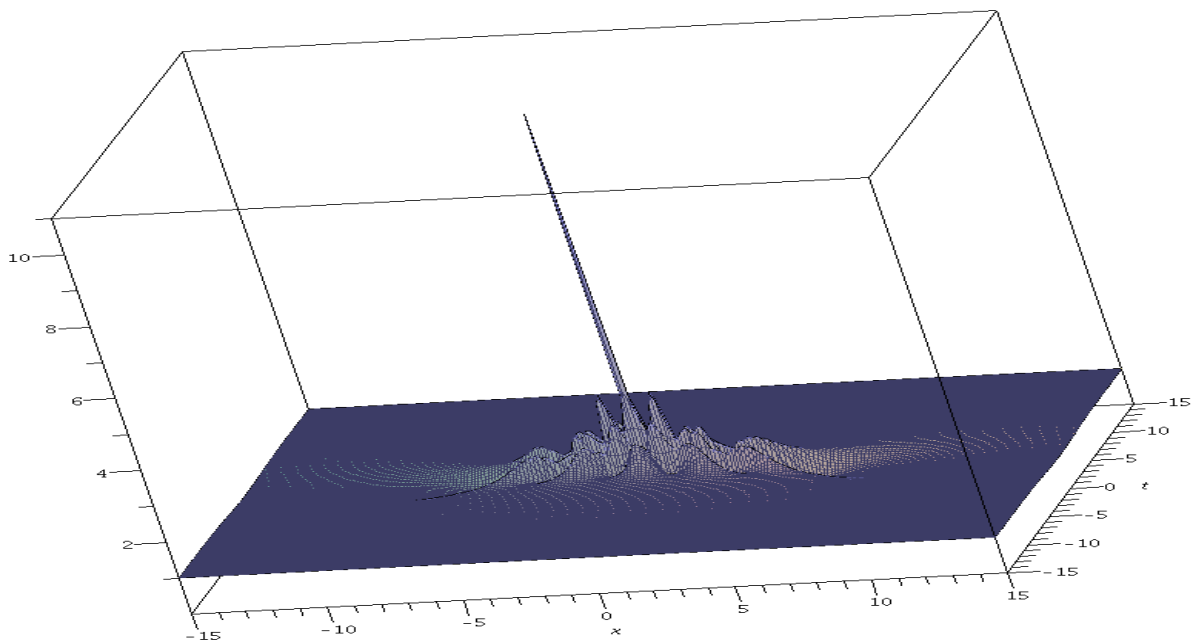

Figure 7: Solution of NLS, N=5, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^5$, we obtain :

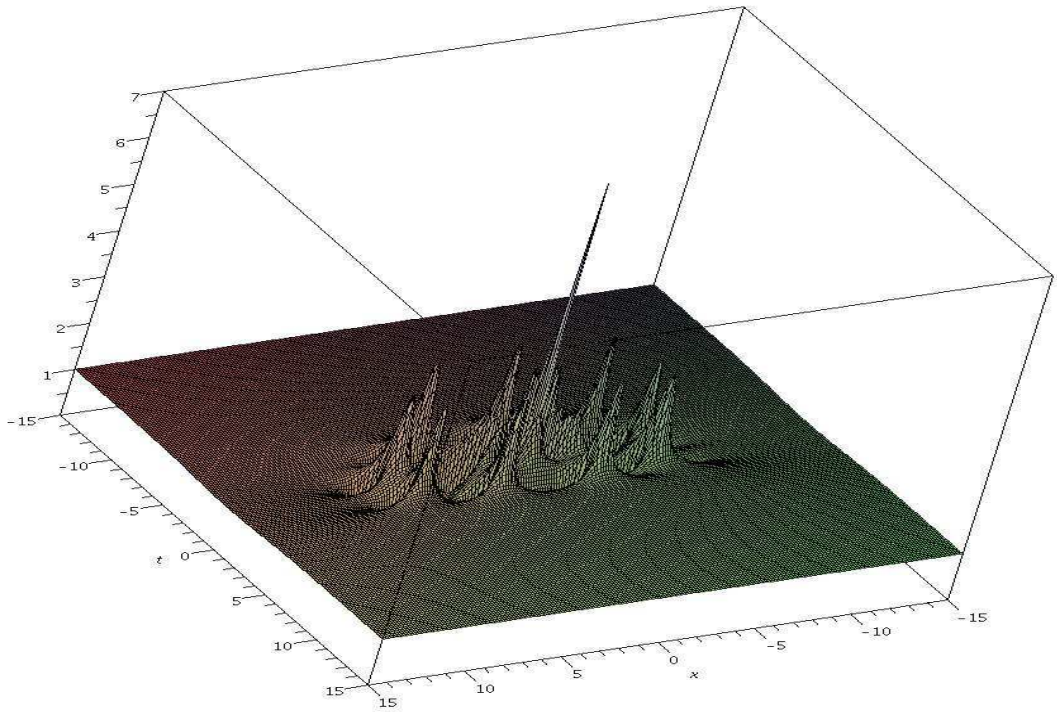

Figure 8: Solution of NLS, N=5, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^5$.

If we choose $\tilde{a}_1 = 10^8$, $\tilde{b}_1 = -10^5$, we have :

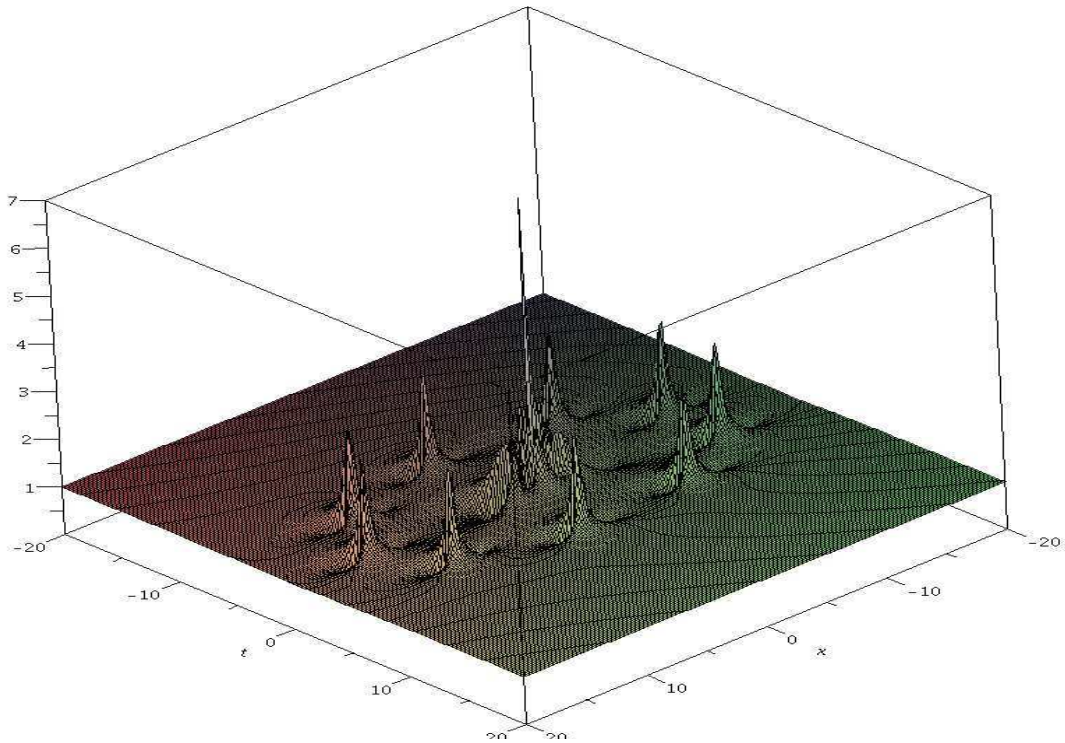

Figure 9: Solution of NLS, N=5, $\tilde{a}_1 = 10^8$, $\tilde{b}_1 = -10^5$.

4.4 Case N=6

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain the following plot :

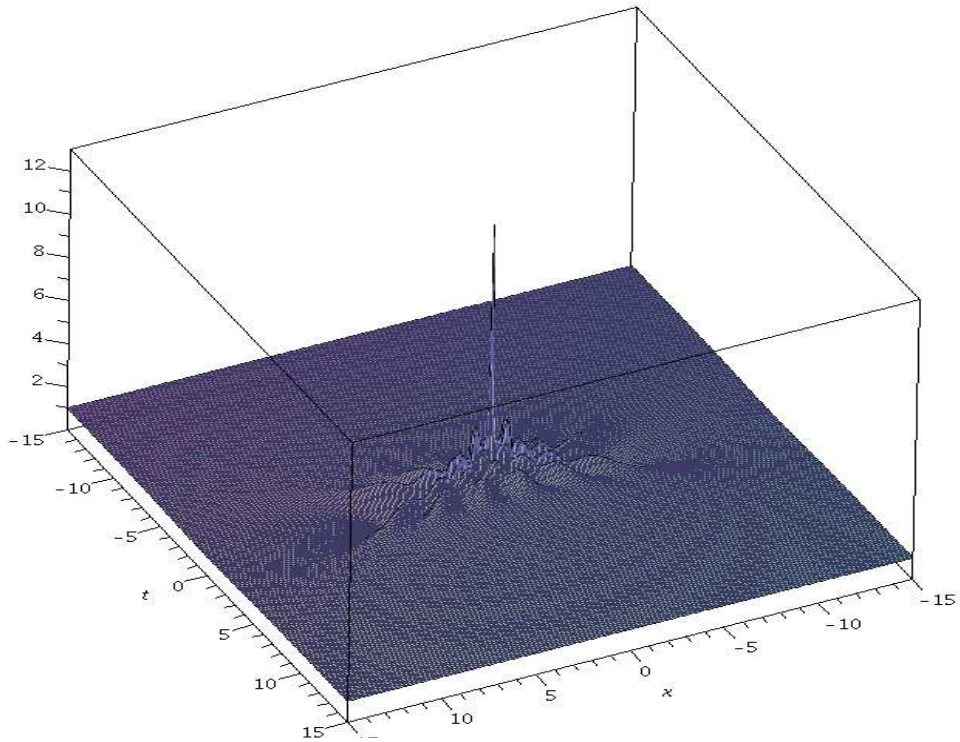

Figure 10: Solution of NLS, N=6, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^6$, we obtain :

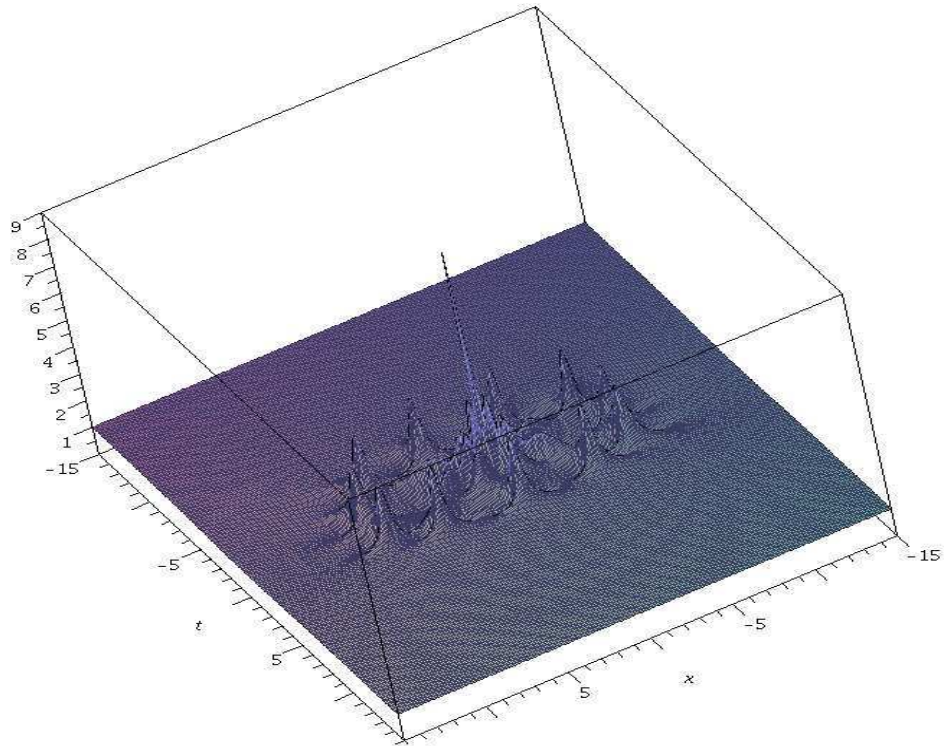

Figure 11: Solution of NLS, N=6, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^6$.

If we choose $\tilde{a}_1 = 10^9$, $\tilde{b}_1 = -10^6$, we obtain the regular figure with 12 peaks :

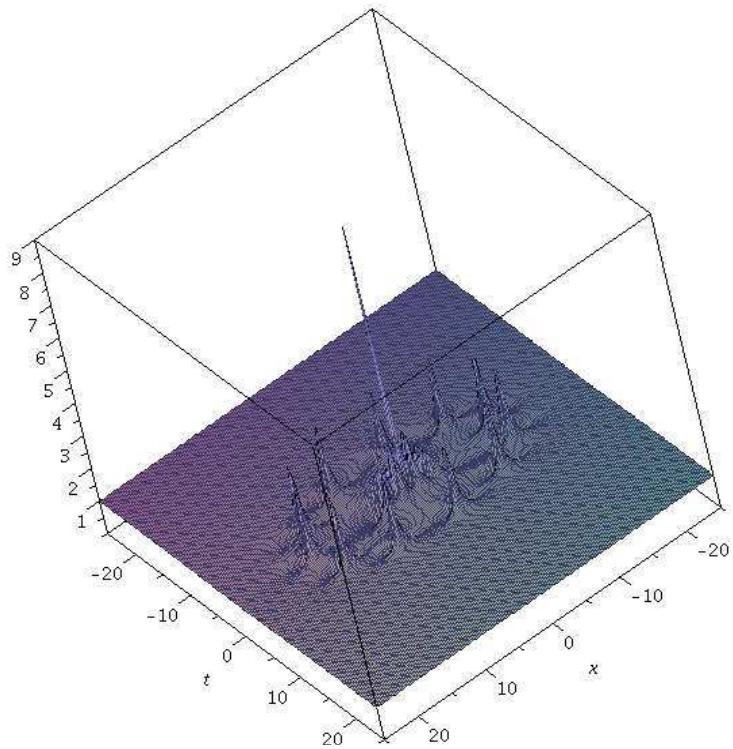

Figure 12: Solution of NLS, N=6, $\tilde{a}_1 = 10^9$, $\tilde{b}_1 = -10^6$.

