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Degenerate determinant representation of solutions of the NLS equation, higher Peregrine breathers and multi-rogue waves.

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Abstract

We present a new representation of solutions of the focusing NLS equation as a quotient of two determinants. This work is based on a recent paper in which we have constructed a multi-parametric family of this equation in terms of Wronskians. This formulation was written in terms of a limit involving a parameter.

Here we give a very compact formulation without presence of a limit. This is a completely new result which gives a very efficient procedure to construct families of quasi-rational solutions of the NLS equation. With this method, we construct Peregrine breathers of orders $N = 4$ to 7 and multi-rogue waves associated by deformation of parameters.

1 Introduction

In 1972 Zakharov and Shabat solved the nonlinear Schrödinger equation (NLS) using the inverse scattering method. The case of periodic and almost periodic algebro-geometric solutions to the focusing NLS equation were first constructed in 1976 by Its and Kotlyarov [15]. The first quasi-rational solutions of NLS equation were constructed in 1983 by Peregrine, nowadays called worldwide Peregrine breathers. In 1986 Eleonski, Akhmediev and Kulagin obtained the two-phase almost periodic solution to the NLS equation

and obtained the first higher order analogue of the Peregrine breather[3]. Other families of higher order were constructed in a series of articles by Akhmediev et al. [1, 2] using Darboux transformations.

In 2010, it has been shown in [7] that rational solutions of the NLS equation can be written as a quotient of two Wronskians. With this formulation, it was possible to recover as a particular case, Akhmediev's quasi-rational solution of the NLS equation.

Recently, it has been constructed in [13] a new representation of the solutions of the NLS equation in terms of a ratio of two Wronskians determinants of even order $2N$ composed of elementary functions; the related solutions of NLS are called of order N . Quasi-rational solutions of the NLS equation were obtained by the passage to the limit when some parameter tended to 0.

Here we obtain a new representation of quasi-rational solutions of NLS in term of a quotient of two determinants different from preceding works which does not involve Wronskians.

With this method, we obtain very easily solutions of NLS equation with parameters of deformations until order 7. The following orders will be able to be the object of other publications.

These results can be compared with those obtain recently by Akhmediev et al. in [5] with Darboux dressing method and numerical approach.

This method appears to be very efficient without common measurement with the preceding one exposed in [13] and leads to new results about NLS equation.

2 Expression of solutions of NLS equation in terms of Wronskian determinant and quasi-rational limit

2.1 Solutions of NLS equation in terms of Wronskian determinant

We recall results obtained in [13]. We consider the focusing NLS equation

$$iv_t + v_{xx} + 2|v|^2v = 0. \tag{1}$$

From [12], the solution of the NLS equation can be written in the form

$$v(x, t) = \frac{\det(I + A_3(x, t))}{\det(I + A_1(x, t))} \exp(2it - i\varphi). \quad (2)$$

In (2), the matrix $A_r = (a_{\nu\mu})_{1 \leq \nu, \mu \leq 2N}$ ($r = 3, 1$) is defined by

$$a_{\nu\mu} = (-1)^{\epsilon_\nu} \prod_{\lambda \neq \mu} \left| \frac{\gamma_\lambda + \gamma_\nu}{\gamma_\lambda - \gamma_\mu} \right| \exp(i\kappa_\nu x - 2\delta_\nu t + x_{r,\nu} + e_\nu). \quad (3)$$

$\kappa_\nu, \delta_\nu, \gamma_\nu$ are functions of the parameters $\lambda_\nu, \nu = 1, \dots, 2N$ satisfying the relations

$$0 < \lambda_j < 1, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \leq j \leq N. \quad (4)$$

They are given by the following equations,

$$\kappa_\nu = 2\sqrt{1 - \lambda_\nu^2}, \quad \delta_\nu = \kappa_\nu \lambda_\nu, \quad \gamma_\nu = \sqrt{\frac{1 - \lambda_\nu}{1 + \lambda_\nu}}, \quad (5)$$

and

$$\kappa_{N+j} = \kappa_j, \quad \delta_{N+j} = -\delta_j, \quad \gamma_{N+j} = 1/\gamma_j, \quad j = 1 \dots N. \quad (6)$$

The terms $x_{r,\nu}$ ($r = 3, 1$) are defined by

$$x_{r,\nu} = (r - 1) \ln \frac{\gamma_\nu - i}{\gamma_\nu + i}, \quad 1 \leq j \leq 2N. \quad (7)$$

The parameters e_ν are defined by

$$e_j = ia_j - b_j, \quad e_{N+j} = ia_j + b_j, \quad 1 \leq j \leq N, \quad (8)$$

where a_j and b_j , for $1 \leq j \leq N$ are arbitrary real numbers.

The terms ϵ_ν are defined by :

$$\begin{aligned} \epsilon_\nu &= 0, & 1 \leq \nu \leq N \\ \epsilon_\nu &= 1, & N + 1 \leq \nu \leq 2N. \end{aligned}$$

We use the following notations :

$$\Theta_{r,\nu} = \kappa_\nu x/2 + i\delta_\nu t - ix_{r,\nu}/2 + \gamma_\nu y - ie_\nu, \quad 1 \leq \nu \leq 2N.$$

We consider the functions

$$\begin{aligned} \phi_{r,\nu}(y) &= \sin \Theta_{r,\nu}, & 1 \leq \nu \leq N, \\ \phi_{r,\nu}(y) &= \cos \Theta_{r,\nu}, & N + 1 \leq \nu \leq 2N. \end{aligned} \quad (9)$$

$W_r(\mathbf{y}) = W(\phi_{r,1}, \dots, \phi_{r,2N})$ is the Wronskian

$$W_r(\mathbf{y}) = \det[(\partial_{\mathbf{y}}^{\mu-1} \phi_{r,\nu})_{\nu, \mu \in [1, \dots, 2N]}]. \quad (10)$$

Then we get the following link between Fredholm and Wronskian determinants [10]

Proposition 2.1

$$\det(I + A_r) = k_r(0) \times W_r(\phi_{r,1}, \dots, \phi_{r,2N})(0), \quad (11)$$

where

$$k_r(\mathbf{y}) = \frac{2^{2N} \exp(i \sum_{\nu=1}^{2N} \Theta_{r,\nu})}{\prod_{\nu=2}^{2N} \prod_{\mu=1}^{\nu-1} (\gamma_\nu - \gamma_\mu)}.$$

In (11), the matrix A_r is defined by (3).

It can be deduced the following result :

Proposition 2.2 *The function v defined by*

$$v(x, t) = \frac{W_3(0)}{W_1(0)} \exp(2it - i\varphi). \quad (12)$$

is solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2v = 0.$$

2.2 Quasi-rational solutions of NLS equation in terms of a limit of a ratio of Wronskian determinants

In the following, we take the limit when the parameters $\lambda_j \rightarrow 1$ for $1 \leq j \leq N$ and $\lambda_j \rightarrow -1$ for $N + 1 \leq j \leq 2N$.

We consider the parameter λ_j written in the form

$$\lambda_j = 1 - 2j^2\epsilon^2, \quad 1 \leq j \leq N. \quad (13)$$

When ϵ goes to 0, we realize limited expansions at order p , for $1 \leq j \leq N$, of the terms

$$\begin{aligned} \kappa_j &= 4j\epsilon(1 - \epsilon^2j^2)^{1/2}, \quad \delta_j = 4j\epsilon(1 - 2\epsilon^2j^2)(1 - \epsilon^2j^2)^{1/2}, \\ \gamma_j &= j\epsilon(1 - \epsilon^2j^2)^{-1/2}, \quad x_{r,j} = (r - 1) \ln \frac{1+i\epsilon j(1-\epsilon^2j^2)^{-1/2}}{1-i\epsilon j(1-\epsilon^2j^2)^{-1/2}}, \end{aligned}$$

$\kappa_{N+j} = 4j\epsilon(1 - \epsilon^2 j^2)^{1/2}$, $\delta_{N+j} = -4j\epsilon(1 - 2\epsilon^2 j^2)(1 - \epsilon^2 j^2)^{1/2}$,
 $\gamma_{N+j} = 1/(j\epsilon)(1 - \epsilon^2 j^2)^{1/2}$, $x_{r,N+j} = (r - 1) \ln \frac{1 - i\epsilon j(1 - \epsilon^2 j^2)^{-1/2}}{1 + i\epsilon j(1 - \epsilon^2 j^2)^{-1/2}}$.
 The parameters a_j and b_j , for $1 \leq N$ are chosen in the form

$$a_j = \hat{a}_j \epsilon^{2N-1}, \quad b_j = \hat{b}_j \epsilon^{2N-1}, \quad 1 \leq j \leq N. \quad (14)$$

We have the result given in [12] :

Proposition 2.3 *With the parameters λ_j defined by (13), a_j and b_j chosen as in (14), for $1 \leq j \leq N$, the function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \lim_{\epsilon \rightarrow 0} \frac{W_3(0)}{W_1(0)}, \quad (15)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2 v = 0,$$

depending on $2N$ parameters $\tilde{a}_j, \tilde{b}_j, 1 \leq j \leq N$.

3 Expression of solutions of NLS equation in terms of a ratio of two determinants

We construct here solutions of the NLS equation which does not involve Wronskian determinant and a passage to the limit, but which is expressed as a quotient of two determinants.

For this we need the following notations :

$$A_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{3,\nu}/2 - ie_\nu/2,$$

$$B_\nu = \kappa_\nu x/2 + i\delta_\nu t - ix_{1,\nu}/2 - ie_\nu/2,$$

for $1 \leq \nu \leq 2N$, with $\kappa_\nu, \delta_\nu, x_{r,\nu}$ defined in (5), (6) and (7).

The parameters e_ν are defined by (8). For simplicity of the reduction, we choose a_j and b_j in the form

$$a_j = \tilde{a}_1 j^{2N-1} \epsilon^{2N-1}, \quad b_j = \tilde{b}_1 j^{2N-1} \epsilon^{2N-1}, \quad 1 \leq j \leq N. \quad (16)$$

Below we use the following notations :

$$\begin{aligned} f_{4j+1,k} &= \gamma_k^{4j-1} \sin A_k, & f_{4j+2,k} &= \gamma_k^{4j} \cos A_k, \\ f_{4j+3,k} &= -\gamma_k^{4j+1} \sin A_k, & f_{4j+4,k} &= -\gamma_k^{4j+2} \cos A_k, \end{aligned}$$

for $1 \leq k \leq N$, and

$$\begin{aligned} f_{4j+1,k} &= \gamma_k^{2N-4j-2} \cos A_k, & f_{4j+2,k} &= -\gamma_k^{2N-4j-3} \sin A_k, \\ f_{4j+3,k} &= -\gamma_k^{2N-4j-4} \cos A_k, & f_{4j+4,k} &= \gamma_k^{2N-4j-5} \sin A_k, \end{aligned}$$

for $N+1 \leq k \leq 2N$.

We define the functions $g_{j,k}$ for $1 \leq j \leq 2N$, $1 \leq k \leq 2N$ in the same way, we replace only the term A_k by B_k .

$$\begin{aligned} g_{4j+1,k} &= \gamma_k^{4j-1} \sin B_k, & g_{4j+2,k} &= \gamma_k^{4j} \cos B_k, \\ g_{4j+3,k} &= -\gamma_k^{4j+1} \sin B_k, & g_{4j+4,k} &= -\gamma_k^{4j+2} \cos B_k, \end{aligned}$$

for $1 \leq k \leq N$, and

$$\begin{aligned} g_{4j+1,k} &= \gamma_k^{2N-4j-2} \cos B_k, & g_{4j+2,k} &= -\gamma_k^{2N-4j-3} \sin B_k, \\ g_{4j+3,k} &= -\gamma_k^{2N-4j-4} \cos B_k, & g_{4j+4,k} &= \gamma_k^{2N-4j-5} \sin B_k, \end{aligned}$$

for $N+1 \leq k \leq 2N$.

Then it is clear that

$$q(x, t) := \frac{W_3(0)}{W_1(0)}$$

can be written as

$$q(x, t) = \frac{\Delta_3}{\Delta_1} = \frac{\det(f_{j,k})_{j,k \in [1,2N]}}{\det(g_{j,k})_{j,k \in [1,2N]}}. \quad (17)$$

We recall that $\lambda_j = 1 - 2j\epsilon^2$. All the functions $f_{j,k}$ and $g_{j,k}$ depend on ϵ . We use the expansions

$$f_{j,k}(x, t, \epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} f_{j,1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad f_{j,1}[l] = \frac{\partial^{2l} f_{j,1}}{\partial \epsilon^{2l}}(x, t, 0),$$

$$f_{j,1}[0] = f_{j,1}(x, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad 1 \leq l \leq N-1,$$

$$f_{j,N+k}(x, t, \epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} f_{j,N+1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad f_{j,N+1}[l] = \frac{\partial^{2l} f_{j,N+1}}{\partial \epsilon^{2l}}(x, t, 0),$$

$$f_{j,N+1}[0] = f_{j,N+1}(x, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad 1 \leq l \leq N-1.$$

We have the same expansions for the functions $g_{j,k}$.

$$g_{j,k}(x, t, \epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} g_{j,1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad g_{j,1}[l] = \frac{\partial^{2l} g_{j,1}}{\partial \epsilon^{2l}}(x, t, 0),$$

$$g_{j,1}[0] = g_{j,1}(x, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad 1 \leq l \leq N-1,$$

$$g_{j,N+k}(x, t, \epsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} g_{j,N+1}[l] k^{2l} \epsilon^{2l} + O(\epsilon^{2N}), \quad g_{j,N+1}[l] = \frac{\partial^{2l} g_{j,N+1}}{\partial \epsilon^{2l}}(x, t, 0),$$

$$g_{j,N+1}[0] = g_{j,N+1}(x, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad N+1 \leq k \leq 2N..$$

Combining the columns of the determinants appearing in $q(x, t)$ successively to eliminate in each column k (or $N+k$) of them the powers of ϵ lower than $2(k-1)$, and factorizing and simplifying each common terms, $q(x, t)$ can be replaced by $Q(x, t)$

$$Q(x, t) := \begin{vmatrix} f_{1,1}[0] & \cdots & f_{1,1}[N-1] & f_{1,N+1}[0] & \cdots & f_{1,N+1}[N-1] \\ f_{2,1}[0] & \cdots & f_{2,1}[N-1] & f_{2,N+1}[0] & \cdots & f_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{2N,1}[0] & \cdots & f_{2N,1}[N-1] & f_{2N,N+1}[0] & \cdots & f_{2N,N+1}[N-1] \\ \hline g_{1,1}[0] & \cdots & g_{1,1}[N-1] & g_{1,N+1}[0] & \cdots & g_{1,N+1}[N-1] \\ g_{2,1}[0] & \cdots & g_{2,1}[N-1] & g_{2,N+1}[0] & \cdots & g_{2,N+1}[N-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{2N,1}[0] & \cdots & g_{2N,1}[N-1] & g_{2N,N+1}[0] & \cdots & g_{2N,N+1}[N-1] \end{vmatrix} \quad (18)$$

Then we get the following result :

Proposition 3.1 *The function v defined by*

$$v(x, t) = \exp(2it - i\varphi) \times Q(x, t) \quad (19)$$

is a quasi-rational solution of the NLS equation (1)

$$iv_t + v_{xx} + 2|v|^2v = 0,$$

where $Q(x, t)$ is defined in (18).

4 Quasi-rational solutions of order N

We have already constructed in [13] solutions for the cases $N = 1, 2, 3$, and this method gives the same results. In all the following, for simplicity we present only the cases $\tilde{a}_1 = 0$ and $\tilde{b}_1 = 0$. It gives exactly Peregrine breathers.

Here, we recall the case $N = 3$ and give new solutions of NLS equation in the cases from $N = 4$ to 7.

Because of the length of the expressions, in the cases $N = 4$ to 7, for the polynomials n and d in the solutions v of the NLS equation defined by

$$v_N(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it - i\varphi) = \left(1 - \alpha_N \frac{G_N(2x, 4t) + iH_N(2x, 4t)}{Q_N(2x, 4t)}\right) e^{2it - i\varphi}$$

with

$$G_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{g}_k(T) X^k$$

$$H_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{h}_k(T) X^k$$

$$Q_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{q}_k(T) X^k$$

we only give in the appendix.

4.1 Case $N=3$

The expressions of the coefficients of the polynomials G , H and Q are given by :

$$\begin{aligned}
\alpha_3 &= 4, & \mathbf{g}_{12} &= 0, & \mathbf{g}_{11} &= 0, & \mathbf{g}_{10} &= 6, & \mathbf{g}_9 &= 0, & \mathbf{g}_8 &= 90T^2 + 90, & \mathbf{g}_7 &= 0, \\
\mathbf{g}_6 &= 300T^4 - 360T^2 + 1260, \\
\mathbf{g}_5 &= 0, \\
\mathbf{g}_4 &= 420T^6 - 900T^4 + 2700T^2 - 2700, \\
\mathbf{g}_3 &= 0, \\
\mathbf{g}_2 &= 270T^8 + 2520T^6 + 40500T^4 - 81000T^2 + 180Tb - 4050, \\
\mathbf{g}_1 &= 0, \\
\mathbf{g}_0 &= 66T^{10} + 2970T^8 + 13140T^6 - 45900T^4 - 12150T^2 + 4050
\end{aligned}$$

$$\begin{aligned}
\mathbf{h}_{12} &= 0, & \mathbf{h}_{11} &= 0, & \mathbf{h}_{10} &= 6T, & \mathbf{h}_9 &= 0, & \mathbf{h}_8 &= 30T^3 - 90T, & \mathbf{h}_7 &= 0, \\
\mathbf{h}_6 &= 60T^5 - 840T^3 - 900T, \\
\mathbf{h}_5 &= 0, \\
\mathbf{h}_4 &= 60T^7 - 1260T^5 - 2700T^3 - 8100T, \\
\mathbf{h}_3 &= 0, \\
\mathbf{h}_2 &= 30T^9 - 360T^7 + 10260T^5 - 37800T^3 + 28350T, \\
\mathbf{h}_1 &= 0, \\
\mathbf{h}_0 &= 6T^{11} + 150T^9 - 5220T^7 - 57780T^5 - 14850T^3 + 28350T
\end{aligned}$$

$$\begin{aligned}
\mathbf{q}_{12} &= 1, & \mathbf{q}_{11} &= 0, & \mathbf{q}_{10} &= 6T^2 + 6, & \mathbf{q}_9 &= 0, & \mathbf{q}_8 &= 15T^4 - 90T^2 + 135, & \mathbf{q}_7 &= 0, \\
\mathbf{q}_6 &= 20T^6 - 180T^4 + 540T^2 + 2340, \\
\mathbf{q}_5 &= 0, \\
\mathbf{q}_4 &= 15T^8 + 60T^6 - 1350T^4 + 13500T^2 + 3375, \\
\mathbf{q}_3 &= 0, \\
\mathbf{q}_2 &= 6T^{10} + 270T^8 + 13500T^6 + 78300T^4 - 36450T^2 + 12150, \\
\mathbf{q}_1 &= 0, \\
\mathbf{q}_0 &= T^{12} + 126T^{10} + 3735T^8 + 15300T^6 + 143775T^4 + 93150T^2 + 2025
\end{aligned}$$

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain the classical Akhmediev's breather
:

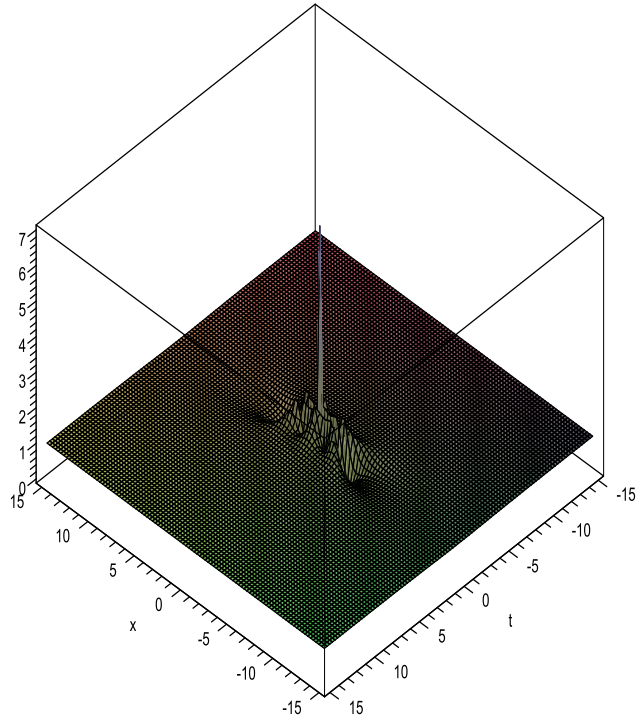


Figure 1: Solution of NLS, $N=3$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$, we obtain the plot :

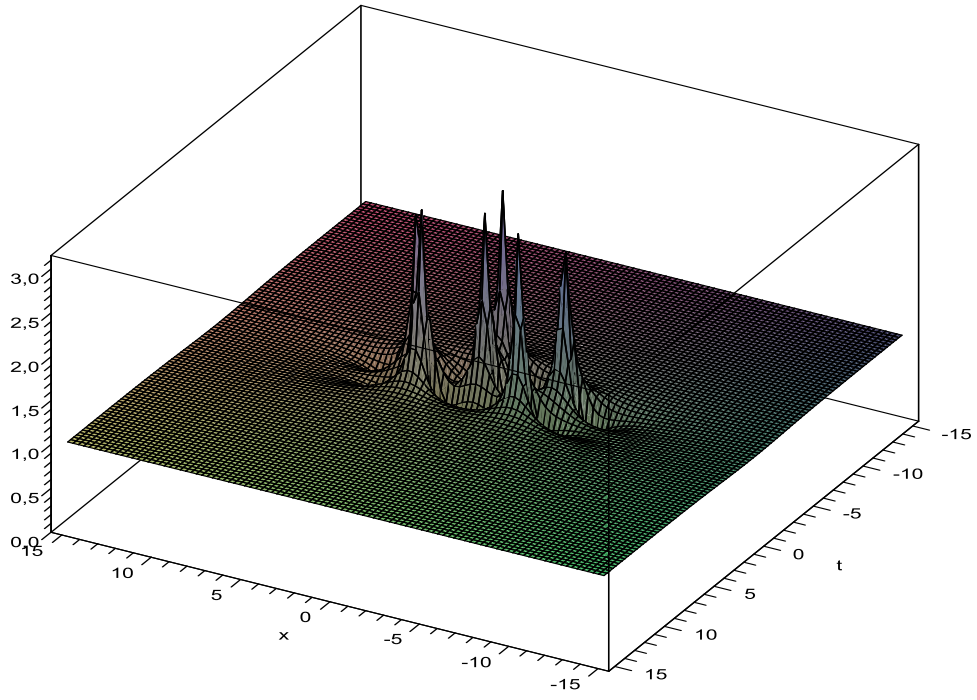


Figure 2: Solution of NLS, $N=3$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$.

If we choose $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$, we obtain the regular plot :

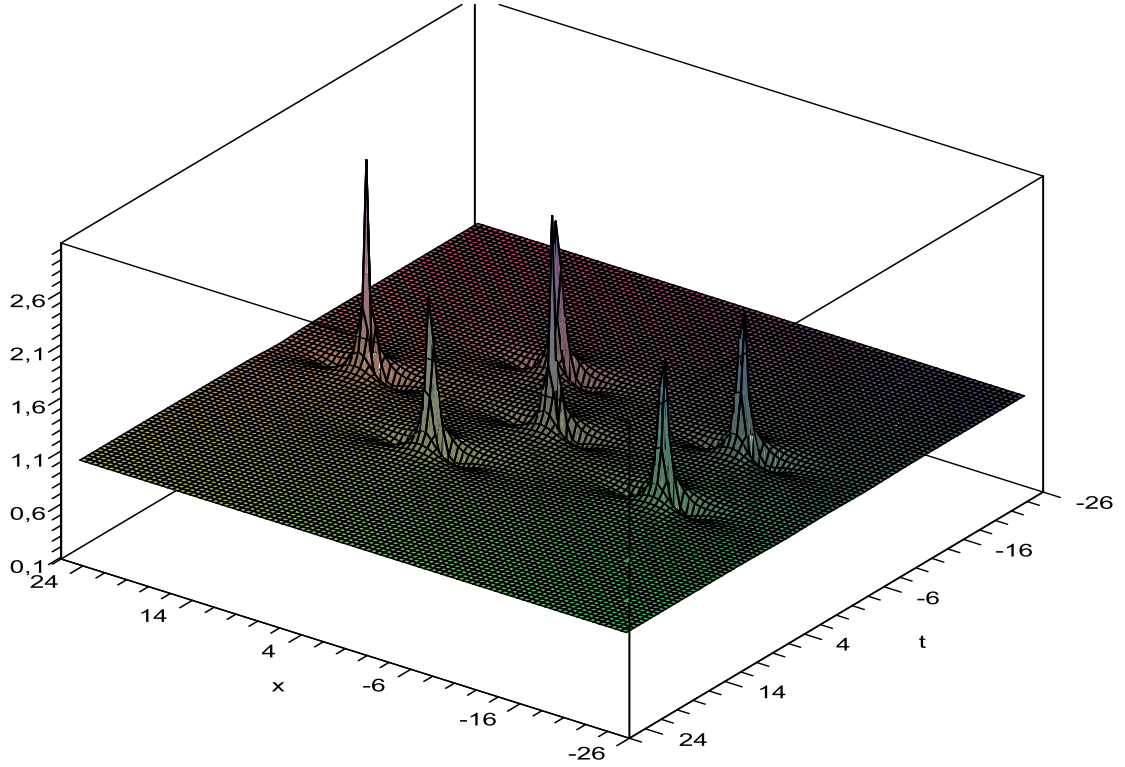


Figure 3: Solution of NLS, $N=3$, $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$.

4.2 Case $N=4$

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain the classical Akhmediev's breather :

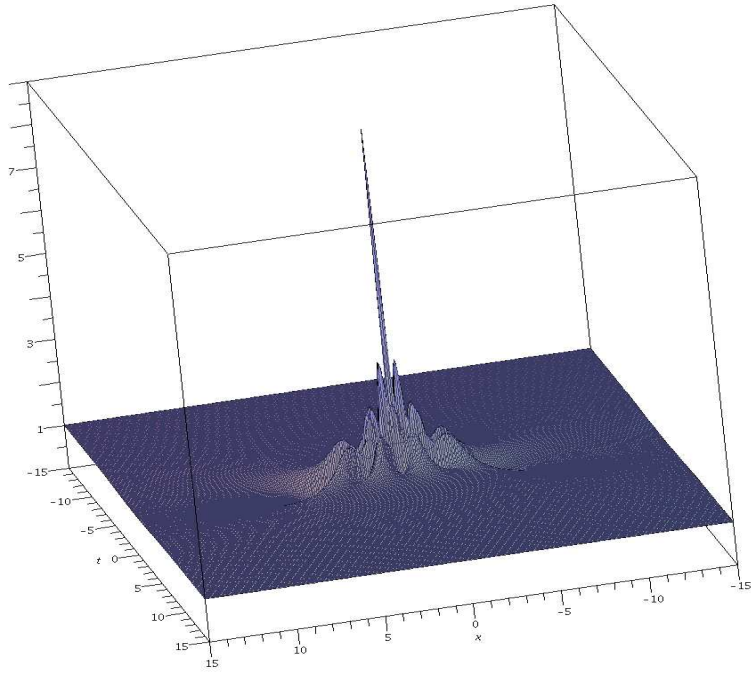


Figure 4: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$, we obtain the plot :

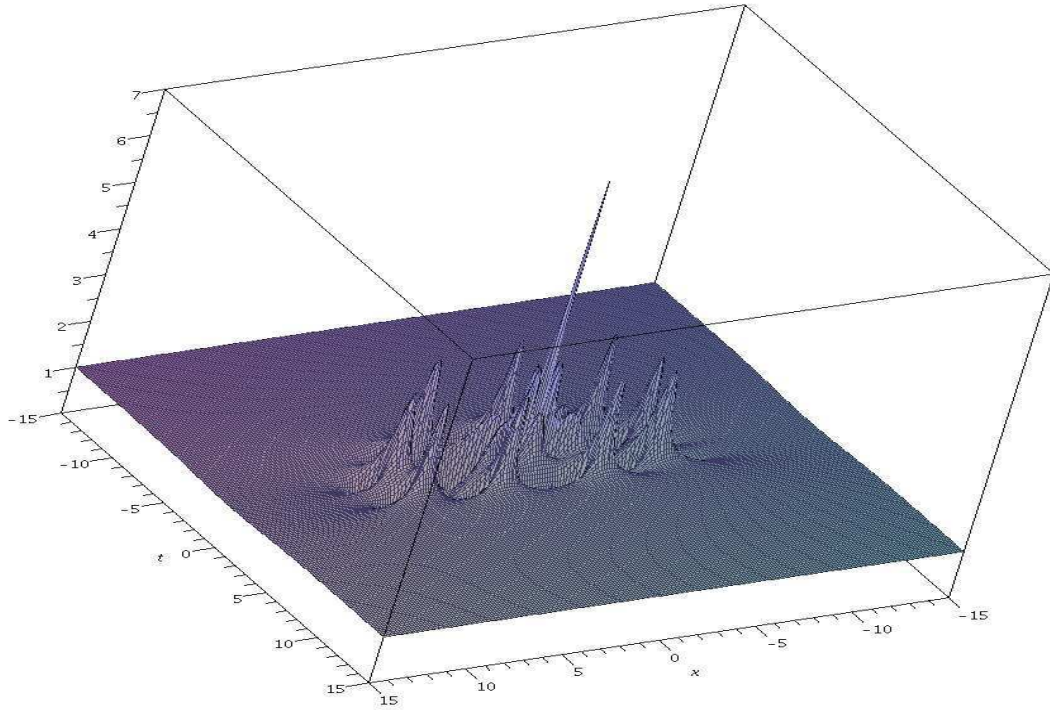


Figure 5: Solution of NLS, $N=4$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^3$.

If we choose $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$, we obtain the regular plot :

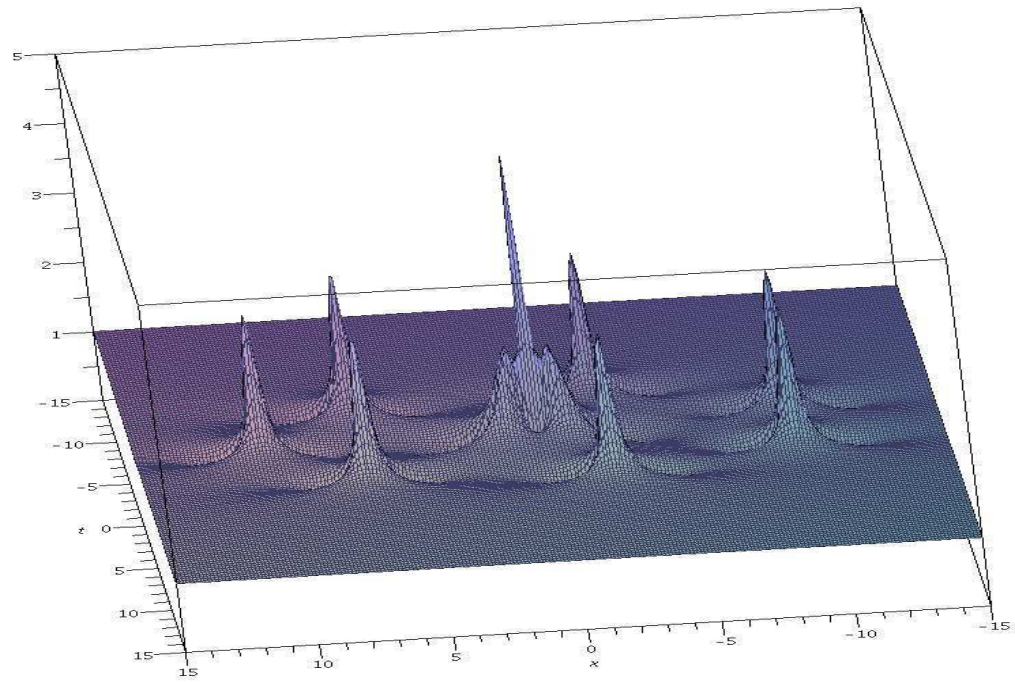


Figure 6: Solution of NLS, $N=4$, $\tilde{a}_1 = 10^6$, $\tilde{b}_1 = -10^2$.

4.3 Case $N=5$

For $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain Akhmediev's breather.

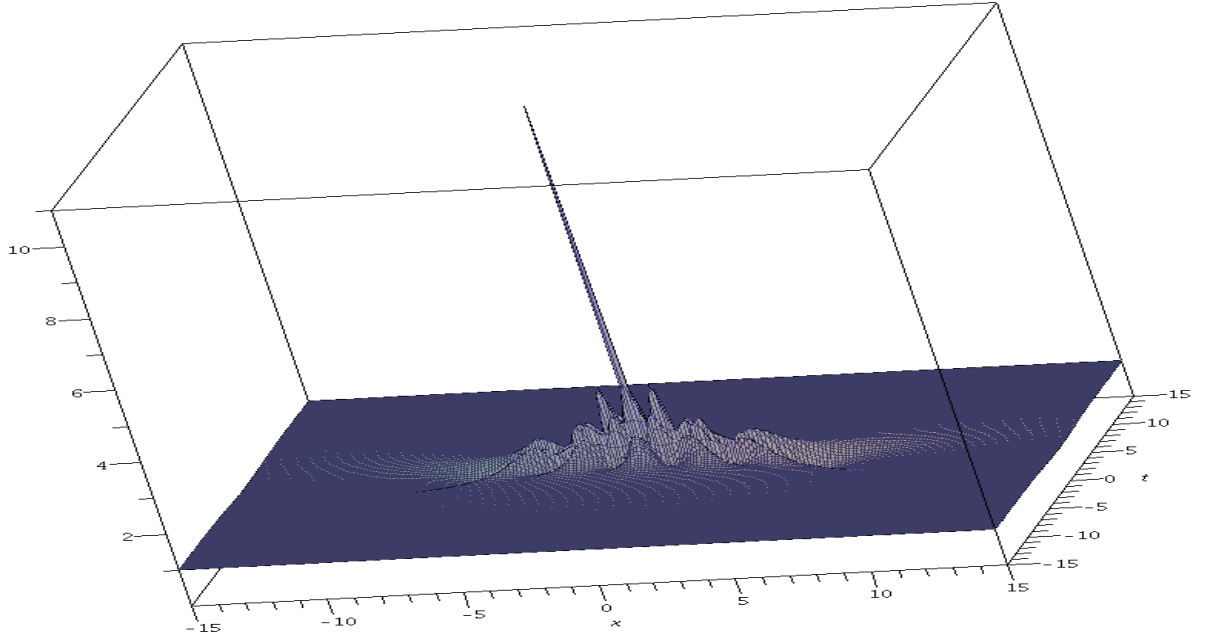


Figure 7: Solution of NLS, $N=5$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^5$, we obtain :

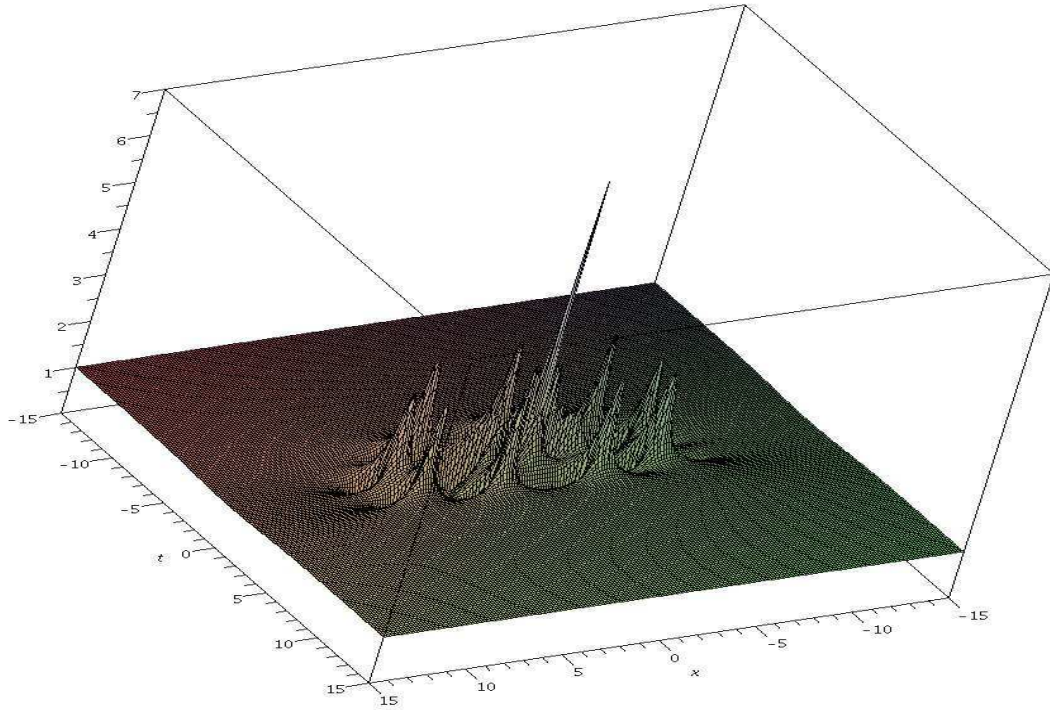


Figure 8: Solution of NLS, $N=5$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^5$.

If we choose $\tilde{a}_1 = 10^8$, $\tilde{b}_1 = -10^5$, we have :

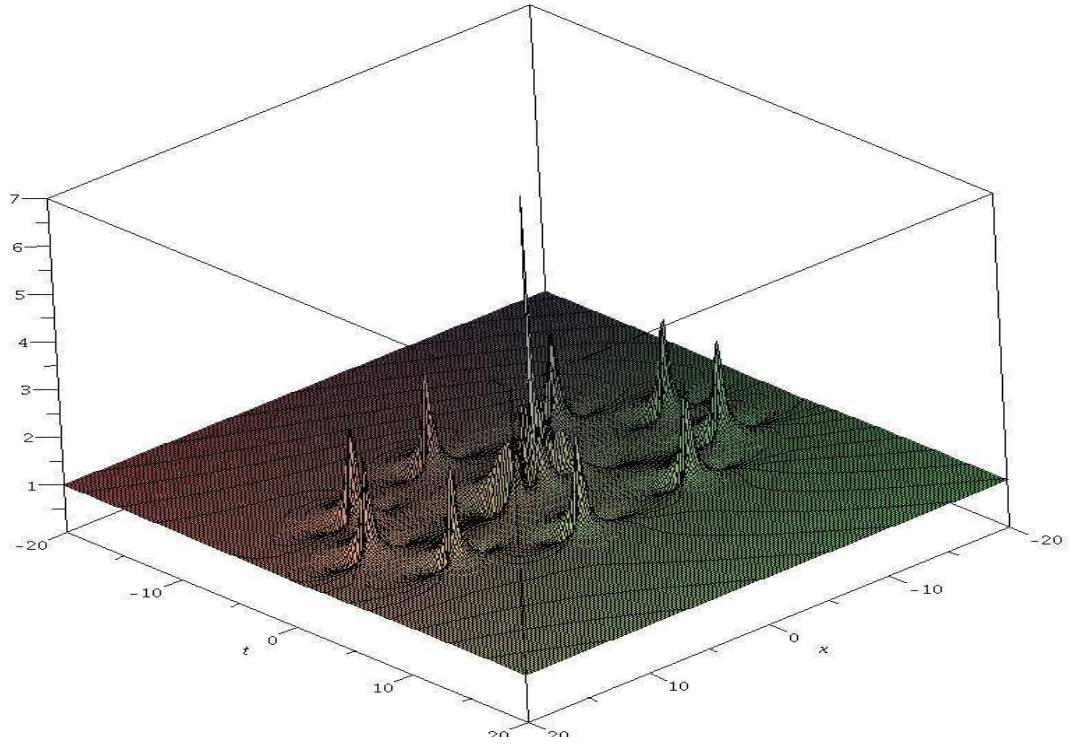


Figure 9: Solution of NLS, $N=5$, $\tilde{a}_1 = 10^8$, $\tilde{b}_1 = -10^5$.

4.4 Case $N=6$

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we obtain the following plot :

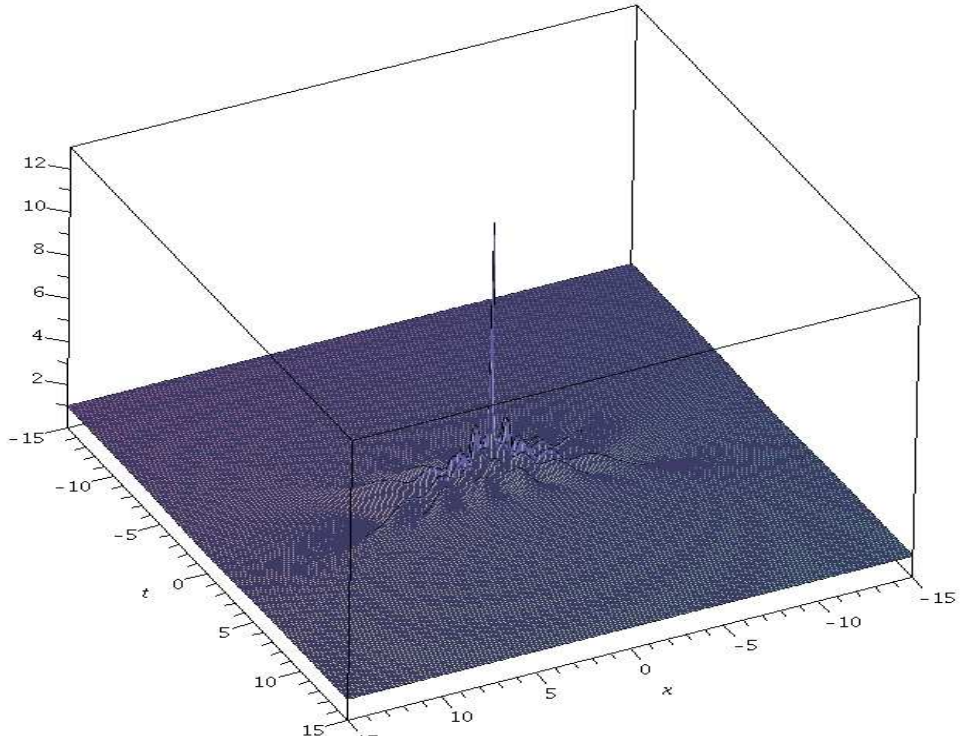


Figure 10: Solution of NLS, $N=6$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^6$, we obtain :

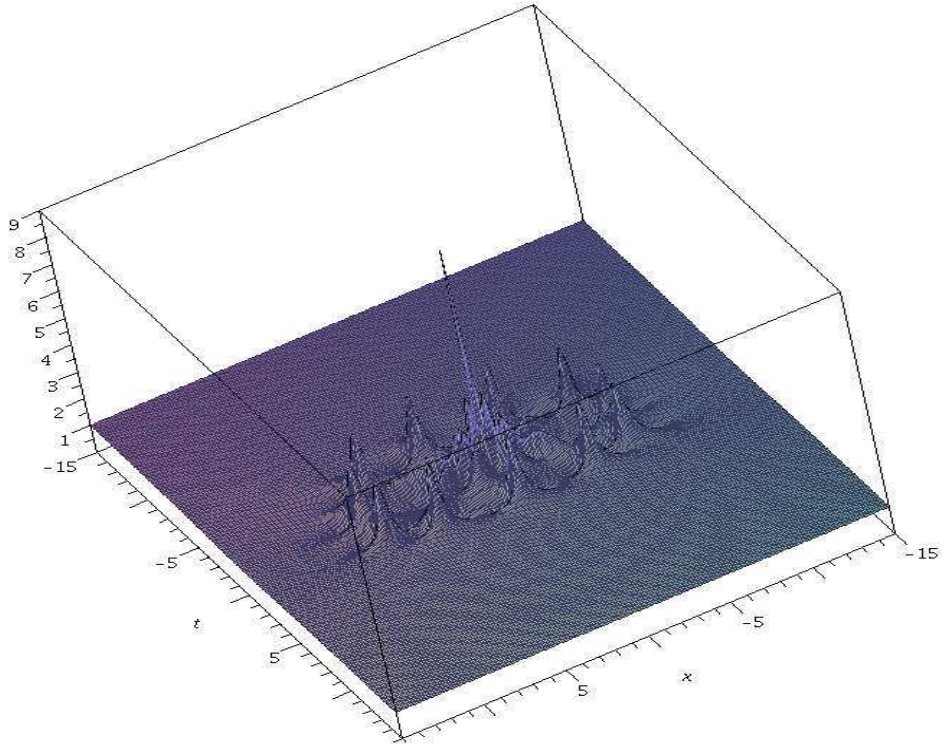


Figure 11: Solution of NLS, $N=6$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^6$.

If we choose $\tilde{a}_1 = 10^9$, $\tilde{b}_1 = -10^6$, we obtain the regular figure with 12 peaks :

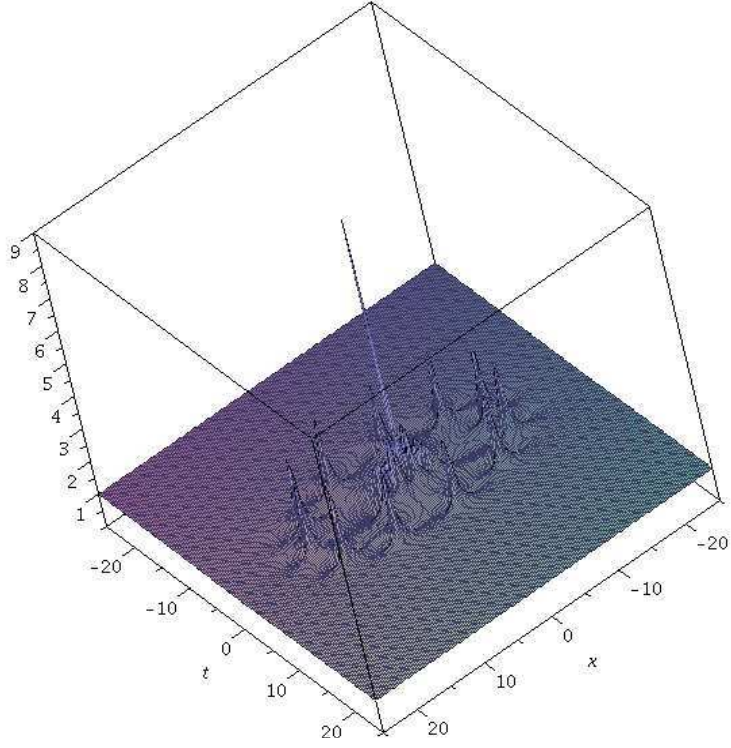


Figure 12: Solution of NLS, $N=6$, $\tilde{a}_1 = 10^9$, $\tilde{b}_1 = -10^6$.

4.5 Case $N=7$

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$, we recognize Akhmediev's breather :

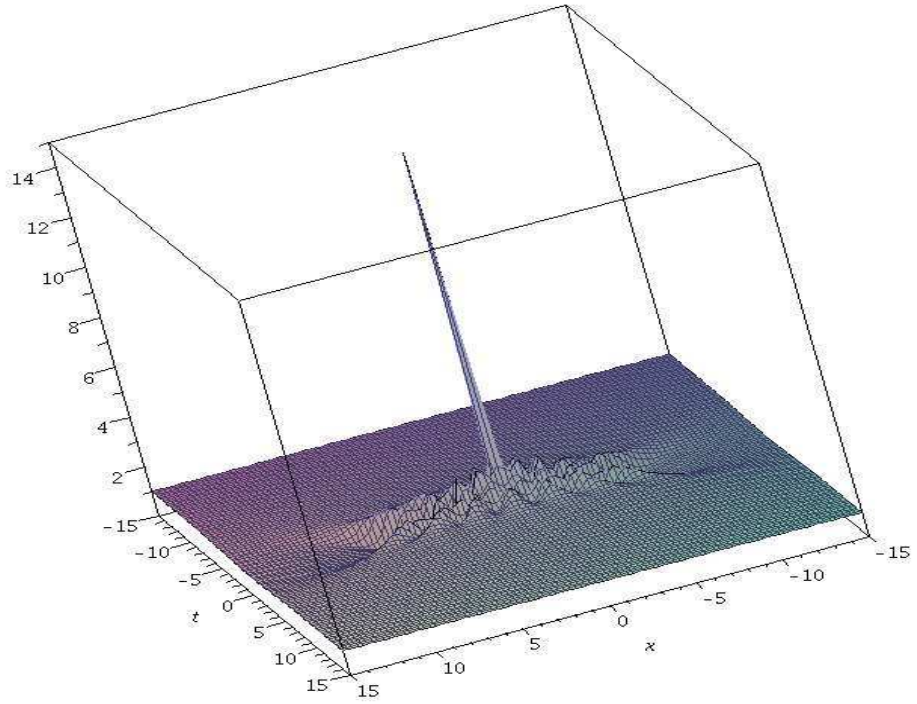


Figure 13: Solution of NLS, $N=7$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 0$.

If we choose $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^8$, we obtain the following plot :

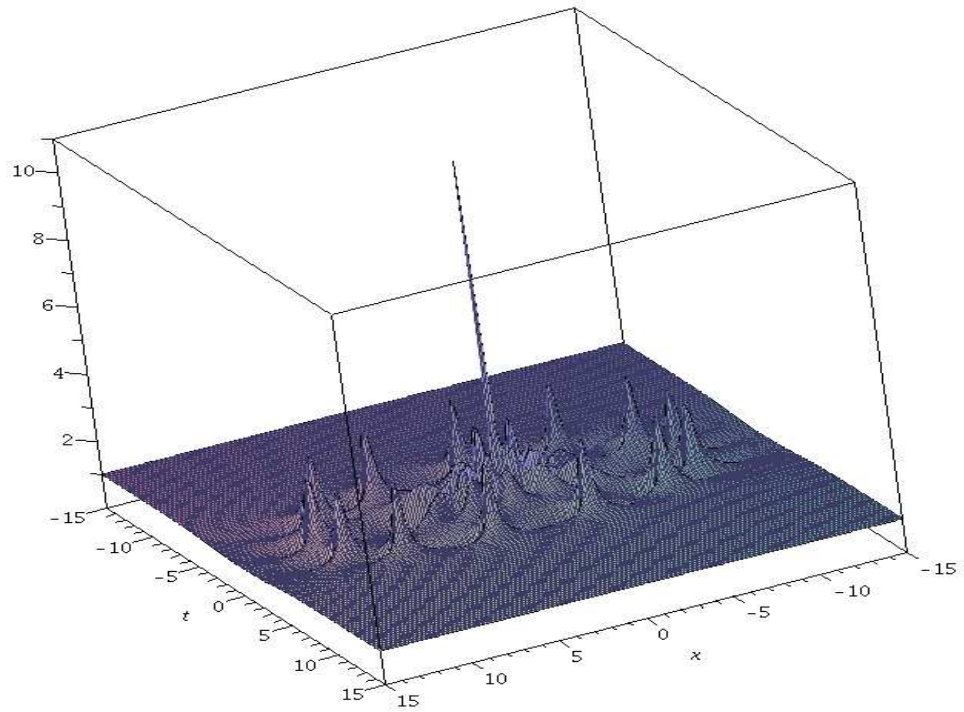


Figure 14: Solution of NLS, $N=7$, $\tilde{a}_1 = 0$, $\tilde{b}_1 = 10^8$.

If we choose $\tilde{a}_1 = 10^{12}$, $\tilde{b}_1 = -10^8$, we obtain the following regular plot :

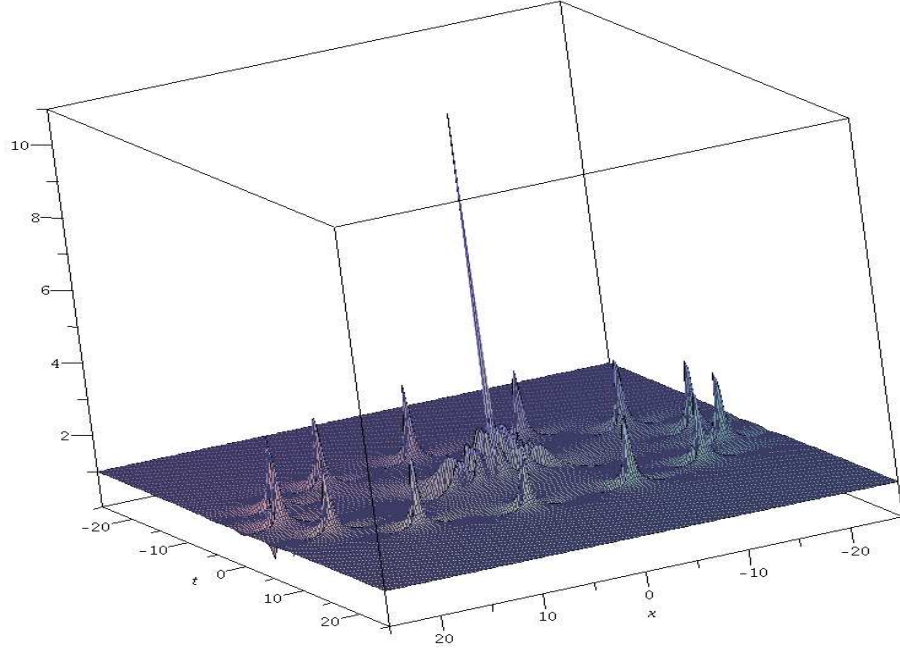


Figure 15: Solution of NLS, $N=7$, $\tilde{a}_1 = 10^{12}$, $\tilde{b}_1 = -10^8$.

5 Conclusion

The method described in the present paper provides a powerful tool to get explicitly solutions of the NLS equation.

As my knowledge, it is the first time that the Peregrine breather of order seven is presented.

It confirms the conjecture about the shape of the breather in the (x, t) coordinates, the maximum of amplitude equal to $2N + 1$ and the degree of polynomials in x and t here equal to $N(N + 1)$. This new formulation gives an infinite set of non singular solution of NLS equation. It opens a large way to future researches in this domain.

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Appendix

In the following, we choose all the parameters a_1 and b_1 equal to 0. The solution of NLS equation takes the form

$$v_N(x, t) = \frac{n(x, t)}{d(x, t)} \exp(2it) = \left(1 - \alpha_N \frac{G_N(2x, 4t) + iH_N(2x, 4t)}{Q_N(2x, 4t)}\right) e^{2it}$$

with

$$G_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{g}_k(T) X^k$$

$$H_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{h}_k(T) X^k$$

$$Q_N(X, T) = \sum_{k=0}^{N(N+1)} \mathbf{q}_k(T) X^k$$

1. Case $N=4$

The coefficients of the polynomials G , H and Q are defined by

$$\begin{aligned}
&\alpha_4 = 4, \quad \mathbf{g}_{20} = 0, \quad \mathbf{g}_{19} = 0, \quad \mathbf{g}_{18} = 10, \quad \mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 270T^2 + 270, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 1800T^4 \\
&\quad - 3600T^2 + 9000, \quad \mathbf{g}_{13} = 0, \quad \mathbf{g}_{12} = 5880T^6 - 54600T^4 - 12600T^2 + 189000, \quad \mathbf{g}_{11} = 0, \\
&\mathbf{g}_{10} = 11340T^8 - 176400T^6 + 189000T^4 - 378000T^2 - 1077300, \quad \mathbf{g}_9 = 0, \\
&\mathbf{g}_8 = 13860T^{10} - 207900T^8 + 2356200T^6 + 1701000T^4 - 56983500T^2 - 4819500, \\
&\mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 10920T^{12} - 18480T^{10} + 6967800T^8 + 56095200T^6 - 342657000T^4 \\
&\quad + 198450000T^2 - 11907000, \quad \mathbf{g}_5 = 0 \quad \mathbf{g}_4 = 5400T^{14} + 163800T^{12} + 9034200T^{10} \\
&\quad + 107919000T^8 - 615195000T^6 + 178605000T^4 + 654885000T^2 + 178605000, \quad \mathbf{g}_3 = 0, \\
&\mathbf{g}_2 = 1530T^{16} + 133200T^{14} + 5506200T^{12} - 116802000T^{10} - 1731334500T^8 \\
&\quad + 2532222000T^6 - 893025000T^4 + 4643730000T^2 + 223256250, \quad \mathbf{g}_1 = 0, \\
&\mathbf{g}_0 = 190T^{18} + 33150T^{16} + 1294200T^{14} + 3288600T^{12} + 48629700T^{10} \\
&\quad - 2015401500T^8 - 1845585000T^6 + 14586075000T^4 + 2098608750T^2 - 44651250,
\end{aligned}$$

$$\begin{aligned}
&\mathbf{h}_{20} = 0, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 10T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 90T^3 - 270T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 360T^5 \\
&\quad - 6000T^3 - 5400T, \quad \mathbf{h}_{13} = 0, \quad \mathbf{h}_{12} = 840T^7 - 29400T^5 + 12600T^3 - 138600T, \quad \mathbf{h}_{11} = 0, \\
&\mathbf{h}_{10} = 1260T^9 - 65520T^7 + 259560T^5 - 529200T^3 - 1984500T, \quad \mathbf{h}_9 = 0, \\
&\mathbf{h}_8 = 1260T^{11} - 77700T^9 + 718200T^7 - 5329800T^5 - 6142500T^3 + 29767500T, \\
&\mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 840T^{13} - 48720T^{11} + 718200T^9 + 2973600T^7 - 72765000T^5 \\
&\quad + 436590000T^3 + 146853000T, \quad \mathbf{h}_5 = 0, \quad \mathbf{h}_4 = 360T^{15} - 12600T^{13} + 138600T^{11} \\
&\quad - 5859000T^9 - 328293000T^7 + 1075599000T^5 + 773955000T^3 + 535815000T, \quad \mathbf{h}_3 = 0, \\
&\mathbf{h}_2 = 90T^{17} + 1200T^{15} - 189000T^{13} - 40143600T^{11} \\
&\quad - 307786500T^9 + 2085426000T^7 - 4465125000T^5 + 4405590000T^3 - 1205583750T, \quad \mathbf{h}_1 = 0, \\
&\mathbf{h}_0 = 10T^{19} + 930T^{17} - 86040T^{15} - 7018200T^{13} - 48100500T^{11} - 542902500T^9 \\
&\quad + 6039117000T^7 + 12942909000T^5 + 937676250T^3, \quad \mathbf{q}_{20} = 1, \quad \mathbf{q}_{19} = 0, \\
&\mathbf{q}_{18} = 10T^2 + 10, \quad \mathbf{q}_{17} = 0, \quad \mathbf{q}_{16} = 45T^4 - 270T^2 + 405, \quad \mathbf{q}_{15} = 0, \\
&\mathbf{q}_{14} = 120T^6 - 1800T^4 + 1800T^2 + 16200, \quad \mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = 210T^8 - 4200T^6 + 6300T^4 \\
&\quad + 113400T^2 + 425250, \quad \mathbf{q}_{11} = 0, \quad \mathbf{q}_{10} = 252T^{10} - 3780T^8 + 63000T^6 \\
&\quad + 718200T^4 + 3005100T^2 + 1644300, \quad \mathbf{q}_9 = 0, \quad \mathbf{q}_8 = 210T^{12} + 1260T^{10} \\
&\quad + 255150T^8 - 567000T^6 + 23388750T^4 - 31468500T^2 + 17435250, \quad \mathbf{q}_7 = 0, \\
&\mathbf{q}_6 = 120T^{14} + 5880T^{12} + 476280T^{10} + 16443000T^8 + 162729000T^6 \\
&\quad - 154791000T^4 + 130977000T^2 + 130977000, \quad \mathbf{q}_5 = 0, \quad \mathbf{q}_4 = 45T^{16} + 5400T^{14} \\
&\quad + 459900T^{12} + 19845000T^{10} + 153798750T^8 + 702513000T^6 - 89302500T^4 \\
&\quad + 1250235000T^2 + 111628125, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 10T^{18} \\
&\quad + 2250T^{16} + 225000T^{14} + 4422600T^{12} - 99508500T^{10} - 224248500T^8 \\
&\quad + 9704205000T^6 + 15181425000T^4 - 1920003750T^2 + 223256250, \quad \mathbf{q}_1 = 0, \\
&\mathbf{q}_0 = T^{20} + 370T^{18} + 44325T^{16} + 2208600T^{14} + 62795250T^{12} + 693384300T^{10} \\
&\quad + 6641129250T^8 + 4346055000T^6 + 14042818125T^4 + 2902331250T^2 + 22325625
\end{aligned}$$