4.5 Case N=7

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we recognize Akhmediev's breather :

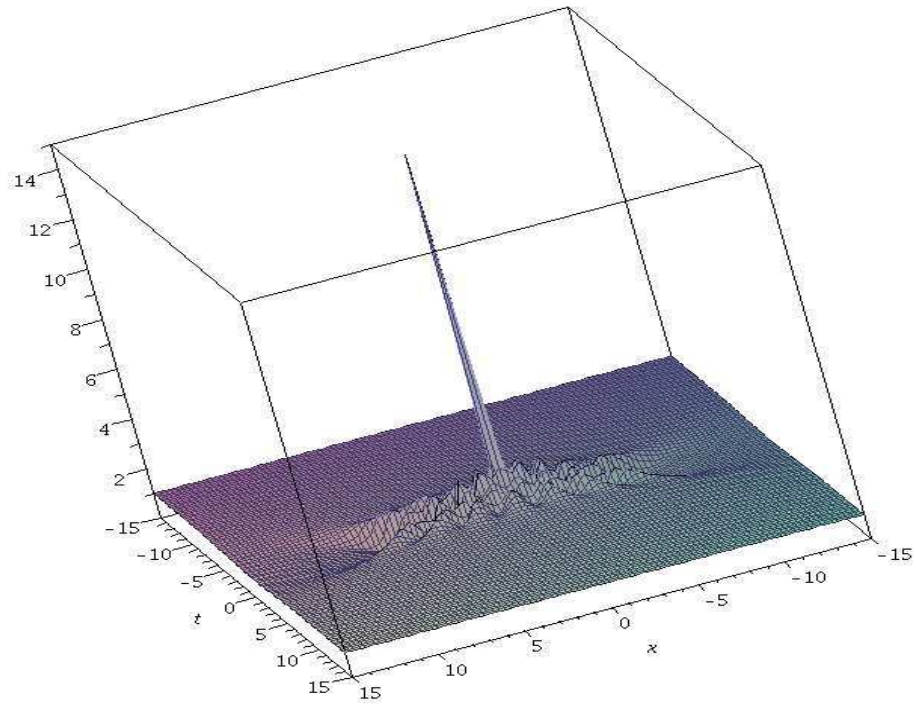

Figure 13: Solution of NLS, N=7, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^8$, we obtain the following plot :

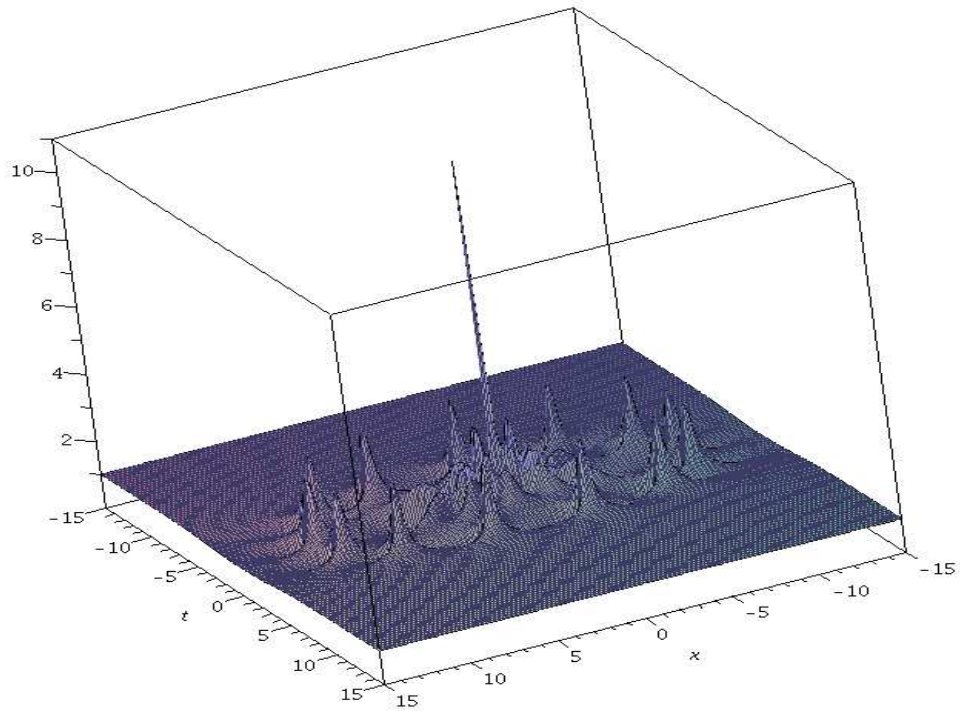

Figure 14: Solution of NLS, N=7, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^8$.

If we choose $\tilde{a}_1 = 10^{12}$, $\tilde{b}_1 = -10^8$, we obtain the following regular plot :

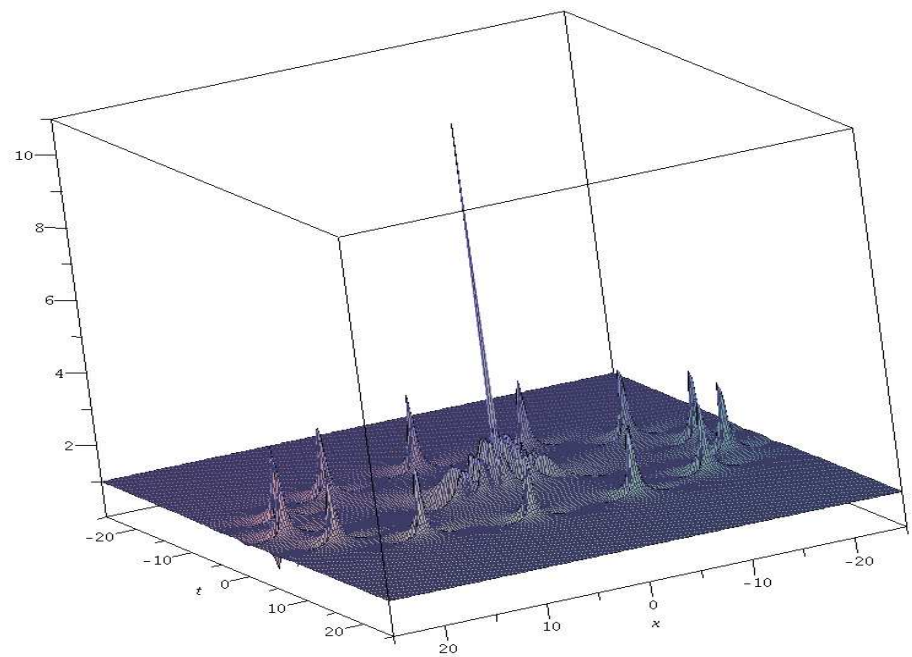

Figure 15: Solution of NLS, N=7, $\tilde{a}_1 = 10^{12}$, $\tilde{b}_1 = -10^8$.

5 Conclusion

The method described in the present paper provides a powerful tool to get explicitly solutions of the NLS equation.
As my knowledge, it is the first time that the Peregrine breather of order seven is presented.
It confirms the conjecture about the shape of the breather in the (x, t) coordinates, the maximum of amplitude equal to $2N + 1$ and the degree of polynomials in x and t here equal to $N(N + 1)$. This new formulation gives an infinite set of non singular solution of NLS equation. It opens a large way to future researches in this domain.

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Appendix

In the following, we choose all the parameters a_1 and b_1 equal to 0.
The solution of NLS equation takes the form

$$v_N(x,t) = \frac{n(x,t)}{d(x,t)} \exp(2it) = (1 - \alpha_N \frac{G_N(2x,4t) + iH_N(2x,4t)}{Q_N(2x,4t)})e^{2it}$$

with

$$\begin{aligned} G_N(X,T) &= \textstyle\sum_{k=0}^{N(N+1)} \mathbf{g}_k(T) X^k \\ H_N(X,T) &= \textstyle\sum_{k=0}^{N(N+1)} \mathbf{h}_k(T) X^k \\ Q_N(X,T) &= \textstyle\sum_{k=0}^{N(N+1)} \mathbf{q}_k(T) X^k \end{aligned}$$

1. Case N=4

The coefficients of the polynomials G, H and Q are defined by

$\alpha_4 = 4, \quad \mathbf{g}_{20} = 0, \quad \mathbf{g}_{19} = 0, \quad \mathbf{g}_{18} = 10, \quad \mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 270\,T^2 + 270, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 1800\,T^4$
$-3600\,T^2 + 9000, \quad \mathbf{g}_{13} = 0, \quad \mathbf{g}_{12} = 5880\,T^6 - 54600\,T^4 - 12600\,T^2 + 189000, \quad \mathbf{g}_{11} = 0,$
$\mathbf{g}_{10} = 11340\,T^8 - 176400\,T^6 + 189000\,T^4 - 378000\,T^2 - 1077300, \quad \mathbf{g}_9 = 0,$
$\mathbf{g}_8 = 13860\,T^{10} - 207900\,T^8 + 2356200\,T^6 + 1701000\,T^4 - 56983500\,T^2 - 4819500,$
$\mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 10920\,T^{12} - 18480\,T^{10} + 6967800\,T^8 + 56095200\,T^6 - 342657000\,T^4$
$+198450000\,T^2 - 11907000, \quad \mathbf{g}_5 = 0 \quad \mathbf{g}_4 = 5400\,T^{14} + 163800\,T^{12} + 9034200\,T^{10}$
$+107919000\,T^8 - 615195000\,T^6 + 178605000\,T^4 + 654885000\,T^2 + 178605000, \quad \mathbf{g}_3 = 0,$
$\mathbf{g}_2 = 1530\,T^{16} + 133200\,T^{14} + 5506200\,T^{12} - 116802000\,T^{10} - 1731334500\,T^8$
$+2532222000\,T^6 - 893025000\,T^4 + 4643730000)T^2 + 223256250, \quad \mathbf{g}_1 = 0,$
$\mathbf{g}_0 = 190\,T^{18} + 33150\,T^{16} + 1294200\,T^{14} + 3288600\,T^{12} + 48629700\,T^{10}$
$-2015401500\,T^8 - 1845585000\,T^6 + 14586075000)T^4 + 2098608750T^2 - 44651250,$

$\mathbf{h}_{20} = 0, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 10\,T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 90\,T^3 - 270\,T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 360\,T^5$
$-6000\,T^3 - 5400\,T, \quad \mathbf{h}_{13} = 0, \quad \mathbf{h}_{12} = 840\,T^7 - 29400\,T^5 + 12600\,T^3 - 138600\,T, \quad \mathbf{h}_{11} = 0,$
$\mathbf{h}_{10} = 1260\,T^9 - 65520\,T^7 + 259560\,T^5 - 529200\,T^3 - 1984500\,T, \quad \mathbf{h}_9 = 0,$
$\mathbf{h}_8 = 1260\,T^{11} - 77700\,T^9 + 718200\,T^7 - 5329800\,T^5 - 6142500\,T^3 + 29767500\,T,$
$\mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 840\,T^{13} - 48720\,T^{11} + 718200\,T^9 + 2973600\,T^7 - 72765000\,T^5$
$+436590000\,T^3 + 146853000\,T, \mathbf{h}_5 = 0, \quad \mathbf{h}_4 = 360\,T^{15} - 12600\,T^{13} + 138600\,T^{11}$
$-5859000\,T^9 - 328293000\,T^7 + 1075599000\,T^5 + 773955000\,T^3 + 535815000T, \quad \mathbf{h}_3 = 0,$
$\mathbf{h}_2 = 90\,T^{17} + 1200\,T^{15} - 189000\,T^{13} - 40143600\,T^{11}$
$-307786500\,T^9 + 2085426000\,T^7 - 4465125000\,T^5 + 4405590000T^3 - 1205583750T, \quad \mathbf{h}_1 = 0,$
$\mathbf{h}_0 = 10\,T^{19} + 930\,T^{17} - 86040\,T^{15} - 7018200\,T^{13} - 48100500\,T^{11} - 542902500\,T^9$
$+6039117000\,T^7 + 12942909000T^5 + 937676250T^3, \quad \mathbf{q}_{20} = 1, \quad \mathbf{q}_{19} = 0,$
$\mathbf{q}_{18} = 10\,T^2 + 10, \quad \mathbf{q}_{17} = 0, \quad \mathbf{q}_{16} = 45\,T^4 - 270\,T^2 + 405, \quad \mathbf{q}_{15} = 0,$
$\mathbf{q}_{14} = 120\,T^6 - 1800\,T^4 + 1800\,T^2 + 16200, \quad \mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = 210\,T^8 - 4200\,T^6 + 6300\,T^4$
$+113400\,T^2 + 425250, \quad \mathbf{q}_{11} = 0, \quad \mathbf{q}_{10} = 252\,T^{10} - 3780\,T^8 + 63000\,T^6$
$+718200\,T^4 + 3005100\,T^2 + 1644300, \quad \mathbf{q}_9 = 0, \quad \mathbf{q}_8 = 210\,T^{12} + 1260\,T^{10}$
$+255150\,T^8 - 567000\,T^6 + 23388750\,T^4 - 31468500\,T^2 + 17435250, \quad \mathbf{q}_7 = 0,$
$\mathbf{q}_6 = 120\,T^{14} + 5880\,T^{12} + 476280\,T^{10} + 16443000\,T^8 + 162729000\,T^6$
$-154791000\,T^4 + 130977000\,T^2 + 130977000, \quad \mathbf{q}_5 = 0, \quad \mathbf{q}_4 = 45\,T^{16} + 5400\,T^{14}$
$+459900\,T^{12} + 19845000\,T^{10} + 153798750\,T^8 + 702513000\,T^6 - 89302500\,T^4$
$+1250235000T^2 + 111628125, \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 10\,T^{18}$
$+2250\,T^{16} + 225000\,T^{14} + 4422600\,T^{12} - 99508500\,T^{10} - 224248500\,T^8$
$+9704205000\,T^6 + 15181425000T^4 - 1920003750T^2 + 223256250, \quad \mathbf{q}_1 = 0,$
$\mathbf{q}_0 = T^{20} + 370\,T^{18} + 44325\,T^{16} + 2208600\,T^{14} + 62795250\,T^{12} + 693384300\,T^{10}$
$+6641129250\,T^8 + 4346055000T^6 + 14042818125)T^4 + 2902331250T^2 + 22325625$

2. Case N=5

The expressions of the coefficients of the polynomials G, H and Q are defined by

$\alpha_5 = 60, \quad \mathbf{g}_{30} = 0, \quad \mathbf{g}_{29} = 0, \quad \mathbf{g}_{28} = 1, \quad \mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 42\,T^2 + 42, \quad \mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 455\,T^4$
$-1050\,T^2 + 2415, \quad \mathbf{g}_{23} = 0, \quad \mathbf{g}_{22} = 2548\,T^6 - 30660\,T^4 - 13860\,T^2 + 119700, \quad \mathbf{g}_{21} = 0,$
$\mathbf{g}_{20} = 9009\,T^8 - 226380\,T^6 + 171990\,T^4 - 343980\,T^2 + 3221505, \quad \mathbf{g}_{19} = 0, \quad \mathbf{g}_{18} = 22022\,T^{10}$
$-838530\,T^8 + 4142460\,T^6 - 44100\,T^4 - 36713250\,T^2 - 40153050, \quad \mathbf{g}_{17} = 0,$
$\mathbf{g}_{16} = 39039\,T^{12} - 1844766\,T^{10} + 22431465\,T^8 - 9075780\,T^6 - 259473375\,T^4 - 2703484350\,T^2$
$-370010025, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 51480\,T^{14} - 2522520\,T^{12} + 61319160\,T^{10}$
$+39803400\,T^8 - 773955000\,T^6 - 21896973000\,T^4 + 33756345000\,T^2 - 2893401000,$
$\mathbf{g}_{13} = 0, \quad \mathbf{g}_{12} = 51051\,T^{16} - 2023560\,T^{14} + 104367060\,T^{12} + 629483400\,T^{10}$
$+6114046050\,T^8 - 132697164600\,T^6 + 554979316500\,T^4 + 319310019000\,T^2 + 30787036875,$
$\mathbf{g}_{11} = 0, \quad \mathbf{g}_{10} = 38038\,T^{18} - 589050\,T^{16} + 124369560\,T^{14} + 1700266680\,T^{12}$
$+37748127060\,T^{10} - 446713728300\,T^8 + 2431707075000\,T^6 + 1380509487000\,T^4$
$+4238859255750\,T^2 + 1299806817750, \quad \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 21021\,T^{20} + 570570\,T^{18}$
$+112372785\,T^{16} + 1735587000\,T^{14} - 43189665750\,T^{12} - 2318934687300\,T^{10}$
$+10714665764250\,T^8 - 20464596621000\,T^6 + 35015175365625\,T^4 + 40381027706250T^2$
$+5540260123125, \quad \mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 8372\,T^{22} + 769692\,T^{20} + 78618540\,T^{18}$
$+570662820\,T^{16} - 223349124600\,T^{14} - 2950615722600\,T^{12} + 16520555280600\,T^{10}$
$-11401393059000\,T^8 + 147193042090500\,T^6 + 422927620447500T^4 - 99598095922500T^2$
$+17840228332500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 2275\,T^{24} + 415380\,T^{22} + 39897270\,T^{20}$
$-30649500\,T^{18} - 148598863875\,T^{16} - 1555875783000\,T^{14} - 2135859799500\,T^{12}$
$+94593530241000\,T^{10} - 98463038821875\,T^8 + 2611250197762500T^6 - 159203362106250T^4$
$-83293781287500T^2 - 21709549378125, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 378\,T^{26} + 114450\,T^{24}$
$+12621420\,T^{22} + 89037900\,T^{20} - 283320450\,T^{18} + 1545272004150\,T^{16}$
$+12633981885000\,T^{14} - 118201467699000\,T^{12} + 1380551057313750\,T^{10}$
$+7814079083238750T^8 + 3521850108367500T^6 + 4776100863187500T^4$
$-1406247137268750T^2 - 13291560843750, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 29\,T^{28} + 13230\,T^{26}$
$+1814295\,T^{24} + 74845260\,T^{22} - 764250795\,T^{20} - 204794909550\,T^{18}$
$-3849793565625\,T^{16} - 34193820087000\,T^{14} + 942733356807375\,T^{12}$
$+1889980437035250T^{10} + 13147594251868125T^8 + 3164572952887500T^6$
$-3369410673890625T^4 - 124940671931250T^2 + 3987468253125,$

$\mathbf{h}_{30} = 0, \quad \mathbf{h}_{29} = 0, \quad \mathbf{h}_{28} = T, \quad \mathbf{h}_{27} = 0, \quad \mathbf{h}_{26} = 14\,T^3 - 42\,T, \quad \mathbf{h}_{25} = 0, \quad \mathbf{h}_{24} = 91\,T^5 - 1610\,T^3$
$-1365\,T, \quad \mathbf{h}_{23} = 0, \quad \mathbf{h}_{22} = 364\,T^7 - 14700\,T^5 + 13860\,T^3 - 64260\,T, \quad \mathbf{h}_{21} = 0, \quad \mathbf{h}_{20} = 1001\,T^9$
$-67452\,T^7 + 411894\,T^5 - 97020\,T^3 - 2546775\,T, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 2002\,T^{11} - 190190\,T^9$
$+2572500\,T^7 - 3342780\,T^5 - 6769350\,T^3 - 39756150\,T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 3003\,T^{13}$
$-358974\,T^{11} + 7821765\,T^9 - 39225060\,T^7 - 73327275\,T^5 + 439963650\,T^3 + 2114980875\,T,$
$\mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 3432\,T^{15} - 471240\,T^{13} + 13736520\,T^{11} - 135513000\,T^9 - 793686600\,T^7$
$+2779093800\,T^5 + 51116751000\,T^3 + 24754653000\,T, \quad \mathbf{h}_{13} = 0, \quad \mathbf{h}_{12} = 3003\,T^{17}$
$-434280\,T^{15} + 14403060\,T^{13} - 248776920\,T^{11} - 793072350\,T^9 - 2707651800\,T^7 + 375945664500\,T^5$
$-81098577000\,T^3 + 297337138875\,T, \quad \mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 2002\,T^{19} - 275814\,T^{17}$
$+8148168\,T^{15} - 362872440\,T^{13} - 114704100\,T^{11} - 90682521300\,T^9 + 1534457471400\,T^7$
$-1772183107800\,T^5 + 1117897625250\,T^3 + 1493718266250)T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 1001\,T^{21}$

$-113190\,T^{19}+836325\,T^{17}-501931080\,T^{15}-15705928350\,T^{13}-400107348900\,T^{11}$
$+4976480045250\,T^{9}-11450902365000\,T^{7}+30510953731125\,T^{5}-5820156483750T^{3}$
$-21110374254375T,\quad \mathbf{h}_7=0,\quad \mathbf{h}_6=364\,T^{23}-24332\,T^{21}-2084460\,T^{19}$
$-528432660\,T^{17}-31926371400\,T^{15}+150244907400\,T^{13}+11823972489000\,T^{11}$
$-3962494809000\,T^{9}+7158970633500\,T^{7}+132364254802500T^{5}-455536249717500T^{3}$
$-63681344842500T,\quad \mathbf{h}_5=0,\quad \mathbf{h}_4=91\,T^{25}+420\,T^{23}-1450890\,T^{21}-337761900\,T^{19}$
$-18543465675\,T^{17}+274020553800\,T^{15}+5724951088500\,T^{13}+48513868893000\,T^{11}$
$+171111381643125\,T^{9}+1334157649492500T^{7}-1694171881946250T^{5}-515712560737500T^{3}$
$-131586452353125)T,\quad \mathbf{h}_3=0,\quad \mathbf{h}_2=14\,T^{27}+1470\,T^{25}-409500\,T^{23}$
$-111637260\,T^{21}-3311799750\,T^{19}+88973271450\,T^{17}-3045655809000\,T^{15}$
$-34947318861000\,T^{13}+1002802178873250\,T^{11}+1999468016831250T^{9}-6800738923597500T^{7}$
$+4269249343012500T^{5}-1666761729806250T^{3}+204690036993750T,\quad \mathbf{h}_1=0,$
$\mathbf{h}_0=T^{29}+238\,T^{27}-43701\,T^{25}-14070420\,T^{23}-1034990775\,T^{21}-32505382350\,T^{19}$
$+259820563275\,T^{17}+13855420996200\,T^{15}+406907765530875\,T^{13}+497730743291250T^{11}$
$+1983581436965625T^{9}-10570073675332500T^{7}-7864084888813125T^{5}-224184326231250T^{3}$
$+73103584640625)T$

$\mathbf{q}_{30}=1,\quad \mathbf{q}_{29}=0,\quad \mathbf{q}_{28}=15\,T^{2}+15,\quad \mathbf{q}_{27}=0,\quad \mathbf{q}_{26}=105\,T^{4}-630\,T^{2}+945,\quad \mathbf{q}_{25}=0,$
$\mathbf{q}_{24}=455\,T^{6}-7875\,T^{4}+4725\,T^{2}+64575,\quad \mathbf{q}_{23}=0,\quad \mathbf{q}_{22}=1365\,T^{8}-39900\,T^{6}$
$+103950\,T^{4}+548100\,T^{2}+3709125,\quad \mathbf{q}_{21}=0,\quad \mathbf{q}_{20}=3003\,T^{10}-114345\,T^{8}+859950\,T^{6}+4035150\,T^{4}$
$+34827975\,T^{2}+133656075,\quad \mathbf{q}_{19}=0,\quad \mathbf{q}_{18}=5005\,T^{12}-200970\,T^{10}+3649275\,T^{8}+220500\,T^{6}$
$+277333875\,T^{4}+959505750\,T^{2}+1115785125,\quad \mathbf{q}_{17}=0,\quad \mathbf{q}_{16}=6435\,T^{14}-204435\,T^{12}+10174815\,T^{10}$
$+42170625\,T^{8}+2030639625\,T^{6}+7693410375\,T^{4}-27357820875\,T^{2}+24214372875,\quad \mathbf{q}_{15}=0,$
$\mathbf{q}_{14}=6435\,T^{16}-59400\,T^{14}+21035700\,T^{12}+451672200\,T^{10}+2902331250\,T^{8}+79622109000\,T^{6}$
$-319613647500\,T^{4}+191285955000\,T^{2}+463546951875,\quad \mathbf{q}_{13}=0,\quad \mathbf{q}_{12}=5005\,T^{18}$
$+155925\,T^{16}+33585300\,T^{14}+1481098500\,T^{12}+42118035750\,T^{10}+639849435750\,T^{8}$
$-1190848837500\,T^{6}+1787210932500\,T^{4}+4850130403125\,T^{2}+5581517878125,\quad \mathbf{q}_{11}=0$
$\mathbf{q}_{10}=3003\,T^{20}+279510\,T^{18}+40951575\,T^{16}+2550025800\,T^{14}+112585249350\,T^{12}$
$+1486454400900\,T^{10}+2935114197750\,T^{8}+10430710605000\,T^{6}+58973741229375\,T^{4}$
$+49590883833750T^{2}+14657286301875,\quad \mathbf{q}_{9}=0,\quad \mathbf{q}_{8}=1365\,T^{22}+246015\,T^{20}$
$+36850275\,T^{18}+2719544625\,T^{16}+98273999250\,T^{14}-830307854250\,T^{12}$
$-8598553724250\,T^{10}+211739487041250\,T^{8}+162726680615625\,T^{6}$
$+731900852746875T^{4}-370328202365625T^{2}+93610564228125,\quad \mathbf{q}_{7}=0,$
$\mathbf{q}_{6}=455\,T^{24}+134820\,T^{22}+23403870\,T^{20}+1942384500\,T^{18}+36981653625\,T^{16}$
$-1371507795000\,T^{14}+3080287318500\,T^{12}+299367020421000\,T^{10}+3135310421315625\,T^{8}$
$+10570433743012500T^{6}-3151872128081250T^{4}+1114718902762500T^{2}$
$+412481438184375,\quad \mathbf{q}_{5}=0,\quad \mathbf{q}_{4}=105\,T^{26}+46725\,T^{24}+9856350\,T^{22}$
$+950244750\,T^{20}+28094731875\,T^{18}+11015463375\,T^{16}-7594552507500\,T^{14}$
$+191792925292500\,T^{12}+6041183185209375\,T^{10}+13451797993921875T^{8}$
$+29454394197768750T^{6}+13424476452187 50T^{4}+5894807234203125T^{2}$
$+232602314765625,\quad \mathbf{q}_{3}=0,\quad \mathbf{q}_{2}=15\,T^{28}+9450\,T^{26}$
$+2461725\,T^{24}+296748900\,T^{22}+21862740375\,T^{20}+1401063945750\,T^{18}$
$+53045439418125\,T^{16}+801963419355000\,T^{14}+7240875247423125\,T^{12}$
$-18225599206106250T^{10}-19943100941990625T^{8}+193179545303062500T^{6}$

$+118075580755453125T^4 - 5861578332093750T^2 + 299060118984375,$
$\mathbf{q}_1 = 0, \quad \mathbf{q}_0 = T^{30} + 855\,T^{28} + 275625\,T^{26} + 44441775\,T^{24} + 4060783125\,T^{22}$
$+207533751075\,T^{20} + 5923312282125\,T^{18} + 77461769896875\,T^{16}$
$+1691986493491875\,T^{14} + 21127132873153125T^{12} + 60580010182426875T^{10}$
$+225021251512378125T^8 + 50098108080234375T^6 + 67806897644390625T^4$
$+5881515673359375T^2 + 19937341265625$

3. Case N=6

The coefficients of the polynomials G, H and Q are defined by

$\alpha_6 = 4, \quad \mathbf{g}_{42} = 0, \quad \mathbf{g}_{41} = 0, \quad \mathbf{g}_{40} = 21, \quad \mathbf{g}_{39} = 0, \quad \mathbf{g}_{38} = 1260\,T^2 + 1260, \quad \mathbf{g}_{37} = 0, \quad \mathbf{g}_{36} = 19950\,T^4$
$-49140\,T^2 + 108990, \quad \mathbf{g}_{35} = 0, \quad \mathbf{g}_{34} = 167580\,T^6 - 2249100\,T^4 - 1190700\,T^2 + 9128700, \quad \mathbf{g}_{33} = 0,$
$\mathbf{g}_{32} = 915705\,T^8 - 27439020\,T^6 + 24409350\,T^4 - 48818700\,T^2 + 589098825, \quad \mathbf{g}_{31} = 0, \quad \mathbf{g}_{30} = 3581424\,T^{10}$
$-178128720\,T^8 + 984947040\,T^6 + 60328800\,T^4 - 4186501200\,T^2 + 18503478000, \quad \mathbf{g}_{29} = 0,$
$\mathbf{g}_{28} = 10581480\,T^{12} - 743616720\,T^{10} + 9464477400\,T^8 - 7045768800\,T^6 - 30612897000\,T^4$
$-520740738000\,T^2 - 459479223000, \quad \mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 24418800\,T^{14} - 2175329520\,T^{12}$
$+47637843120\,T^{10} - 159272794800\,T^8 - 224542206000\,T^6 - 4616367714000\,T^4 - 32428595430000\,T^2$
$-7322876442000, \quad \mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 44971290\,T^{16} - 4661420400\,T^{14} + 152359061400\,T^{12}$
$-1026048416400\,T^{10} - 337581310500\,T^8 - 30371208714000\,T^6 - 186218752545000\,T^4$
$+1100164292010000\,T^2 - 108079377603750, \quad \mathbf{g}_{23} = 0, \quad \mathbf{g}_{22} = 67016040\,T^{18} - 7469967960\,T^{16}$
$+337998679200\,T^{14} - 3317725202400\,T^{12} + 14701701495600\,T^{10} - 134995160034000\,T^8$
$-1193237286444000\,T^6 + 29149053992100000\,T^4 + 20986218417465000\,T^2 + 1152340349265000,$
$\mathbf{g}_{21} = 0, \quad \mathbf{g}_{20} = 81477396\,T^{20} - 8962513560\,T^{18} + 547168642020\,T^{16} - 6239220372000\,T^{14}$
$+108187552103400\,T^{12} - 282142532826000\,T^{10} - 9222667785639000\,T^8 + 257357195058348000\,T^6$
$+18279145208362500\,T^4 + 529488288837585000\,T^2 + 156886140925552500, \quad \mathbf{g}_{19} = 0$
$\mathbf{g}_{18} = 81124680\,T^{22} - 7847559720\,T^{20} + 671406535800\,T^{18} - 6955900320600\,T^{16} + 296997420258000\,T^{14}$
$+830239889094000\,T^{12} - 53402987374506000\,T^{10} + 1441119106966770000\,T^8 - 2734739470632015000\,T^6$
$+4283688113241075000\,T^4 + 11213622020930175000\,T^2 + 4883826305513925000, \quad \mathbf{g}_{17} = 0,$
$\mathbf{g}_{16} = 66134250\,T^{24} - 4572870120\,T^{22} + 650465836020\,T^{20} - 3956090430600\,T^{18}$
$+265628919921750\,T^{16} - 8374317575778000\,T^{14} - 49449016042618500 0\,T^{12} + 7083936614255886000\,T^{10}$
$-27625636046797346250\,T^8 + 58715278628789475000\,T^6 + 100296773020980112500\,T^4$
$+109063413373951575000T^2 + 48008615346625106250, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 43953840\,T^{26}$
$-1085994000\,T^{24} + 525422570400\,T^{22} + 283015101600\,T^{20} - 440867272230000\,T^{18}$
$-70625220688638000\,T^{16} - 1840700971022760000\,T^{14} + 23136462127298520000\,T^{12}$
$-77288502059715870000\,T^{10} + 203718051732368250000\,T^8 + 1193114598558012900000\,T^6$
$+3826674393311385000000T^4 - 17785540707534022500 00T^2 + 45058359360566475 0000, \quad \mathbf{g}_{13} = 0,$
$\mathbf{g}_{12} = 23604840\,T^{28} + 923030640\,T^{26} + 375409452600\,T^{24} + 2624172616800\,T^{22}$
$-1510226919293400\,T^{20} - 169673051037798000\,T^{18} - 1533701946518973000\,T^{16}$
$+6017129050632084000 0\,T^{14} - 81243452836954485000\,T^{12} + 989384777473422330000\,T^{10}$
$+9218597475208223025000\,T^8 + 11767748128231995900000T^6 - 40367065478563333125000T^4$
$-84257542439914882500 00T^2 + 20972827756431416250 00, \quad \mathbf{g}_{11} = 0, \quad \mathbf{g}_{10} = 10093104\,T^{30}$
$+1259753040\,T^{28} + 242036287920\,T^{26} + 3006820731600\,T^{24} - 1903047548547600\,T^{22}$
$-177522454448262000\,T^{20} + 1673066563892262000\,T^{18} + 8752837151032266 6000\,T^{16}$
$+6620696554494330000\,T^{14} + 58071156589382636700 00\,T^{12} + 2323938029547478113000 0\,T^{10}$
$+137613317496661002150000T^8 - 2103183399902364895500 00T^6 + 1183354860628415475000 0T^4$
$-8309908689414653475000 0T^2 - 994815686313620325000 0, \quad \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 3357585\,T^{32}$
$+788855760\,T^{30} + 133834793400\,T^{28} + 2706551593200\,T^{26} - 1301950524262500\,T^{24}$
$-74974735386558000\,T^{22} + 3528441325033317000\,T^{20} - 9148978382135130000\,T^{18}$
$-917328476828617796250\,T^{16} + 20331186148107969750000\,T^{14} + 697010070670980239250 00\,T^{12}$
$+887836329346915768050000T^{10} - 5177725230698592350625 00T^8 + 2152850692307163675750000T^6$
$-6395113427142586781250 00T^4 - 7976155443395234287500 00T^2 - 2126309102201586585937 5, \quad \mathbf{g}_7 = 0,$
$\mathbf{g}_6 = 837900\,T^{34} + 311725260\,T^{32} + 57825260640\,T^{30} + 2004411679200\,T^{28} - 486305908326000\,T^{26}$
$-2096884640214000\,T^{24} + 2746634505228180000\,T^{22} + 35193597983772228000\,T^{20}$
$+535094835158922885000\,T^{18} + 1283077121350172072 5000\,T^{16} + 3830217467903955513000 00\,T^{14}$
$+2069806537668865079700000T^{12} - 2878103885627077118550000T^{10} + 1375299493382801118825000 0T^8$
$+8319513974090622187500000T^6 - 1186111617859045820250000 0T^4$