2. Case N=5

The expressions of the coefficients of the polynomials G , H and Q are defined by

$$\begin{aligned}
& \alpha_5 = 60, \quad \mathbf{g}_{30} = 0, \quad \mathbf{g}_{29} = 0, \quad \mathbf{g}_{28} = 1, \quad \mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 42T^2 + 42, \quad \mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 455T^4 \\
& -1050T^2 + 2415, \quad \mathbf{g}_{23} = 0, \quad \mathbf{g}_{22} = 2548T^6 - 30660T^4 - 13860T^2 + 119700, \quad \mathbf{g}_{21} = 0, \\
& \mathbf{g}_{20} = 9009T^8 - 226380T^6 + 171990T^4 - 343980T^2 + 3221505, \quad \mathbf{g}_{19} = 0, \quad \mathbf{g}_{18} = 22022T^{10} \\
& -838530T^8 + 4142460T^6 - 44100T^4 - 36713250T^2 - 40153050, \quad \mathbf{g}_{17} = 0, \\
& \mathbf{g}_{16} = 39039T^{12} - 1844766T^{10} + 22431465T^8 - 9075780T^6 - 259473375T^4 - 2703484350T^2 \\
& -370010025, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 51480T^{14} - 2522520T^{12} + 61319160T^{10} \\
& +39803400T^8 - 773955000T^6 - 21896973000T^4 + 33756345000T^2 - 2893401000, \\
& \mathbf{g}_{13} = 0, \quad \mathbf{g}_{12} = 51051T^{16} - 2023560T^{14} + 104367060T^{12} + 629483400T^{10} \\
& +6114046050T^8 - 132697164600T^6 + 554979316500T^4 + 319310019000T^2 + 30787036875, \\
& \mathbf{g}_{11} = 0, \quad \mathbf{g}_{10} = 38038T^{18} - 589050T^{16} + 124369560T^{14} + 1700266680T^{12} \\
& +37748127060T^{10} - 446713728300T^8 + 2431707075000T^6 + 1380509487000T^4 \\
& +4238859255750T^2 + 1299806817750, \quad \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 21021T^{20} + 570570T^{18} \\
& +112372785T^{16} + 1735587000T^{14} - 43189665750T^{12} - 2318934687300T^{10} \\
& +10714665764250T^8 - 20464596621000T^6 + 35015175365625T^4 + 40381027706250T^2 \\
& +5540260123125, \quad \mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 8372T^{22} + 769692T^{20} + 78618540T^{18} \\
& +570662820T^{16} - 223349124600T^{14} - 2950615722600T^{12} + 16520555280600T^{10} \\
& -11401393059000T^8 + 147193042090500T^6 + 422927620447500T^4 - 99598095922500T^2 \\
& +17840228332500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 2275T^{24} + 415380T^{22} + 39897270T^{20} \\
& -30649500T^{18} - 148598863875T^{16} - 1555875783000T^{14} - 2135859799500T^{12} \\
& +94593530241000T^{10} - 98463038821875T^8 + 2611250197762500T^6 - 159203362106250T^4 \\
& -83293781287500T^2 - 21709549378125, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 378T^{26} + 114450T^{24} \\
& +12621420T^{22} + 89037900T^{20} - 283320450T^{18} + 1545272004150T^{16} \\
& +12633981885000T^{14} - 118201467699000T^{12} + 1380551057313750T^{10} \\
& +7814079083238750T^8 + 3521850108367500T^6 + 4776100863187500T^4 \\
& -1406247137268750T^2 - 13291560843750, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 29T^{28} + 13230T^{26} \\
& +1814295T^{24} + 74845260T^{22} - 764250795T^{20} - 204794909550T^{18} \\
& -3849793565625T^{16} - 34193820087000T^{14} + 942733356807375T^{12} \\
& +1889980437035250T^{10} + 13147594251868125T^8 + 3164572952887500T^6 \\
& -3369410673890625T^4 - 124940671931250T^2 + 3987468253125,
\end{aligned}$$

$$\begin{aligned}
& \mathbf{h}_{30} = 0, \quad \mathbf{h}_{29} = 0, \quad \mathbf{h}_{28} = T, \quad \mathbf{h}_{27} = 0, \quad \mathbf{h}_{26} = 14T^3 - 42T, \quad \mathbf{h}_{25} = 0, \quad \mathbf{h}_{24} = 91T^5 - 1610T^3 \\
& -1365T, \quad \mathbf{h}_{23} = 0, \quad \mathbf{h}_{22} = 364T^7 - 14700T^5 + 13860T^3 - 64260T, \quad \mathbf{h}_{21} = 0, \quad \mathbf{h}_{20} = 1001T^9 \\
& -67452T^7 + 411894T^5 - 97020T^3 - 2546775T, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 2002T^{11} - 190190T^9 \\
& +2572500T^7 - 3342780T^5 - 6769350T^3 - 39756150T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 3003T^{13} \\
& -358974T^{11} + 7821765T^9 - 39225060T^7 - 73327275T^5 + 439963650T^3 + 2114980875T, \\
& \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 3432T^{15} - 471240T^{13} + 13736520T^{11} - 135513000T^9 - 793686600T^7 \\
& +2779093800T^5 + 51116751000T^3 + 24754653000T, \quad \mathbf{h}_{13} = 0, \quad \mathbf{h}_{12} = 3003T^{17} \\
& -434280T^{15} + 14403060T^{13} - 248776920T^{11} - 793072350T^9 - 2707651800T^7 + 375945664500T^5 \\
& -81098577000T^3 + 297337138875T, \quad \mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 2002T^{19} - 275814T^{17} \\
& +8148168T^{15} - 362872440T^{13} - 114704100T^{11} - 90682521300T^9 + 1534457471400T^7 \\
& -1772183107800T^5 + 1117897625250T^3 + 1493718266250T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 1001T^{21}
\end{aligned}$$

$$\begin{aligned}
& -113190 T^{19} + 836325 T^{17} - 501931080 T^{15} - 15705928350 T^{13} - 400107348900 T^{11} \\
& + 4976480045250 T^9 - 11450902365000 T^7 + 30510953731125 T^5 - 5820156483750 T^3 \\
& - 21110374254375 T, \quad \mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 364 T^{23} - 24332 T^{21} - 2084460 T^{19} \\
& - 528432660 T^{17} - 31926371400 T^{15} + 150244907400 T^{13} + 11823972489000 T^{11} \\
& - 3962494809000 T^9 + 7158970633500 T^7 + 132364254802500 T^5 - 455536249717500 T^3 \\
& - 63681344842500 T, \quad \mathbf{h}_5 = 0, \quad \mathbf{h}_4 = 91 T^{25} + 420 T^{23} - 1450890 T^{21} - 337761900 T^{19} \\
& - 18543465675 T^{17} + 274020553800 T^{15} + 5724951088500 T^{13} + 48513868893000 T^{11} \\
& + 171111381643125 T^9 + 1334157649492500 T^7 - 1694171881946250 T^5 - 515712560737500 T^3 \\
& - 131586452353125 T, \quad \mathbf{h}_3 = 0, \quad \mathbf{h}_2 = 14 T^{27} + 1470 T^{25} - 409500 T^{23} \\
& - 111637260 T^{21} - 3311799750 T^{19} + 88973271450 T^{17} - 3045655809000 T^{15} \\
& - 34947318861000 T^{13} + 1002802178873250 T^{11} + 1999468016831250 T^9 - 6800738923597500 T^7 \\
& + 4269249343012500 T^5 - 1666761729806250 T^3 + 204690036993750 T, \quad \mathbf{h}_1 = 0, \\
& \mathbf{h}_0 = T^{29} + 238 T^{27} - 43701 T^{25} - 14070420 T^{23} - 1034990775 T^{21} - 32505382350 T^{19} \\
& + 259820563275 T^{17} + 13855420996200 T^{15} + 406907765530875 T^{13} + 497730743291250 T^{11} \\
& + 1983581436965625 T^9 - 10570073675332500 T^7 - 786408488813125 T^5 - 224184326231250 T^3 \\
& + 73103584640625 T
\end{aligned}$$

$$\begin{aligned}
& \mathbf{q}_{30} = 1, \quad \mathbf{q}_{29} = 0, \quad \mathbf{q}_{28} = 15 T^2 + 15, \quad \mathbf{q}_{27} = 0, \quad \mathbf{q}_{26} = 105 T^4 - 630 T^2 + 945, \quad \mathbf{q}_{25} = 0, \\
& \mathbf{q}_{24} = 455 T^6 - 7875 T^4 + 4725 T^2 + 64575, \quad \mathbf{q}_{23} = 0, \quad \mathbf{q}_{22} = 1365 T^8 - 39900 T^6 \\
& + 103950 T^4 + 548100 T^2 + 3709125, \quad \mathbf{q}_{21} = 0, \quad \mathbf{q}_{20} = 3003 T^{10} - 114345 T^8 + 859950 T^6 + 4035150 T^4 \\
& + 34827975 T^2 + 133656075, \quad \mathbf{q}_{19} = 0, \quad \mathbf{q}_{18} = 5005 T^{12} - 200970 T^{10} + 3649275 T^8 + 220500 T^6 \\
& + 277333875 T^4 + 959505750 T^2 + 1115785125, \quad \mathbf{q}_{17} = 0, \quad \mathbf{q}_{16} = 6435 T^{14} - 204435 T^{12} + 10174815 T^{10} \\
& + 42170625 T^8 + 2030639625 T^6 + 7693410375 T^4 - 27357820875 T^2 + 24214372875, \quad \mathbf{q}_{15} = 0, \\
& \mathbf{q}_{14} = 6435 T^{16} - 59400 T^{14} + 21035700 T^{12} + 451672200 T^{10} + 2902331250 T^8 + 79622109000 T^6 \\
& - 319613647500 T^4 + 191285955000 T^2 + 463546951875, \quad \mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = 5005 T^{18} \\
& + 155925 T^{16} + 33585300 T^{14} + 1481098500 T^{12} + 42118035750 T^{10} + 639849435750 T^8 \\
& - 1190848837500 T^6 + 1787210932500 T^4 + 4850130403125 T^2 + 5581517878125, \quad \mathbf{q}_{11} = 0 \\
& \mathbf{q}_{10} = 3003 T^{20} + 279510 T^{18} + 40951575 T^{16} + 2550025800 T^{14} + 112585249350 T^{12} \\
& + 1486454400900 T^{10} + 2935114197750 T^8 + 10430710605000 T^6 + 58973741229375 T^4 \\
& + 49590883833750 T^2 + 14657286301875, \quad \mathbf{q}_9 = 0, \quad \mathbf{q}_8 = 1365 T^{22} + 246015 T^{20} \\
& + 36850275 T^{18} + 2719544625 T^{16} + 98273999250 T^{14} - 830307854250 T^{12} \\
& - 8598553724250 T^{10} + 211739487041250 T^8 + 162726680615625 T^6 \\
& + 731900852746875 T^4 - 370328202365625 T^2 + 93610564228125, \quad \mathbf{q}_7 = 0, \\
& \mathbf{q}_6 = 455 T^{24} + 134820 T^{22} + 23403870 T^{20} + 1942384500 T^{18} + 36981653625 T^{16} \\
& - 1371507795000 T^{14} + 3080287318500 T^{12} + 299367020421000 T^{10} + 3135310421315625 T^8 \\
& + 10570433743012500 T^6 - 3151872128081250 T^4 + 1114718902762500 T^2 \\
& + 412481438184375, \quad \mathbf{q}_5 = 0, \quad \mathbf{q}_4 = 105 T^{26} + 46725 T^{24} + 9856350 T^{22} \\
& + 950244750 T^{20} + 28094731875 T^{18} + 11015463375 T^{16} - 7594552507500 T^{14} \\
& + 191792925292500 T^{12} + 6041183185209375 T^{10} + 13451797993921875 T^8 \\
& + 29454394197768750 T^6 + 1342447645218750 T^4 + 5894807234203125 T^2 \\
& + 232602314765625, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 15 T^{28} + 9450 T^{26} \\
& + 2461725 T^{24} + 296748900 T^{22} + 21862740375 T^{20} + 1401063945750 T^{18} \\
& + 53045439418125 T^{16} + 801963419355000 T^{14} + 7240875247423125 T^{12} \\
& - 18225599206106250 T^{10} - 19943100941990625 T^8 + 193179545303062500 T^6
\end{aligned}$$

$$\begin{aligned}
& +118075580755453125T^4 - 5861578332093750T^2 + 299060118984375, \\
\mathbf{q}_1 = 0, \quad \mathbf{q}_0 = & T^{30} + 855T^{28} + 275625T^{26} + 44441775T^{24} + 4060783125T^{22} \\
& +207533751075T^{20} + 5923312282125T^{18} + 77461769896875T^{16} \\
& +1691986493491875T^{14} + 21127132873153125T^{12} + 60580010182426875T^{10} \\
& +225021251512378125T^8 + 50098108080234375T^6 + 67806897644390625T^4 \\
& +5881515673359375T^2 + 19937341265625
\end{aligned}$$

3. Case N=6

The coefficients of the polynomials G , H and Q are defined by

$$\begin{aligned}
& \alpha_6 = 4, \quad \mathbf{g}_{42} = 0, \quad \mathbf{g}_{41} = 0, \quad \mathbf{g}_{40} = 21, \quad \mathbf{g}_{39} = 0, \quad \mathbf{g}_{38} = 1260 T^2 + 1260, \quad \mathbf{g}_{37} = 0, \quad \mathbf{g}_{36} = 19950 T^4 \\
& - 49140 T^2 + 108990, \quad \mathbf{g}_{35} = 0, \quad \mathbf{g}_{34} = 167580 T^6 - 2249100 T^4 - 1190700 T^2 + 9128700, \quad \mathbf{g}_{33} = 0, \\
& \mathbf{g}_{32} = 915705 T^8 - 27439020 T^6 + 24409350 T^4 - 48818700 T^2 + 589098825, \quad \mathbf{g}_{31} = 0, \quad \mathbf{g}_{30} = 3581424 T^{10} \\
& - 178128720 T^8 + 984947040 T^6 + 60328800 T^4 - 4186501200 T^2 + 18503478000, \quad \mathbf{g}_{29} = 0, \\
& \mathbf{g}_{28} = 10581480 T^{12} - 743616720 T^{10} + 9464477400 T^8 - 7045768800 T^6 - 30612897000 T^4 \\
& - 520740738000 T^2 - 459479223000, \quad \mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 24418800 T^{14} - 2175329520 T^{12} \\
& + 47637843120 T^{10} - 159272794800 T^8 - 224542206000 T^6 - 4616367714000 T^4 - 32428595430000 T^2 \\
& - 7322876442000, \quad \mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 44971290 T^{16} - 4661420400 T^{14} + 152359061400 T^{12} \\
& - 1026048416400 T^{10} - 337581310500 T^8 - 30371208714000 T^6 - 186218752545000 T^4 \\
& + 1100164292010000 T^2 - 108079377603750, \quad \mathbf{g}_{23} = 0, \quad \mathbf{g}_{22} = 67016040 T^{18} - 7469967960 T^{16} \\
& + 337998679200 T^{14} - 3317725202400 T^{12} + 14701701495600 T^{10} - 134995160034000 T^8 \\
& - 1193237286444000 T^6 + 29149053992100000 T^4 + 20986218417465000 T^2 + 1152340349265000, \\
& \mathbf{g}_{21} = 0, \quad \mathbf{g}_{20} = 81477396 T^{20} - 8962513560 T^{18} + 547168642020 T^{16} - 6239220372000 T^{14} \\
& + 108187552103400 T^{12} - 282142532826000 T^{10} - 9222667785639000 T^8 + 257357195058348000 T^6 \\
& + 18279145208362500 T^4 + 529488288837585000 T^2 + 156886140925552500, \quad \mathbf{g}_{19} = 0 \\
& \mathbf{g}_{18} = 81124680 T^{22} - 7847559720 T^{20} + 671406535800 T^{18} - 6955900320600 T^{16} + 296997420258000 T^{14} \\
& + 830239889094000 T^{12} - 53402987374506000 T^{10} + 1441119106966770000 T^8 - 2734739470632015000 T^6 \\
& + 4283688113241075000 T^4 + 11213622020930175000 T^2 + 4883826305513925000, \quad \mathbf{g}_{17} = 0, \\
& \mathbf{g}_{16} = 66134250 T^{24} - 4572870120 T^{22} + 650465836020 T^{20} - 3956090430600 T^{18} \\
& + 265628919921750 T^{16} - 8374317575778000 T^{14} - 494490160426185000 T^{12} + 7083936614255886000 T^{10} \\
& - 27625636046797346250 T^8 + 58715278628789475000 T^6 + 100296773020980112500 T^4 \\
& + 109063413373951575000 T^2 + 48008615346625106250, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 43953840 T^{26} \\
& - 1085994000 T^{24} + 525422570400 T^{22} + 283015101600 T^{20} - 440867272230000 T^{18} \\
& - 70625220688638000 T^{16} - 1840700971022760000 T^{14} + 23136462127298520000 T^{12} \\
& - 77288502059715870000 T^{10} + 203718051732368250000 T^8 + 1193114598558012900000 T^6 \\
& + 382667439331138500000 T^4 - 1778554070753402250000 T^2 + 450583593605664750000, \quad \mathbf{g}_{13} = 0, \\
& \mathbf{g}_{12} = 23604840 T^{28} + 923030640 T^{26} + 375409452600 T^{24} + 2624172616800 T^{22} \\
& - 1510226919293400 T^{20} - 169673051037798000 T^{18} - 1533701946518973000 T^{16} \\
& + 60171290506320840000 T^{14} - 81243452836954485000 T^{12} + 989384777473422330000 T^{10} \\
& + 9218597475208223025000 T^8 + 11767748128231995900000 T^6 - 40367065478563333125000 T^4 \\
& - 8425754243991488250000 T^2 + 2097282775643141625000, \quad \mathbf{g}_{11} = 0, \quad \mathbf{g}_{10} = 10093104 T^{30} \\
& + 1259753040 T^{28} + 242036287920 T^{26} + 3006820731600 T^{24} - 1903047548547600 T^{22} \\
& - 177522454448262000 T^{20} + 1673066563892262000 T^{18} + 87528371510322666000 T^{16} \\
& + 6620696554494330000 T^{14} + 5807115658938263670000 T^{12} + 23239380295474781130000 T^{10} \\
& + 137613317496661002150000 T^8 - 210318339990236489550000 T^6 + 11833548606284154750000 T^4 \\
& - 83099086894146534750000 T^2 - 9948156863136203250000, \quad \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 3357585 T^{32} \\
& + 788855760 T^{30} + 133834793400 T^{28} + 2706551593200 T^{26} - 1301950524262500 T^{24} \\
& - 74974735386558000 T^{22} + 3528441325033317000 T^{20} - 9148978382135130000 T^{18} \\
& - 917328476828617796250 T^{16} + 20331186148107969750000 T^{14} + 69701007067098023925000 T^{12} \\
& + 887836329346915768050000 T^{10} - 517772523069859235062500 T^8 + 2152850692307163675750000 T^6 \\
& - 639511342714258678125000 T^4 - 797615544339523428750000 T^2 - 21263091022015865859375, \quad \mathbf{g}_7 = 0, \\
& \mathbf{g}_6 = 837900 T^{34} + 311725260 T^{32} + 57825260640 T^{30} + 2004411679200 T^{28} - 486305908326000 T^{26} \\
& - 2096884640214000 T^{24} + 2746634505228180000 T^{22} + 35193597983772228000 T^{20} \\
& + 535094835158922885000 T^{18} + 12830771213501350725000 T^{16} + 38302174679039555130000 T^{14} \\
& + 2069806537668865079700000 T^{12} - 2878103885627077118550000 T^{10} + 13752994933828011188250000 T^8 \\
& + 8319513974090622187500000 T^6 - 11861116178590458202500000 T^4
\end{aligned}$$