$+1576218096385317649687500T^2 - 26235193683001885312500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 147630\,T^{36}$
$+79512300\,T^{34} + 17590409550\,T^{32} + 1021528519200\,T^{30} - 81006309657000\,T^{28} + 1634306691906000\,T^{26}$
$+606189343425435000\,T^{24} + 11528510197530060000\,T^{22} - 396162299973691597500\,T^{20}$
$+14798306913954275250 00\,T^{18} + 581811329651584456462500\,T^{16} + 12479230555322630715000 00T^{14}$
$+384265929974146076587500 00T^{12} + 7389770241237094037250000T^{10} + 282582672524661537393750 00T^8$
$-571168891143567468675000 00T^6 + 33890316803279260242187 50T^4 + 2208926709003114942187500T^2$
$+110589958773237874218750, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 16380\,T^{38} + 12074580\,T^{36} + 3288316500\,T^{34}$
$+300491005500\,T^{32} - 226761670800\,T^{30} - 2504351229534000\,T^{28} - 460421913549654000\,T^{26}$
$-26192900747252610000\,T^{24} - 1105363357063008750 00\,T^{22} + 1167000373860202381500 0\,T^{20}$
$+4172050048409443257750 00\,T^{18} + 37391568087958236652500 0T^{16} + 82084510186033493407500 00T^{14}$
$+97521767433772016816250000T^{12} - 248372033459815147632750000T^{10}$
$-439480474711801012586250000T^8 + 507566738749878299559375 00T^6 - 116535964348837936546875 00T^4$
$+65923105294696343203125 00T^2 + 6032179569449338593750 0, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 861\,T^{40}$
$+835380\,T^{38} + 282059190\,T^{36} + 38110734900\,T^{34} + 757788568425\,T^{32} - 357432351318000\,T^{30}$
$-33079684204539000\,T^{28} - 671694631354782000\,T^{26} + 47369156210904386250\,T^{24}$
$+1080980353533726135000\,T^{22} + 57329971310306073802500\,T^{20} + 1450451085746969465775000T^{18}$
$+57350647803714791396062 50T^{16} + 319705794407152763032500 00T^{14} - 700649607133638933828750 00T^{12}$
$-551776429312196509575000 00T^{10} - 2035703097778171709327343 75)T^8 - 2096651569905713443218750 0T^6$
$+4223818308522276159609375 0)T^4 + 1146114118195374332812500)T^2 - 646304953869571992187 5,$

$\mathbf{h}_{42} = 0, \quad \mathbf{h}_{41} = 0, \quad \mathbf{h}_{40} = 21\,T, \quad \mathbf{h}_{39} = 0, \quad \mathbf{h}_{38} = 420\,T^3 - 1260\,T, \quad \mathbf{h}_{37} = 0, \quad \mathbf{h}_{36} = 3990\,T^5$
$-72660\,T^3 - 59850\,T, \quad \mathbf{h}_{35} = 0, \quad \mathbf{h}_{34} = 23940\,T^7 - 1031940\,T^5 + 1190700\,T^3 - 4365900\,T, \quad \mathbf{h}_{33} = 0,$
$\mathbf{h}_{32} = 101745\,T^9 - 7646940\,T^7 + 55526310\,T^5 + 6747300\,T^3 - 308688975\,T, \quad \mathbf{h}_{31} = 0, \quad \mathbf{h}_{30} = 325584\,T^{11}$
$-36414000\,T^9 + 606039840\,T^7 - 487075680\,T^5 - 442940400\,T^3 - 15569593200\,T, \quad \mathbf{h}_{29} = 0,$
$\mathbf{h}_{28} = 813960\,T^{13} - 122608080\,T^{11} + 3460174200\,T^9 - 15134364000\,T^7 - 6941781000\,T^5 + 41650686000\,T^3$
$-133275051000\,T, \quad \mathbf{h}_{27} = 0, \quad \mathbf{h}_{26} = 1627920\,T^{15} - 307676880\,T^{13} + 12556328400\,T^{11}$
$-123859789200\,T^9 - 414363600\,T^7 + 625617594000\,T^5 + 14773276854000\,T^3 + 40215059010000\,T,$
$\mathbf{h}_{25} = 0, \quad \mathbf{h}_{24} = 2645370\,T^{17} - 593676720\,T^{15} + 31525767000\,T^{13} - 527395957200\,T^{11}$
$+752464849500\,T^9 + 2323222398000\,T^7 + 161180546247000\,T^5 + 1150723795410000\,T^3$
$+884942275376250\,T, \quad \mathbf{h}_{23} = 0, \quad \mathbf{h}_{22} = 3527160\,T^{19} - 897198120\,T^{17} + 57072768480\,T^{15}$
$-1415227917600\,T^{13} + 5710878205200\,T^{11} - 31038334110000\,T^9 + 1020560829372000\,T^7$
$+10607242786812000\,T^5 - 10549431670365000\,T^3 + 21705296078655000\,T, \quad \mathbf{h}_{21} = 0,$
$\mathbf{h}_{20} = 3879876\,T^{21} - 1072071000\,T^{19} + 75633978420\,T^{17} - 2635403934240\,T^{15}$
$+18071080057800\,T^{13} - 232668433443600\,T^{11} + 3887539470045000\,T^9 + 73903432233708000\,T^7$
$-332535925127983500\,T^5 - 35419823925255000\,T^3 + 373647041815162500\,T, \quad \mathbf{h}_{19} = 0,$
$\mathbf{h}_{18} = 3527160\,T^{23} - 1014893880\,T^{21} + 72635963400\,T^{19} - 3661723989000\,T^{17} + 22961577870000\,T^{15}$
$-558100069758000\,T^{13} + 8233601126058000\,T^{11} + 401324717365110000\,T^9 - 2629541354603625000\,T^7$
$+1891946762885025000\,T^5 - 2077818312668175000\,T^3 + 469617427731375000T, \quad \mathbf{h}_{17} = 0,$
$\mathbf{h}_{16} = 2645370\,T^{25} - 756854280\,T^{23} + 47845898100\,T^{21} - 4100304839400\,T^{19} - 18503101143450\,T^{17}$
$-1692739675242000\,T^{15} - 39034514859417000\,T^{13} + 1908000279128790000\,T^{11}$
$-13226094269284916250\,T^9 + 41333515336516095000\,T^7 - 30934512876526875 00\,T^5$
$-152813978846731125000T^3 - 136885438789385343750)T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 1627920\,T^{27}$
$-437739120\,T^{25} + 17295314400\,T^{23} - 4002542359200\,T^{21} - 118543877298000\,T^{19}$
$-5096839239954000\,T^{17} + 6829841768904000\,T^{15} + 8476647700225320000\,T^{13}$
$-3966200331257241000 0\,T^{11} + 1985918854192465500 00\,T^9 + 3386764257251067000 00\,T^7$
$-1458984265260221700000T^5 - 3931624238736174750000T^3 - 1056176028083310750000T,$

$\mathbf{h}_{13} = 0, \quad \mathbf{h}_{12} = 813960\,T^{29} - 190123920\,T^{27} - 2343879720\,T^{25} - 3521465085600\,T^{23}$
$-214336185247800\,T^{21} - 7685261724294000\,T^{19} + 559933161897879000\,T^{17}$
$+18300777177408264000\,T^{15} - 673194046229293050 00\,T^{13} + 7020284323949612100 00\,T^{11}$
$+96525620962655692500 0\,T^{9} - 501622106907599370000 0T^{7} - 410281094967542048250 00T^{5}$
$+508850425604834775000 0T^{3} - 765950527810686487500 0T, \quad \mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 325584\,T^{31}$
$-57834000\,T^{29} - 7480453680\,T^{27} - 2671612327440\,T^{25} - 227871922345200\,T^{23} - 4515437156907600\,T^{21}$
$+1063591395097770000\,T^{19} + 15434675218020366000\,T^{17} - 89580321136341882000\,T^{15}$
$+204118336030921389000 0\,T^{13} + 472319063299643175000 0\,T^{11} + 116641003453739572500 00T^{9}$
$-254778861744583996050000T^{7} + 117211546169871233850000T^{5} - 262892994004804432500 00T^{3}$
$-190865517523236517500 00T, \quad \mathbf{h}_{9} = 0, \quad \mathbf{h}_{8} = 101745\,T^{33} - 10024560\,T^{31} - 5022145800\,T^{29}$
$-1608119730000\,T^{27} - 161174098606500\,T^{25} + 335632587066000\,T^{23} + 736078113424269000\,T^{21}$
$-7565148948169890000\,T^{19} - 106455713823272756250\,T^{17} + 581150502429384195 0000\,T^{15}$
$+677936548901582932500 0\,T^{13} + 154597964806237664250000T^{11} - 152695649554003974206 2500T^{9}$
$+132286655446844542875 0000T^{7} - 166506393999097888312 5000)T^{5} + 333570867196457756250000T^{3}$
$+248146906256610376640625T, \quad \mathbf{h}_{7} = 0, \quad \mathbf{h}_{6} = 23940\,T^{35} + 149940\,T^{33} - 1927346400\,T^{31}$
$-708549055200\,T^{29} - 79214033883600\,T^{27} + 823565301174000\,T^{25} + 200617502001084000\,T^{23}$
$-8279951353262820000\,T^{21} - 777838836351965850 00\,T^{19} + 298838300272420639500 0\,T^{17}$
$+903984977552590395000 0\,T^{15} - 415079152248697744500000T^{13} - 626463443444424218325 0000T^{11}$
$-711055582862691008250000T^{9} - 175642478756527825350000 0T^{7} - 332389052423667681750000 0T^{5}$
$+743158139697354864656250 0T^{3} + 510341541351824995312500T, \quad \mathbf{h}_{5} = 0, \quad \mathbf{h}_{4} = 3990\,T^{37}$
$+555660\,T^{35} - 451870650\,T^{33} - 210601490400\,T^{31} - 27164808405000\,T^{29} - 494049363822000\,T^{27}$
$-131254421193000\,T^{25} - 2331351766748340000\,T^{23} - 57607676795659567500\,T^{21}$
$+3182842545894136125000\,T^{19} + 12147149609360590151250 0\,T^{17} - 730868754303114203700000T^{15}$
$-395217077024913282562500 0T^{13} - 188173836741059476807500 00T^{11} - 246009274469149934531250 00T^{9}$
$-620845964457093958275000 00T^{7} + 389620393368373874798437 50T^{5} + 586270404821242860468750 0T^{3}$
$+690828184025031396093750T, \quad \mathbf{h}_{3} = 0, \quad \mathbf{h}_{2} = 420\,T^{39} + 127260\,T^{37} - 59773140\,T^{35} - 37389170700\,T^{33}$
$-6082652847600\,T^{31} - 456491304234000\,T^{29} - 20958512959818000\,T^{27} + 1456163148715002000\,T^{25}$
$+201140982748216035000\,T^{23} + 4864337674818781005000\,T^{21} + 60289385834313075825000\,T^{19}$
$-641663910397442064825000T^{17} + 3860956945797085030050000T^{15} + 17024232728719929462750000T^{13}$
$-174686108305029234779250000T^{11} - 133153380406498251318750000T^{9}$
$+252883712879152705566562500T^{7} - 662913554950789171115625 00T^{5} + 124349073124505651296875 00T^{3}$
$-784183344028414017187500T, \quad \mathbf{h}_{1} = 0, \quad \mathbf{h}_{0} = 21\,T^{41} + 10500\,T^{39} - 3356010\,T^{37}$
$-2977396380\,T^{35} - 678988637775\,T^{33} - 67009019137200\,T^{31} - 1544931642555000\,T^{29}$
$+190273318160058000\,T^{27} + 15779576666742320250\,T^{25} + 302225493684212595000\,T^{23}$
$+1075022773008800096250 0\,T^{21} + 16316397675737800987500 0T^{19} - 134533446161898484614375 0T^{17}$
$-104030328791024514301500 00T^{15} - 115925574335468992738875 00T^{13}$
$-521032994713865609167500 00T^{11} - 863233355083699959433593 75T^{9} + 255531634533965133599062500T^{7}$
$+853527556878926628135937 50T^{5} + 115760398404194450156250 0T^{3} - 196045836007103504296875T$

$\mathbf{q}_{42} = 1, \quad \mathbf{q}_{41} = 0, \quad \mathbf{q}_{40} = 21\,T^{2} + 21, \quad \mathbf{q}_{39} = 0, \quad \mathbf{q}_{38} = 210\,T^{4} - 1260\,T^{2} + 1890, \quad \mathbf{q}_{37} = 0,$
$\mathbf{q}_{36} = 1330\,T^{6} - 24570\,T^{4} + 10710\,T^{2} + 193410, \quad \mathbf{q}_{35} = 0, \quad \mathbf{q}_{34} = 5985\,T^{8} - 202860\,T^{6} + 595350\,T^{4}$
$+1984500\,T^{2} + 18356625, \quad \mathbf{q}_{33} = 0, \quad \mathbf{q}_{32} = 20349\,T^{10} - 1012095\,T^{8} + 8136450\,T^{6} + 17662050\,T^{4}$
$+215219025\,T^{2} + 1383295725, \quad \mathbf{q}_{31} = 0, \quad \mathbf{q}_{30} = 54264\,T^{12} - 3444336\,T^{10} + 53290440\,T^{8} - 20109600\,T^{6}$
$+1912264200\,T^{4} + 15369555600\,T^{2} + 65128908600, \quad \mathbf{q}_{29} = 0, \quad \mathbf{q}_{28} = 116280\,T^{14} - 8439480\,T^{12}$
$+217024920\,T^{10} - 694575000\,T^{8} + 14832153000\,T^{6} + 133703703000\,T^{4} + 386251173000\,T^{2}$
$+1034587323000, \quad \mathbf{q}_{27} = 0, \quad \mathbf{q}_{26} = 203490\,T^{16} - 15165360\,T^{14} + 622815480\,T^{12}$