$$\begin{aligned}
&+1576218096385317649687500T^2 - 26235193683001885312500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 147630T^{36} \\
&+79512300T^{34} + 17590409550T^{32} + 1021528519200T^{30} - 81006309657000T^{28} + 1634306691906000T^{26} \\
&+606189343425435000T^{24} + 11528510197530060000T^{22} - 396162299973691597500T^{20} \\
&+1479830691395427525000T^{18} + 581811329651584456462500T^{16} + 1247923055532263071500000T^{14} \\
&+3842659299741460765875000T^{12} + 7389770241237094037250000T^{10} + 28258267252466153739375000T^8 \\
&-57116889114356746867500000T^6 + 3389031680327926024218750T^4 + 2208926709003114942187500T^2 \\
&+110589958773237874218750, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 16380T^{38} + 12074580T^{36} + 3288316500T^{34} \\
&+300491005500T^{32} - 226761670800T^{30} - 2504351229534000T^{28} - 460421913549654000T^{26} \\
&-26192900747252610000T^{24} - 110536335706300875000T^{22} + 11670003738602023815000T^{20} \\
&+417205004840944325775000T^{18} + 373915680879582366525000T^{16} + 8208451018603349340750000T^{14} \\
&+97521767433772016816250000T^{12} - 248372033459815147632750000T^{10} \\
&-439480474711801012586250000T^8 + 50756673874987829955937500T^6 - 11653596434883793654687500T^4 \\
&+6592310529469634320312500T^2 + 60321795694493385937500, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 861T^{40} \\
&+835380T^{38} + 282059190T^{36} + 38110734900T^{34} + 757788568425T^{32} - 357432351318000T^{30} \\
&-33079684204539000T^{28} - 671694631354782000T^{26} + 47369156210904386250T^{24} \\
&+1080980353533726135000T^{22} + 57329971310306073802500T^{20} + 1450451085746969465775000T^{18} \\
&+5735064780371479139606250T^{16} + 31970579440715276303250000T^{14} - 70064960713363893382875000T^{12} \\
&-5517764293121965095750000T^{10} - 203570309777817170932734375)T^8 - 20966515699057134432187500T^6 \\
&+42238183085222761596093750)T^4 + 1146114118195374332812500)T^2 - 6463049538695719921875,
\end{aligned}$$

$$\begin{aligned}
\mathbf{h}_{42} &= 0, \quad \mathbf{h}_{41} = 0, \quad \mathbf{h}_{40} = 21T, \quad \mathbf{h}_{39} = 0, \quad \mathbf{h}_{38} = 420T^3 - 1260T, \quad \mathbf{h}_{37} = 0, \quad \mathbf{h}_{36} = 3990T^5 \\
&-72660T^3 - 59850T, \quad \mathbf{h}_{35} = 0, \quad \mathbf{h}_{34} = 23940T^7 - 1031940T^5 + 1190700T^3 - 4365900T, \quad \mathbf{h}_{33} = 0, \\
\mathbf{h}_{32} &= 101745T^9 - 7646940T^7 + 55526310T^5 + 6747300T^3 - 308688975T, \quad \mathbf{h}_{31} = 0, \quad \mathbf{h}_{30} = 325584T^{11} \\
&-36414000T^9 + 606039840T^7 - 487075680T^5 - 442940400T^3 - 15569593200T, \quad \mathbf{h}_{29} = 0, \\
\mathbf{h}_{28} &= 813960T^{13} - 122608080T^{11} + 3460174200T^9 - 15134364000T^7 - 6941781000T^5 + 41650686000T^3 \\
&-133275051000T, \quad \mathbf{h}_{27} = 0, \quad \mathbf{h}_{26} = 1627920T^{15} - 307676880T^{13} + 12556328400T^{11} \\
&-123859789200T^9 - 414363600T^7 + 625617594000T^5 + 14773276854000T^3 + 40215059010000T, \\
\mathbf{h}_{25} &= 0, \quad \mathbf{h}_{24} = 2645370T^{17} - 593676720T^{15} + 31525767000T^{13} - 527395957200T^{11} \\
&+752464849500T^9 + 2323222398000T^7 + 161180546247000T^5 + 1150723795410000T^3 \\
&+884942275376250T, \quad \mathbf{h}_{23} = 0, \quad \mathbf{h}_{22} = 3527160T^{19} - 897198120T^{17} + 57072768480T^{15} \\
&-1415227917600T^{13} + 5710878205200T^{11} - 31038334110000T^9 + 1020560829372000T^7 \\
&+10607242786812000T^5 - 10549431670365000T^3 + 21705296078655000T, \quad \mathbf{h}_{21} = 0, \\
\mathbf{h}_{20} &= 3879876T^{21} - 1072071000T^{19} + 75633978420T^{17} - 2635403934240T^{15} \\
&+18071080057800T^{13} - 232668433443600T^{11} + 3887539470045000T^9 + 73903432233708000T^7 \\
&-332535925127983500T^5 - 35419823925255000T^3 + 373647041815162500T, \quad \mathbf{h}_{19} = 0, \\
\mathbf{h}_{18} &= 3527160T^{23} - 1014893880T^{21} + 72635963400T^{19} - 3661723989000T^{17} + 22961577870000T^{15} \\
&-558100069758000T^{13} + 8233601126058000T^{11} + 401324717365110000T^9 - 2629541354603625000T^7 \\
&+1891946762885025000T^5 - 2077818312668175000T^3 + 4696174277313750000T, \quad \mathbf{h}_{17} = 0, \\
\mathbf{h}_{16} &= 2645370T^{25} - 756854280T^{23} + 47845898100T^{21} - 4100304839400T^{19} - 18503101143450T^{17} \\
&-1692739675242000T^{15} - 39034514859417000T^{13} + 1908000279128790000T^{11} \\
&-13226094269284916250T^9 + 41333515336516095000T^7 - 3093451287652687500T^5 \\
&-152813978846731125000T^3 - 136885438789385343750)T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 1627920T^{27} \\
&-437739120T^{25} + 17295314400T^{23} - 4002542359200T^{21} - 118543877298000T^{19} \\
&-5096839239954000T^{17} + 6829841768904000T^{15} + 8476647700225320000T^{13} \\
&-39662003312572410000T^{11} + 198591885419246550000T^9 + 338676425725106700000T^7 \\
&-1458984265260221700000T^5 - 3931624238736174750000T^3 - 10561760280833107500000T,
\end{aligned}$$

$$\begin{aligned}
& \mathbf{h}_{13} = 0, \quad \mathbf{h}_{12} = 813960 T^{29} - 190123920 T^{27} - 2343879720 T^{25} - 3521465085600 T^{23} \\
& - 214336185247800 T^{21} - 7685261724294000 T^{19} + 559933161897879000 T^{17} \\
& + 18300777177408264000 T^{15} - 67319404622929305000 T^{13} + 702028432394961210000 T^{11} \\
& + 965256209626556925000 T^9 - 5016221069075993700000 T^7 - 41028109496754204825000 T^5 \\
& + 5088504256048347750000 T^3 - 7659505278106864875000 T, \quad \mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 325584 T^{31} \\
& - 57834000 T^{29} - 7480453680 T^{27} - 2671612327440 T^{25} - 227871922345200 T^{23} - 4515437156907600 T^{21} \\
& + 1063591395097770000 T^{19} + 15434675218020366000 T^{17} - 89580321136341882000 T^{15} \\
& + 2041183360309213890000 T^{13} + 4723190632996431750000 T^{11} + 11664100345373957250000 T^9 \\
& - 254778861744583996050000 T^7 + 117211546169871233850000 T^5 - 26289299400480443250000 T^3 \\
& - 19086551752323651750000 T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 101745 T^{33} - 10024560 T^{31} - 5022145800 T^{29} \\
& - 1608119730000 T^{27} - 161174098606500 T^{25} + 335632587066000 T^{23} + 736078113424269000 T^{21} \\
& - 7565148948169890000 T^{19} - 106455713823272756250 T^{17} + 5811505024293841950000 T^{15} \\
& + 6779365489015829325000 T^{13} + 154597964806237664250000 T^{11} - 1526956495540039742062500 T^9 \\
& + 1322866554468445428750000 T^7 - 1665063939990978883125000 T^5 + 333570867196457756250000 T^3 \\
& + 248146906256610376640625 T, \quad \mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 23940 T^{35} + 149940 T^{33} - 1927346400 T^{31} \\
& - 708549055200 T^{29} - 79214033883600 T^{27} + 823565301174000 T^{25} + 200617502001084000 T^{23} \\
& - 8279951353262820000 T^{21} - 77783883635196585000 T^{19} + 2988383002724206395000 T^{17} \\
& + 90398497755259039500000 T^{15} - 415079152248697744500000 T^{13} - 6264634434444242183250000 T^{11} \\
& - 711055582862691008250000 T^9 - 1756424787565278253500000 T^7 - 3323890524236676817500000 T^5 \\
& + 7431581396973548646562500 T^3 + 510341541351824995312500 T, \quad \mathbf{h}_5 = 0, \quad \mathbf{h}_4 = 3990 T^{37} \\
& + 555660 T^{35} - 451870650 T^{33} - 210601490400 T^{31} - 27164808405000 T^{29} - 494049363822000 T^{27} \\
& - 131254421193000 T^{25} - 2331351766748340000 T^{23} - 57607676795659567500 T^{21} \\
& + 3182842545894136125000 T^{19} + 121471496093605901512500 T^{17} - 730868754303114203700000 T^{15} \\
& - 3952170770249132825625000 T^{13} - 18817383674105947680750000 T^{11} - 24600927446914993453125000 T^9 \\
& - 62084596445709395827500000 T^7 + 38962039336837387479843750 T^5 + 5862704048212428604687500 T^3 \\
& + 690828184025031396093750 T, \quad \mathbf{h}_3 = 0, \quad \mathbf{h}_2 = 420 T^{39} + 127260 T^{37} - 59773140 T^{35} - 37389170700 T^{33} \\
& - 6082652847600 T^{31} - 456491304234000 T^{29} - 20958512959818000 T^{27} + 1456163148715002000 T^{25} \\
& + 201140982748216035000 T^{23} + 4864337674818781005000 T^{21} + 60289385834313075825000 T^{19} \\
& - 641663910397442064825000 T^{17} + 3860956945797085030050000 T^{15} + 17024232728719929462750000 T^{13} \\
& - 174686108305029234779250000 T^{11} - 133153380406498251318750000 T^9 \\
& + 252883712879152705566562500 T^7 - 66291355495078917111562500 T^5 + 12434907312450565129687500 T^3 \\
& - 784183344028414017187500 T, \quad \mathbf{h}_1 = 0, \quad \mathbf{h}_0 = 21 T^{41} + 10500 T^{39} - 3356010 T^{37} \\
& - 2977396380 T^{35} - 678988637775 T^{33} - 67009019137200 T^{31} - 1544931642555000 T^{29} \\
& + 190273318160058000 T^{27} + 15779576666742320250 T^{25} + 302225493684212595000 T^{23} \\
& + 10750227730088000962500 T^{21} + 163163976757378009875000 T^{19} - 1345334461618984846143750 T^{17} \\
& - 10403032879102451430150000 T^{15} - 115925574335468992738875000 T^{13} \\
& - 52103299471386560916750000 T^{11} - 86323335508369995943359375 T^9 + 255531634533965133599062500 T^7 \\
& + 85352755687892662813593750 T^5 + 1157603984041944501562500 T^3 - 196045836007103504296875 T
\end{aligned}$$

$$\begin{aligned}
& \mathbf{q}_{42} = 1, \quad \mathbf{q}_{41} = 0, \quad \mathbf{q}_{40} = 21 T^2 + 21, \quad \mathbf{q}_{39} = 0, \quad \mathbf{q}_{38} = 210 T^4 - 1260 T^2 + 1890, \quad \mathbf{q}_{37} = 0, \\
& \mathbf{q}_{36} = 1330 T^6 - 24570 T^4 + 10710 T^2 + 193410, \quad \mathbf{q}_{35} = 0, \quad \mathbf{q}_{34} = 5985 T^8 - 202860 T^6 + 595350 T^4 \\
& + 1984500 T^2 + 18356625, \quad \mathbf{q}_{33} = 0, \quad \mathbf{q}_{32} = 20349 T^{10} - 1012095 T^8 + 8136450 T^6 + 17662050 T^4 \\
& + 215219025 T^2 + 1383295725, \quad \mathbf{q}_{31} = 0, \quad \mathbf{q}_{30} = 54264 T^{12} - 3444336 T^{10} + 53290440 T^8 - 20109600 T^6 \\
& + 1912264200 T^4 + 15369555600 T^2 + 65128908600, \quad \mathbf{q}_{29} = 0, \quad \mathbf{q}_{28} = 116280 T^{14} - 8439480 T^{12} \\
& + 217024920 T^{10} - 694575000 T^8 + 14832153000 T^6 + 133703703000 T^4 + 386251173000 T^2 \\
& + 1034587323000, \quad \mathbf{q}_{27} = 0, \quad \mathbf{q}_{26} = 203490 T^{16} - 15165360 T^{14} + 622815480 T^{12}
\end{aligned}$$

$$\begin{aligned}
& -2210256720 T^{10} + 56135551500 T^8 + 1192724190000 T^6 + 1433519451000 T^4 - 27693705438000 T^2 \\
& + 38780976953250, \quad \mathbf{q}_{25} = 0, \quad \mathbf{q}_{24} = 293930 T^{18} - 19631430 T^{16} + 1366583400 T^{14} \\
& + 2951613000 T^{12} + 363193267500 T^{10} + 10089021379500 T^8 + 22829041053000 T^6 \\
& - 475525632015000 T^4 + 342500315456250 T^2 + 1376457162806250, \quad \mathbf{q}_{23} = 0, \quad \mathbf{q}_{22} = 352716 T^{20} \\
& - 16521960 T^{18} + 2444960700 T^{16} + 41186426400 T^{14} + 2435977719000 T^{12} + 35777824966800 T^{10} \\
& + 398543162031000 T^8 - 3822883567020000 T^6 + 7873515567697500 T^4 + 16086079864575000 T^2 \\
& + 38577274289347500, \quad \mathbf{q}_{21} = 0, \quad \mathbf{q}_{20} = 352716 T^{22} - 4288284 T^{20} + 3742158420 T^{18} \\
& + 149684881500 T^{16} + 9553331403000 T^{14} + 212261247613800 T^{12} + 3849610940821800 T^{10} \\
& - 15394646399517000 T^8 + 71758478683237500 T^6 + 204046725448912500 T^4 + 436091149100722500 T^2 \\
& + 674506306475347500, \quad \mathbf{q}_{19} = 0, \quad \mathbf{q}_{18} = 293930 T^{24} + 11695320 T^{22} + 5003418420 T^{20} \\
& + 334607427000 T^{18} + 23101428561750 T^{16} + 807591346254000 T^{14} + 16185920631111000 T^{12} \\
& - 8352665505954000 T^{10} + 576663818982693750 T^8 + 1242059144478075000 T^6 + 5668373975877112500 T^4 \\
& + 3761501085532575000 T^2 + 4325812023235106250, \quad \mathbf{q}_{17} = 0, \quad \mathbf{q}_{16} = 203490 T^{26} + 22694490 T^{24} \\
& + 5786087580 T^{22} + 534631523580 T^{20} + 37855900293750 T^{18} + 1433816319300750 T^{16} \\
& + 1641456285777000 T^{14} - 265649781021471000 T^{12} + 5360993203091352750 T^{10} + 6909604613062593750 T^8 \\
& + 80880110043223837500 T^6 + 42236970373512337500 T^4 - 113560599693873543750 T^2 \\
& + 60012457876086581250, \quad \mathbf{q}_{15} = 0, \quad \mathbf{q}_{14} = 116280 T^{28} + 23904720 T^{26} + 5624073000 T^{24} \\
& + 647566164000 T^{22} + 45746991675000 T^{20} + 1145438659134000 T^{18} - 60947538804999000 T^{16} \\
& - 613314917245800000 T^{14} + 13256411880737025000 T^{12} + 146890015244661150000 T^{10} \\
& + 484908475289220675000 T^8 + 1198562394651167700000 T^6 - 2003316012774759375000 T^4 \\
& + 716725088677001250000 T^2 + 708119334693106875000, \quad \mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = 54264 T^{30} \\
& + 17607240 T^{28} + 4448719800 T^{26} + 609209483400 T^{24} + 44277505939800 T^{22} + 412689587725800 T^{20} \\
& - 84804973421553000 T^{18} - 250432798766535000 T^{16} + 22882838948853345000 T^{14} \\
& + 466654428328441815000 T^{12} + 3892228471345297545000 T^{10} + 20229343597701089775000 T^8 \\
& - 11143853714235193875000 T^6 + 11154104591287840875000 T^4 + 10381186823997961125000 T^2 \\
& + 5208039143851957875000, \quad \mathbf{q}_{11} = 0, \quad \mathbf{q}_{10} = 20349 T^{32} + 9579024 T^{30} + 2778050520 T^{28} \\
& + 446310874800 T^{26} + 36326975409900 T^{24} + 602640375094800 T^{22} - 16736996613922200 T^{20} \\
& + 1698321128858046000 T^{18} + 59188148168115408750 T^{16} + 972699066504389070000 T^{14} \\
& + 10020895210571390145000 T^{12} + 56146858034606462970000 T^{10} + 104753437906970047387500 T^8 \\
& + 83635145538228726750000 T^6 + 196955754351328578375000 T^4 + 58611690264525032250000 T^2 \\
& + 9836753798777262328125, \quad \mathbf{q}_9 = 0, \quad \mathbf{q}_8 = 5985 T^{34} + 3871665 T^{32} + 1327301640 T^{30} \\
& + 250676937000 T^{28} + 24928524967500 T^{26} + 1277755297843500 T^{24} + 75968161579863000 T^{22} \\
& + 3075359436148923000 T^{20} + 26011936101356493750 T^{18} + 1168303051165440093750 T^{16} \\
& + 25639663936063282875000 T^{14} - 68682733785579184425000 T^{12} - 266323431588369421612500 T^{10} \\
& + 2701519751233743319687500 T^8 + 1281120041892573815625000 T^6 + 1364503639750540183125000 T^4 \\
& - 323501276433699733359375 T^2 + 42533021249892785390625, \quad \mathbf{q}_7 = 0, \quad \mathbf{q}_6 = 1330 T^{36} \\
& + 1137780 T^{34} + 466317810 T^{32} + 104251764960 T^{30} + 13208232681000 T^{28} + 1193020745742000 T^{26} \\
& + 99210798874341000 T^{24} + 5805544496212692000 T^{22} + 214383530642638549500 T^{20} \\
& + 4530497152000032675000 T^{18} + 46797298481837387137500 T^{16} - 217436851956849317100000 T^{14} \\
& + 19692526781721872925000 T^{12} + 5336577984490937657550000 T^{10} + 21530470535065521500625000 T^8 \\
& + 32380484598451903567500000 T^6 - 4144298861975805116718750 T^4 + 730946965605972235312500 T^2 \\
& + 125335286609669590781250, \quad \mathbf{q}_5 = 0, \quad \mathbf{q}_4 = 210 T^{38} + 230790 T^{36} + 113314950 T^{34} \\
& + 30177299250 T^{32} + 4848169221000 T^{30} + 561695828967000 T^{28} + 50626283202243000 T^{26} \\
& + 3225603873293865000 T^{24} + 122692742315543137500 T^{22} + 2347942991384156512500 T^{20} \\
& + 36794456407482655612500 T^{18} + 266523839860686468637500 T^{16} - 90441117924876093375000 T^{14}
\end{aligned}$$