$-2210256720\,T^{10}+56135551500\,T^{8}+1192724190000\,T^{6}+1433519451000\,T^{4}-27693705438000\,T^{2}$
$+38780976953250,\quad \mathbf{q}_{25}=0,\quad \mathbf{q}_{24}=293930\,T^{18}-19631430\,T^{16}+1366583400\,T^{14}$
$+2951613000\,T^{12}+363193267500\,T^{10}+10089021379500\,T^{8}+22829041053000\,T^{6}$
$-475525632015000\,T^{4}+342500315456250\,T^{2}+1376457162806250,\quad \mathbf{q}_{23}=0,\quad \mathbf{q}_{22}=352716\,T^{20}$
$-16521960\,T^{18}+2444960700\,T^{16}+41186426400\,T^{14}+2435977719000\,T^{12}+35777824966800\,T^{10}$
$+398543162031000\,T^{8}-3822883567020000\,T^{6}+7873515567697500\,T^{4}+16086079864575000\,T^{2}$
$+38577274289347500,\quad \mathbf{q}_{21}=0,\quad \mathbf{q}_{20}=352716\,T^{22}-4288284\,T^{20}+3742158420\,T^{18}$
$+149684881500\,T^{16}+9553331403000\,T^{14}+212261247613800\,T^{12}+3849610940821800\,T^{10}$
$-15394646399517000\,T^{8}+71758478683237500\,T^{6}+204046725448912500\,T^{4}+436091149100722500\,T^{2}$
$+674506306475347500,\quad \mathbf{q}_{19}=0,\quad \mathbf{q}_{18}=293930\,T^{24}+11695320\,T^{22}+5003418420\,T^{20}$
$+334607427000\,T^{18}+23101428561750\,T^{16}+807591346254000\,T^{14}+16185920631111000\,T^{12}$
$-8352665505954000\,T^{10}+576663818982693750\,T^{8}+1242059144478075000\,T^{6}+5668373975877112500\,T^{4}$
$+37615010855325750000T^{2}+4325812023235106250,\quad \mathbf{q}_{17}=0,\quad \mathbf{q}_{16}=203490\,T^{26}+22694490\,T^{24}$
$+5786087580\,T^{22}+534631523580\,T^{20}+37855900293750\,T^{18}+1433816319300750\,T^{16}$
$+1641456285777000\,T^{14}-265649781021471000\,T^{12}+5360993203091352750\,T^{10}+69096046130625937 50\,T^{8}$
$+80880110043223837500\,T^{6}+422369703735123375 00T^{4}-11356059969387354375 0T^{2}$
$+60012457876086581250,\quad \mathbf{q}_{15}=0,\quad \mathbf{q}_{14}=116280\,T^{28}+23904720\,T^{26}+5624073000\,T^{24}$
$+647566164000\,T^{22}+45746991675000\,T^{20}+1145438659134000\,T^{18}-60947538804999000\,T^{16}$
$-613314917245800000\,T^{14}+13256411880737025000\,T^{12}+146890015244661150000\,T^{10}$
$+484908475289220675000\,T^{8}+1198562394651167700000T^{6}-2003316012774759375000T^{4}$
$+7167250886770012500 00T^{2}+708119334693106875000,\quad \mathbf{q}_{13}=0,\quad \mathbf{q}_{12}=54264\,T^{30}$
$+17607240\,T^{28}+4448719800\,T^{26}+609209483400\,T^{24}+44277505939800\,T^{22}+412689587725800\,T^{20}$
$-84804973421553000\,T^{18}-250432798766535000\,T^{16}+2288283894885334 5000\,T^{14}$
$+466654428328441815000\,T^{12}+3892228471345297545000\,T^{10}+20229343597701089775000T^{8}$
$-111438537142351938750 00T^{6}+111541045912878408750 00T^{4}+103811868239979611250 00T^{2}$
$+520803914385195787 5000,\quad \mathbf{q}_{11}=0,\quad \mathbf{q}_{10}=20349\,T^{32}+9579024\,T^{30}+2778050520\,T^{28}$
$+446310874800\,T^{26}+36326975409900\,T^{24}+602640375094800\,T^{22}-16736996613922200\,T^{20}$
$+1698321128858046000\,T^{18}+5918814816811540875 0\,T^{16}+9726990665043890700 00\,T^{14}$
$+10020895210571390145000\,T^{12}+56146858034606462970000T^{10}+104753437906970047387500T^{8}$
$+83635145538228726750000T^{6}+196955754351328578375000T^{4}+586116902645250322500 00T^{2}$
$+9836753798777262328125,\quad \mathbf{q}_{9}=0,\quad \mathbf{q}_{8}=5985\,T^{34}+3871665\,T^{32}+1327301640\,T^{30}$
$+250676937000\,T^{28}+24928524967500\,T^{26}+1277755297843500\,T^{24}+75968161579863000\,T^{22}$
$+3075359436148923000\,T^{20}+26011936101356493750\,T^{18}+116830305116544009 3750\,T^{16}$
$+2563966393606328287 5000\,T^{14}-6868273378557918442 5000T^{12}-2663234315883694216125 00T^{10}$
$+2701519751233743319687500T^{8}+1281120041892573815625000T^{6}+1364503639750540183125000T^{4}$
$-3235012764336997333593 75T^{2}+4253302124989278539062 5,\quad \mathbf{q}_{7}=0,\quad \mathbf{q}_{6}=1330\,T^{36}$
$+1137780\,T^{34}+466317810\,T^{32}+104251764960\,T^{30}+13208232681000\,T^{28}+1193020745742000\,T^{26}$
$+99210798874341000\,T^{24}+5805544496212692000\,T^{22}+214383530642638549500\,T^{20}$
$+4530497152000032675000\,T^{18}+46797298481837387137500\,T^{16}-2174368519568493171000 00T^{14}$
$+196925267817218729250 00T^{12}+533657798449093765755000 0T^{10}+215304705350655215006250 00T^{8}$
$+323804845984519035675000 00T^{6}-414429886197580511671875 0T^{4}+730946965605972235312500T^{2}$
$+1253352866096695907812 50,\quad \mathbf{q}_{5}=0,\quad \mathbf{q}_{4}=210\,T^{38}+230790\,T^{36}+113314950\,T^{34}$
$+30177299250\,T^{32}+4848169221000\,T^{30}+561695828967000\,T^{28}+50626283202243000\,T^{26}$
$+3225603873293865000\,T^{24}+122692742315543137500\,T^{22}+2347942991384156512500\,T^{20}$
$+36794456407482655612500\,T^{18}+266523839860686468637500T^{16}-904411179248760933750 00T^{14}$

$+5001462949574321566875000T^{12} + 5969745826984951833937500 0T^{10} + 5558835396123469541812500 0)T^8$
$+627175284723853208732812 50T^6 + 2437287792703697046093750T^4 + 2506226987783118058593750)T^2$
$+5026816307874448828125 0, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 21\,T^{40} + 28980\,T^{38} + 17000550\,T^{36} + 5412525300\,T^{34}$
$+1069516209825\,T^{32} + 138706771971600\,T^{30} + 10810972830861000\,T^{28} + 389665850910210000\,T^{26}$
$-659267012964183750\,T^{24} - 4544658515342165850 00\,T^{22} - 9204165852885701775 00\,T^{20}$
$+2216315860557603477750 00T^{18} + 5984763190950898799156250T^{16} + 3244404240239514261525000 0T^{14}$
$+1356612363411351198941250 00T^{12} - 1194731497595965722277500 00T^{10}$
$-605387147867884252483593 75T^8 + 2541529600596704902078125 00T^6$
$+7386978376083043616484375 0T^4 - 1749332075140308192187500T^2$
$+4524134677087003945312 5, \quad \mathbf{q}_1 = 0, \quad \mathbf{q}_0 = T^{42} + 1701\,T^{40} + 1186290\,T^{38} + 451782450\,T^{36}$
$+105515964225\,T^{34} + 15941427079725\,T^{32} + 1619444848404600\,T^{30} + 114621189126003000\,T^{28}$
$+5904248149699121250\,T^{26} + 207513330336568226250\,T^{24} + 5039110473448289647500\,T^{22}$
$+110469853983534408247500T^{20} + 137570392751193044960625 0)T^{18} + 3784181946524764546781250T^{16}$
$+36528049590131262499875000T^{14} + 218877641452798644652875000T^{12}$
$+2547163560565688529573281 25T^{10} + 4398434108492295477916406 25T^8$
$+4412156359523972157703125 0T^6 + 2920005781582726260703125 0T^4$
$+1251677260660737758203125T^2 + 2154349846231906640625$

4. Case N=7

We give the coefficients of the polynomials G, H and Q

$\alpha_7 = 4, \quad \mathbf{g}_{56} = 0, \quad \mathbf{g}_{55} = 0, \quad \mathbf{g}_{54} = 28, \quad \mathbf{g}_{53} = 0, \quad \mathbf{g}_{52} = 2268\,T^2 + 2268, \quad \mathbf{g}_{51} = 0, \quad \mathbf{g}_{50} = 49140\,T^4$
$-125496\,T^2 + 272916, \quad \mathbf{g}_{49} = 0, \quad \mathbf{g}_{48} = 573300\,T^6 - 8158500\,T^4 - 4630500\,T^2 + 33736500, \quad \mathbf{g}_{47} = 0,$
$\mathbf{g}_{46} = 4422600\,T^8 - 145000800\,T^6 + 144471600\,T^4 - 288943200\,T^2 + 3645923400, \quad \mathbf{g}_{45} = 0,$
$\mathbf{g}_{44} = 24864840\,T^{10} - 1405819800\,T^8 + 8654642640\,T^6 + 1328821200\,T^4 - 27569467800\,T^2 + 285603683400,$
$\mathbf{g}_{43} = 0, \quad \mathbf{g}_{42} = 107747640\,T^{12} - 9018923760\,T^{10} + 129131097480\,T^8 - 92144939040\,T^6$
$-160080089400\,T^4 - 4068407574000\,T^2 + 9697930011000, \quad \mathbf{g}_{41} = 0, \quad \mathbf{g}_{40} = 372972600\,T^{14}$
$-41946985080\,T^{12} + 1042152554520\,T^{10} - 3985886824920\,T^8 - 2424305683800\,T^6 - 38041300420200\,T^4$
$-532602960535800\,T^2 - 462639709917000, \quad \mathbf{g}_{39} = 0, \quad \mathbf{g}_{38} = 1056755700\,T^{16} - 148997772000\,T^{14}$
$+5529226071600\,T^{12} - 48020254682400\,T^{10} + 9567791263800\,T^8 - 277615181844000\,T^6$
$-3997430875746000\,T^4 - 27691049938860000\,T^2 - 11433617241007500, \quad \mathbf{g}_{37} = 0, \quad \mathbf{g}_{36} = 2493390900\,T^{18}$
$-417526886700\,T^{16} + 21097721106000\,T^{14} - 312305697673200\,T^{12} + 750925867368600\,T^{10}$
$-1296117088497000\,T^8 - 19549525119894000\,T^6 + 178015776175314000\,T^4 + 2547484073859532500\,T^2$
$-257622602070487500, \quad \mathbf{g}_{35} = 0, \quad \mathbf{g}_{34} = 4960535580\,T^{20} - 942501760200\,T^{18}$
$+60895186381260\,T^{16} - 1320509170965600\,T^{14} + 8073640409095800\,T^{12} - 10433330397428400\,T^{10}$
$-94059388591245000\,T^8 + 4212714864528324000\,T^6 + 81525901261652347500\,T^4$
$+838801914831445950000\,T^2 + 4532925200381077500, \quad \mathbf{g}_{33} = 0, \quad \mathbf{g}_{32} = 8396404380\,T^{22}$
$-1736314646220\,T^{20} + 137179010388420\,T^{18} - 3955742649118260\,T^{16} + 44742788370264600\,T^{14}$
$-87890985649467000\,T^{12} - 581266358785035000\,T^{10} + 3448942848604014300 0\,T^8$
$+916422481982789743500\,T^6 - 2671282998913965975 00\,T^4 + 3533027762243314912500\,T^2$
$+1052175273585966337500, \quad \mathbf{g}_{31} = 0, \quad \mathbf{g}_{30} = 12168702000\,T^{24} - 2628289863360\,T^{22}$
$+246156751098720\,T^{20} - 8802207660818880\,T^{18} + 156743234946421200\,T^{16}$
$-400285925208892800\,T^{14} - 4082398971145387200\,T^{12} + 18229901239236456000 0\,T^{10}$
$+699360189556446217800 0\,T^8 - 2913627155426338428000 0\,T^6 + 2270533993414592598000 0\,T^4$
$+1336444408876356133200 0\,T^2 + 6972010067071453635000 0, \quad \mathbf{g}_{29} = 0, \quad \mathbf{g}_{28} = 15164074800\,T^{26}$
$-3271508730000\,T^{24} + 356805698608800\,T^{22} - 14963164581103200\,T^{20} + 373275824011952400\,T^{18}$
$-1646661408941934000\,T^{16} - 23044246961534568000\,T^{14} + 647137266547990680000\,T^{12}$
$+4444033279186848501000 0\,T^{10} - 36852426606838995999000 0\,T^8 + 515157395379005679300000\,T^6$
$+964818990199979907300000\,T^4 + 3432008280166633455750000\,T^2 + 2522450003846320374750000,$
$\mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 16287339600\,T^{28} - 3325987072800\,T^{26} + 422344463612400\,T^{24}$
$-19765833506798400\,T^{22} + 609616438648232400\,T^{20} - 8426887778118348000\,T^{18}$
$-198426421456868682000\,T^{16} - 2190017982667295088000\,T^{14} + 26060637624337581111000 0\,T^{12}$
$-266754017632065488910000 0\,T^{10} + 884720913835781473605000 0\,T^8 + 1309601491514514849420000 0\,T^6$
$+658174386522489491655000 0\,T^4 + 1102541264086366947150000 0T^2 + 4729380278457221419725000 0,$
$\mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 15089190480\,T^{30} - 2709127486800\,T^{28} + 412978164031440\,T^{26} - 20496272265378000\,T^{24}$
$+625906405138669200\,T^{22} - 35718951161305011600\,T^{20} - 1333506673014425910000\,T^{18}$
$-344187966373120249380 00\,T^{16} + 12443394473998069081500 00\,T^{14} - 11990961186295394893590000\,T^{12}$
$+659455199484762629648700 00\,T^{10} + 16015024255554115216425000 0\,T^8$
$+911384950419785072731500 00\,T^6 - 15549080022877618905187500 00T^4$
$-2392952896647698174039250000T^2 + 8897390928562945861552500 00, \quad \mathbf{g}_{23} = 0,$
$\mathbf{g}_{22} = 12047014980\,T^{32} - 1685346505920\,T^{30} + 339670088966880\,T^{28} - 16767007875455040\,T^{26}$
$+195015256305433200\,T^{24} - 10311510218482418400 0\,T^{22} - 496316409038089239600 0\,T^{20}$
$-733500169199927243496000\,T^{18} + 5521291279776971601903000\,T^{16} - 39856907136517355724840000\,T^{14}$
$+376003222446071990870100000\,T^{12} + 7188527388604277509794000 00\,T^{10}$
$+317624301594610650094455000 0\,T^8 - 1799058528663058779856744000 00T^6$
$-6777620806040176174132050000 00T^4 - 2713698765859343749987500000 00T^2$