$$\begin{aligned}
& +5001462949574321566875000T^{12} + 59697458269849518339375000T^{10} + 55588353961234695418125000)T^8 \\
& +62717528472385320873281250T^6 + 2437287792703697046093750T^4 + 2506226987783118058593750)T^2 \\
& +50268163078744488281250, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 21T^{40} + 28980T^{38} + 17000550T^{36} + 5412525300T^{34} \\
& +1069516209825T^{32} + 138706771971600T^{30} + 10810972830861000T^{28} + 389665850910210000T^{26} \\
& -659267012964183750T^{24} - 454465851534216585000T^{22} - 920416585288570177500T^{20} \\
& +221631586055760347775000T^{18} + 5984763190950898799156250T^{16} + 32444042402395142615250000T^{14} \\
& +135661236341135119894125000T^{12} - 119473149759596572227750000T^{10} \\
& -60538714786788425248359375T^8 + 254152960059670490207812500T^6 \\
& +73869783760830436164843750T^4 - 1749332075140308192187500T^2 \\
& +45241346770870039453125, \quad \mathbf{q}_1 = 0, \quad \mathbf{q}_0 = T^{42} + 1701T^{40} + 1186290T^{38} + 451782450T^{36} \\
& +105515964225T^{34} + 15941427079725T^{32} + 1619444848404600T^{30} + 114621189126003000T^{28} \\
& +5904248149699121250T^{26} + 207513330336568226250T^{24} + 5039110473448289647500T^{22} \\
& +110469853983534408247500T^{20} + 1375703927511930449606250)T^{18} + 3784181946524764546781250T^{16} \\
& +36528049590131262499875000T^{14} + 218877641452798644652875000T^{12} \\
& +254716356056568852957328125T^{10} + 439843410849229547791640625T^8 \\
& +44121563595239721757031250T^6 + 29200057815827262607031250T^4 \\
& +1251677260660737758203125T^2 + 2154349846231906640625
\end{aligned}$$

4. Case N=7

We give the coefficients of the polynomials G , H and Q

$$\begin{aligned}
& \alpha_7 = 4, \quad \mathbf{g}_{56} = 0, \quad \mathbf{g}_{55} = 0, \quad \mathbf{g}_{54} = 28, \quad \mathbf{g}_{53} = 0, \quad \mathbf{g}_{52} = 2268T^2 + 2268, \quad \mathbf{g}_{51} = 0, \quad \mathbf{g}_{50} = 49140T^4 \\
& -125496T^2 + 272916, \quad \mathbf{g}_{49} = 0, \quad \mathbf{g}_{48} = 573300T^6 - 8158500T^4 - 4630500T^2 + 33736500, \quad \mathbf{g}_{47} = 0, \\
& \mathbf{g}_{46} = 4422600T^8 - 145000800T^6 + 144471600T^4 - 288943200T^2 + 3645923400, \quad \mathbf{g}_{45} = 0, \\
& \mathbf{g}_{44} = 24864840T^{10} - 1405819800T^8 + 8654642640T^6 + 1328821200T^4 - 27569467800T^2 + 285603683400, \\
& \mathbf{g}_{43} = 0, \quad \mathbf{g}_{42} = 107747640T^{12} - 9018923760T^{10} + 129131097480T^8 - 92144939040T^6 \\
& -160080089400T^4 - 4068407574000T^2 + 9697930011000, \quad \mathbf{g}_{41} = 0, \quad \mathbf{g}_{40} = 372972600T^{14} \\
& -41946985080T^{12} + 1042152554520T^{10} - 3985886824920T^8 - 2424305683800T^6 - 38041300420200T^4 \\
& -532602960535800T^2 - 462639709917000, \quad \mathbf{g}_{39} = 0, \quad \mathbf{g}_{38} = 1056755700T^{16} - 148997772000T^{14} \\
& +5529226071600T^{12} - 48020254682400T^{10} + 9567791263800T^8 - 277615181844000T^6 \\
& -3997430875746000T^4 - 27691049938860000T^2 - 11433617241007500, \quad \mathbf{g}_{37} = 0, \quad \mathbf{g}_{36} = 2493390900T^{18} \\
& -417526886700T^{16} + 21097721106000T^{14} - 312305697673200T^{12} + 750925867368600T^{10} \\
& -1296117088497000T^8 - 19549525119894000T^6 + 178015776175314000T^4 + 2547484073859532500T^2 \\
& -257622602070487500, \quad \mathbf{g}_{35} = 0, \quad \mathbf{g}_{34} = 4960535580T^{20} - 942501760200T^{18} \\
& +60895186381260T^{16} - 1320509170965600T^{14} + 8073640409095800T^{12} - 10433330397428400T^{10} \\
& -94059388591245000T^8 + 4212714864528324000T^6 + 81525901261652347500T^4 \\
& +83880191483144595000T^2 + 4532925200381077500, \quad \mathbf{g}_{33} = 0, \quad \mathbf{g}_{32} = 8396404380T^{22} \\
& -1736314646220T^{20} + 137179010388420T^{18} - 3955742649118260T^{16} + 44742788370264600T^{14} \\
& -87890985649467000T^{12} - 581266358785035000T^{10} + 34489428486040143000T^8 \\
& +916422481982789743500T^6 - 267128299891396597500T^4 + 3533027762243314912500T^2 \\
& +1052175273585966337500, \quad \mathbf{g}_{31} = 0, \quad \mathbf{g}_{30} = 12168702000T^{24} - 2628289863360T^{22} \\
& +246156751098720T^{20} - 8802207660818880T^{18} + 156743234946421200T^{16} \\
& -400285925208892800T^{14} - 4082398971145387200T^{12} + 182299012392364560000T^{10} \\
& +6993601895564462178000T^8 - 29136271554263384280000T^6 + 22705339934145925980000T^4 \\
& +133644440887635613320000T^2 + 69720100670714536350000, \quad \mathbf{g}_{29} = 0, \quad \mathbf{g}_{28} = 15164074800T^{26} \\
& -3271508730000T^{24} + 356805698608800T^{22} - 14963164581103200T^{20} + 373275824011952400T^{18} \\
& -1646661408941934000T^{16} - 23044246961534568000T^{14} + 647137266547990680000T^{12} \\
& +44440332791868485010000T^{10} - 368524266068389959990000T^8 + 515157395379005679300000T^6 \\
& +964818990199979907300000T^4 + 3432008280166633455750000T^2 + 2522450003846320374750000, \\
& \mathbf{g}_{27} = 0, \quad \mathbf{g}_{26} = 16287339600T^{28} - 3325987072800T^{26} + 422344463612400T^{24} \\
& -19765833506798400T^{22} + 609616438648232400T^{20} - 8426887778118348000T^{18} \\
& -198426421456868682000T^{16} - 2190017982667295088000T^{14} + 260606376243375811110000T^{12} \\
& -2667540176320654889100000T^{10} + 8847209138357814736050000T^8 + 13096014915145148494200000T^6 \\
& +6581743865224894916550000T^4 + 11025412640863669471500000T^2 + 47293802784572214197250000, \\
& \mathbf{g}_{25} = 0, \quad \mathbf{g}_{24} = 15089190480T^{30} - 2709127486800T^{28} + 412978164031440T^{26} - 20496272265378000T^{24} \\
& +625906405138669200T^{22} - 35718951161305011600T^{20} - 1333506673014425910000T^{18} \\
& -34418796637312024938000T^{16} + 1244339447399806908150000T^{14} - 11990961186295394893590000T^{12} \\
& +65945519948476262964870000T^{10} + 160150242555541152164250000T^8 \\
& +91138495041978507273150000T^6 - 1554908002287761890518750000T^4 \\
& -2392952896647698174039250000T^2 + 889739092856294586155250000, \quad \mathbf{g}_{23} = 0, \\
& \mathbf{g}_{22} = 12047014980T^{32} - 1685346505920T^{30} + 339670088966880T^{28} - 16767007875455040T^{26} \\
& +195015256305433200T^{24} - 103115102184824184000T^{22} - 4963164090380892396000T^{20} \\
& -73350016919927243496000T^{18} + 5521291279776971601903000T^{16} - 39856907136517355724840000T^{14} \\
& +376003222446071990870100000T^{12} + 718852738860427750979400000T^{10} \\
& +3176243015946106500944550000T^8 - 17990585286630587798567400000T^6 \\
& -67776208060401761741320500000T^4 - 27136987658593437499875000000T^2
\end{aligned}$$

$$\begin{aligned}
& +12087940146214297919616562500, \quad \mathbf{g}_{21} = 0, \quad \mathbf{g}_{20} = 8267559300 T^{34} - 689768832060 T^{32} \\
& +242818935559200 T^{30} - 10771451892321120 T^{28} - 609328959012219600 T^{26} \\
& -203253573603293413200 T^{24} - 10395034180133435632800 T^{22} + 165116685980326092012000 T^{20} \\
& +17170916828136913280439000 T^{18} - 102607297371198744509385000 T^{16} \\
& +1635023327366588192773740000 T^{14} + 2860768497736673150887260000 T^{12} \\
& +45606671281786995394764750000 T^{10} - 80947367097903820900836450000 T^8 \\
& -700286840100765740788465500000 T^6 + 130801873396469808053020500000 T^4 \\
& -447283391478568632911861437500 T^2 + 56768583214608413567481562500, \quad \mathbf{g}_{19} = 0, \\
& \mathbf{g}_{18} = 4855550700 T^{36} - 40277853000 T^{34} + 158719690968300 T^{32} - 5194244162923200 T^{30} \\
& -1360528622797251600 T^{28} - 281596511270242476000 T^{26} - 11850683160765907458000 T^{24} \\
& +920175424144902165240000 T^{22} + 25468628785050976429905000 T^{20} \\
& -310836193534541858413230000 T^{18} + 5807378097843796606757625000 T^{16} \\
& +15634234802719441673364600000 T^{14} + 278887408065007528609572750000 T^{12} \\
& +52000683646906354993132500000 T^{10} - 4439081082994819230111392250000 T^8 \\
& +8438816045170493665968285000000 T^6 - 3001122035235356030511325312500 T^4 \\
& -7391743593040300996805428125000 T^2 - 1054152239431812041879170312500, \\
& \mathbf{g}_{17} = 0, \quad \mathbf{g}_{16} = 2424321900 T^{38} + 211776711300 T^{36} + 99477568505700 T^{34} \\
& -1389209187712500 T^{32} - 1621264449868232400 T^{30} - 280260753940566630000 T^{28} \\
& -4787780690058717582000 T^{26} + 1656692481162132035622000 T^{24} + 5482826579394141962985000 T^{22} \\
& -565790679793850662876725000 T^{20} + 14714148688801952105590875000 T^{18} \\
& +102902614988568912796304025000 T^{16} + 981244617028989252062473350000 T^{14} \\
& +3169479578193458904435311250000 T^{12} - 24931373650539092246792031750000 T^{10} \\
& +12085551247727524652824418750000 T^8 - 53956818495735676912368790312500 T^6 \\
& -74996679144078506620333965937500 T^4 - 61056959125851227464611623437500 T^2 \\
& -5245972489497170017957682812500, \quad \mathbf{g}_{15} = 0, \quad \mathbf{g}_{14} = 1019458440 T^{40} + 209361952800 T^{38} \\
& +59799859202160 T^{36} + 597930018285600 T^{34} - 1327830040347010200 T^{32} - 203680458354584534400 T^{30} \\
& +5023675724251310280000 T^{28} + 1627805433319326998352000 T^{26} - 25078906119364614947430000 T^{24} \\
& -752646251179376198484840000 T^{22} + 17704585224888137245690020000 T^{20} \\
& +466071240006558080789895000000 T^{18} + 2225469927681285651457141050000 T^{16} \\
& +24422914134784165137114978000000 T^{14} - 209128252918688749663068051000000 T^{12} \\
& +868839416613022428214659018000000 T^{10} + 38899879539858038162626790625000 T^8 \\
& -954575061914762846454243847500000 T^6 - 411166271799374891645351756250000 T^4 \\
& +783333878093152389727736512500000 T^2 - 20793848949925378145840896875000, \quad \mathbf{g}_{13} = 0, \\
& \mathbf{g}_{12} = 356396040 T^{42} + 123877270440 T^{40} + 32610284395920 T^{38} + 1216306932881040 T^{36} \\
& -784563891537195000 T^{34} - 110761814779398783000 T^{32} + 7937796416339529432000 T^{30} \\
& +978775172774012225304000 T^{28} - 21292516324448140972662000 T^{26} + 161923476389289550585650000 T^{24} \\
& +34432591861323164507298300000 T^{22} + 897588666218543018306687100000 T^{20} \\
& +10399172873750652367819292850000 T^{18} + 12734590641823469076245037450000 T^{16} \\
& -1056247157487187710224365185000000 T^{14} + 1832056249038285845965084695000000 T^{12} \\
& +2959286708382169575258073850625000 T^{10} - 344028559792148330017460416875000 T^8 \\
& -12140076235349263409707132383750000 T^6 + 21659795132230009469150342156250000 T^4 \\
& +3602055625460117946415936265625000 T^2 + 61049167368118769138877703125000, \quad \mathbf{g}_{11} = 0, \\
& \mathbf{g}_{10} = 101719800 T^{44} + 52472463120 T^{42} + 15005352063240 T^{40} + 1021506129594720 T^{38}
\end{aligned}$$

$$\begin{aligned}
& -331006205910303000 T^{36} - 47731541013103978800 T^{34} + 3909120362191961947800 T^{32} \\
& + 302275195306483257936000 T^{30} - 3397149051079935164706000 T^{28} \\
& + 1063471996099453676509620000 T^{26} + 35282865975833690856131490000 T^{24} \\
& + 1009461102866985900033662760000 T^{22} + 28108708355943404795842686450000 T^{20} \\
& - 85747360020983326179491877900000 T^{18} - 1491463246568322076250046095250000 T^{16} \\
& + 2775278138548494265001581266000000 T^{14} - 18540128543730526297289281181625000 T^{12} \\
& - 21944626454220073070250932898750000 T^{10} - 80973913998934031957478576354375000 T^8 \\
& + 148114784421877559463874334137500000 T^6 + 31126078957047574894186796128125000 T^4 \\
& + 20673599609280586675821470606250000 T^2 + 1398479083269650104689513421875000, \\
& \mathbf{g}_9 = 0, \quad \mathbf{g}_8 = 23095800 T^{46} + 16444701000 T^{44} + 5475036936600 T^{42} \\
& + 566976092898600 T^{40} - 93441539643190200 T^{38} - 17935367491764261000 T^{36} \\
& - 34358185653025311000 T^{34} - 84340417236266770425000 T^{32} - 6247372784431620892050000 T^{30} \\
& + 79423291493938895975010000 T^{28} - 6517484194549893449935050000 T^{26} \\
& + 1157932673902046452477117050000 T^{24} + 48145337637619603341498788250000 T^{22} \\
& - 238274280945074139753039302250000 T^{20} + 1614124491632111637086543885250000 T^{18} \\
& + 30463748856261460407343234308750000 T^{16} - 118755251721826815926043684095625000 T^{14} \\
& - 48773592689963392959811287159375000 T^{12} - 1738438081260949813862744404903125000 T^{10} \\
& + 1572448964546520061265933822278125000 T^8 - 673017920052474095253612817734375000 T^6 \\
& + 326795486450765136002224260609375000 T^4 + 79407567379987217482876217578125000 T^2 \\
& + 2882552147106173292709630546875000, \quad \mathbf{g}_7 = 0, \quad \mathbf{g}_6 = 4013100 T^{48} \\
& + 3764023200 T^{46} + 1497943465200 T^{44} + 217159761218400 T^{42} - 14221439411394600 T^{40} \\
& - 6062992711379604000 T^{38} - 868091861569403394000 T^{36} - 137559109216434396876000 T^{34} \\
& - 9260888323767068417467500 T^{32} + 83092589684282324500200000 T^{30} \\
& + 45951189509544653085374700000 T^{28} + 2043751713304237133419114200000 T^{26} \\
& + 39823148838347256006248674050000 T^{24} - 107645574930283654095040209000000 T^{22} \\
& + 5704220460549886178354194243500000 T^{20} + 47477860076491575514786707705000000 T^{18} \\
& + 96760004701426703802202262537812500 T^{16} - 2746609454519613461575760731267500000 T^{14} \\
& - 5055092442696255371030072885681250000 T^{12} + 7685340668503114406318287145887500000 T^{10} \\
& - 2527654302854142287937009789515625000 T^8 + 2549540272922798483165127227437500000 T^6 \\
& + 1681003830938891487377650144593750000 T^4 - 119038912584481624295814690937500000 T^2 \\
& + 4248891856349740702685512617187500, \quad \mathbf{g}_5 = 0, \quad \mathbf{g}_4 = 501228 T^{50} + 599186700 T^{48} \\
& + 286948221840 T^{46} + 55465811326800 T^{44} + 225191978134200 T^{42} - 1592867890825734600 T^{40} \\
& - 378975454715995830000 T^{38} - 49698971523700677486000 T^{36} - 2416070416982018904607500 T^{34} \\
& + 163449545090537650746532500 T^{32} + 22630419780133976885691540000 T^{30} \\
& + 658066531416844960184890500000 T^{28} + 17958744056622134874813046650000 T^{26} \\
& + 265938368682687944669400224250000 T^{24} + 3373546912165127652278618794500000 T^{22} \\
& + 49992632917159774293562606996500000 T^{20} + 243622840935340303792373213507812500 T^{18} \\
& - 3472965533440290793642368859414687500 T^{16} - 3909930410917269828231259536168750000 T^{14} \\
& - 2373854538016335651007933345893750000 T^{12} + 22505528586240563125522950136321875000 T^{10} \\
& + 1725433101762243379165143781171875000 T^8 + 20602166714282000488422754253531250000 T^6 \\
& - 256192442301384365332296834843750000 T^4 - 79260192530178496317997648945312500 T^2 \\
& - 3304693666049798324310954257812500, \quad \mathbf{g}_3 = 0, \quad \mathbf{g}_2 = 40068 T^{52} \\
& + 59556168 T^{50} + 34248844980 T^{48} + 8538182019360 T^{46} + 489064062475800 T^{44} \\
& - 249702181686342000 T^{42} - 69203448982691793000 T^{40} - 5213133014477381532000 T^{38} \\
& + 438714917019434320303500 T^{36} + 84931805893311124163955000 T^{34}
\end{aligned}$$