$+120879401462142979196165625 00, \quad \mathbf{g}_{21} = 0, \quad \mathbf{g}_{20} = 8267559300\,T^{34} - 689768832060\,T^{32}$
$+242818935559200\,T^{30} - 10771451892321120\,T^{28} - 609328959012219600\,T^{26}$
$-20325357360329341320 0\,T^{24} - 1039503418013343563280 0\,T^{22} + 16511668598032609201 2000\,T^{20}$
$+17170916828136913280439000\,T^{18} - 10260729737119874450938500 0\,T^{16}$
$+16350233273665881927737400 00\,T^{14} + 28607684977366731508872600 00\,T^{12}$
$+45606671281786995394764750000\,T^{10} - 80947367097903820900836450000T^{8}$
$-70028684010076574078846550000 0T^{6} + 13080187339646980805302050000 0T^{4}$
$-44728339147856863291186143750 0T^{2} + 5676858321460841356748156250 0, \quad \mathbf{g}_{19} = 0,$
$\mathbf{g}_{18} = 4855550700\,T^{36} - 40277853000\,T^{34} + 158719690968300\,T^{32} - 5194244162923200\,T^{30}$
$-1360528622797251600\,T^{28} - 281596511270242476000\,T^{26} - 11850683160765907458000\,T^{24}$
$+920175424144902165240000\,T^{22} + 25468628785050976429905000\,T^{20}$
$-310836193534541858413230000\,T^{18} + 5807378097843796606757625000\,T^{16}$
$+156342348027194416733646000 00\,T^{14} + 27888740806500752860957275 0000\,T^{12}$
$+520006836469063549931325000 00T^{10} - 443908108299481923011139225 0000T^{8}$
$+843881604517049366596828500 0000T^{6} - 300112203523535603051132531 2500T^{4}$
$-739174359304030099680542812 5000T^{2} - 105415223943181204187917031 2500,$
$\mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 2424321900\,T^{38} + 211776711300\,T^{36} + 99477568505700\,T^{34}$
$-1389209187712500\,T^{32} - 1621264449868232400\,T^{30} - 280260753940566630000\,T^{28}$
$-4787780690058717582000\,T^{26} + 165669248116213203562 2000\,T^{24} + 548282657939414196298 5000\,T^{22}$
$-565790679793850662876725000\,T^{20} + 147141486888019521055908750 00\,T^{18}$
$+102902614988568912796304025000\,T^{16} + 981244617028989252062473350000\,T^{14}$
$+316947957819345890443531125 0000T^{12} - 249313736505390922467920317 50000T^{10}$
$+120855512477727524652824418750 000T^{8} - 539568184957356769123687903 12500T^{6}$
$-749966791440785066203339659375 00T^{4} - 610569591258512274646116234375 00T^{2}$
$-524597248949717001795768281 2500, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 1019458440\,T^{40} + 209361952800\,T^{38}$
$+59799859202160\,T^{36} + 597930018285600\,T^{34} - 1327830040347010200\,T^{32} - 203680458354584534400\,T^{30}$
$+502367572425131028000 0\,T^{28} + 162780543331932699835 2000\,T^{26} - 250789061193646149474300 00\,T^{24}$
$-752646251179376198484840000\,T^{22} + 177045852248881372456900200 00\,T^{20}$
$+466071240006558080789895000000\,T^{18} + 222546992768128565145714105 0000\,T^{16}$
$+244229141347841651371149780000 00T^{14} - 209128252918688749663068051 0000 00T^{12}$
$+868839416613022428214659018000 000T^{10} + 388998795398580381626267906 25000T^{8}$
$-954575061914762846454243847500 000T^{6} - 411166271799374891645351756 250000T^{4}$
$+783333878093152389727736512500 000T^{2} - 207938489499253781458408968 75000, \quad \mathbf{g}_{13} = 0,$
$\mathbf{g}_{12} = 356396040\,T^{42} + 123877270440\,T^{40} + 32610284395920\,T^{38} + 1216306932881040\,T^{36}$
$-784563891537195000\,T^{34} - 1107618147939878300 0\,T^{32} + 7937796416339529432000\,T^{30}$
$+9787751727740122253040 00\,T^{28} - 2129251632444814097266 2000\,T^{26} + 1619234763892895505856 50000\,T^{24}$
$+3443259186132316450729830 0000\,T^{22} + 8975886662185430183066871 00000\,T^{20}$
$+1039917287375065236781929285 0000\,T^{18} + 1273459064182346907624503745 0000T^{16}$
$-1056247157487187710224365185 000000T^{14} + 1832056249038285845965084695 000000T^{12}$
$+2959286708382169575258073850 625000T^{10} - 3440285597921483300174604168 75000T^{8}$
$-1214007623534926340970713238 3750000T^{6} + 2165979513223000946915034215 6250000T^{4}$
$+3602055625460117946415936265 625000T^{2} + 6104916736811876913887770312 5000, \quad \mathbf{g}_{11} = 0,$
$\mathbf{g}_{10} = 101719800\,T^{44} + 52472463120\,T^{42} + 15005352063240\,T^{40} + 1021506129594720\,T^{38}$

$-331006205910303000\,T^{36} - 47731541013103978800\,T^{34} + 3909120362191961947800\,T^{32}$
$+302275195306483257936000\,T^{30} - 33971490510799351647060 00\,T^{28}$
$+1063471996099453676509620000\,T^{26} + 35282865975833690856131490000\,T^{24}$
$+1009461102866985900033662760000\,T^{22} + 28108708355943404795842686450000\,T^{20}$
$-857473600209833261794918779000 00T^{18} - 1491463246568322076250046095250000T^{16}$
$+2775278138548494265001581266000000T^{14} - 1854012854373052629728928118162 5000)T^{12}$
$-2194462645422007307025093289875000 0T^{10} - 8097391399893403195747857635437 5000T^{8}$
$+1481147844218775594638743341375000 00T^{6} + 3112607895704757489418679612812 5000T^{4}$
$+2067359960928058667582147060625000 0T^{2} + 1398479083269650104689513421875000,$
$\mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 23095800\,T^{46} + 16444701000\,T^{44} + 5475036936600\,T^{42}$
$+566976092898600\,T^{40} - 93441539643190200\,T^{38} - 17935367491764261000\,T^{36}$
$-3435818565302531100 0\,T^{34} - 843404172362667704250 00\,T^{32} - 6247372784431620892050000\,T^{30}$
$+7942329149393889597501000 0\,T^{28} - 6517484194549893449935050000\,T^{26}$
$+1157932673902046452477117050000\,T^{24} + 4814533763761960334149878825 0000\,T^{22}$
$-2382742809450741397530393022500 00)T^{20} + 1614124491632111637086543885250000T^{18}$
$+3046374885626146040734323430875000 0T^{16} - 1187552517218268159260436840956250 00T^{14}$
$-4877359268999633929598112871593750 00T^{12} - 1738438081260949813862744404903125000T^{10}$
$+1572448964546520061265933822278125000T^{8} - 673017920052474095253612817734375000T^{6}$
$+3267954864507651360022242606093750 00T^{4} + 7940756737998721748287621757812500 0T^{2}$
$+2882552147106173292709630546875000, \quad \mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 4013100\,T^{48}$
$+3764023200\,T^{46} + 1497943465200\,T^{44} + 217159761218400\,T^{42} - 14221439411394600\,T^{40}$
$-6062992711379604000\,T^{38} - 868091861569403394000\,T^{36} - 137559109216434396876000\,T^{34}$
$-9260888323767068417467500\,T^{32} + 830925896842823245002000 00\,T^{30}$
$+4595118950954465308537470000 0\,T^{28} + 2043751713304237133419114200000\,T^{26}$
$+3982314883834725600624867405000 0\,T^{24} - 1076455749302836540950402090000 00T^{22}$
$+5704220460549886178354194243500000T^{20} + 4747786007649157551478670770500000 0T^{18}$
$+9676000470142670380220226253781250 0T^{16} - 2746609454519613461575760731267500000T^{14}$
$-5055092442696255371030072885681250000T^{12} + 7685340668503114406318287145887500000T^{10}$
$-2527654302854142287937009789515625000T^{8} + 2549540272922798483165127227437500000T^{6}$
$+1681003830938891487377650144593750000T^{4} - 1190389125844816242958146909375000 00T^{2}$
$+4248891856349740702685512617187500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 501228\,T^{50} + 599186700\,T^{48}$
$+286948221840\,T^{46} + 55465811326800\,T^{44} + 225191978134200\,T^{42} - 1592867890825734600\,T^{40}$
$-3789754547159958300 00\,T^{38} - 4969897152370067748600 0\,T^{36} - 2416070416982018904607500\,T^{34}$
$+1634495450905376507465325 00\,T^{32} + 2263041978013397688569154000 0\,T^{30}$
$+6580665314168449601848905000 00\,T^{28} + 1795874405662213487481304665000\,T^{26}$
$+2659383686826879446694002242500 00T^{24} + 3373546912165127652278618794500000T^{22}$
$+4999263291715977429356260699650000 0T^{20} + 2436228409353403037923732135078125 00)T^{18}$
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$+1725433101762243379165143781171875000T^{8} + 2060216671428200048842275425353125000 0T^{6}$
$-2561924423013843653322968348437500000T^{4} - 7926019253017849631799764894531250 0T^{2}$
$-3304693666049798324310954257812500, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 40068\,T^{52}$
$+59556168\,T^{50} + 34248844980\,T^{48} + 8538182019360\,T^{46} + 489064062475800\,T^{44}$
$-249702181686342000\,T^{42} - 69203448982691793000\,T^{40} - 5213133014477381532000\,T^{38}$
$+438714917019434320303500\,T^{36} + 84931805893311124163955000\,T^{34}$

$+4218956433098176838806237500\,T^{32} + 3110727694471338353241060000 0\,T^{30}$
$+3271649949938812288254997050000\,T^{28} + 2018492352419479645355488611000 00)T^{26}$
$+4953109227394118516246184914250000T^{24} - 2563054142874596676007460257500000 0T^{22}$
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$+1250917932579608790870875793760218750 0T^{16} + 1717369346635390314494911719686250000 0T^{14}$
$-3642120166769105746171171513851562500 0T^{12} + 1303483866198931493501291240436562500 00T^{10}$
$+1355273196885974780570524869594843750 00T^{8} + 1226984848888708231851532384631250000 0T^{6}$
$+4223241138846592268073003781757812500T^{4} - 3748466815490771242146996686718750 00T^{2}$
$-1101564555349932774770318085937500, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 1540\,T^{54} + 2791404\,T^{52}$
$+1916196156\,T^{50} + 600986059380\,T^{48} + 60893832396600\,T^{46} - 14981298604738200\,T^{44}$
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$+2321568723921179741662500\,T^{36} + 4831536749377859928947925 00\,T^{34}$
$+3443219629285717817642625750 0\,T^{32} + 1329648591046758598027462050000\,T^{30}$
$+4803046543523552046156981855000 0T^{28} + 8711445191741896717720715377500 00T^{26}$
$+8622871735649253726149298292500 00T^{24} + 3773339925953260189690516897293750 0T^{22}$
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$-1331277192341743800516529380745781250 0T^{16} - 2173440108663513814024339191565312500 0T^{14}$
$+1626020722335937093897303769102156250 00T^{12} + 5720394783589844294508224729639062500 0T^{10}$
$+9547974794218227759757086701667187500 0T^{8} + 4823751187877355620719222898320312500T^{6}$
$-1973321762271862907337361717851562500T^{4} - 2092972655164872272063604363281250 0T^{2}$
$+1573663650499903963957597265625 00,$

$\mathbf{h}_{56} = 0, \quad \mathbf{h}_{55} = 0, \quad \mathbf{h}_{54} = 28T, \quad \mathbf{h}_{53} = 0, \quad \mathbf{h}_{52} = 756\,T^3 - 2268\,T, \quad \mathbf{h}_{51} = 0, \quad \mathbf{h}_{50} = 9828\,T^5$
$-181944\,T^3 - 147420\,T, \quad \mathbf{h}_{49} = 0, \quad \mathbf{h}_{48} = 81900\,T^7 - 3660300\,T^5 + 4630500\,T^3 - 15214500\,T, \quad \mathbf{h}_{47} = 0,$
$\mathbf{h}_{46} = 491400\,T^9 - 39160800\,T^7 + 310428720\,T^5 + 79380000\,T^3 - 1631259000\,T, \quad \mathbf{h}_{45} = 0,$
$\mathbf{h}_{44} = 2260440\,T^{11} - 275020200\,T^9 + 5106674160\,T^7 - 3889302480\,T^5 + 392931000\,T^3 - 148377889800\,T,$
$\mathbf{h}_{43} = 0, \quad \mathbf{h}_{42} = 8288280\,T^{13} - 1398806640\,T^{11} + 45284861160\,T^9 - 207951559200\,T^7 - 42200789400\,T^5$
$+253204736400\,T^3 - 8291708548200\,T, \quad \mathbf{h}_{41} = 0, \quad \mathbf{h}_{40} = 24864840\,T^{15} - 5449225320\,T^{13}$
$+264073498920\,T^{11} - 2825524908360\,T^9 + 1514969046360\,T^7 + 6573185526600\,T^5 + 137993987993400\,T^3$
$+149047765713000\,T, \quad \mathbf{h}_{39} = 0, \quad \mathbf{h}_{38} = 62162100\,T^{17} - 16844335200\,T^{15} + 1110003879600\,T^{13}$
$-20635793085600\,T^{11} + 60341881030200\,T^9 + 76117021596000\,T^7 + 2171428651854000\,T^5$
$+25299618434244000\,T^3 + 62466943452952500\,T, \quad \mathbf{h}_{37} = 0, \quad \mathbf{h}_{36} = 131231100\,T^{19}$
$-42306887700\,T^{17} + 3541157207280\,T^{15} - 98466907770000\,T^{13} + 661042062865800\,T^{11}$
$-16499618011800\,T^9 + 18885110233374000\,T^7 + 314318361244734000\,T^5$
$+1773658200690337500\,T^3 + 2282181897675487500\,T, \quad \mathbf{h}_{35} = 0, \quad \mathbf{h}_{34} = 236215980\,T^{21}$
$-87763321800\,T^{19} + 8848450735740\,T^{17} - 335891899617120\,T^{15} + 3894124207743000\,T^{13}$
$-10086562817377200\,T^{11} + 107633107470735000\,T^9 + 2631781328889636000\,T^7$
$+12651337028921629500\,T^5 - 44712998532434925000\,T^3 + 94738725025613707500\,T, \quad \mathbf{h}_{33} = 0,$
$\mathbf{h}_{32} = 365061060\,T^{23} - 152080693380\,T^{21} + 17658627372540\,T^{19} - 861904089654300\,T^{17}$
$+14783141715388200\,T^{15} - 91110040017935400\,T^{13} + 397997325082942200\,T^{11}$
$+1775611760340518100 0\,T^{9} + 6999251051550161250 0\,T^{7} - 1798102986888203572500\,T^{5}$
$-6639275783651620725 00\,T^{3} + 3221445733967484292500\,T, \quad \mathbf{h}_{31} = 0, \quad \mathbf{h}_{30} = 486748080\,T^{25}$
$-221807355840\,T^{23} + 28457171376480\,T^{21} - 1718595944936640\,T^{19} + 39120073399970640\,T^{17}$
$-415203708424982400\,T^{15} + 817743548090145600\,T^{13} + 9869011247369736000 0\,T^{11}$
$+5251698420476909940 00\,T^{9}$

$-2072933840607831986400 0\,T^{7} + 1094655184911042678000 0\,T^{5} - 3043091493763018236000 0\,T^{3}$
$+5775864371659343115000 0\,T, \quad \mathbf{h}_{29} = 0, \quad \mathbf{h}_{28} = 561632400\,T^{27} - 273514978800\,T^{25}$
$+37167025240800\,T^{23} - 2726075908432800\,T^{21} + 74609232551607600\,T^{19} - 1241876967913890000\,T^{17}$
$+2806282455935112000\,T^{15} + 49009199291011159200 0\,T^{13} + 43353078145533179100 00\,T^{11}$
$-1501369487987887772100 00\,T^{9} + 5171375788944821379000 00\,T^{7} - 2215960777147657617000 00\,T^{5}$
$-2219284940307326995950000\,T^{3} - 1317730766813851524750000T, \quad \mathbf{h}_{27} = 0,$
$\mathbf{h}_{26} = 561632400\,T^{29} - 285683680800\,T^{27} + 39160805609520\,T^{25} - 3512223888264000\,T^{23}$
$+102167332243376400\,T^{21} - 2763398175005714400\,T^{19} + 16607639793911166000\,T^{17}$
$+1283509837177544592000\,T^{15} + 3336159563178067395000 0\,T^{13} - 8458294842388638185400 00\,T^{11}$
$+5929023781997105418090000\,T^{9} - 2765051286836672017800000\,T^{7} - 2714891693581746274365000 0\,T^{5}$
$-1159024796130068700525000 00T^{3} - 1049839255615202158837500 00T, \quad \mathbf{h}_{25} = 0,$
$\mathbf{h}_{24} = 486748080\,T^{31} - 252547369200\,T^{29} + 32714359619760\,T^{27} - 3765812506026480\,T^{25}$
$+93108670306470000\,T^{23} - 4900225415888602800\,T^{21} + 73064176305750630000\,T^{19}$
$+3317316129490661298000\,T^{17} + 24009223969558718490600 0\,T^{15} - 38209392559955217100500 00\,T^{13}$
$+36755694097751666247210000\,T^{11} - 28748698362619854869250000\,T^{9}$
$-22728345620200455350115000 0\,T^{7} - 14157788153628199150570500 00T^{5}$
$-26498410703382151221187500 00T^{3} - 16156607242862936392042500 00T, \quad \mathbf{h}_{23} = 0,$
$\mathbf{h}_{22} = 365061060\,T^{33} - 188259180480\,T^{31} + 20712348488160\,T^{29}$
$-3475991858875200\,T^{27} + 34258831092486000\,T^{25} - 7053263233444209600\,T^{23}$
$+2541555050183597448 00\,T^{21} + 2196390982272615928800 0\,T^{19}$
$+1097079878815670724975000\,T^{17} - 1398726157872854102292000\,T^{15}$
$+1563743168288717496280200 00\,T^{13} - 2332877100532471852959600 00\,T^{11}$
$-4974465160593469787746500 00\,T^{9} - 1365057391464090933909300 0000T^{7}$
$-2231005361402477972362050 0000T^{5} + 2584293392336928413040900 0000T^{3}$
$-2500523594007687138684543 7500T, \quad \mathbf{h}_{21} = 0, \quad \mathbf{h}_{20} = 236215980\,T^{35} - 117484137540\,T^{33}$
$+8643874266720\,T^{31} - 2885517466745760\,T^{29} - 53721876349920240\,T^{27} - 8070710526481402800\,T^{25}$
$+7034000251082228424 00\,T^{23} + 8773846095839389765200 0\,T^{21} + 2207780690580728420157000\,T^{19}$
$-4715949826852164601400 7000\,T^{17} + 4965672567955952890837800 00\,T^{15}$
$-9786098933313967371907800 00\,T^{13} + 1437856090599426449801850000\,T^{11}$
$-9273217121912252102028435 0000T^{9} - 2409241723532733246025869000 00T^{7}$
$+8821730425318301656640475 0000T^{5} + 1507151362901343475803361875 00T^{3}$
$-2454705596311823545581515625 00T, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 131231100\,T^{37} - 60628768200\,T^{35}$
$+713083902300\,T^{33} - 2235366203716800\,T^{31} - 128502034840040400\,T^{29} - 7113116608897068000\,T^{27}$
$+1435474058223587382000\,T^{25} + 17923602568257497311200 0\,T^{23}$
$+5849582789677353977250 00\,T^{21} - 1145645258129651902322700 00\,T^{19}$
$+1365628046065395347760525000\,T^{17} - 4320053654794030396643400000\,T^{15}$
$+2417892228564416341072335000 0\,T^{13} - 5261700874603616143896195000 00T^{11}$
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$-3234615439021971072725219062 5000T^{5} + 4466772809861963778344041875 000T^{3}$
$+6287661093121959787095234375 00T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 62162100\,T^{39} - 25347791700\,T^{37}$
$-2330970600420\,T^{35} - 1619478410665500\,T^{33} - 157961463444615600\,T^{31} - 4857821543087528400\,T^{29}$
$+205711353550422027000 0\,T^{27} + 2012578212714911461620 00\,T^{25} - 4914048257261533415025000\,T^{23}$
$-1041466990110732087452550 00\,T^{21} + 3611762205877271065424805000\,T^{19}$
$-6525946067600925299859525000\,T^{17} - 9037211655634256219780655000 0\,T^{15}$
$-1707727156947430436644076250 0000T^{13} - 1991451415394763389307465225 0000T^{11}$

$+604415043226045927854200722500000T^9 - 1219906629052273105079032771875 00T^7$
$+320247996564416600379171346875 00T^5 + 1333471596996664870237852734375 00T^3$
$+508978551659201475553342265625 00T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 24864840\,T^{41} - 8275528800\,T^{39}$
$-2290606108560\,T^{37} - 1056111041351520\,T^{35} - 139995360430695000\,T^{33} - 3012968529441763200\,T^{31}$
$+2016715929359309640000\,T^{29} + 1227769195451387006880 00\,T^{27} - 8724617561136462399414000\,T^{25}$
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$+3429199255041238829049900000 0\,T^{19} - 7421978126634509002009945500 00\,T^{17}$
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$+2489803928401440229271833140000 00T^{11} - 8182992645167470409898212343750 00T^9$
$-4767134190048367516575971775000 00T^7 + 1758336091397966713046828163750 00T^5$
$+1769350768557853190505858562500000T^3 + 2801600688772617226043580281250 00T \quad \mathbf{h}_{13} = 0,$
$\mathbf{h}_{12} = 8288280\,T^{43} - 1950296040\,T^{41} - 1303260734160\,T^{39} - 587448080326800\,T^{37}$
$-96181467152740200\,T^{35} - 2333545862710847400\,T^{33} + 1303888057145840635200\,T^{31}$
$+3454936837605734896800 0\,T^{29} - 6423142555199586204450000\,T^{27} + 8499183090859997434899000 0\,T^{25}$
$+5196658815883842091604820000\,T^{23} + 7121626762830638339315034000 0\,T^{21}$
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$+1046540842509229121719952161875000 0T^7 + 2494068577939442043736705884375000 0T^5$
$-1891607794892583614798379553125000T^3 + 1383666073198822939633454221875000T$
$\mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 2260440\,T^{45} - 256299120\,T^{43} - 520433325720\,T^{41} - 265398457595040\,T^{39}$
$-51963689173971960\,T^{37} - 2078330803534062000\,T^{35} + 5044141942583063166 00\,T^{33}$
$+2354920000955531856000\,T^{31} - 1514616736300246258218000\,T^{29} + 1313719878449668060574280 00\,T^{27}$
$+2424830356405951553429370000\,T^{25} + 7403990470828582252353732000 0\,T^{23}$
$+9960482786674699261404922500 00\,T^{21} - 1098417180548725077136523427000 00T^{19}$
$-5271393160657170402296771830500 00T^{17} + 9486298267101269022382433460000 00T^{15}$
$-1573801048886049179108335936012500 0T^{13} - 2119540516756692868912335136875000 0T^{11}$
$+1413630315663142779897801919312500 0T^9 + 2161279290686024527799718912975000 00T^7$
$-4756167746159140432967649240937500 0T^5 + 4018232176163239787982535556250000T^3$
$+2017953028228342458755361234375000T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 491400\,T^{47} + 17098200\,T^{45}$
$-150724589400\,T^{43} - 93307202742600\,T^{41} - 21786354176617800\,T^{39} - 1497011705550207000\,T^{37}$
$+74558739462560199000\,T^{35} + 2282444312171889789000\,T^{33} + 1284486001836552906930000\,T^{31}$
$+1587642839682158528091900 00\,T^{29} + 5961808009517232701540250000\,T^{27}$
$+3001705429391265689668075500 00\,T^{25} + 3037790768826275206178984550000\,T^{23}$
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$+3211142985153408253476494216250000T^{17} - 8287354492183606815543822191437500 0T^{15}$
$+4377452093259998766770113044187500 0T^{13} - 1507874749558693647488658269718750 00T^{11}$
$+2021475165637011479161476994659375000T^9 - 8180856179751501804722896137656250 00T^7$
$+5578492764051161148372683035781250 00T^5 - 7798744001369481737656612345312500 0T^3$
$-2384475112328902101577654523437500 0T, \quad \mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 81900\,T^{49} + 17287200\,T^{47}$
$-31138868880\,T^{45} - 24379010964000\,T^{43} - 6845248148092200\,T^{41} - 741093475985200800\,T^{39}$
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$+1755495169772525151339600 00\,T^{31} + 9472551469732244487395340000\,T^{29}$
$+1418722717682415315997134000 00\,T^{27} - 2611527288780357895294497150000\,T^{25}$
$-1598766284575464225791620050000 00T^{23} + 9048210812916061710010802715000 00T^{21}$

$+2911105425590491035120995781000000T^{19} - 5785697135055638012845077413718750 0T^{17}$
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$+1215468390419467016280272788713750000 0T^{11} + 1386390082538832027630690125784375000T^{9}$
$+1098571541450992569418057287187500000T^{7} + 607947066875348083972181687343750000T^{5}$
$-947695220634386609405575242187500000T^{3} - 357571351696922622921476267578125 00T,$
$\mathbf{h}_5 = 0, \quad \mathbf{h}_4 = 9828\,T^{51} + 4101300\,T^{49} - 4340604240\,T^{47} - 4436265289680\,T^{45}$
$-1514282595903000\,T^{43} - 234322521203647800\,T^{41} - 15633540600684930000\,T^{39}$
$+1586816675525592126000\,T^{37} + 603412183386569812909500\,T^{35}$
$+70495161343635553258387500\,T^{33} + 3381977148695834843343420000\,T^{31}$
$+231448024917777432853935000 00\,T^{29} - 50800168749634781211853965000 0\,T^{27}$
$-453171790304591580578226370500 00T^{25} - 639392657721512904650453512500000T^{23}$
$-633453674783199881723813200500000T^{21} - 873081560973221793073084970390625 00T^{19}$
$-135597933263429225386890056746031250 0T^{17} + 441154023530676509027137970181375000 0T^{15}$
$+100576038323625894420416296208437500 00T^{13} + 269112955305549301444897456213406250 00T^{11}$
$+171259183984891014166539116763281250 00T^{9} + 189542820919638521650628546670937500 00T^{7}$
$-666031808189044242935763839690625000 0T^{5} - 552093664050382974021790374023437500T^{3}$
$-344632339459478968106713801171875 00T, \quad \mathbf{h}_3 = 0, \quad \mathbf{h}_2 = 756\,T^{53} + 495432\,T^{51}$
$-362210940\,T^{49} - 500636959200\,T^{47} - 209842694598600\,T^{45} - 42645497546559600\,T^{43}$
$-3498152086126744200\,T^{41} + 37728102434746323600 0\,T^{39} + 109988509720439880337500\,T^{37}$
$+74210605724192824482750 00\,T^{35} + 35933479335640711244227500\,T^{33}$
$-11661105491106988983093720000\,T^{31} + 4397410808791438188597808500 00\,T^{29}$
$+197837117462090424654577395000 00T^{27} - 53756632570817276374301527500 00T^{25}$
$-224935691902189962116165452710000 00T^{23} - 186281502287073108403281941459812500T^{21}$
$-105908482233497400480206702831812500 0T^{19} + 632915394944505431499782008311093750 0T^{17}$
$-149187694958394656543606193942375000 00T^{15} - 293735485131814269479008139848031250 00T^{13}$
$+152565680355947759587851167072956250000T^{11} + 545410125243004477841976493871718750 00T^{9}$
$-624014289314629918251889789321875000 00T^{7} + 819831552000934968102989447472656250 0T^{5}$
$-834776111135182389414040096171875000T^{3} + 297422429944481849187985883203125 00T,$
$\mathbf{h}_1 = 0, \quad \mathbf{h}_0 = 28\,T^{55} + 25956\,T^{53} - 13362300\,T^{51} - 26340465060\,T^{49} - 13674573352440\,T^{47}$
$-3395567206168200\,T^{45} - 366196973040498600\,T^{43} + 12483830514741201000\,T^{41}$
$+8840223660767963656500\,T^{39} + 1211740695196554052183500\,T^{37}$
$+95626598418360513349807500\,T^{35} + 4134578042051998735362052500\,T^{33}$
$+798514861428847857206335500 00\,T^{31} + 6647342629495538303341816500 00T^{29}$
$-978345676892655526516091761500 00T^{27} - 3703009772457379493336677700250000T^{25}$
$-1447589445778846579114620908943750 0T^{23} - 2268108628672565514203261033765625 00T^{21}$
$-1684904275255975197128945251574062500T^{19} + 9448363637590584401034224408586562500T^{17}$
$+2854859659504129786637121338657812500T^{15} + 14622515167606893864630168525357187500 0T^{13}$
$+3132604549971562810367834584410937500 0T^{11} + 2509283159118743384382267075876562500 0T^{9}$
$-4491839496089722368372463352200781250 0T^{7} - 7823888482100405864538916831992187500T^{5}$
$-5712399051314651389166078074218750 0T^{3} + 5507822776749663873851590429687500)T,$

$\mathbf{q}_{56} = 1, \quad \mathbf{q}_{55} = 0, \quad \mathbf{q}_{54} = 28\,T^2 + 28, \quad \mathbf{q}_{53} = 0, \quad \mathbf{q}_{52} = 378\,T^4 - 2268\,T^2 + 3402, \quad \mathbf{q}_{51} = 0,$
$\mathbf{q}_{50} = 3276\,T^6 - 62748\,T^4 + 21924\,T^2 + 483084, \quad \mathbf{q}_{49} = 0, \quad \mathbf{q}_{48} = 20475\,T^8 - 749700\,T^6$

$+2315250\,T^4 + 5953500\,T^2 + 66976875, \quad \mathbf{q}_{47} = 0, \quad \mathbf{q}_{46} = 98280\,T^{10} - 5556600\,T^8 + 48157200\,T^6$
$+65091600\,T^4 + 959704200\,T^2 + 8132481000, \quad \mathbf{q}_{45} = 0, \quad \mathbf{q}_{44} = 376740\,T^{12} - 29037960\,T^{10}$
$+479058300\,T^8 - 442940400\,T^6 + 9798270300\,T^4 + 117272043000\,T^2 + 765518890500, \quad \mathbf{q}_{43} = 0,$
$\mathbf{q}_{42} = 1184040\,T^{14} - 114123240\,T^{12} + 2988021960\,T^{10} - 13647803400\,T^8 + 81493889400\,T^6$
$+1166454966600\,T^4 + 9193406607000\,T^2 + 45889232697000, \quad \mathbf{q}_{41} = 0, \quad \mathbf{q}_{40} = 3108105\,T^{16}$
$-350020440\,T^{14} + 13152128220\,T^{12} - 117779757480\,T^{10} + 606076420950\,T^8 + 10517426904600\,T^6$
$+74383606489500\,T^4 + 157761403569000\,T^2 + 1272788332979625, \quad \mathbf{q}_{39} = 0, \quad \mathbf{q}_{38} = 6906900\,T^{18}$
$-855924300\,T^{16} + 43638210000\,T^{14} - 571636018800\,T^{12} + 4633520938200\,T^{10} + 91699486263000\,T^8$
$+670471524954000\,T^6 - 2950507091070000\,T^4 - 33200878716967500\,T^2 + 74380367755732500,$
$\mathbf{q}_{37} = 0, \quad \mathbf{q}_{36} = 13123110\,T^{20} - 1685814900\,T^{18} + 114021611550\,T^{16} - 1773231490800\,T^{14}$
$+32095272062700\,T^{12} + 488569383779400\,T^{10} + 7285991633959500\,T^8 - 37353863706390000\,T^6$
$-712242268565231250\,T^4 + 766523807513887500\,T^2 + 4232772093141033750, \quad \mathbf{q}_{35} = 0,$
$\mathbf{q}_{34} = 21474180\,T^{22} - 2671057620\,T^{20} + 242800796700\,T^{18} - 3452383449900\,T^{16}$
$+173542141218600\,T^{14} + 2616311024019000\,T^{12} + 72996496011747000\,T^{10} - 112701870219015000\,T^8$
$-761923508007642750 0\,T^6 + 2639787596347497750 0\,T^4 + 5626654247479938750 0\,T^2$
$+207628508681964562500, \quad \mathbf{q}_{33} = 0, \quad \mathbf{q}_{32} = 30421755\,T^{24} - 3341712780\,T^{22}$
$+434948276070\,T^{20} - 2762553786300\,T^{18} + 707628327869925\,T^{16} + 16452498066820200\,T^{14}$
$+420040677679276500\,T^{12} + 1905343386976953000\,T^{10} - 55819031246158516875\,T^8$
$+383292609490411582500\,T^6 + 758856538412299923750\,T^4 + 2764219001238915412500\,T^2$
$+781861632663095479687 5, \quad \mathbf{q}_{31} = 0, \quad \mathbf{q}_{30} = 37442160\,T^{26} - 3107699280\,T^{24}$
$+676957508640\,T^{22} + 7303633777440\,T^{20} + 2213013639790800\,T^{18} + 80237629739005200\,T^{16}$
$+2078146188047160000\,T^{14} + 28199965799057976000\,T^{12} - 257869432151320374000\,T^{10}$
$+3386896499825497890000\,T^8 + 50088823581176641800 00\,T^6 + 3549366460196788770000 0\,T^4$
$+7978546632407395275000 0\,T^2 + 1910316848952680437500 00, \quad \mathbf{q}_{29} = 0, \quad \mathbf{q}_{28} = 40116600\,T^{28}$
$-1684897200\,T^{26} + 943969761000\,T^{24} + 36029988237600\,T^{22} + 5459806605468600\,T^{20}$
$+266337649042542000\,T^{18} + 8144921696066001000\,T^{16} + 188833010171624280000\,T^{14}$
$-7493759121967791750 00\,T^{12} + 2659388768874029115000 0\,T^{10} + 1385765247135305767500 0\,T^8$
$+4900198063442630949000 00\,T^6 + 8470916381200810556250 00\,T^4 + 7623168468808263525000 0)T^2$
$+2590948189562408935875000, \quad \mathbf{q}_{27} = 0, \quad \mathbf{q}_{26} = 37442160\,T^{30} + 561632400\,T^{28}$
$+1204554985200\,T^{26} + 88218653418000\,T^{24} + 10930955933444400\,T^{22} + 618649682178805200\,T^{20}$
$+19905633824246478000\,T^{18} + 212421481759657170000\,T^{16} - 7909388178373163790000\,T^{14}$
$+190838377838882403630000\,T^{12} + 2466489477655456060500 00\,T^{10} + 5106259955755665733950000\,T^8$
$+1450929802719031121025000 0\,T^6 - 1682277582817370610225000 0T^4 - 5737774189726202275575000 0T^2$
$+648936169642401153117500 00, \quad \mathbf{q}_{25} = 0, \quad \mathbf{q}_{24} = 30421755\,T^{32} + 2733277680\,T^{30}$
$+1411247917800\,T^{28} + 156504460165200\,T^{26} + 18220186626232500\,T^{24} + 1063917237344598000\,T^{22}$
$+24341468738209623000\,T^{20} - 1458407669898168990000\,T^{18} - 38195651665360037598750\,T^{16}$
$+704274365043021222450000\,T^{14} + 5599053528058930314375000\,T^{12}$
$+37978367503214552163750000\,T^{10} + 257388209612512632466312500\,T^8$
$-768270961945263904627500 00T^6 - 1281634404886971940179375000T^4$
$+831668698074883802883750000)T^2 + 1475186709752105729050546875, \quad \mathbf{q}_{23} = 0,$
$\mathbf{q}_{22} = 21474180\,T^{34} + 3959508420\,T^{32} + 1496491506720\,T^{30} + 220136214564000\,T^{28}$
$+25819817141790000\,T^{26} + 1464130513297998000\,T^{24} + 431038469000412000\,T^{22}$
$-5567239710825100308000\,T^{20} - 5197270234035582202500 0\,T^{18} + 2111596128156974433975000\,T^{16}$
$+2611045474660783750590000 0\,T^{14} + 3708376601445201660975000 00\,T^{12}$