$$\begin{aligned}
& +4218956433098176838806237500 T^{32} + 31107276944713383532410600000 T^{30} \\
& +3271649949938812288254997050000 T^{28} + 201849235241947964535548861100000 T^{26} \\
& +4953109227394118516246184914250000 T^{24} - 25630541428745966760074602575000000 T^{22} \\
& -412192793582829745372981826294062500 T^{20} - 4700092011184358352745951843198125000 T^{18} \\
& +12509179325796087908708757937602187500 T^{16} + 17173693466353903144949117196862500000 T^{14} \\
& -36421201667691057461711715138515625000 T^{12} + 130348386619893149350129124043656250000 T^{10} \\
& +135527319688597478057052486959484375000 T^8 + 12269848488887082318515323846312500000 T^6 \\
& +4223241138846592268073003781757812500 T^4 - 374846681549077124214699668671875000 T^2 \\
& -1101564555349932774770318085937500, \quad \mathbf{g}_1 = 0, \quad \mathbf{g}_0 = 1540 T^{54} + 2791404 T^{52} \\
& +1916196156 T^{50} + 600986059380 T^{48} + 60893832396600 T^{46} - 14981298604738200 T^{44} \\
& -5448980120702995800 T^{42} - 677699093948317749000 T^{40} - 28109651584658009404500 T^{38} \\
& +2321568723921179741662500 T^{36} + 483153674937785992894792500 T^{34} \\
& +34432196292857178176426257500 T^{32} + 1329648591046758598027462050000 T^{30} \\
& +48030465435235520461569818550000 T^{28} + 871144519174189671772071537750000 T^{26} \\
& +862287173564925372614929829250000 T^{24} + 37733399259532601896905168972937500 T^{22} \\
& -97131347340613907133312877564687500 T^{20} - 6464853953097864723190272724680937500 T^{18} \\
& -13312771923417438005165293807457812500 T^{16} - 21734401086635138140243391915653125000 T^{14} \\
& +162602072233593709389730376910215625000 T^{12} + 57203947835898442945082247296390625000 T^{10} \\
& +95479747942182277597570867016671875000 T^8 + 4823751187877355620719222898320312500 T^6 \\
& -1973321762271862907337361717851562500 T^4 - 20929726551648722720636043632812500 T^2 \\
& +157366365049990396395759726562500,
\end{aligned}$$

$$\begin{aligned}
& \mathbf{h}_{56} = 0, \quad \mathbf{h}_{55} = 0, \quad \mathbf{h}_{54} = 28T, \quad \mathbf{h}_{53} = 0, \quad \mathbf{h}_{52} = 756 T^3 - 2268 T, \quad \mathbf{h}_{51} = 0, \quad \mathbf{h}_{50} = 9828 T^5 \\
& -181944 T^3 - 147420 T, \quad \mathbf{h}_{49} = 0, \quad \mathbf{h}_{48} = 81900 T^7 - 3660300 T^5 + 4630500 T^3 - 15214500 T, \quad \mathbf{h}_{47} = 0, \\
& \mathbf{h}_{46} = 491400 T^9 - 39160800 T^7 + 310428720 T^5 + 79380000 T^3 - 1631259000 T, \quad \mathbf{h}_{45} = 0, \\
& \mathbf{h}_{44} = 2260440 T^{11} - 275020200 T^9 + 5106674160 T^7 - 3889302480 T^5 + 392931000 T^3 - 148377889800 T, \\
& \mathbf{h}_{43} = 0, \quad \mathbf{h}_{42} = 8288280 T^{13} - 1398806640 T^{11} + 45284861160 T^9 - 207951559200 T^7 - 42200789400 T^5 \\
& +253204736400 T^3 - 8291708548200 T, \quad \mathbf{h}_{41} = 0, \quad \mathbf{h}_{40} = 24864840 T^{15} - 5449225320 T^{13} \\
& +264073498920 T^{11} - 2825524908360 T^9 + 1514969046360 T^7 + 6573185526600 T^5 + 137993987993400 T^3 \\
& +149047765713000 T, \quad \mathbf{h}_{39} = 0, \quad \mathbf{h}_{38} = 62162100 T^{17} - 16844335200 T^{15} + 1110003879600 T^{13} \\
& -20635793085600 T^{11} + 60341881030200 T^9 + 76117021596000 T^7 + 2171428651854000 T^5 \\
& +25299618434244000 T^3 + 62466943452952500 T, \quad \mathbf{h}_{37} = 0, \quad \mathbf{h}_{36} = 131231100 T^{19} \\
& -42306887700 T^{17} + 3541157207280 T^{15} - 98466907770000 T^{13} + 661042062865800 T^{11} \\
& -16499618011800 T^9 + 18885110233374000 T^7 + 314318361244734000 T^5 \\
& +1773658200690337500 T^3 + 2282181897675487500 T, \quad \mathbf{h}_{35} = 0, \quad \mathbf{h}_{34} = 236215980 T^{21} \\
& -87763321800 T^{19} + 8848450735740 T^{17} - 335891899617120 T^{15} + 3894124207743000 T^{13} \\
& -10086562817377200 T^{11} + 107633107470735000 T^9 + 2631781328889636000 T^7 \\
& +12651337028921629500 T^5 - 44712998532434925000 T^3 + 94738725025613707500 T, \quad \mathbf{h}_{33} = 0, \\
& \mathbf{h}_{32} = 365061060 T^{23} - 152080693380 T^{21} + 17658627372540 T^{19} - 861904089654300 T^{17} \\
& +14783141715388200 T^{15} - 91110040017935400 T^{13} + 397997325082942200 T^{11} \\
& +17756117603405181000 T^9 + 69992510515501612500 T^7 - 1798102986888203572500 T^5 \\
& -663927578365162072500 T^3 + 3221445733967484292500 T, \quad \mathbf{h}_{31} = 0, \quad \mathbf{h}_{30} = 486748080 T^{25} \\
& -221807355840 T^{23} + 28457171376480 T^{21} - 1718595944936640 T^{19} + 39120073399970640 T^{17} \\
& -415203708424982400 T^{15} + 817743548090145600 T^{13} + 98690112473697360000 T^{11} \\
& +525169842047690994000 T^9
\end{aligned}$$

$$\begin{aligned}
& -20729338406078319864000 T^7 + 10946551849110426780000 T^5 - 30430914937630182360000 T^3 \\
& + 57758643716593431150000 T, \quad \mathbf{h}_{29} = 0, \quad \mathbf{h}_{28} = 561632400 T^{27} - 273514978800 T^{25} \\
& + 37167025240800 T^{23} - 2726075908432800 T^{21} + 74609232551607600 T^{19} - 1241876967913890000 T^{17} \\
& + 2806282455935112000 T^{15} + 490091992910111592000 T^{13} + 4335307814553317910000 T^{11} \\
& - 150136948798788777210000 T^9 + 517137578894482137900000 T^7 - 221596077714765761700000 T^5 \\
& - 2219284940307326995950000 T^3 - 1317730766813851524750000 T, \quad \mathbf{h}_{27} = 0, \\
\mathbf{h}_{26} & = 561632400 T^{29} - 285683680800 T^{27} + 39160805609520 T^{25} - 3512223888264000 T^{23} \\
& + 102167332243376400 T^{21} - 2763398175005714400 T^{19} + 16607639793911166000 T^{17} \\
& + 1283509837177544592000 T^{15} + 33361595631780673950000 T^{13} - 845829484238863818540000 T^{11} \\
& + 5929023781997105418090000 T^9 - 2765051286836672017800000 T^7 - 27148916935817462743650000 T^5 \\
& - 115902479613006870052500000 T^3 - 104983925561520215883750000 T, \quad \mathbf{h}_{25} = 0, \\
\mathbf{h}_{24} & = 486748080 T^{31} - 252547369200 T^{29} + 32714359619760 T^{27} - 3765812506026480 T^{25} \\
& + 93108670306470000 T^{23} - 4900225415888602800 T^{21} + 73064176305750630000 T^{19} \\
& + 3317316129490661298000 T^{17} + 240092239695587184906000 T^{15} - 3820939255995521710050000 T^{13} \\
& + 36755694097751666247210000 T^{11} - 28748698362619854869250000 T^9 \\
& - 227283456202004553501150000 T^7 - 1415778815362819915057050000 T^5 \\
& - 2649841070338215122118750000 T^3 - 1615660724286293639204250000 T, \quad \mathbf{h}_{23} = 0, \\
\mathbf{h}_{22} & = 365061060 T^{33} - 188259180480 T^{31} + 20712348488160 T^{29} \\
& - 3475991858875200 T^{27} + 34258831092486000 T^{25} - 7053263233444209600 T^{23} \\
& + 254155505018359744800 T^{21} + 21963909822726159288000 T^{19} \\
& + 1097079878815670724975000 T^{17} - 13987261578728541022920000 T^{15} \\
& + 156374316828871749628020000 T^{13} - 233287710053247185295960000 T^{11} \\
& - 497446516059346978774650000 T^9 - 13650573914640909339093000000 T^7 \\
& - 22310053614024779723620500000 T^5 + 25842933923369284130409000000 T^3 \\
& - 25005235940076871386845437500 T, \quad \mathbf{h}_{21} = 0, \quad \mathbf{h}_{20} = 236215980 T^{35} - 117484137540 T^{33} \\
& + 8643874266720 T^{31} - 2885517466745760 T^{29} - 53721876349920240 T^{27} - 8070710526481402800 T^{25} \\
& + 703400025108222842400 T^{23} + 87738460958393897652000 T^{21} + 2207780690580728420157000 T^{19} \\
& - 47159498268521646014007000 T^{17} + 496567256795595289083780000 T^{15} \\
& - 978609893331396737190780000 T^{13} + 1437856090599426449801850000 T^{11} \\
& - 92732171219122521020284350000 T^9 - 240924172353273324602586900000 T^7 \\
& + 882173042531830165664047500000 T^5 + 1507151362901343475803361875000 T^3 \\
& - 245470559631182354558151562500 T, \quad \mathbf{h}_{19} = 0, \quad \mathbf{h}_{18} = 131231100 T^{37} - 60628768200 T^{35} \\
& + 713083902300 T^{33} - 2235366203716800 T^{31} - 128502034840040400 T^{29} - 7113116608897068000 T^{27} \\
& + 1435474058223587382000 T^{25} + 179236025682574973112000 T^{23} \\
& + 584958278967735397725000 T^{21} - 114564525812965190232270000 T^{19} \\
& + 1365628046065395347760525000 T^{17} - 4320053654794030396643400000 T^{15} \\
& + 24178922285644163410723350000 T^{13} - 526170087460361614389619500000 T^{11} \\
& - 2187241005250348610938625250000 T^9 + 8215068646799644855790565000000 T^7 \\
& - 3234615439021971072725219062500 T^5 + 4466772809861963778344041875000 T^3 \\
& + 628766109312195978709523437500 T, \quad \mathbf{h}_{17} = 0, \quad \mathbf{h}_{16} = 62162100 T^{39} - 25347791700 T^{37} \\
& - 2330970600420 T^{35} - 1619478410665500 T^{33} - 157961463444615600 T^{31} - 4857821543087528400 T^{29} \\
& + 2057113535504220270000 T^{27} + 201257821271491146162000 T^{25} - 4914048257261533415025000 T^{23} \\
& - 104146699011073208745255000 T^{21} + 3611762205877271065424805000 T^{19} \\
& - 6525946067600925299859525000 T^{17} - 90372116556342562197806550000 T^{15} \\
& - 1707727156947430436644076250000 T^{13} - 19914514153947633893074652250000 T^{11}
\end{aligned}$$

$$\begin{aligned}
&+60441504322604592785420072250000T^9 - 121990662905227310507903277187500T^7 \\
&+32024799656441660037917134687500T^5 + 133347159699666487023785273437500T^3 \\
&+50897855165920147555334226562500T, \quad \mathbf{h}_{15} = 0, \quad \mathbf{h}_{14} = 24864840T^{41} - 8275528800T^{39} \\
&-2290606108560T^{37} - 1056111041351520T^{35} - 139995360430695000T^{33} - 3012968529441763200T^{31} \\
&+2016715929359309640000T^{29} + 122776919545138700688000T^{27} - 8724617561136462399414000T^{25} \\
&-8776215034264551673800000T^{23} + 5320414469496529196653860000T^{21} \\
&+34291992550412388290499000000T^{19} - 742197812663450900200994550000T^{17} \\
&-99692057280974811892755516000000T^{15} - 161880090490472685603186915000000T^{13} \\
&+248980392840144022927183314000000T^{11} - 818299264516747040989821234375000T^9 \\
&-476713419004836751657597177500000T^7 + 1758336091397966713046828163750000T^5 \\
&+1769350768557853190505858562500000T^3 + 280160068877261722604358028125000T \quad \mathbf{h}_{13} = 0, \\
\mathbf{h}_{12} = 8288280T^{43} - 1950296040T^{41} - 1303260734160T^{39} - 587448080326800T^{37} \\
&-96181467152740200T^{35} - 2333545862710847400T^{33} + 1303880857145840635200T^{31} \\
&+34549368376057348968000T^{29} - 6423142555199586204450000T^{27} + 84991830908599974348990000T^{25} \\
&+5196658815883842091604820000T^{23} + 71216267628306383393150340000T^{21} \\
&-537186993036056736281959050000T^{19} - 49680822174712697320494392250000T^{17} \\
&-510089192998572154824243387000000T^{15} + 648961140374231120423300313000000T^{13} \\
&-4595842673483238620153978392125000T^{11} - 2306917135543113677700059068125000T^9 \\
&+10465408425092291217199521618750000T^7 + 24940685779394420437367058843750000T^5 \\
&-1891607794892583614798379553125000T^3 + 1383666073198822939633454221875000T \\
\mathbf{h}_{11} = 0, \quad \mathbf{h}_{10} = 2260440T^{45} - 256299120T^{43} - 520433325720T^{41} - 265398457595040T^{39} \\
&-51963689173971960T^{37} - 2078330803534062000T^{35} + 504414194258306316600T^{33} \\
&+2354920000955531856000T^{31} - 1514616736300246258218000T^{29} + 131371987844966806057428000T^{27} \\
&+2424830356405951553429370000T^{25} + 74039904708285822523537320000T^{23} \\
&+996048278667469926140492250000T^{21} - 109841718054872507713652342700000T^{19} \\
&-527139316065717040229677183050000T^{17} + 948629826710126902238243346000000T^{15} \\
&-15738010488860491791083359360125000T^{13} - 21195405167566928689123351368750000T^{11} \\
&+14136303156631427798978019193125000T^9 + 216127929068602452779971891297500000T^7 \\
&-47561677461591404329676492409375000T^5 + 4018232176163239787982535556250000T^3 \\
&+2017953028228342458755361234375000T, \quad \mathbf{h}_9 = 0, \quad \mathbf{h}_8 = 491400T^{47} + 17098200T^{45} \\
&-150724589400T^{43} - 93307202742600T^{41} - 21786354176617800T^{39} - 1497011705550207000T^{37} \\
&+74558739462560199000T^{35} + 2282444312171889789000T^{33} + 1284486001836552906930000T^{31} \\
&+158764283968215852809190000T^{29} + 5961808009517232701540250000T^{27} \\
&+300170542939126568966807550000T^{25} + 3037790768826275206178984550000T^{23} \\
&-178058394981408498570197934750000T^{21} + 706918769910782227397813964750000T^{19} \\
&+3211142985153408253476494216250000T^{17} - 82873544921836068155438221914375000T^{15} \\
&+43774520932599987667701130441875000T^{13} - 150787474955869364748865826971875000T^{11} \\
&+2021475165637011479161476994659375000T^9 - 818085617975150180472289613765625000T^7 \\
&+557849276405116114837268303578125000T^5 - 77987440013694817376566123453125000T^3 \\
&-23844751123289021015776545234375000T, \quad \mathbf{h}_7 = 0, \quad \mathbf{h}_6 = 81900T^{49} + 17287200T^{47} \\
&-31138868880T^{45} - 24379010964000T^{43} - 6845248148092200T^{41} - 741093475985200800T^{39} \\
&-25555815521743122000T^{37} + 3158961025226410404000T^{35} + 1425135751337418094012500T^{33} \\
&+175549516977252515133960000T^{31} + 9472551469732244487395340000T^{29} \\
&+141872271768241531599713400000T^{27} - 2611527288780357895294497150000T^{25} \\
&-159876628457546422579162005000000T^{23} + 904821081291606171001080271500000T^{21}
\end{aligned}$$

$$\begin{aligned}
&+2911105425590491035120995781000000T^{19} - 57856971350556380128450774137187500T^{17} \\
&-597240785276937569608838354857500000T^{15} + 2056519193919467475565331182968750000T^{13} \\
&+12154683904194670162802727887137500000T^{11} + 13863900825388320276306901257843750000T^9 \\
&+1098571541450992569418057287187500000T^7 + 607947066875348083972181687343750000T^5 \\
&-947695220634386609405575242187500000T^3 - 35757135169692262292147626757812500T, \\
\mathbf{h}_5 = 0, \quad \mathbf{h}_4 = 9828 T^{51} + 4101300 T^{49} - 4340604240 T^{47} - 4436265289680 T^{45} \\
&-1514282595903000 T^{43} - 234322521203647800 T^{41} - 15633540600684930000 T^{39} \\
&+1586816675525592126000 T^{37} + 603412183386569812909500 T^{35} \\
&+70495161343635553258387500 T^{33} + 3381977148695834843343420000 T^{31} \\
&+2314480249177743285393500000 T^{29} - 508001687496347812118539650000 T^{27} \\
&-45317179030459158057822637050000T^{25} - 639392657721512904650453512500000T^{23} \\
&-633453674783199881723813200500000T^{21} - 87308156097322179307308497039062500T^{19} \\
&-1355979332634292253868900567460312500T^{17} + 4411540235306765090271379701813750000T^{15} \\
&+10057603832362589442041629620843750000T^{13} + 26911295530554930144489745621340625000T^{11} \\
&+17125918398489101416653911676328125000T^9 + 18954282091963852165062854667093750000T^7 \\
&-6660318081890442429357638396906250000T^5 - 552093664050382974021790374023437500T^3 \\
&-34463233945947896810671380117187500T, \quad \mathbf{h}_3 = 0, \quad \mathbf{h}_2 = 756 T^{53} + 495432 T^{51} \\
&-362210940 T^{49} - 500636959200 T^{47} - 209842694598600 T^{45} - 42645497546559600 T^{43} \\
&-3498152086126744200 T^{41} + 377281024347463236000 T^{39} + 109988509720439880337500 T^{37} \\
&+7421060572419282448275000 T^{35} + 35933479335640711244227500 T^{33} \\
&-11661105491106988983093720000 T^{31} + 439741080879143818859780850000 T^{29} \\
&+19783711746209042465457739500000T^{27} - 5375663257081727637430152750000T^{25} \\
&-22493569190218996211616545271000000T^{23} - 186281502287073108403281941459812500T^{21} \\
&-1059084822334974004802067028318125000T^{19} + 6329153949445054314997820083110937500T^{17} \\
&-14918769495839465654360619394237500000T^{15} - 29373548513181426947900813984803125000T^{13} \\
&+152565680355947795987851167072956250000T^{11} + 54541012524300447784197649387171875000T^9 \\
&-62401428931462991825188978932187500000T^7 + 8198315520009349681029894474726562500T^5 \\
&-834776111135182389414040096171875000T^3 + 29742242994448184918798588320312500T, \\
\mathbf{h}_1 = 0, \quad \mathbf{h}_0 = 28 T^{55} + 25956 T^{53} - 13362300 T^{51} - 26340465060 T^{49} - 13674573352440 T^{47} \\
&-3395567206168200 T^{45} - 366196973040498600 T^{43} + 12483830514741201000 T^{41} \\
&+8840223660767963656500 T^{39} + 1211740695196554052183500 T^{37} \\
&+95626598418360513349807500 T^{35} + 4134578042051998735362052500 T^{33} \\
&+79851486142884785720633550000 T^{31} + 664734262949553830334181650000T^{29} \\
&-97834567689265552651609176150000T^{27} - 3703009772457379493336677700250000T^{25} \\
&-14475894457788465791146209089437500T^{23} - 226810862867256551420326103376562500T^{21} \\
&-1684904275255975197128945251574062500T^{19} + 9448363637590584401034224408586562500T^{17} \\
&+28548596595041297866371213386578125000T^{15} + 146225151676068938646301685253571875000T^{13} \\
&+31326045499715628103678345844109375000T^{11} + 25092831591187433843822670758765625000T^9 \\
&-44918394960897223683724633522007812500T^7 - 7823888482100405864538916831992187500T^5 \\
&-57123990513146513891660780742187500T^3 + 5507822776749663873851590429687500)T,
\end{aligned}$$