$+2211046006855432481484750000\,T^{10} + 3768821426548444795278750000T^{8}$
$-1505754075738033621249750000T^{6} + 2416629013911563882681250000T^{4}$
$+2272736420351957712558656250 0T^{2} + 2603625516384061399136156250 0,$
$\mathbf{q}_{21} = 0, \quad \mathbf{q}_{20} = 13123110\,T^{36} + 3942989820\,T^{34} + 1403651839590\,T^{32}$
$+254097412763040\,T^{30} + 31521548873014200\,T^{28} + 1794671024233626000\,T^{26}$
$-35581106620452393000\,T^{24} - 815251872735928227600 0\,T^{22} + 806310558838226607165 00\,T^{20}$
$+593272844955395919814500 0\,T^{18} + 825786369251531620057125 00\,T^{16}$
$+154333578310677281265750000 0\,T^{14} + 142909887632599777673733750 00\,T^{12}$
$+80369144237433402713612250000T^{10} - 65257978237151882814757125000T^{8}$
$+29832464730100964181500250000 0T^{6} + 42380328342387497713485609375 0T^{4}$
$+35732841858966526717536843750 0T^{2} + 28995094378965325214087859375 0, \quad \mathbf{q}_{19} = 0,$
$\mathbf{q}_{18} = 6906900\,T^{38} + 3038504700\,T^{36} + 1138241065500\,T^{34} + 243867643658100\,T^{32}$
$+33248280773307600\,T^{30} + 2138061788277030000\,T^{28} - 18711013316805810000\,T^{26}$
$-356548986988305111000 0\,T^{24} + 3757844271838685178150 00\,T^{22} + 858963493672981132936500 0\,T^{20}$
$+177884937944352775261125000\,T^{18} + 502218893474399009245837500 0\,T^{16}$
$+347677008074746543776082500 00\,T^{14} + 346392882720924307150632750000T^{12}$
$+72984030680438047980584775000 0T^{10} + 33498697122187512728825812500 00T^{8}$
$+50161640835685729420148203125 00T^{6} + 66821497895664776492212359375 00T^{4}$
$+13089935286296859344713734375 00T^{2} + 14387082829305460052982328125 00, \quad \mathbf{q}_{17} = 0,$
$\mathbf{q}_{16} = 3108105\,T^{40} + 1879208100\,T^{38} + 782694539550\,T^{36} + 194946808587300\,T^{34}$
$+30068684997553125\,T^{32} + 2434041863757781200\,T^{30} + 797989252618161450 00\,T^{28}$
$+803595355979738553000 0\,T^{26} + 754836005796200279201250\,T^{24}$
$+171251797864943776330350 00\,T^{22} + 613061169627775399022992500\,T^{20}$
$+132139755475143130003038750 00\,T^{18} + 825826158807091124327957812 50\,T^{16}$
$-277980267954943276814841750000T^{14} - 394850784054988870478507137500 0T^{12}$
$+556867506960269496317418592500 00T^{10} + 426153720673097409298732207031 25T^{8}$
$+151368250313086725334355001562500T^{6} + 171419859272111289234074367187 50T^{4}$
$-425620693423325735645169234375 00)T^{2} + 140361186516690352537140222656 25,$
$\mathbf{q}_{15} = 0, \quad \mathbf{q}_{14} = 1184040\,T^{42} + 942951240\,T^{40} + 448820341200\,T^{38} + 129061854306000\,T^{36}$
$+22950971378735400\,T^{34} + 2420746857044901000\,T^{32} + 189431470627899192000\,T^{30}$
$+19740165844212003960000\,T^{28} + 1288418099780576436210000\,T^{26} + 39634356458318874582330000\,T^{24}$
$+932948607189954436181100000\,T^{22} + 24964784950300898037835500000\,T^{20}$
$+336813899817214828406648250000\,T^{18} - 427706011087250123803650075000 0T^{16}$
$-157932260765418584767852530000 00T^{14} + 182147996494954471930263315000000T^{12}$
$+957228897383337168450769033125000T^{10} + 165922946506816274672435601562500 0T^{8}$
$+125099437867260449534404588125000 0T^{6} - 104621898052002385300633261875000 0T^{4}$
$+233488960703366614557013828125000)T^{2} + 114491186085844621649987015625000,$
$\mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = 376740\,T^{44} + 383173560\,T^{42} + 211091556060\,T^{40} + 69936531310800\,T^{38}$
$+14478829033214700\,T^{36} + 1951093276439491800\,T^{34} + 219781394892621292500\,T^{32}$
$+23753718553640461848000\,T^{30} + 1737564922252746032985000\,T^{28} + 85455658543552552907190000\,T^{26}$
$+3155694614350839848822535000\,T^{24} + 83228531888006342213676300000\,T^{22}$
$+980904596645249105852397375000\,T^{20} - 806997217932225026660178225000 0T^{18}$
$+585205373634730438621193662500 0T^{16} + 470130559276041642828328479000000T^{14}$

$+3826968310962656491781882545312500T^{12} + 1440016892511060994440768395437500 0T^{10}$
$+3542298189441221448935528550468750 0T^{8} - 8045800083267738229862592918750000T^{6}$
$+5267694033280469186100991223437500T^{4} + 2241399641805817999957998421875000T^{2}$
$+580522174177198819681404351562500, \quad \mathbf{q}_{11} = 0, \quad \mathbf{q}_{10} = 98280\,T^{46} + 124671960\,T^{44}$
$+79862456520\,T^{42} + 30469610860920\,T^{40} + 7363086516739800\,T^{38} + 1221618341096029800\,T^{36}$
$+165380901580623267000\,T^{34} + 18523557951212005461000\,T^{32} + 1521418822317545771466000\,T^{30}$
$+94208790464887079928870000\,T^{28} + 4142878394428875557173410000\,T^{26}$
$+108603899031880110735226350000\,T^{24} + 1586101557851603861258940750000\,T^{22}$
$+5626299013637437585429229250000T^{20} + 9594266908184703048895165575000 0T^{18}$
$+16007562012607609796835085102500 00T^{16} + 12940726819204628290681342633125000T^{14}$
$+4177173029720423647670023956187500 0T^{12} + 1180106119864826783100046631306250 00T^{10}$
$+154559704011983463621551783034375000T^{8} + 53946862592799320363224126996875000T^{6}$
$+6013682091731523655309734182812500 0T^{4} + 818871284123621984377258073437500 0T^{2}$
$+82496610842079621560274464062500 0, \quad \mathbf{q}_{9} = 0, \quad \mathbf{q}_{8} = 20475\,T^{48} + 31789800\,T^{46} + 23681605500\,T^{44}$
$+10399502698200\,T^{42} + 2927281902340950\,T^{40} + 577627006715451000\,T^{38} + 87402057712417255500\,T^{36}$
$+9919859107719402765000\,T^{34} + 800327687122633415563125\,T^{32} + 42769550356395608294010000\,T^{30}$
$+1015850709135001823928075000\,T^{28} - 9859626751919274812824650000\,T^{26}$
$-1299408346157936382586968750 0\,T^{24} + 2001628870256144118014400375000 0T^{22}$
$+6797834694793857435246044268750 00T^{20} + 2547659051955188113304398286250000T^{18}$
$+2393652068714912372408850066070312 5T^{16} + 1865931621178589375031049210031250 00T^{14}$
$-2367353428015619118020800682953125 00T^{12} - 4138818145363189936325786139281250 00T^{10}$
$+2319699480672946592561913286558593750T^{8} + 6451331552497958675251462603593750 00T^{6}$
$+2725710336606349530968523352734375 00T^{4} - 3455565546192963720220539773437500 0T^{2}$
$+2590924796001627597801615498046875, \quad \mathbf{q}_{7} = 0, \quad \mathbf{q}_{6} = 3276\,T^{50} + 6129900\,T^{48}$
$+5297821200\,T^{46} + 2674898024400\,T^{44} + 872848514676600\,T^{42} + 199686264011249400\,T^{40}$
$+33197856289357914000\,T^{38} + 3837403641408383970000\,T^{36} + 276908778929881432372500\,T^{34}$
$+9159559733209296842392500\,T^{32} - 1312905925839327790231800 00\,T^{30}$
$-1503203831382209860542870000 0\,T^{28} + 2715226929373058958829702500 00\,T^{26}$
$+2372710094964422600873911425000 00T^{24} + 7683099993212680146888803025000 00T^{22}$
$+1290215252590509994001138135250000 0T^{20} + 1205042486245182898196761155403125 00T^{18}$
$+6456225926436114593964321575578125 00T^{16} - 1119578306397850896579839825718750000T^{14}$
$+1031933068873327421979388835625000 00T^{12} + 6342730065378280526865194003596875000T^{10}$
$+1213260337028046794285660078404687500 00T^{8} + 9468629274147888824201932240781250000T^{6}$
$-6159669231090068538033137385937500 00T^{4} + 6048113963421297568143698824218750 0T^{2}$
$+5507822776749663873851590429687500, \quad \mathbf{q}_{5} = 0, \quad \mathbf{q}_{4} = 378\,T^{52} + 841428\,T^{50}$
$+840568050\,T^{48} + 487096318800\,T^{46} + 183138162788700\,T^{44} + 47831592399129000\,T^{42}$
$+8850859090368025500\,T^{40} + 1129359739079859210000\,T^{38} + 93052542964654276368750\,T^{36}$
$+4403146806459838814107500\,T^{34} + 89202381643571262806643750\,T^{32}$
$-1179658500799710888841500000\,T^{30} + 2343258324572452063007962 5000\,T^{28}$
$+1057809450096985623169515075000 00T^{26} + 4993394575482908245623648926250000T^{24}$
$+1073592166456872548044178870250000 0T^{22} + 7579037128816657231278730413609375 0T^{20}$
$+6085241566662574845352471194421875 00T^{18} + 2677685608955016886758750786536718750T^{16}$
$+6724863048315369636753548036062500 00T^{14}$

$+913174000744707946586289556202343750 0T^{12} + 4547519229727268816963700022326562500 0T^{10}$
$+2136921583893000145545909038027343750 0T^{8} + 1350895824135137558819759796703125000 0T^{6}$
$+348382904493170405887479407988281250T^{4} + 146822818591641039837243824882812500T^{2}$
$+16523468330248991621554771289062 50, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 28\,T^{54} + 73332\,T^{52} + 84306852\,T^{50}$
$+55935513900\,T^{48} + 24073903614600\,T^{46} + 7132291668797400\,T^{44} + 1498506622541775000\,T^{42}$
$+230871155311572237000\,T^{40} + 27771970845932873752500\,T^{38} + 2783359442956321295797500\,T^{36}$
$+231023465050074485008027500\,T^{34} + 14304208964922571107875962500\,T^{32}$
$+609392496844662576050832750000\,T^{30} + 17180550932698920283485680250000T^{28}$
$+351839533333824978599094572250000T^{26} + 2915178549212598578022656292750000)T^{24}$
$-324989572803787691073543028134375 00T^{22} - 102838157202293944963833561572812500T^{20}$
$+161515079295495739927781991917343750 0T^{18} + 28742506839178493428111859797795312500T^{16}$
$+7290583586399364125316807575522812500 0T^{14} + 1583686189395997336438649237721093750 00T^{12}$
$-651460611996332570268047587782656250 00T^{10} - 170579194762621034500250814892968750 00T^{8}$
$+37949266119990300735095714833242187500T^{6} + 5943570241573087281471449112539062500T^{4}$
$-760079543191453614591519479296875 00T^{2} + 110156455534993277477031808593750 0, \quad \mathbf{q}_1 = 0,$
$\mathbf{q}_0 = T^{56} + 3052\,T^{54} + 4019274\,T^{52} + 3044365884\,T^{50} + 1490175636075\,T^{48} + 501200329379400\,T^{46}$
$+120293856369130500\,T^{44} + 21109132070495469000\,T^{42} + 2743830229192474739625\,T^{40}$
$+264032656532001259402500\,T^{38} + 18705499355839478697033750\,T^{36}$
$+967356830845193024425612500\,T^{34} + 36317543301591844669914796875\,T^{32}$
$+1182862195373087251907172750000T^{30} + 43265271525507252011412727875000T^{28}$
$+1146101963874736797501493362750000T^{26} + 18763596889105886868604052170546875T^{24}$
$+156011886701245799140278339790312500T^{22} + 162197305529451370281237507857859375 0)T^{20}$
$+103341068282332109678135420749640625 00T^{18} + 11130688397493816047672073880222265625T^{16}$
$+539329113026185201177963453229531250 00T^{14} + 171043092322485209840747207763351562500T^{12}$
$+991606521316115862961986175274531250 00T^{10} + 905410659781058287129595165104980468 75T^{8}$
$+476825331646972567425698496808593750 0T^{6} + 172386984594011979731734992462890625 0T^{4}$
$+407578885479475126665017691796875 00T^{2} + 39341591262497599098939931640625.$