$$\begin{aligned}
\mathbf{q}_{56} = 1, \quad \mathbf{q}_{55} = 0, \quad \mathbf{q}_{54} = 28 T^2 + 28, \quad \mathbf{q}_{53} = 0, \quad \mathbf{q}_{52} = 378 T^4 - 2268 T^2 + 3402, \quad \mathbf{q}_{51} = 0, \\
\mathbf{q}_{50} = 3276 T^6 - 62748 T^4 + 21924 T^2 + 483084, \quad \mathbf{q}_{49} = 0, \quad \mathbf{q}_{48} = 20475 T^8 - 749700 T^6
\end{aligned}$$

$$\begin{aligned}
&+2315250 T^4 + 5953500 T^2 + 66976875, \quad \mathbf{q}_{47} = 0, \quad \mathbf{q}_{46} = 98280 T^{10} - 5556600 T^8 + 48157200 T^6 \\
&+65091600 T^4 + 959704200 T^2 + 8132481000, \quad \mathbf{q}_{45} = 0, \quad \mathbf{q}_{44} = 376740 T^{12} - 29037960 T^{10} \\
&+479058300 T^8 - 442940400 T^6 + 9798270300 T^4 + 117272043000 T^2 + 765518890500, \quad \mathbf{q}_{43} = 0, \\
&\mathbf{q}_{42} = 1184040 T^{14} - 114123240 T^{12} + 2988021960 T^{10} - 13647803400 T^8 + 81493889400 T^6 \\
&+1166454966600 T^4 + 9193406607000 T^2 + 45889232697000, \quad \mathbf{q}_{41} = 0, \quad \mathbf{q}_{40} = 3108105 T^{16} \\
&-350020440 T^{14} + 13152128220 T^{12} - 117779757480 T^{10} + 606076420950 T^8 + 10517426904600 T^6 \\
&+74383606489500 T^4 + 157761403569000 T^2 + 1272788332979625, \quad \mathbf{q}_{39} = 0, \quad \mathbf{q}_{38} = 6906900 T^{18} \\
&-855924300 T^{16} + 43638210000 T^{14} - 571636018800 T^{12} + 4633520938200 T^{10} + 91699486263000 T^8 \\
&+670471524954000 T^6 - 2950507091070000 T^4 - 33200878716967500 T^2 + 74380367755732500, \\
&\mathbf{q}_{37} = 0, \quad \mathbf{q}_{36} = 13123110 T^{20} - 1685814900 T^{18} + 114021611550 T^{16} - 1773231490800 T^{14} \\
&+32095272062700 T^{12} + 488569383779400 T^{10} + 7285991633959500 T^8 - 37353863706390000 T^6 \\
&-712242268565231250 T^4 + 766523807513887500 T^2 + 4232772093141033750, \quad \mathbf{q}_{35} = 0, \\
&\mathbf{q}_{34} = 21474180 T^{22} - 2671057620 T^{20} + 242800796700 T^{18} - 3452383449900 T^{16} \\
&+173542141218600 T^{14} + 2616311024019000 T^{12} + 72996496011747000 T^{10} - 112701870219015000 T^8 \\
&-7619235080076427500 T^6 + 26397875963474977500 T^4 + 56266542474799387500 T^2 \\
&+207628508681964562500, \quad \mathbf{q}_{33} = 0, \quad \mathbf{q}_{32} = 30421755 T^{24} - 3341712780 T^{22} \\
&+434948276070 T^{20} - 2762553786300 T^{18} + 707628327869925 T^{16} + 16452498066820200 T^{14} \\
&+420040677679276500 T^{12} + 1905343386976953000 T^{10} - 55819031246158516875 T^8 \\
&+383292609490411582500 T^6 + 758856538412299923750 T^4 + 2764219001238915412500 T^2 \\
&+7818616326630954796875, \quad \mathbf{q}_{31} = 0, \quad \mathbf{q}_{30} = 37442160 T^{26} - 3107699280 T^{24} \\
&+676957508640 T^{22} + 7303633777440 T^{20} + 2213013639790800 T^{18} + 80237629739005200 T^{16} \\
&+2078146188047160000 T^{14} + 28199965799057976000 T^{12} - 257869432151320374000 T^{10} \\
&+3386896499825497890000 T^8 + 5008882358117664180000 T^6 + 35493664601967887700000 T^4 \\
&+79785466324073952750000 T^2 + 191031684895268043750000, \quad \mathbf{q}_{29} = 0, \quad \mathbf{q}_{28} = 40116600 T^{28} \\
&-1684897200 T^{26} + 943969761000 T^{24} + 36029988237600 T^{22} + 5459806605468600 T^{20} \\
&+266337649042542000 T^{18} + 8144921696066001000 T^{16} + 188833010171624280000 T^{14} \\
&-749375912196779175000 T^{12} + 26593887688740291150000 T^{10} + 13857652471353057675000 T^8 \\
&+490019806344263094900000 T^6 + 847091638120081055625000 T^4 + 76231684688082635250000 T^2 \\
&+2590948189562408935875000, \quad \mathbf{q}_{27} = 0, \quad \mathbf{q}_{26} = 37442160 T^{30} + 561632400 T^{28} \\
&+1204554985200 T^{26} + 88218653418000 T^{24} + 10930955933444400 T^{22} + 618649682178805200 T^{20} \\
&+19905633824246478000 T^{18} + 212421481759657170000 T^{16} - 7909388178373163790000 T^{14} \\
&+190838377838882403630000 T^{12} + 246648947765545606050000 T^{10} + 5106259955755665733950000 T^8 \\
&+14509298027190311210250000 T^6 - 16822775828173706102250000 T^4 - 57377741897262022755750000 T^2 \\
&+64893616962420115311750000, \quad \mathbf{q}_{25} = 0, \quad \mathbf{q}_{24} = 30421755 T^{32} + 2733277680 T^{30} \\
&+1411247917800 T^{28} + 156504460165200 T^{26} + 18220186626232500 T^{24} + 1063917237344598000 T^{22} \\
&+24341468738209623000 T^{20} - 1458407669898168990000 T^{18} - 38195651665360037598750 T^{16} \\
&+704274365043021222450000 T^{14} + 5599053528058930314375000 T^{12} \\
&+37978367503214552163750000 T^{10} + 257388209612512632466312500 T^8 \\
&-76827096194526390462750000 T^6 - 1281634404886971940179375000 T^4 \\
&+831668698074883802883750000 T^2 + 1475186709752105729050546875, \quad \mathbf{q}_{23} = 0, \\
&\mathbf{q}_{22} = 21474180 T^{34} + 3959508420 T^{32} + 1496491506720 T^{30} + 220136214564000 T^{28} \\
&+25819817141790000 T^{26} + 1464130513297998000 T^{24} + 431038469000412000 T^{22} \\
&-5567239710825100308000 T^{20} - 51972702340355822025000 T^{18} + 2111596128156974433975000 T^{16} \\
&+26110454746607837505900000 T^{14} + 370837660144520166097500000 T^{12}
\end{aligned}$$

$$\begin{aligned}
& +2211046006855432481484750000 T^{10} + 3768821426548444795278750000 T^8 \\
& -15057540757380336212497500000 T^6 + 24166290139115638826812500000 T^4 \\
& +22727364203519577125586562500 T^2 + 26036255163840613991361562500, \\
\mathbf{q}_{21} = 0, \quad \mathbf{q}_{20} = & 13123110 T^{36} + 3942989820 T^{34} + 1403651839590 T^{32} \\
& +254097412763040 T^{30} + 31521548873014200 T^{28} + 1794671024233626000 T^{26} \\
& -35581106620452393000 T^{24} - 8152518727359282276000 T^{22} + 80631055883822660716500 T^{20} \\
& +5932728449553959198145000 T^{18} + 82578636925153162005712500 T^{16} \\
& +1543335783106772812657500000 T^{14} + 14290988763259977767373375000 T^{12} \\
& +80369144237433402713612250000 T^{10} - 65257978237151882814757125000 T^8 \\
& +298324647301009641815002500000 T^6 + 423803283423874977134856093750 T^4 \\
& +357328418589665267175368437500 T^2 + 289950943789653252140878593750, \quad \mathbf{q}_{19} = 0, \\
\mathbf{q}_{18} = & 6906900 T^{38} + 3038504700 T^{36} + 1138241065500 T^{34} + 243867643658100 T^{32} \\
& +33248280773307600 T^{30} + 2138061788277030000 T^{28} - 18711013316805810000 T^{26} \\
& -3565489869883051110000 T^{24} + 375784427183868517815000 T^{22} + 8589634936729811329365000 T^{20} \\
& +177884937944352775261125000 T^{18} + 5022188934743990092458375000 T^{16} \\
& +34767700807474654377608250000 T^{14} + 346392882720924307150632750000 T^{12} \\
& +7298403068043804798058477500000 T^{10} + 3349869712218751272882581250000 T^8 \\
& +5016164083568572942014820312500 T^6 + 6682149789566477649221235937500 T^4 \\
& +1308993528629685934471373437500 T^2 + 1438708282930546005298232812500, \quad \mathbf{q}_{17} = 0, \\
\mathbf{q}_{16} = & 3108105 T^{40} + 1879208100 T^{38} + 782694539550 T^{36} + 194946808587300 T^{34} \\
& +30068684997553125 T^{32} + 2434041863757781200 T^{30} + 79798925261816145000 T^{28} \\
& +8035953559797385530000 T^{26} + 754836005796200279201250 T^{24} \\
& +17125179786494377633035000 T^{22} + 613061169627775399022992500 T^{20} \\
& +13213975547514313000303875000 T^{18} + 82582615880709112432795781250 T^{16} \\
& -277980267954943276814841750000 T^{14} - 3948507840549888704785071375000 T^{12} \\
& +55686750696026949631741859250000 T^{10} + 42615372067309740929873220703125 T^8 \\
& +151368250313086725334355001562500 T^6 + 17141985927211128923407436718750 T^4 \\
& -42562069342332573564516923437500 T^2 + 14036118651669035253714022265625, \\
\mathbf{q}_{15} = 0, \quad \mathbf{q}_{14} = & 1184040 T^{42} + 942951240 T^{40} + 448820341200 T^{38} + 129061854306000 T^{36} \\
& +22950971378735400 T^{34} + 2420746857044901000 T^{32} + 189431470627899192000 T^{30} \\
& +19740165844212003960000 T^{28} + 1288418099780576436210000 T^{26} + 39634356458318874582330000 T^{24} \\
& +932948607189954436181100000 T^{22} + 24964784950300898037835500000 T^{20} \\
& +336813899817214828406648250000 T^{18} - 4277060110872501238036500750000 T^{16} \\
& -15793226076541858476785253000000 T^{14} + 182147996494954471930263315000000 T^{12} \\
& +957228897383337168450769033125000 T^{10} + 1659229465068162746724356015625000 T^8 \\
& +1250994378672604495344045881250000 T^6 - 1046218980520023853006332618750000 T^4 \\
& +233488960703366614557013828125000 T^2 + 114491186085844621649987015625000, \\
\mathbf{q}_{13} = 0, \quad \mathbf{q}_{12} = & 376740 T^{44} + 383173560 T^{42} + 211091556060 T^{40} + 69936531310800 T^{38} \\
& +14478829033214700 T^{36} + 1951093276439491800 T^{34} + 219781394892621292500 T^{32} \\
& +23753718553640461848000 T^{30} + 1737564922252746032985000 T^{28} + 85455658543552552907190000 T^{26} \\
& +3155694614350839848822535000 T^{24} + 83228531888006342213676300000 T^{22} \\
& +980904596645249105852397375000 T^{20} - 8069972179322250266601782250000 T^{18} \\
& +5852053736347304386211936625000 T^{16} + 470130559276041642828328479000000 T^{14}
\end{aligned}$$

$$\begin{aligned}
&+3826968310962656491781882545312500T^{12} + 14400168925110609944407683954375000T^{10} \\
&+35422981894412214489355285504687500T^8 - 8045800083267738229862592918750000T^6 \\
&+5267694033280469186100991223437500T^4 + 2241399641805817999957998421875000T^2 \\
&+580522174177198819681404351562500, \mathbf{q}_{11} = 0, \mathbf{q}_{10} = 98280T^{46} + 124671960T^{44} \\
&+79862456520T^{42} + 30469610860920T^{40} + 7363086516739800T^{38} + 1221618341096029800T^{36} \\
&+165380901580623267000T^{34} + 18523557951212005461000T^{32} + 1521418822317545771466000T^{30} \\
&+94208790464887079928870000T^{28} + 4142878394428875557173410000T^{26} \\
&+108603899031880110735226350000T^{24} + 1586101557851603861258940750000T^{22} \\
&+5626299013637437585429229250000T^{20} + 95942669081847030488951655750000T^{18} \\
&+1600756201260760979683508510250000T^{16} + 12940726819204628290681342633125000T^{14} \\
&+41771730297204236476700239561875000T^{12} + 118010611986482678310004663130625000T^{10} \\
&+154559704011983463621551783034375000T^8 + 53946862592799320363224126996875000T^6 \\
&+60136820917315236553097341828125000T^4 + 8188712841236219843772580734375000T^2 \\
&+824966108420796215602744640625000, \mathbf{q}_9 = 0, \mathbf{q}_8 = 20475T^{48} + 31789800T^{46} + 23681605500T^{44} \\
&+10399502698200T^{42} + 2927281902340950T^{40} + 577627006715451000T^{38} + 87402057712417255500T^{36} \\
&+9919859107719402765000T^{34} + 800327687122633415563125T^{32} + 42769550356395608294010000T^{30} \\
&+1015850709135001823928075000T^{28} - 9859626751919274812824650000T^{26} \\
&-12994083461579363825869687500T^{24} + 20016288702561441180144003750000T^{22} \\
&+679783469479385743524604426875000T^{20} + 2547659051955188113304398286250000T^{18} \\
&+23936520687149123724088500660703125T^{16} + 186593162117858937503104921003125000T^{14} \\
&-236735342801561911802080068295312500T^{12} - 413881814536318993632578613928125000T^{10} \\
&+2319699480672946592561913286558593750T^8 + 645133155249795867525146260359375000T^6 \\
&+272571033660634953096852335273437500T^4 - 34555655461929637202205397734375000T^2 \\
&+2590924796001627597801615498046875, \mathbf{q}_7 = 0, \mathbf{q}_6 = 3276T^{50} + 6129900T^{48} \\
&+5297821200T^{46} + 2674898024400T^{44} + 872848514676600T^{42} + 199686264011249400T^{40} \\
&+33197856289357914000T^{38} + 3837403641408383970000T^{36} + 276908778929881432372500T^{34} \\
&+9159559733209296842392500T^{32} - 131290592583932779023180000T^{30} \\
&-15032038313822098605428700000T^{28} + 271522692937305895882970250000T^{26} \\
&+23727100949644226008739114250000T^{24} + 768309999321268014688880302500000T^{22} \\
&+12902152525905099940011381352500000T^{20} + 120504248624518289819676115540312500T^{18} \\
&+645622592643611459396432157557812500T^{16} - 1119578306397850896579839825718750000T^{14} \\
&+10319330688733274219793888356250000T^{12} + 6342730065378280526865194003596875000T^{10} \\
&+12132603370280467942856600784046875000T^8 + 9468629274147888824201932240781250000T^6 \\
&-615966923109006853803313738593750000T^4 + 60481139634212975681436988242187500T^2 \\
&+5507822776749663873851590429687500, \mathbf{q}_5 = 0, \mathbf{q}_4 = 378T^{52} + 841428T^{50} \\
&+840568050T^{48} + 487096318800T^{46} + 183138162788700T^{44} + 47831592399129000T^{42} \\
&+8850859090368025500T^{40} + 1129359739079859210000T^{38} + 93052542964654276368750T^{36} \\
&+4403146806459838814107500T^{34} + 89202381643571262806643750T^{32} \\
&-1179658500799710888841500000T^{30} + 23432583245724520630079625000T^{28} \\
&+10578094500969856231695150750000T^{26} + 499339457548290824562364892625000T^{24} \\
&+10735921664568725480441788702500000T^{22} + 75790371288166572312787304136093750T^{20} \\
&+608524156666257484535247119442187500T^{18} + 2677685608955016886758750786536718750T^{16} \\
&+672486304831536963675354803606250000T^{14}
\end{aligned}$$

$$\begin{aligned}
&+9131740007447079465862895562023437500T^{12} + 45475192297272688169637000223265625000T^{10} \\
&+21369215838930001455459090380273437500T^8 + 13508958241351375588197597967031250000T^6 \\
&+348382904493170405887479407988281250T^4 + 146822818591641039837243824882812500T^2 \\
&+1652346833024899162155477128906250, \quad \mathbf{q}_3 = 0, \quad \mathbf{q}_2 = 28T^{54} + 73332T^{52} + 84306852T^{50} \\
&+55935513900T^{48} + 24073903614600T^{46} + 7132291668797400T^{44} + 1498506622541775000T^{42} \\
&+230871155311572237000T^{40} + 27771970845932873752500T^{38} + 2783359442956321295797500T^{36} \\
&+231023465050074485008027500T^{34} + 14304208964922571107875962500T^{32} \\
&+609392496844662576050832750000T^{30} + 17180550932698920283485680250000T^{28} \\
&+351839533333824978599094572250000T^{26} + 2915178549212598578022656292750000)T^{24} \\
&-32498957280378769107354302813437500T^{22} - 102838157202293944963833561572812500T^{20} \\
&+1615150792954957399277819919173437500T^{18} + 28742506839178493428111859797795312500T^{16} \\
&+72905835863993641253168075755228125000T^{14} + 158368618939599733643864923772109375000T^{12} \\
&-65146061199633257026804758778265625000T^{10} - 17057919476262103450025081489296875000T^8 \\
&+37949266119990300735095714833242187500T^6 + 5943570241573087281471449112539062500T^4 \\
&-76007954319145361459151947929687500T^2 + 1101564555349932774770318085937500, \quad \mathbf{q}_1 = 0, \\
&\mathbf{q}_0 = T^{56} + 3052T^{54} + 4019274T^{52} + 3044365884T^{50} + 1490175636075T^{48} + 501200329379400T^{46} \\
&+120293856369130500T^{44} + 21109132070495469000T^{42} + 2743830229192474739625T^{40} \\
&+264032656532001259402500T^{38} + 18705499355839478697033750T^{36} \\
&+967356830845193024425612500T^{34} + 36317543301591844669914796875T^{32} \\
&+1182862195373087251907172750000T^{30} + 43265271525507252011412727875000T^{28} \\
&+1146101963874736797501493362750000T^{26} + 18763596889105886868604052170546875T^{24} \\
&+156011886701245799140278339790312500T^{22} + 1621973055294513702812375078578593750)T^{20} \\
&+10334106828233210967813542074964062500T^{18} + 11130688397493816047672073880222265625T^{16} \\
&+53932911302618520117796345322953125000T^{14} + 171043092322485209840747207763351562500T^{12} \\
&+99160652131611586296198617527453125000T^{10} + 90541065978105828712959516510498046875T^8 \\
&+4768253316469725674256984968085937500T^6 + 1723869845940119797317349924628906250T^4 \\
&+40757888547947512666501769179687500T^2 + 39341591262497599098939931640625.
\end{aligned}$$